



Università degli Studi di Cagliari

Facoltà di Scienze Matematiche, Fisiche e Naturali

Laurea Magistrale in Fisica

Thesis Title

Relatore:

Prof. Riccardo Murgia

Co-relatori:

Prof. Thomas Kupfer

Weitian Yu

Candidato:

Gabriele Todde

ANNO ACCADEMICO 2024/2025

Contents

| | | |
|----------|---|-----------|
| 1 | Gravitational Waves Theory | 7 |
| 1.1 | Gravitational Waves as Perturbations | 7 |
| 1.1.1 | Harmonic gauge | 9 |
| 1.1.2 | The TT gauge | 9 |
| 1.2 | Motion and geodesics | 9 |
| 1.2.1 | The motion deviation | 9 |
| 1.2.2 | The geodesic deviation | 9 |
| 1.3 | The Quadrupole Approximation | 9 |
| 1.3.1 | The weak-field, slow-motion approximation | 9 |
| 1.3.2 | The quadrupole formula | 9 |
| 1.3.3 | Transform to the TT gauge | 9 |
| 1.4 | Gravitational waves from a binary system | 9 |
| 1.4.1 | General solution for circular orbits | 9 |
| 1.5 | Energy carried by a gravitational wave | 9 |
| 1.5.1 | Stress-energy pseudo-tensor | 9 |
| 1.5.2 | Gravitational wave luminosity | 9 |
| 1.6 | Evolution of a compact binary system | 9 |
| 1.6.1 | Signal from inspiralling compact objects | 9 |
| 1.6.2 | ASD and multiple sources | 9 |
| 2 | LISA | 11 |
| 2.1 | interferometers | 11 |
| 3 | COSMIC and Stellar Populations Synthesis | 13 |
| 4 | GWGC and Galaxy Properies | 15 |
| 5 | Total Gravitational Wave Signal | 17 |
| 6 | Results | 19 |
| 7 | Conclusions and Future Perspectives | 21 |
| A | Appendix | 23 |

Introduction

In the context of the gravitational waves study, there are mainly two possible paths: the first is the **analysis of existing data** from experiments like Ligo, Virgo and Kagra, with the goal of detecting and characterizing the observable sources within their relative frequency bands; the second approach is the **forecasting approach**, which aims to characterize future experiments in order to better understand what types of sources they could detect and how well.

One of the most important future detectors is the European *Laser Interferometer Space Antenna* (LISA), the first space-based gravitational wave detector. LISA will be arranged as an equilateral triangle with 2.5 million kilometers long arms, placed in a heliocentric orbit, and will operate within the frequency range from approximately 0.1mHz to 1Hz. Among the typical sources that emit gravitational wave signal in this range are the **compact binaries**, and in our particular case we are interested in **double white dwarf binaries** (WDBs).

The goal of this work is to estimate the gravitational wave background produced by the extragalactic WDBs in the local universe, by using the **COSMIC** code to generate synthetic astrophysical populations to represent the galaxies listed in the **Gravitational Wave Galaxy Catalog** (GWGC).

In **Chapter 1** we will introduce the theoretical foundations of gravitational waves, derive the amplitude of the signal generated by a binary system, and define the most important parameters. Finally, we discuss how to combine the signals from multiple sources, to find a cumulative background.

In **Chapter 2** we will give a brief overview of how gravitational wave detectors work, trying to better understand what kinds of sources LISA will be able to see, what resolution and what sensitivity it will have and why in particular we are interested in WDBs.

In **Chapter 3** we introduce the concept of stellar population synthesis and the code COSMIC used for this purpose. We will introduce its features, main parameters, and pipeline, and explain how it is used to generate full-size astrophysical populations of WDBs.

In **Chapter 4** we introduce the GWGC, list the key information it provides and explain how we move from there to infer the remaining parameters that we need.

In **Chapter 5** we use the obtained information to compute the total gravitational wave signal summing the contribution from all the simulated sources, taking into account their spatial distribution, LISA's frequency resolution, and the *zone of avoidance* caused by the milky way.

In **Chapter 6** we plot the total resulting signal on the LISA sensitivity curve, and discuss the results and their possible implications.

Finally, in **Chapter 7** we will draw some conclusions from the work as a whole, discussing its limitations, assumptions and its possible extensions and follow-ups.

Chapter 1

Gravitational Waves Theory

After considering, in 1905, the problem of the apparently *instantaneous* propagation of light, with the theory of Special Relativity, in 1916 Albert Einstein considered the problem of the apparently *instantaneous* propagation of gravity through *long distances*, in his theory of General Relativity. Einstein showed that long-distance interaction arises from the deformation of spacetime caused by massive objects. Hence, in the "static case", the deviated motion apparently caused by the interaction between two distant masses really is, in fact, a manifestation of spacetime curvature nearby, generated by the presence of the two objects. The "static case" just depicted, though, treats the curvature as if it had always been there, and doesn't take into account of any variation in the masses, positions or velocities of the two objects, that would induce an evolution to the curvature itself. In truth, after a change in the mass-energy distribution, the corresponding curvature variation requires its time to reach far distances, and a fascinating prediction of General Relativity is that it propagates in the form of a wave, that travels at the speed of light.

1.1 Gravitational Waves as Perturbations

The typical approach to the study of gravitational waves is to derive them as small perturbations of the background from the Einstein's equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R, \quad (1.1)$$

which can be conveniently written as

$$R_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right), \quad (1.2)$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric of spacetime, and $T_{\mu\nu}$ is the stress-energy tensor. As a background solution we can consider the flat spacetime described by the metric $\eta_{\mu\nu}$, to which the perturbation term appears as a fluctuation in the metric $|h_{\mu\nu}| \ll |\eta_{\mu\nu}|$, known

as *weak field* approximation. Thus, the perturbed spacetime can be written as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll |\eta_{\mu\nu}|.$$

With this metric, the equations 1.2 becomes

$$\{\square_F h_{\mu\nu} - \left[\frac{\partial^2}{\partial x^\lambda \partial x^\mu} h_\nu^\lambda + \frac{\partial^2}{\partial x^\lambda \partial x^\nu} h_\mu^\lambda + \frac{\partial^2}{\partial x^\nu \partial x^\mu} h_\lambda^\lambda \right]\} = -\frac{16\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T \right).$$

Now, by requiring that the *weak-field* approximation remains satisfied for infinitesimal diffeomorphisms, and by choosing a coordinate system in which the *harmonic gauge condition*¹ is satisfied, we can find that, up to first order in $h_{\mu\nu}$, the harmonic gauge condition is equivalent to

$$\frac{\partial}{\partial x^\mu} h_\rho^\mu = \frac{1}{2} \frac{\partial}{\partial x^\rho} h, \quad h = \eta^{\mu\nu} h_{\mu\nu} \equiv h_\nu^\nu, \quad (1.3)$$

and after defining the tensor²

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h,$$

we can finally write the linearized Einstein equations as

$$\begin{cases} \square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \\ \partial^\mu \bar{h}_{\mu\nu} = 0. \end{cases}$$

This form, and its twin with $T_{\mu\nu} = 0$, where the first equation becomes the D'Alembert equation, are relevant because they show that *a perturbation of a flat spacetime propagates as a wave travelling at the speed of light*.

¹It is an arbitrary coordinate condition which makes it possible to solve the Einstein field equations. It can be found by requiring that the linearized Einstein equations satisfy the D'Alembert equation.

²Also known as *trace-reversed* perturbation tensor, since $\bar{h} = \eta^{\mu\nu} \bar{h}_{\mu\nu} = -h$.

1.1.1 Harmonic gauge

1.1.2 The TT gauge

1.2 Motion and geodesics

1.2.1 The motion deviation

1.2.2 The geodesic deviation

1.3 The Quadrupole Approximation

1.3.1 The weak-field, slow-motion approximation

1.3.2 The quadrupole formula

1.3.3 Transform to the TT gauge

1.4 Gravitational waves from a binary system

1.4.1 General solution for circular orbits

Up to 13.86/7 at page 255 of the book.

1.5 Energy carried by a gravitational wave

1.5.1 Stress-energy pseudo-tensor

1.5.2 Gravitational wave luminosity

1.6 Evolution of a compact binary system

1.6.1 Signal from inspiralling compact objects

Here we get to the actual amplitude we used, and the parameters involved.

1.6.2 ASD and multiple sources

Chapter 2

LISA

2.1 interferometers

- Instrument description (what is an interferometer, why in space, how it will be made, orbit) - frequency band - what will it see? - frequency resolution - sensibility curve and ASD meaning - Why WD choice in particular?

Chapter 3

COSMIC and Stellar Populations Synthesis

- COSMIC introduction (reference Breivik, BSE, general information) - COSMIC pipeline (why it's quick, intro to fixed population idea, match conditions, ...) - important parameters in COSMIC (the ones user chooses - stress on metallicity relevance) - population's parameters (the ones COSMIC gives to user) - parameter distribution graphs - Scaling to astrophysical population, what is needed (refer to next chapter)

Chapter 4

GWGC and Galaxy Properties

- Catalog introduction and description (what it has - what we need) - How we'll get what we need (brief description of how we use catalog info to get what we want) - T value to Class - Class to Mass-Luminosity relation -> Mass (Faber & Gallagher)
- Finding each galaxy's metallicity from the mass (Tremonti - Allende Prieto) - Metallicity bins for GWGC galaxies

Chapter 5

Total Gravitational Wave Signal

- For each galaxy: - Compute right N_{astro} - compute each binary's GW signal; - bin it to LISA's frequency sensibility bins - Plot it on LISA's sensibility curve - Zone of avoidance: how to consider all the sky

Chapter 6

Results

- Plot of spectral distribution of the computed signal - Analysis of the distribution of the sources - Eventual implications (none, since it shouldn't be visible)

Chapter 7

Conclusions and Future Perspectives

Recap of the whole Work - Limits of the Work, assumptions and approximations used - Possible extensions

Appendix A

Appendix

Dettagli tecnici sul codice.

Tabelle di parametri.

Ulteriori grafici.

Script di calcolo, se rilevante.

Bibliography

- K. A. Arnaud. XSPEC: The First Ten Years. In George H. Jacoby and Jeannette Barnes, editors, *Astronomical Data Analysis Software and Systems V*, volume 101 of *Astronomical Society of the Pacific Conference Series*, page 17, January 1996.
- Isaiah Beauchamp, Stefania Belvedere, Mitchell Hernandez, Morgan Himes, Trevor McCaffrey, Kelly Sanderson , and Bailee Wolfe. VLA Radio Detection of MAXI J1816-195. *The Astronomer's Telegram*, 15481:1, June 2022.
- Lars Bildsten. Gravitational Radiation and Rotation of Accreting Neutron Stars. *ApJ*, 501(1):L89–L93, July 1998. doi: 10.1086/311440.
- Joe Bright, Tom Russell, Evangelia Tremou, Rob Fender, Patrick Woudt, James Miller-Jones, Melania Del Santo, and Alessio Marino. Further radio detections of the AMXP MAXI J1816-195 from MeerKAT and ATCA. *The Astronomer's Telegram*, 15484:1, June 2022.
- R. Buccheri, K. Bennett, G. F. Bignami, J. B. G. M. Bloemen , V. Boriakoff, P. A. Caraveo, W. Hermsen, G. Kanbach, R. N. Manchester, J. L. Masnou, H. A. Mayer-Hasselwander, M. E. Özel, J. A. Paul, B. Sacco, L. Scarsi, and A. W. Strong. Search for pulsed γ -ray emission from radio pulsars in the COS-B data. *A&A*, 128:245–251, December 1983.
- P. M. Bult, M. Ng, W. Iwakiri, D. Altamirano, D. Chakrabarty, K. C. Gendreau, T. E. Strohmayer, Z. Arzoumanian, A. Sanna, S. Guillot, P. S. Ray, T. Mihara, T. Enoto, J. Homan, and E. C. Ferrara . NICER detects 528 Hz pulsations and a thermonuclear X-ray burst from MAXI J1816-195. *The Astronomer's Telegram*, 15425:1, June 2022a.
- P. M. Bult, A. Sanna, M. Ng, D. Chakrabarty, S. Guillot, K. C. Gendreau, Z. Arzoumanian, T. E. Strohmayer, J. Chenevez, D. Altamirano, G. K. Jaisawal, and E. C. Ferrara. NICER measures the binary orbit of MAXI J1816-195. *The Astronomer's Telegram*, 15431:1, June 2022b.
- Peter Bult, Deepto Chakrabarty, Zaven Arzoumanian, Keith C. Gendreau, Sebastien Guillot, Christian Malacaria, Paul. S. Ray, and Tod E. Strohmayer. Timing the Pulsations of the Accreting Millisecond Pulsar SAX J1808.4-3658 during Its 2019 Outburst. *ApJ*, 898(1):38, July 2020. doi: 10.3847/1538-4357/ab9827.

- Peter Bult, Diego Altamirano, Zaven Arzoumanian, Deepto Chakrabarty, Jérôme Chenevez, Elizabeth C. Ferrara, Keith C. Gendreau, Sebastien Guillot, Tolga Güver, Wataru Iwakiri, Gaurava K. Jaisawal, Giulio C. Mancuso, Christian Malacaria, Mason Ng, Andrea Sanna, Tod E. Strohmayer, Zorawar Wadiasingh, and Michael T. Wolff. The Discovery of the 528.6 Hz Accreting Millisecond X-Ray Pulsar MAXI J1816-195. *ApJ*, 935(2):L32, August 2022c. doi: 10.3847/2041-8213/ac87f9.
- L. Burderi, T. Di Salvo, M. T. Menna, A. Riggio, and A. Papitto. Order in the Chaos: Spin-up and Spin-down during the 2002 Outburst of SAX J1808.4-3658. *ApJ*, 653(2):L133–L136, December 2006. doi: 10.1086/510666.
- L. Burderi, T. Di Salvo, G. Lavagetto, M. T. Menna, A. Papitto, A. Riggio, R. Iaria, F. D’Antona, N. R. Robba, and L. Stella. Timing an Accreting Millisecond Pulsar: Measuring the Accretion Torque in IGR J00291+5934. *ApJ*, 657(2):961–966, March 2007. doi: 10.1086/510659.
- Y. Cavecchi, D. K. Galloway, A. J. Goodwin, Z. Johnston, and A. Heger. The efficiency of nuclear burning during thermonuclear (Type I) bursts as a function of accretion rate. *MNRAS*, 499(2):2148–2156, December 2020. doi: 10.1093/mnras/staa2858.
- Jaiverdhan Chauhan, Anne Lohfink, Priya Bharali, Manoj Mandal, Paul Draghis, Sabyasachi Pal, and Andrea Sanna. Detection of X-ray reflection in MAXI J1816-195 with the NuSTAR. *The Astronomer’s Telegram*, 15470:1, June 2022.
- J. Chenevez, D. Altamirano, D. K. Galloway, J. J. M. in’t Zand, E. Kuulkers, N. Degenaar, M. Falanga, E. Del Monte, Y. Evangelista, M. Feroci, and E. Costa. Puzzling thermonuclear burst behaviour from the transient low-mass X-ray binary IGR J17473-2721. *MNRAS*, 410(1):179–189, January 2011. doi: 10.1111/j.1365-2966.2010.17433.x.
- D. de Martino, P. D’Avanzo, F. Ambrosino, A. Miraval Zanon, A. Papitto, S. Campana, M. C. Baglio, and A. Sanna. New optical and IR counterpart of MAXIJ1816-195. *The Astronomer’s Telegram*, 15479:1, June 2022.
- Tiziana Di Salvo and Andrea Sanna. Accretion powered X-ray millisecond pulsars. *arXiv e-prints*, art. arXiv:2010.09005, October 2020. doi: 10.48550/arXiv.2010.09005.
- P. P. Eggleton. Approximations to the radii of Roche lobes. *ApJ*, 268:368–369, May 1983. doi: 10.1086/160960.
- Masayuki Y. Fujimoto. Angular Distribution of Radiation from Low-Mass X-Ray Binaries. *ApJ*, 324:995, January 1988. doi: 10.1086/165955.

- E. Kuulkers, P. R. den Hartog, J. J. M. in't Zand, F. W. M. Verbunt, W. E. Harris, and M. Cocchi. Photospheric radius expansion X-ray bursts as standard candles. *A&A*, 399:663–680, February 2003. doi: 10.1051/0004-6361:20021781.
- Jean-Pierre Lasota. The disc instability model of dwarf novae and low-mass X-ray binary transients. *New A Rev.*, 45(7):449–508, June 2001. doi: 10.1016/S1387-6473(01)00112-9.
- D. A. Leahy, W. Darbro, R. F. Elsner, M. C. Weisskopf, P. G. Sutherland, S. Kahn, and J. E. Grindlay. On searches for pulsed emission with application to four globular cluster X-ray sources : NGC 1851, 6441, 6624 and 6712. *ApJ*, 266:160–170, March 1983. doi: 10.1086/160766.
- Walter H. G. Lewin, Jan van Paradijs, and Ronald E. Taam. X-Ray Bursts. *Space Sci. Rev.*, 62(3-4):223–389, September 1993. doi: 10.1007/BF00196124.
- Sara Mascia. Analisi temporale del pulsatore X al millisecondo XTE J0929-314, 2018/19.
- National Aeronautics and Space Administration. Neutron Star Interior Composition Explorer, 2023.
- H. Negoro, M. Serino, W. Iwakiri, M. Nakajima, K. Kobayashi, M. Tanaka, Y. Soejima, T. Mihara, T. Kawamuro, S. Yamada, T. Tamagawa, M. Matsuoka, T. Sakamoto, S. Sugita, H. Hiramatsu, A. Yoshida, Y. Tsuboi, J. Kohara, M. Shidatsu, M. Iwasaki, N. Kawai, M. Niwano, R. Hosokawa, Y. Imai, N. Ito, Y. Takamatsu, S. Nakahira, S. Ueno, H. Tomida, M. Ishikawa, M. Tominaga, T. Nagatsuka, T. Kurihara, Y. Ueda, S. Ogawa, K. Setoguchi, T. Yoshitake, K. Inaba, H. Tsunemi, M. Yamauchi, T. Sato, R. Hatsuda, R. Fukuoka, Y. hagiwara, Y. Umeki, K. Yamaoka, Y. Kawakubo, and M. Sugizaki. MAXI/GSC discovered a new X-ray transient MAXI J1816-195. *The Astronomer's Telegram*, 15418:1, June 2022.
- Alessandro Patruno, Rudy Wijnands, and Michiel van der Klis. An Alternative Interpretation of the Timing Noise in Accreting Millisecond Pulsars. *ApJ*, 698(1):L60–L63, June 2009. doi: 10.1088/0004-637X/698/1/L60.
- Juri Poutanen and Marek Gierliński. On the nature of the X-ray emission from the accreting millisecond pulsar SAX J1808.4-3658. *MNRAS*, 343(4):1301–1311, August 2003. doi: 10.1046/j.1365-8711.2003.06773.x.
- Dimitrios Psaltis and Deepto Chakrabarty. The Disk-Magnetosphere Interaction in the Accretion-powered Millisecond Pulsar SAX J1808.4-3658. *ApJ*, 521(1):332–340, August 1999. doi: 10.1086/307525.
- S. A. Rappaport, J. M. Fregeau, and H. Spruit. Accretion onto Fast X-Ray Pulsars. *ApJ*, 606(1):436–443, May 2004. doi: 10.1086/382863.

- Ronald A. Remillard, Michael Loewenstein, James F. Steiner, Gregory Y. Prigozhin, Beverly LaMarr, Teruaki Enoto, Keith C. Gendreau, Zaven Arzoumanian, Craig Markwardt, Arkadip Basak, Abigail L. Stevens, Paul S. Ray, Diego Altamirano, and Douglas J. K. Buisson. An Empirical Background Model for the NICER X-Ray Timing Instrument. *AJ*, 163(3):130, March 2022. doi: 10.3847/1538-3881/ac4ae6.
- A. Sanna, L. Burderi, K. C. Gendreau, T. Di Salvo, P. S. Ray, A. Riggio, A. F. Gambino, R. Iaria, L. Piga, C. Malacaria, and G. K. Jaisawal. Timing of the accreting millisecond pulsar IGR J17591-2342: evidence of spin-down during accretion. *MNRAS*, 495(2):1641–1649, June 2020. doi: 10.1093/mnras/staa1253.
- Simone Sartori. *Fisica dell’accrescimento*, 2018/19.
- Christopher A. Tout, Onno R. Pols, Peter P. Eggleton, and Zhanwen Han. Zero-age main-sequence radii and luminosities as analytic functions of mass and metallicity. *MNRAS*, 281(1):257–262, July 1996. doi: 10.1093/mnras/281.1.257.
- Michiel Van der Klis. Fourier techniques for X-ray timing, 1988. Second, slightly updated version, May 1994 Proceedings title: Proceedings of the NATO Advanced Study Institute Timing Neutron Stars , Çeşme, Turkey, 4 - 15 April 1988, NATO ASI Series C, Vol. 262 Publisher: Kluwer Academic Publishers Place of publication: Dordrecht ISBN: 0377-2071 Editors: H. Ögelman, E.P.J. van den Heuvel.
- Rudy Wijnands and Michiel van der Klis. The Broadband Power Spectrum of SAX J1808.4-3658. *ApJ*, 507(1):L63–L66, November 1998. doi: 10.1086/311676.
- Wikipedia. Neutron Star Interior Composition Explorer, 2023a.
- Wikipedia. Accretion (astrophysics), 2023b.
- S. E. Woosley, A. Heger, A. Cumming, R. D. Hoffman, J. Pruet, T. Rauscher, J. L. Fisker, H. Schatz, B. A. Brown, and M. Wiescher. Models for Type I X-Ray Bursts with Improved Nuclear Physics. *ApJS*, 151(1):75–102, March 2004. doi: 10.1086/381533.
- Hauke Worpel, Duncan K. Galloway, and Daniel J. Price. Evidence for Accretion Rate Change during Type I X-Ray Bursts. *ApJ*, 772(2):94, August 2013. doi: 10.1088/0004-637X/772/2/94.
- A. A. Zdziarski, W. N. Johnson, and P. Magdziarz. Broad-band γ -ray and X-ray spectra of NGC 4151 and their implications for physical processes and geometry. *MNRAS*, 283(1):193–206, November 1996. doi: 10.1093/mnras/283.1.193.
- Chao Zuo, Lei Huang, Minliang Zhang, Qian Chen, and Anand Asundi. Temporal phase unwrapping algorithms for fringe projection profilometry: A comparative review. *Optics and Lasers in Engineering*, 85:84–103, 2016. ISSN

0143-8166. doi: <https://doi.org/10.1016/j.optlaseng.2016.04.022>. URL <https://www.sciencedirect.com/science/article/pii/S0143816616300653>.

Piotr T. Życki, Chris Done, and David A. Smith. The 1989 May outburst of the soft X-ray transient GS 2023+338 (V404 Cyg). *MNRAS*, 309(3):561–575, November 1999. doi: 10.1046/j.1365-8711.1999.02885.x.