3.10

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$\psi_1(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)$$

$$\hat{p}\,\psi_1(x) = \frac{\hbar}{i}\frac{\partial}{\partial x}\left(\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)\right) = \frac{\hbar\pi}{ia}\left(\sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right)\right)$$

ψ_1 is not eigenfunction under the momentum operator

$\Rightarrow$ cannot find the momentum

3.14

a) $$[\hat{A}+\hat{B},\hat{C}]\Psi = (\hat{A}+\hat{B})\hat{C}\Psi - \hat{C}(\hat{A}+\hat{B})\Psi$$
$$= \hat{A}\hat{C}\Psi + \hat{B}\hat{C}\Psi - \hat{C}\hat{A}\Psi - \hat{C}\hat{B}\Psi$$
$$= \hat{A}\hat{C}\Psi - \hat{C}\hat{A}\Psi + \hat{B}\hat{C}\Psi - \hat{C}\hat{B}\Psi$$
$$= [\hat{A},\hat{C}]\Psi + [\hat{B},\hat{C}]\Psi$$

$$[\hat{A}\hat{B},\hat{C}]\Psi = (\hat{A}\hat{B})\hat{C}\Psi - \hat{C}(\hat{A}\hat{B})\Psi$$
$$= \hat{A}\hat{B}\hat{C}\Psi - \hat{C}\hat{A}\hat{B}\Psi + \hat{A}\hat{C}\hat{B}\Psi - \hat{A}\hat{C}\hat{B}\Psi$$
$$= \hat{A}(\hat{B}\hat{C} - \hat{C}\hat{B})\Psi + \hat{B}(\hat{A}\hat{C} - \hat{C}\hat{A})\Psi$$
$$= \hat{A}[\hat{B},\hat{C}]\Psi + \hat{B}[\hat{A},\hat{C}]\Psi$$
$$\Rightarrow [\hat{A}\hat{B},C] = \hat{A}[\hat{B},C] + [\hat{A},\hat{C}]\hat{B}$$

b) $$[x^n,\hat{p}]\Psi = x^n\hat{p}\Psi - \hat{p}x^n\Psi$$
$$= x^n\left(-i\hbar\frac{\partial}{\partial x}\right)\Psi - \left(-i\hbar\frac{\partial}{\partial x}\right)x^n\Psi$$
$$= -i\hbar x^n\frac{\partial\Psi}{\partial x} + i\hbar x^n\frac{\partial\Psi}{\partial x} + i\hbar\Psi n x^{n-1}$$
$$= +i\hbar\Psi n x^{n-1}$$

c) $$[f(x),\hat{b}]\Psi = f(x)\hat{b}\Psi - \hat{b}f(x)\Psi$$
$$= f\hat{b}\Psi - \hat{b}f\Psi$$

$$= f\left(-i\hbar \frac{\partial}{\partial x}\right)\psi - \left(-i\hbar \frac{\partial}{\partial x}\right) f\psi$$

$$= -i\hbar f \frac{\partial \psi}{\partial x} + i\hbar f \frac{\partial \psi}{\partial x} + i\hbar \psi \frac{df}{dx}$$

$$= i\hbar \frac{\partial f}{\partial x} \psi$$

d) $[\hat{H}, \hat{a}_\pm]\psi = \hat{H}\hat{a}_\pm \psi - \hat{a}_\pm \hat{H} \psi$

$$= \hbar w\left(\hat{a}_- \hat{a}_+ - \frac{1}{2}\right) a_\pm \psi - a_\pm \hbar w \left(\hat{a}_- \hat{a}_+ - \frac{1}{2}\right)\psi$$

$$= \hbar w\, \hat{a}_- \hat{a}_+ \hat{a}_\pm \psi - \hbar w\, \hat{a}_\pm \hat{a}_- \hat{a}_+ \psi$$

$$= \hbar w \left[\hat{a}_- \hat{a}_+ \hat{a}_\pm - \hat{a}_\pm \hat{a}_- \hat{a}_+ \right] \psi$$

if a_+ only then

$$= \hbar w \left[\hat{a}_- \hat{a}_+ \hat{a}_+ - \hat{a}_+ \hat{a}_- \hat{a}_+ \right] \psi$$

$$= \hbar w \left([\hat{a}_-, \hat{a}_+]\, \hat{a}_+ \right) \psi$$

$$= \hbar w (1)\, \hat{a}_+ \psi = \hbar w\, \hat{a}_+ \psi$$

if a only then

$$= \hbar w \left[\hat{a}_- (\hat{a}_+, \hat{a}_-) \right] \psi$$

$$= -\hbar w\, \hat{a}_- \psi$$

$$\Rightarrow [\hat{H}, \hat{a}_\pm] = \pm \hbar w\, \hat{a}_\pm$$

3.16 $$\begin{aligned}[\hat{P}, \hat{Q}] f &= [\hat{P}, \hat{Q}] \sum c_n f_n \\ &= (\hat{P}\hat{Q} - \hat{Q}\hat{P}) \sum c_n f_n \\ &= \hat{P}\hat{Q} \sum c_n f_n - \hat{Q}\hat{P} \sum c_n f_n \\ &= \hat{P} \sum c_n \hat{Q} f_n - \hat{Q} \sum c_n \hat{P} f_n \\ &= \hat{P} \sum c_n \mu_n f_n - \hat{Q} \sum c_n \lambda_n f_n \\ &= \sum c_n \mu_n \hat{P} f_n - \sum c_n \lambda_n \hat{Q} f_n \\ &= \sum c_n \mu_n \lambda_n f_n - \sum c_n \lambda_n \mu_n f_n = 0\end{aligned}$$