Fuzzy Logic and Fuzzy Control System

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Logic for Artificial Intelligence seminar

Presentation Outline

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   - Membership Functions
   - Fuzzy Inference Systems
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Fuzzy Logic Overview

Definition

Fuzzy logic is a form of many-valued logic in which the truth value of variables may be any real number between 0 and 1. It is employed to handle the concept of partial truth, where the truth value may range between completely true and completely false.

Fuzzy logic is based on the observation that people make decisions based on imprecise and non-numerical information. Fuzzy models or fuzzy sets are mathematical means of representing vagueness and imprecise information.

Fuzzy Sets

They are characterized by a membership function in which:
The values assigned to the elements fall within a specified range and indicate the membership grade of their elements in the set.
Larger values denote higher degrees of set membership.
The most commonly used range of values of membership functions is the unit interval [0,1].

A fuzzy set A is defined by a membership function:

$$\mu_A : X \to [0, 1]$$

Several fuzzy sets representing linguistic concepts such as low, medium, high, and so on are often employed to define states of a variable. Such a variable is usually called a fuzzy variable.

Membership Functions

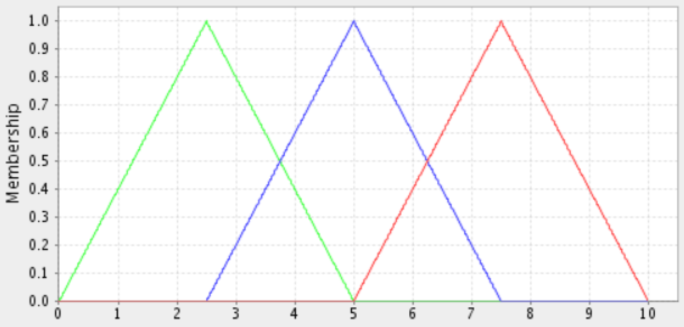
A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1.

Membership functions are broadly divided into two sub classes: continuous and discrete. One variable can only have either continuous or discrete membership functions.

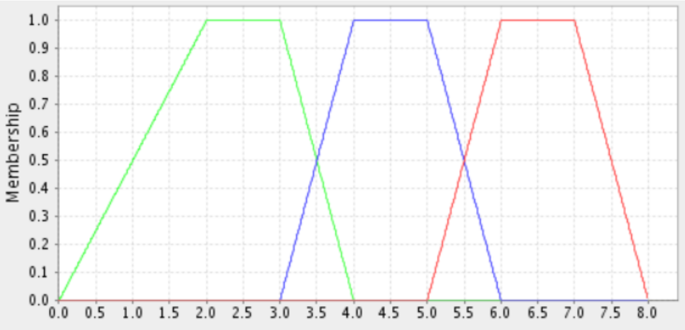
The most common functions are:

- Triangular,
- Trapetzoidal,
- Gaussian,
- Generalized Bell,
- Sigmoidal,
- Singleton,
- Piece-wise Linear.

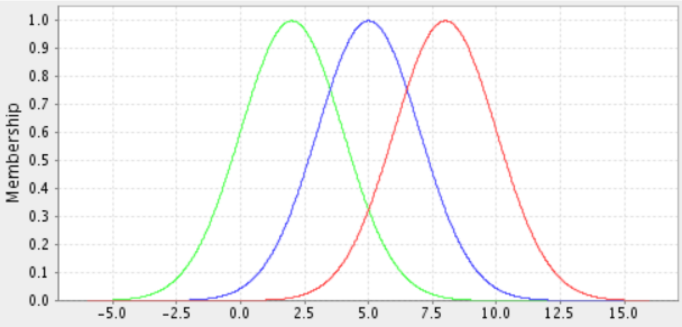
Triangular MF



Trapetzoidal MF

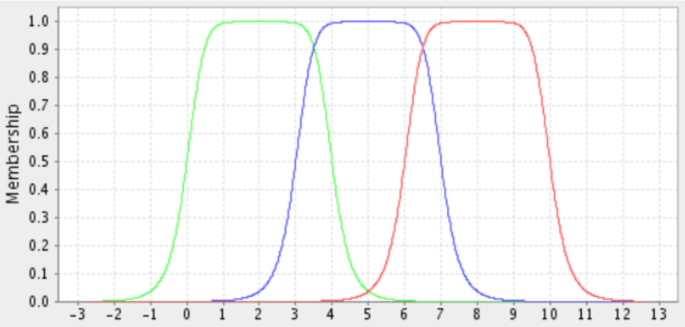


Gaussian MF

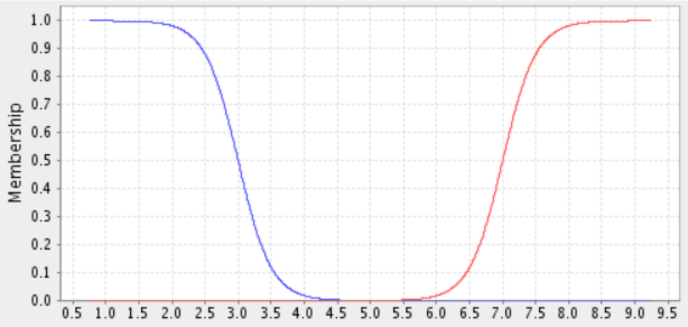


Membership
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-5.0
-2.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0

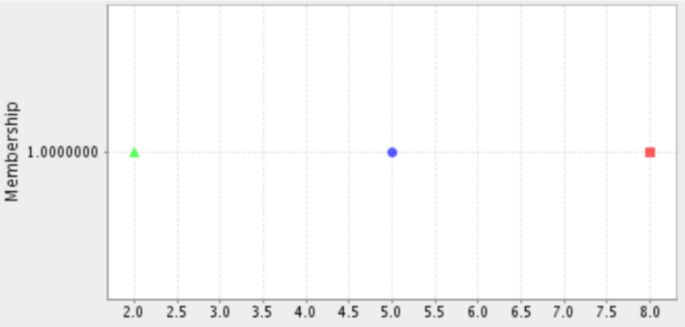
GENERALIZED BELL MF



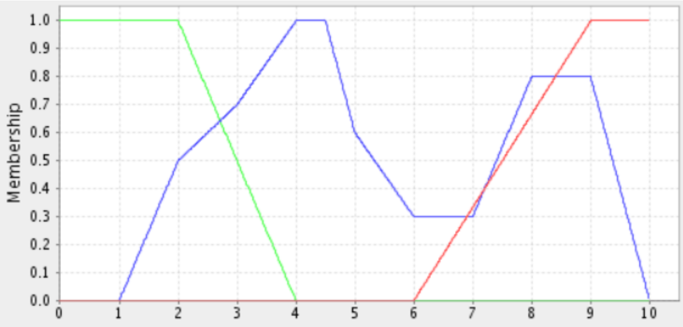
SIGMOIDAL MF



Singleton MF



Piece-wise Linear MF



Inference System

A Fuzzy Inference System (FIS) is a computational framework based on the concepts of fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning. It maps inputs to outputs using fuzzy logic, allowing for approximate reasoning in decision-making processes.

There are mainly two FIS type:

- Mamdani Fuzzy Inference System:
  - Uses fuzzy sets and fuzzy logic operators for both antecedents and consequents.
  - The output fuzzy set needs to be defuzzified.
- Sugeno Fuzzy Inference System:
  - Uses fuzzy sets for antecedents and mathematical functions for consequents.
  - Often used in adaptive and dynamic systems due to its computational efficiency.

Mamdani FIS

Mamdani fuzzy inference was first introduced as a method to create a control system by synthesizing a set of linguistic control rules obtained from experienced human operators.
Since Mamdani systems have more intuitive and easier to understand rule bases, they are well-suited to expert system applications where the rules are created from human expert knowledge.

The output of each rule is a fuzzy set derived from the output membership function and the implication method of the FIS. These output fuzzy sets are combined into a single fuzzy set using the aggregation method of the FIS. Then, to compute a final crisp output value, the combined output fuzzy set is defuzzified.

Rule example: IF temperature IS hot THEN fan_speed is high

Sugeno FIS

Sugeno fuzzy inference, also referred to as Takagi-Sugeno-Kang fuzzy inference, uses singleton output membership functions that are either constant or a linear function of the input values. The defuzzification process for a Sugeno system is more computationally efficient compared to that of a Mamdani system, since it uses a weighted average or weighted sum of a few data points.

Because of the linear dependence of each rule on the input variables, the Sugeno method is ideal for acting as an interpolating supervisor of multiple linear controllers that are to be applied, respectively, to different operating conditions of a dynamic nonlinear system.

Rule example:
IF temp IS cold AND humidity IS high = 2 * temp + 1 * humidity

Defuzzification

Transform fuzzy set output variables into crisp values.

The most used methods are:

- Center Of Gravity (COG) / Center Of Area (COA)
  - Center Of Gravity for Singletons (COGS)
  - Center Of Gravity for Functions (COGF)
- ”Maximum” methods:
  - Mean of Maximum (MM)
  - Leftmost Maximum (LM)
  - Rightmost Maximum (RM)

Center Of Gravity

The COG method is highly valued for its intuitive representation of the ”average” output, making it a popular choice in many fuzzy control applications.

Objective: To find a single crisp value (output) that represents the center of mass of the aggregated fuzzy set.
Approach: The COG method calculates the weighted average of all possible output values, where the weights are the membership values.

Characteristics:
Balanced Output: Takes into account the entire shape of the fuzzy set.
Computational Complexity: Can be intensive, especially for complex or high-resolution membership functions.
Smoothness: Provides smooth and balanced control output, making it suitable for control systems.

Center Of Gravity for Singletons

Objective: To simplify the COG computation for discrete fuzzy sets where the membership functions are represented by singletons.
Approach: Summation over discrete points rather than integration over a continuous range.

Characteristics:
Simplicity: Easier and faster to compute compared to the continuous COG method.
Applicability: Only applicable to discrete fuzzy sets (e.g., singletons).

Center of Gravity for Functions

Objective: To find the center of mass (centroid) of a fuzzy set represented by continuous membership functions.
Approach: Similar to the COG method, it involves integrating over the range of the fuzzy set to determine the weighted average of all possible output values.

Characteristics:
Balanced Output: Provides a single crisp value that considers the entire shape of the membership function.
Continuous Integration: Uses integration to handle continuous functions, which is more precise than summation over discrete points.
Applicability: Suitable for systems where the membership functions are continuous and can be integrated analytically or numerically.

Mean of Maximum

Based on the maximum membership values of the aggregated fuzzy set.

Objective: To find the average of all output values that have the highest membership value.
Approach: This method calculates the mean of the values where the membership function reaches its maximum value.

Characteristics:
Simplicity: Easy to compute, involves only finding the maximum membership values and averaging them.
Smoothness: Generally provides a smooth output, but can be less representative of the overall fuzzy set shape compared to COA.
Applicability: Suitable for systems where the peak values are of primary interest.

Leftmost Maximum

Objective: To find the smallest output value that has the highest membership value.
Approach: This method selects the leftmost value (minimum z) where the membership function reaches its maximum value.

Characteristics:
Simplicity: Very straightforward, involves finding the minimum value among the maximum membership values.
Decisiveness: Provides a crisp and decisive output, which might be useful in certain applications where the earliest maximum value is preferred. Useful for conservative or early action requirements.

Rightmost Maximum

Objective: To find the largest output value that has the highest membership value.
Approach: This method selects the rightmost value (maximum z) where the membership function reaches its maximum value.

Characteristics:
Simplicity: Easy to compute, involves finding the maximum value among the maximum membership values.
Decisiveness: Provides a crisp and decisive output, which might be useful in applications where the latest maximum value is preferred. Useful for aggressive or delayed action requirements.

Rule Accumulation

Rule Accumulation refers to the method used to combine the outputs of multiple fuzzy rules. When multiple rules contribute to the same fuzzy output variable, their effects must be aggregated to form a single fuzzy set.

The supported functions are:
Sum, Max, Bounded Sum, Probabilistic OR

Rule Activation

Rule Activation involves applying the degree of truth of the rule's antecedent to its consequent. This determines the strength with which a rule should influence the output.

The only two supported functions are Min and Product.

Rule Connection

Rule Connection refers to the logical operators used to combine multiple conditions (antecedents) within a single fuzzy rule.

Note that in order to fulfill de Morgan's Law, the algorithms for operators AND and OR shall be used pair-wise, e.g. MAX shall be used for OR if MIN is used for AND.

The supported functions are:
AND: Bounded Difference, Drastic t-norm, Hamacher product, Minimum, Nilpotent minimum, Product;
OR: Bounded Sum, Drastic t-conorm, Einstein Sum, Maximum, Nilpotent maximum, Probabilistic OR

APPLICATION SCENARIO

Short Description

The considered scenario involves the quality control of mass production mechanical pieces for aerospace field.

We have three main parameters to consider:

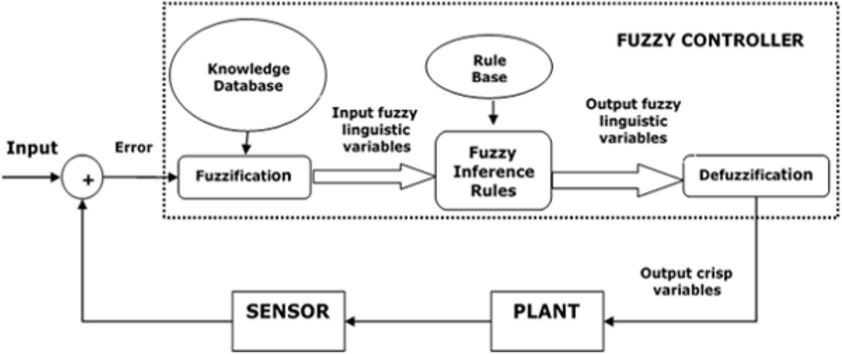
1. Dimensions: Small, Medium, Large;
2. Surface: Smooth, Rough;
3. Tolerance: Low, Medium, High.

And then the outcome are:

- Quality: Low, Medium, High;
- Result: Discarded, Production.

The simulations will take place using a Java library specialized in Fuzzy Logic and Fuzzy Control Systems called jFuzzyLogic.

Control System Example

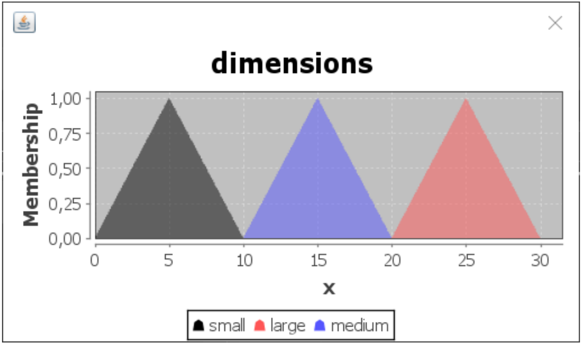


DIMENSIONS

The possible values are:

- Small: 5 mm
- Medium: 10 mm
- Large: 25 mm

(modeled with Triangular Membership Function)

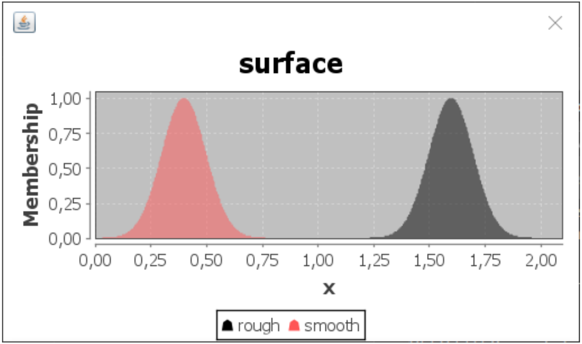


Surface

The possible values are:

- Smooth $< 0{,}4\ \mu$m
- Rough $\geq 1,6\ \mu$m

(modeled with Gaussian Membership Function)



Surface: Average Roughness

The surface imperfections are measured with Arithmetic Average Roughness (Ra) value: Ra is the arithmetic mean of the absolute values of the surface height deviations measured from the mean plane.
Equation:

$$Ra = \frac{1}{L} \int_0^L |z(x)| dx$$

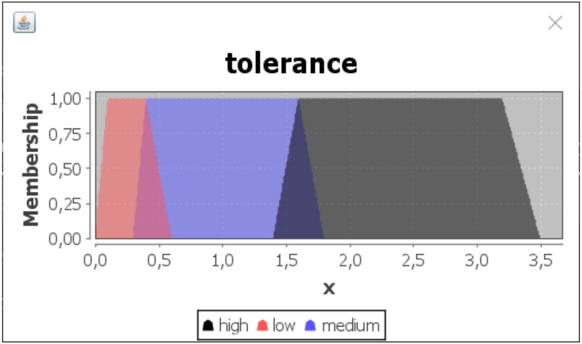
where L is the length over which the measurement is made, and z(x) is the height deviation at position x.

Tolerance

The possible values are:

- Low: from 0,1 mm to 0,4 mm
- Medium: from 0,4 mm to 1,6 mm
- High: from 1,6 mm to 3,2 mm

(modeled with Trapetzoidal Membership Function)

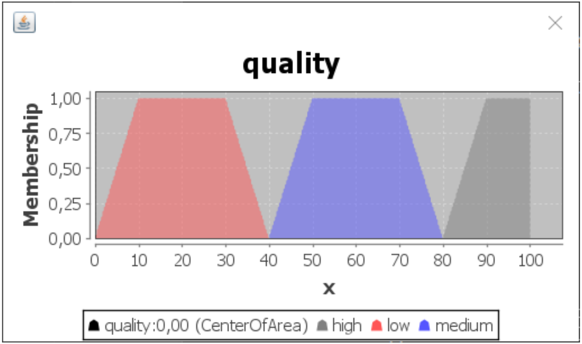


Quality

The possible values are (arbitrary scale):

- Low: from 0 to 40
- Medium: from 40 to 80
- High: from 80 to 100

(modeled with Trapetzoidal Membership Function)

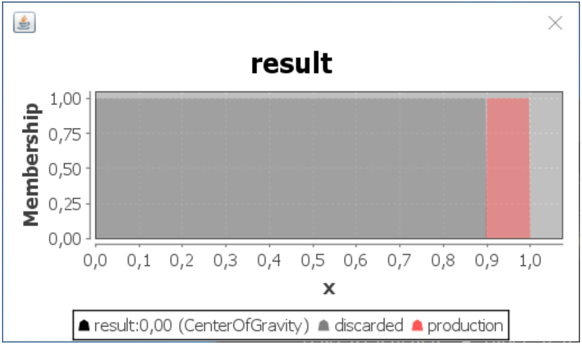


Result

The possible values are:

- Discarded: from 0 to 0.9
- Production: from 0.9 to 1

(modeled with Trapetzoidal Membership Function)



Rules

- RULE 1 : IF surface IS rough OR dimensions IS large OR tolerance IS high THEN quality IS low, result IS discarded;
- RULE 2 : IF surface IS smooth AND tolerance IS low AND dimensions IS small THEN quality IS high, result IS production;
- RULE 3 : IF surface IS smooth AND tolerance IS low AND dimensions IS medium THEN quality IS high, result IS production;
- RULE 4 : IF surface IS smooth AND tolerance IS medium AND dimensions IS medium THEN quality IS medium, result IS production;
- RULE 5 : IF surface IS smooth AND tolerance IS medium AND dimensions IS small THEN quality IS medium, result IS production;

SIMULATIONS

Testing Defuzzification Methods

	COG / COA
{D:4, S:0.3, T:0.2}	{Q:91.4, R:P}
{D:5, S:0.35, T:1}	{Q:60, R:P}
{D, S:2, T}	{Q:20, R:D}

Note that it was not possible testing COGS nor COGF since the membership functions and the defuzzify methods needs to be all discrete or all continuos.

	MM	LM	RM
{D:4, S:0.3, T:0.2}	{Q:0.72, R:D}	{Q:86.1, R:P}	{Q:99.9, R:P}
{D:5, S:0.35, T:1}	{Q:0.32, R:D}	{Q:48.9, R:P}	{Q:71.1, R:P}
{D, S:2, T}	{Q:0.1 R:D}	{Q:0.1, R:D}	{Q:39.9, R:D}

Testing Inference Systems

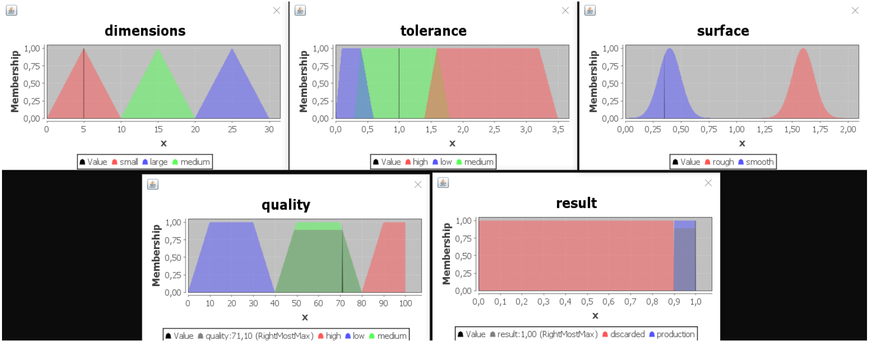
	MaxMin	MaxProd
{D:4, S:0.3, T:0.2}	{Q:99.9, R:P}	{Q:99.9, R:P}
{D:5, S:0.35, T:1}	{Q:71.1, R:P}	{Q:69.9, R:P}
{D, S:2, T}	{Q:39.9, R:D}	{Q:29.9, R:P}

Testing Rule Connections

	BDIFF / BSUM	MIN / MAX
{D:4, S:0.3, T:0.2}	{Q:99.9, R:P}	{Q:99.9, R:P}
{D:5, S:0.35, T:1}	{Q:71.1, R:P}	{Q:69.9, R:P}
{D, S:2, T}	{Q:39.9, R:D}	{Q:39.9, R:P}

	DMIN / DMAX	NIPMIN / NIPMAX
{D:4, S:0.3, T:0.2}	{Q:39.9, R:P}	{Q:99.9, R:P}
{D:5, S:0.35, T:1}	{Q:71.1, R:P}	{Q:71.1, R:P}
{D, S:2, T}	{Q:39.9, R:D}	{Q:39.9, R:P}

EXAMPLE OUTPUT



Thank you for the attention!