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MODULE *ViewstampedReplication*

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EXTENDS *Integers, Sequences, FiniteSets*

CONSTANTS *Replica, Quorum*

*Replica Status*

CONSTANTS *Normal, ViewChange, Recovering*

*Client operation*

CONSTANT *Operation*

*types of log blocks*

CONSTANTS *RequestBlock, ViewBlock*

*Special value*

CONSTANT *None*

*Message types for processing logs*

CONSTANTS *DownloadChunk, StartDownload, PrepareOk, Commit*

*Message types for view changing*

CONSTANTS *StartViewChange, DoViewChange, StartView*

*Sequence with all replicas (for view selection)*

CONSTANT *ReplicaSequence*

*For state space limitation*

CONSTANT *MaxRequests, MaxViews*

*State on each replica*

VARIABLE *replicaState*

VARIABLE *msgs*

$vars \triangleq \langle replicaState, msgs \rangle$

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$LogEntry \triangleq [type : \{RequestBlock\}, opNumber : Nat, op : Operation]$   
 $\cup [type : \{ViewBlock\}, view : Nat]$

*All possible messages*

$Message \triangleq [type : \{DownloadChunk\}, v : Nat, m : LogEntry, n : Nat, k : Nat, i : Replica]$   
 $\cup [type : \{StartDownload\}, v : Nat, n : Nat, src : Replica]$   
 $\cup [type : \{PrepareOk\}, v : Nat, n : Nat, i : Replica]$   
 $\cup [type : \{Commit\}, v : Nat, k : Nat]$   
 $\cup [type : \{StartViewChange\}, v : Nat, i : Replica]$   
 $\cup [type : \{DoViewChange\}, v : Nat, vv : Nat,$   
 $n : Nat, k : Nat, i : Replica]$   
 $\cup [type : \{StartView\}, v : Nat, n : Nat, k : Nat]$

$$\begin{aligned}
Send(m) &\triangleq msgs' = msgs \cup \{m\} \\
SendAll(ms) &\triangleq \wedge msgs' = msgs \cup ms \\
Statuses &\triangleq \{Normal, ViewChange, Recovering\} \\
TypeOK &\triangleq \wedge replicaState \in [ \\
&\quad Replica \rightarrow [ \\
&\quad \quad viewNumber : Nat, \\
&\quad \quad status : Statuses, \\
&\quad \quad log : Seq(LogEntry), \\
&\quad \quad downloadReplica : Replica \cup \{None\}, \\
&\quad \quad commitNumber : Nat \\
&\quad ] \\
&] \\
&\wedge msgs \in SUBSET Message \\
ASSUME QuorumAssumption &\triangleq \wedge \forall Q \in Quorum : Q \subseteq Replica \\
&\quad \wedge \forall Q1, Q2 \in Quorum : Q1 \cap Q2 \neq \{\} \\
ASSUME IsFiniteSet(Replica) \\
Max(S) &\triangleq CHOOSE x \in S : \forall y \in S : y \leq x \\
Min(S) &\triangleq CHOOSE x \in S : \forall y \in S : x \leq y \\
lastOpNumber(l) &\triangleq IF l = \langle \rangle THEN 0 ELSE l[Len(l)].opNumber
\end{aligned}$$


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$$\begin{aligned}
Init &\triangleq \wedge replicaState = [r \in Replica \mapsto [ \\
&\quad viewNumber \mapsto 0, \\
&\quad status \mapsto Normal, \\
&\quad log \mapsto \langle [type \mapsto ViewBlock, view \mapsto 0] \rangle, \\
&\quad downloadReplica \mapsto None, \\
&\quad commitNumber \mapsto 0 \\
&\quad ] \\
&] \\
&\wedge msgs = \{\}
\end{aligned}$$


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#### Getters

$$\begin{aligned}
ViewNumber(r) &\triangleq replicaState[r].viewNumber \\
Status(r) &\triangleq replicaState[r].status \\
Log(r) &\triangleq replicaState[r].log
\end{aligned}$$

$$\text{LogLen}(r) \triangleq \text{Len}(\text{Log}(r))$$

$$\text{LastNormalView}(r) \triangleq \text{Max}(\{\text{Log}(r)[v].\text{view} : v \in \{i \in 1 \dots \text{LogLen}(r) : \text{Log}(r)[i].\text{type} = \text{ViewBlock}\}\})$$

$$\text{OpNumber}(r) \triangleq \text{LogLen}(r)$$

$$\text{DownloadReplica}(r) \triangleq \text{replicaState}[r].\text{downloadReplica}$$

$$\text{CommitNumber}(r) \triangleq \text{replicaState}[r].\text{commitNumber}$$

#### Helpful functions

$$\text{ReplicaIndex}(r) \triangleq \text{CHOOSE } i \in 1 \dots \text{Cardinality}(\text{Replica}) : \text{ReplicaSequence}[i] = r$$

$$\text{PrimaryReplicaInView}(v) \triangleq \text{ReplicaSequence}[(v \% \text{Len}(\text{ReplicaSequence})) + 1]$$

$$\text{IsPrimaryInView}(r, v) \triangleq \text{PrimaryReplicaInView}(v) = r$$

$$\text{IsPrimary}(r) \triangleq \text{IsPrimaryInView}(r, \text{ViewNumber}(r))$$

$$\begin{aligned} \text{IsDownloading}(r) &\triangleq \\ &\wedge \text{replicaState}[r].\text{downloadReplica} \neq \text{None} \end{aligned}$$

$$\text{RequestBlockCount}(\log) \triangleq \text{Cardinality}(\{i \in \text{DOMAIN } \log : \log[i].\text{type} = \text{RequestBlock}\})$$

$$\text{ViewBlockCount}(\log) \triangleq \text{Cardinality}(\{i \in \text{DOMAIN } \log : \log[i].\text{type} = \text{ViewBlock}\})$$

$$\begin{aligned} \text{AddClientRequest}(r, m) &\triangleq \\ &\wedge \text{replicaState}' = [\text{replicaState} \text{ EXCEPT } ![r].\log = \text{Append}(@, m)] \end{aligned}$$

$$\begin{aligned} \text{RecieveClientRequest}(p, op) &\triangleq \\ &\wedge \text{RequestBlockCount}(\text{Log}(p)) < \text{MaxRequests} \\ &\wedge \text{IsPrimary}(p) \\ &\wedge \text{Status}(p) = \text{Normal} \\ &\wedge \neg \text{IsDownloading}(p) \\ &\wedge \text{AddClientRequest}(p, [\text{type} \mapsto \text{RequestBlock}, \\ &\quad \text{opNumber} \mapsto \text{OpNumber}(p) + 1, \\ &\quad \text{op} \mapsto op]) \\ &\wedge \text{Send}([\text{type} \mapsto \text{DownloadChunk}, \\ &\quad v \mapsto \text{ViewNumber}(p), m \mapsto \text{Log}(p)'[\text{OpNumber}(p) + 1], \\ &\quad n \mapsto \text{OpNumber}(p) + 1, k \mapsto \text{CommitNumber}(p), i \mapsto p]) \end{aligned}$$

$$\begin{aligned} \text{RecievePrepare}(r, m) &\triangleq \\ &\wedge \text{RequestBlockCount}(\text{Log}(r)) < \text{MaxRequests} \\ &\wedge \neg \text{IsPrimary}(r) \\ &\wedge \text{Status}(r) = \text{Normal} \\ &\wedge \neg \text{IsDownloading}(r) \\ &\wedge m.\text{type} = \text{DownloadChunk} \end{aligned}$$

$$\begin{aligned}
& \wedge m.v = \text{ViewNumber}(r) \\
& \wedge m.n = \text{OpNumber}(r) + 1 \\
& \wedge m.i = \text{PrimaryReplicaInView}(\text{ViewNumber}(r)) \\
& \wedge \text{AddClientRequest}(r, m.m) \\
& \wedge \text{Send}([type \mapsto \text{PrepareOk}, \\
& \quad v \mapsto \text{ViewNumber}(r), n \mapsto m.n, i \mapsto r])
\end{aligned}$$

$$\begin{aligned}
\text{PrepareOperation}(r) & \triangleq \\
& \wedge \neg \text{IsPrimary}(r) \\
& \wedge \text{Status}(r) = \text{Normal} \\
& \wedge \neg \text{IsDownloading}(r) \\
& \wedge \text{LET } \text{maxPreparedOpNum} \triangleq \text{Max}(\{0\} \cup \{m.n : m \in \{m \in \text{msgs} : m.type = \text{PrepareOk} \wedge m.i = r \wedge m.v = \text{ViewNumber}(r)\}\}) \\
& \quad \text{IN } \wedge \text{LogLen}(r) > \text{maxPreparedOpNum} \\
& \quad \wedge \text{Send}([type \mapsto \text{PrepareOk}, v \mapsto \text{ViewNumber}(r), \\
& \quad \quad n \mapsto \text{maxPreparedOpNum} + 1, i \mapsto r]) \\
& \wedge \text{UNCHANGED } \langle \text{replicaState} \rangle
\end{aligned}$$

$$\begin{aligned}
\text{AchievePrepareOkFromQuorum}(p) & \triangleq \\
& \wedge \text{IsPrimary}(p) \\
& \wedge \text{Status}(p) = \text{Normal} \\
& \wedge \neg \text{IsDownloading}(p) \\
& \wedge \text{LET } \text{newCommit} \triangleq \text{CommitNumber}(p) + 1 \\
& \quad \text{IN } \wedge \exists Q \in \text{Quorum} : \\
& \quad \quad \forall r \in Q : \\
& \quad \quad \quad \vee Q \subseteq \{r\} \cup \{m.i : m \in \{m \in \text{msgs} : m.type = \text{PrepareOk} \wedge m.v = \text{ViewNumber}(p) \wedge m.n = \text{maxPreparedOpNum}\}\} \\
& \quad \quad \quad \vee r = p \\
& \quad \wedge \text{replicaState}' = [\text{replicaState} \text{ EXCEPT } ![p].\text{commitNumber} = \text{newCommit}] \\
& \quad \wedge \text{Send}([type \mapsto \text{Commit}, v \mapsto \text{ViewNumber}(p), k \mapsto \text{replicaState}[p].\text{commitNumber}'])
\end{aligned}$$

$$\begin{aligned}
\text{RecieveCommit}(r, m) & \triangleq \\
& \wedge \neg \text{IsPrimary}(r) \\
& \wedge \text{Status}(r) = \text{Normal} \\
& \wedge \neg \text{IsDownloading}(r) \\
& \wedge m.type = \text{Commit} \\
& \wedge m.v = \text{ViewNumber}(r) \\
& \wedge m.k > \text{CommitNumber}(r) \\
& \wedge \text{replicaState}' = [\text{replicaState} \text{ EXCEPT } ![r].\text{commitNumber} = m.k] \\
& \wedge \text{UNCHANGED } \langle \text{msgs} \rangle
\end{aligned}$$


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#### View Changing

$$\begin{aligned}
\text{TimeoutStartViewChanging}(r) & \triangleq \\
& \wedge \text{ViewNumber}(r) + 1 < \text{MaxViews} \\
& \wedge \text{Status}(r) = \text{Normal} \\
& \wedge \text{replicaState}' = [\text{replicaState} \text{ EXCEPT } ![r].\text{downloadReplica} = \text{None},
\end{aligned}$$



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IN   $\wedge \exists Q \in \text{Quorum} : \wedge p \in Q$ 
       $\wedge Q \subseteq \{m.i : m \in \text{recieved}\}$ 

 $\wedge \text{LET } \text{maxVV} \triangleq \text{Max}(\{m.vv : m \in \text{recieved}\})$ 
       $\text{maxN} \triangleq \text{Max}(\{m.n : m \in \{m \in \text{recieved} : m.vv = \text{maxVV}\}\})$ 
       $\text{maxReplicaIndex} \triangleq \text{Max}(\{\text{ReplicaIndex}(m.i) : m \in \{m \in \text{recieved} : m.vv = \text{maxVV} \wedge m.n = \text{maxN}\}\})$ 
       $\text{maxReplica} \triangleq \text{IF } \wedge \text{maxVV} = \text{LastNormalView}(p)$ 
         $\wedge \text{maxN} = \text{OpNumber}(p)$ 
        THEN  $p$ 
        ELSE  $(\text{CHOOSE } m \in \text{recieved} : \text{ReplicaIndex}(m.i) = \text{maxReplicaIndex}).i$ 
IN   $\wedge \text{replicaState}' = [\text{replicaState} \text{ EXCEPT } ![p].\text{downloadReplica} = \text{IF } \text{maxReplica} = p$ 
      THEN  $\text{None}$ 
      ELSE  $\text{maxReplica},$ 
       $![p].\text{log} = \text{IF } \text{maxReplica} = p$ 
        THEN  $\text{Append}(@, [\text{type} \mapsto \text{ViewBlock},$ 
        ELSE  $@,$ 
         $![p].\text{status} = \text{Normal}]$ 
       $\wedge \text{IF } \text{maxReplica} = p$ 
        THEN  $\text{Send}([\text{type} \mapsto \text{StartView}, v \mapsto \text{ViewNumber}(p), n \mapsto \text{OpNumber}(p)', k \mapsto \text{replicaS}$ 
        ELSE  $\text{Send}([\text{type} \mapsto \text{StartDownload}, v \mapsto \text{ViewNumber}(p), n \mapsto \text{CommitNumber}(p) + 1,$ 

 $\text{SendDownloadChunks}(r) \triangleq$ 
   $\wedge \text{Status}(r) \neq \text{Recovering}$ 
   $\wedge \exists m \in \text{msgs} :$ 
     $\wedge m.\text{type} = \text{StartDownload}$ 
     $\wedge m.\text{src} = r$ 
     $\wedge m.v = \text{ViewNumber}(r)$ 
     $\wedge \text{SendAll}(\{[\text{type} \mapsto \text{DownloadChunk},$ 
       $v \mapsto \text{ViewNumber}(r), m \mapsto \text{Log}(r)[\text{opNum}],$ 
       $n \mapsto \text{opNum}, k \mapsto \text{CommitNumber}(r), i \mapsto r] : \text{opNum} \in m.n \dots \text{LogLen}(r)\})$ 
     $\wedge \text{UNCHANGED } \langle \text{replicaState} \rangle$ 

 $\text{Mc} \rightarrow \text{Mc} / \text{Mc} \rightarrow M$ 
 $\text{MasterDownloadBeforeView}(p) \triangleq$ 
   $\wedge \text{IsPrimary}(p)$ 
   $\wedge \text{Status}(p) \neq \text{Recovering}$ 
   $\wedge \text{IsDownloading}(p)$ 
   $\wedge \text{LET } \text{msgsToDownload} \triangleq \{ \text{msg} \in \text{msgs} :$ 
     $\wedge \text{msg.type} = \text{DownloadChunk}$ 
     $\wedge \text{msg.v} = \text{ViewNumber}(p)$ 
     $\wedge \text{msg.i} = \text{DownloadReplica}(p)$ 
     $\wedge \vee \text{LogLen}(p) + 1 = \text{msg.n}$ 
     $\vee \wedge \text{LogLen}(p) \geq \text{msg.n}$ 
     $\wedge \text{Log}(p)[\text{msg.n}] \neq \text{msg.m} \}$ 
IN   $\wedge \text{msgsToDownload} \neq \{\}$ 
   $\wedge \text{LET } \text{doViewChangeReceived} \triangleq \{m \in \text{msgs} : m.\text{type} = \text{DoViewChange} \wedge m.v = \text{ViewNumber}(p)$ 

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IN  $\wedge \text{msgsToDownload} \neq \{\}$   
 $\wedge \text{LET } \text{MinOpNum} \triangleq \text{Min}(\{\text{msg}.n : \text{msg} \in \text{msgsToDownload}\})$   
 $\text{MinMsg} \triangleq \text{CHOOSE } \text{msg} \in \text{msgsToDownload} : \text{msg}.n = \text{MinOpNum}$   
 $\text{finished} \triangleq \text{MinMsg}.m = [\text{type} \mapsto \text{ViewBlock}, \text{view} \mapsto \text{ViewNumber}(r)]$   
 IN  $\wedge$  all previous chunks are exist (or else it is another download)  
 $\forall \text{prevPos} \in \text{CommitNumber}(r) + 1 \dots \text{MinMsg}.n - 1 :$   
 $\exists \text{prev} \in \text{allMsgsToDownload} :$   
 $\wedge \text{prev.type} = \text{DownloadChunk}$   
 $\wedge \text{prev.v} = \text{ViewNumber}(r)$   
 $\wedge \text{prev.i} = \text{DownloadReplica}(r)$   
 $\wedge \text{prev.n} = \text{prevPos}$   
 $\wedge \text{replicaState}' = [\text{replicaState} \text{ EXCEPT } ![r].\text{log} = \text{Append}(\text{SubSeq}(@, 1, \text{MinOpNum} - 1), M$   
 $![r].\text{downloadReplica} =$   
 $\text{IF } \text{finished}$   
 $\text{THEN } \text{None}$   
 $\text{ELSE } @]$   
 $\wedge \text{UNCHANGED } \langle \text{msgs} \rangle$

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$\text{Finishing} \triangleq$   
 $\wedge \text{LET } r \triangleq \text{ReplicaSequence}[1]$   
 $\text{All Committed}$   
 IN  $\wedge \text{CommitNumber}(r) = \text{OpNumber}(r)$   
 $\text{MaxRequests}$  commands are stored  
 $\wedge \text{RequestBlockCount}(\text{Log}(r)) = \text{MaxRequests}$   
 $\text{All replicas equal}$   
 $\wedge \forall r1 \in \text{Replica} :$   
 $\wedge \text{Log}(r1) = \text{Log}(r)$   
 $\wedge \text{CommitNumber}(r1) = \text{CommitNumber}(r)$   
 $\wedge \text{ViewNumber}(r1) = \text{ViewNumber}(r)$   
 $\wedge \text{Status}(r1) = \text{Normal}$   
 $\wedge \text{DownloadReplica}(r1) = \text{None}$   
 $\wedge \text{UNCHANGED } \langle \text{replicaState}, \text{msgs} \rangle$

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$\text{NormalOperationProtocol} \triangleq$   
 $\vee \exists r \in \text{Replica}, op \in \text{Operation} : \text{RecieveClientRequest}(r, op)$   
 $\vee \exists r \in \text{Replica}, m \in \text{msgs} : \text{RecievePrepare}(r, m)$   
 $\vee \exists r \in \text{Replica} : \text{PrepareOperation}(r)$   
 $\vee \exists p \in \text{Replica} : \text{AchievePrepareOkFromQuorum}(p)$   
 $\vee \exists r \in \text{Replica}, m \in \text{msgs} : \text{RecieveCommit}(r, m)$

$\text{ViewChangeProtocol} \triangleq$   
 $\vee \exists r \in \text{Replica} : \text{TimeoutStartViewChanging}(r)$



$$\begin{aligned}
& \vee \exists r \in \text{Replica}, m \in \text{msgs} : \text{RecieveStartViewChange}(r, m) \\
& \vee \exists p \in \text{Replica}, m \in \text{msgs} : \text{RecieveDoViewChange}(p, m) \\
& \vee \exists r \in \text{Replica} : \text{AchieveDoViewChangeFromQuorum}(r) \\
& \vee \exists r \in \text{Replica} : \text{SendDownloadChunks}(r) \\
& \vee \exists p \in \text{Replica} : \text{MasterDownloadBeforeView}(p) \\
& \vee \exists r \in \text{Replica}, m \in \text{msgs} : \text{RecieveStartView}(r, m) \\
& \vee \exists r \in \text{Replica} : \text{ReplicaDownloadBeforeView}(r)
\end{aligned}$$

$$\begin{aligned}
\text{Next} & \triangleq \vee \text{NormalOperationProtocol} \\
& \vee \text{ViewChangeProtocol} \\
& \vee \text{Finishing}
\end{aligned}$$


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#### Full Spec

$$\begin{aligned}
\text{Spec} & \triangleq \text{Init} \wedge \Box[\text{Next}]_{\text{vars}} \\
\text{FullSpec} & \triangleq \wedge \text{Init} \\
& \wedge \Box[\text{Next}]_{\text{vars}} \\
& \wedge \text{WF}_{\langle \text{vars} \rangle}(\text{Next})
\end{aligned}$$


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$$\text{VRNoMsgs} \triangleq \text{INSTANCE } \text{VR\_without\_message}$$

$$\text{THEOREM } \text{Spec} \Rightarrow \text{VRNoMsgs!Spec}$$


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#### INVARIANTS

$$\begin{aligned}
\text{CommittedLogsPreficesAreEqual} & \triangleq \\
& \forall r1, r2 \in \text{Replica} : \\
& \quad \forall i \in \text{DOMAIN } \text{Log}(r1) \\
& \quad \quad \cap \text{DOMAIN } \text{Log}(r2) \\
& \quad \quad \cap 1 \dots \text{Min}(\{\text{CommitNumber}(r1), \\
& \quad \quad \quad \text{CommitNumber}(r2)\}) : \\
& \quad \quad \text{Log}(r1)[i] = \text{Log}(r2)[i] \\
\text{KeepMaxRequests} & \triangleq \forall r \in \text{Replica} : \text{RequestBlockCount}(\text{Log}(r)) \leq \text{MaxRequests} \\
\text{KeepMaxViews} & \triangleq \forall r \in \text{Replica} : \text{ViewNumber}(r) + 1 \leq \text{MaxViews}
\end{aligned}$$


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#### Properties

$$\text{EventuallyFinished} \triangleq \Diamond(\text{ENABLED } \text{Finishing})$$

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\\* Modification History  
\\* Last modified *Mon* May 15 09:33:48 *MSK* 2023 by *tycoon*  
\\* Created *Mon Nov* 07 20:04:34 *MSK* 2022 by *tycoon*