Problem Set 6
Physics 104A

Due Thursday November 4 11:59 PM

Topics: Matrices and Fourier expansions

1. 0) (This part does not need to be turned in) Make sure you understand why Hermitian and real symmetric matrices have real eigenvalues. Furthermore, eigenvectors corresponding to two different eigenvalues are orthogonal. In case of degeneracies one can construct orthonormal eigenstates by the Gramm Schmidt process.

a) Let U be a unitary matrix with eigenvectors $|v\rangle$ and eigenvalues λ. Act $U^\dagger$ on the equation $U|v\rangle = \lambda|v\rangle$ to show that $U^\dagger$ has the same eigenvectors as U. What are the corresponding eigenvalues?

b) Evaluate $\langle Uv|v\rangle \equiv (Uv)^\dagger v$ in two different ways to find a condition on the values λ. What is the corresponding condition for orthogonal matrices?

2. a) Find the exponential Fourier series on $[-1,1]$ for $g(x) = x$.

b) Find the exponential Fourier series on $[-1,1]$ for $h(x) = |x|$.

c) Find the exponential Fourier series on $[-1,1]$ for the function $f(x) = 0(x < 0), x(x \geq 0)$ in two ways. First, try to write down the series using your answers to a) and b). Then check your answer by integrating directly.

d) After you're sure you understand part c), use your expansions for g and h to write down the Fourier series of $q(x) = -x/2(x < 0), 3x(x \geq 0)$ on $[-1,1]$. (No integrals needed for this part!)

3. a) Expand $g(x) = \cos x + \sqrt{7}\sin 5x$ in a Fourier series of complex exponentials on the interval $[0, 2\pi]$, and also in a mixed sine-cosine series on the same interval.

b) Expand $f(x) = e^{ix/3}$ in a Fourier series (of exponentials) on $[0, 2\pi]$.

c) Use your answer from b) to expand $f(x) = \sin\frac{x}{3}$ in terms of $\sin nx$ and $\cos nx$, for integer n. Why is the expansion not simply $f(x) = \sin\frac{x}{3}$ itself, since the function is sinusoidal?

4. a) Expand the Dirac delta function $\delta(x)$ in an exponential Fourier series on $[-\pi, \pi]$.

b) Show that $\delta(\phi_1 - \phi_2) = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} e^{im(\phi_1-\phi_2)}$ is a Dirac delta function by showing that it satisfies the definition of a Dirac delta function, $\int_{-\pi}^{\pi} f(\phi_1)\frac{1}{2\pi}\sum_{m=-\infty}^{\infty} e^{im(\phi_1-\phi_2)}d\phi_1 = f(\phi_2)$. (Hint: represent $f(\phi_1)$ by an exponential Fourier series.)

5. a) Do problems 11.5, 11.6, 11.7 and 11.8 in Boas book Chapter 7 (third edition, pages 377, 378). You need to use Parseval's theorem and a suitable function as stated in the book to sum infinite series.