Problem Set 8, Physics 104A

Due Wednesday November 17 at 11:59 PM

Late homework accepted for half-credit until November 24

Topics: Legendre series

1. a) Find the Legendre series of $x^3 + 2x^2 - 1$ in two ways. First do it only with algebra, by starting with the x^3 term, then treating the x^2 term, and so on. Second, use the integral formulas to calculate the coefficients a_0 through a_3.

b) Expand x^5 in a Legendre series.

2. a) Calculate the Legendre polynomial $P_2(x)$ from the Rodrigues formula.

b) Show by explicit integration that the Legendre polynomials $P_2(x)$ and $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$ are orthogonal. Also calculate explicitly that the normalization of $P_4(x)$ obeys the formula from class. (It's equation 8.1, chapter 12, in Boas.)

3. a) Evaluate $\int_{-1}^{1} P_l(x) P'_{l-1}(x) dx$.

b) Explain in one sentence how, without any calculation, you know that $\int_{-1}^{1} P_3(x)[P_4(x)]^2 dx = 0$.

4. Expand the Dirac delta function $\delta(x)$ in a Legendre series on [-1,1].

5. Calculate the coefficients of the first four Legendre polynomials (P_o through P_3) in the Legendre expansion of $\sin 2\pi x$ on [-1,1].

6. Find a non-trivial arrangement of charges along the z-axis such that, very far from the charges, the electrostatic potential falls off as $(\frac{1}{r})^4$. What is the angular dependence of the potential?