Problem Set 5, Physics 104A

Due Saturday October 30 at 11:59 PM

Late homework accepted until Saturday November 6

Primary topic: Eigenvectors, Operators and Gramm-Schmidt ortho-normalization

1. Let $A = \begin{pmatrix} 1 & 0 & 4 & 0 \\ 0 & 2 & 0 & 3 \\ 4 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 \end{pmatrix}$.

a) Find the eigenvalues of A.

b) Find two eigenvectors of A which are independent.

Hint: This problem would be much easier if you can recognize that it is really two separate 2x2 problems.

2. a) Let A, B be operators with $[A, B] = I$, where I is the identity operator. Find $[A^2, B]$.

b) Let $A = \partial/\partial x$, $B = x_{op}$. (Here B is an OPERATOR: B takes a function $f(x)$ to another function $g(x) = xf(x)$.) Calculate $[\partial^2/\partial x^2, x_{op}]$ by comparing $\partial^2/\partial x^2(xf(x))$ and $x\partial^2/\partial x^2 f(x)$.

c) Verify that your answer from b) agrees with the general formula from part a).

3. a) Find the eigenvalues and eigenfunctions of the operator $\frac{d}{dx}$. (i.e., the solutions of $\frac{df}{dx} = \lambda f$.)

b) Repeat for the operator $x_{op}\frac{d}{dx}$.

4. a) Find an orthonormal basis by the Gram-Schmidt method, starting with the three vectors (1,1,0), (1,1,-1), and (3,0,4).

b) Construct orthonormal functions $g_1(x)$ and $g_2(x)$ on [0,1] from $F_1(x) = 1$, and $F_2(x) = e^{-x}$. What is the "angle" between F_1 and F_2?

c) Write $h(x) = 3e^{-x} - 2$ in terms of your orthonormal set from b).