Problem Set 9, Physics 104A

Due Nov 24 11:59 PM

Late HW accepted for half credit until Dec 1.

Primary topic: PDE

1. Find whether each of the following partial differential equations is separable in the given variables. If it is, find the resulting ordinary differential equations.

a) $\frac{\partial^2 f}{\partial\theta^2} + r\frac{\partial f}{\partial r} = 0$; can you find $f(r,\theta) = f_1(r)f_2(\theta)$?

b) $\frac{\partial^2 f}{\partial\theta^2} + \theta\frac{\partial f}{\partial r} = 0$; can you find $f(r,\theta) = f_1(r)f_2(\theta)$?

c) $x^2\frac{\partial^2 f}{\partial x^2} + (x+y)\frac{\partial f}{\partial y} = 0$; can you find $f(x,y) = f_1(x)f_2(y)$?

d) $\frac{\partial^2 f}{\partial x\partial y} + e^{xy}\frac{\partial f}{\partial y} = 0$; can you find $f(x,y) = f_1(x)f_2(y)$?

e) $\frac{\partial^2 f}{\partial x\partial y} + xy\frac{\partial^2 f}{\partial y^2} = 0$; can you find $f(x,y) = f_1(x)f_2(y)$?

f) $\frac{\partial^2 f}{\partial r\partial\theta} + r\theta\frac{\partial f}{\partial\theta} + r^2 f = 0$; can you find $f(r,\theta) = f_1(r)f_2(\theta)$?

2. a) A function $f(x,y,z)$ satisfies $\nabla^2 f + Cf = 0$. If you separate f into $f(x,y,z) = f_1(x)f_2(y)f_3(z)$, what differential equation must $f_1(x)$ obey? (You will have to define a separation constant.) List all solutions $f_1(x)$. (Treat separately the three cases of positive, negative, and zero separation constant.)

b) Which of your solutions $f_1(x)$ in a) meet the boundary conditions $\frac{df_1}{dx}|_{x=0} = \frac{df_1}{dx}|_{x=1} = 0$? In other words, which values of the separation constant are "allowed"–i.e., for which values is there some solution, other than $f_1(x) = 0$, that meets the boundary conditions?

c) Going back to three dimensions, consider boundary conditions $\frac{\partial f}{\partial x}|_{(0,y,z)} = \frac{\partial f}{\partial x}|_{(1,y,z)} = \frac{\partial f}{\partial y}|_{(x,0,z)} = \frac{\partial f}{\partial y}|_{(x,1,z)} = \frac{\partial f}{\partial z}|_{(x,y,0)} = \frac{\partial f}{\partial z}|_{(x,y,1)} = 0$. List all $C < 100$ for which solutions exist, and list all combinations of solutions for each C.

3. Find the steady-state temperature distribution in a plate with boundary temperatures $T = 30°$ for $x = 0$ and $y = 3$, and $T = 20°$ for $y = 0$ and $x = 5$.

4. A string is stretched between 0 and L, with the ends fixed in place (i.e., $f(0,t) = f(L,t) = 0$). Its motion obeys the wave equation for $0 \le x \le L$. Find the motion of the string for $t > 0$ for the following situations. For parts b) and c) you may leave the coefficients as integral expressions.

a) The previously stationary and flat string is suddenly whacked at $t = 0$, so $\frac{\partial f}{\partial t}(x,0) = L\delta(x-a)$.

b) For $t < 0$ the string is clamped in the middle. The right half is motionless while the left half vibrates as $f(x,t) = \sin\frac{4\pi}{L}x\sin\frac{4\pi}{L}vt$. At time $t = 0$ the clamp is suddenly removed.

c) The string is released from the position $y(x,0) = 0(x < L/4, x > 3L/4), 15(L/4 \le x \le 3L/4)$ at $t = 0$.