Problem Set 7, Physics 104A

Due Wednesday November 10 at 11:59 PM

Late homework accepted for half-credit until November 17

Topics: Greens functions, Series solutions, Legendre series

0. (Need not be turned in) Make sure you understand the solution to the Greens function equation discussed in class: $\frac{d^2}{dt^2}G(t,t') + \omega^2 G(t,t') = \delta(t-t')$, is given by $G(t,t') = 0$ for $t < t'$ and $G(t,t') = \frac{1}{\omega}\sin(\omega(t-t'))$ for $t > t'$.

1. Using the Greens function above in problem 1, find the solution to the differential equation $\frac{d^2y(t)}{dt^2} + \omega^2 y(t) = \sin\alpha t$, for $t > 0$ (assume no force and $y = 0$ for $t < 0$). What happens to the solution when α approaches ω? (Recall the phenomena of resonance in classical mechanics.)

2. Using the Greens function above in problem 1, find the solution to the differential equation $\frac{d^2y(t)}{dt^2} + \omega^2 y(t) = e^{-t}$, for $t > 0$ (assume no force and $y = 0$ for $t < 0$).

3. Use the series solution method to solve the following differential equations. Check your solution by solving the differential equations directly:

a) $xy' = y$

b) $y' = 3x^2 y$

4. Use the series solution method to solve the following differential equation:

a) $y'' - 4xy' + (4x^2 - 2)y = 0$

5. Write down the first five normalized Legendre series: $P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)$ and $P_4(x)$. Explain why all the even Legendre series are even functions of x and all the odd ones are odd functions of x.