

Problem Set 3

Physics 104A

Due Wednesday October 13 at 11:59 PM

Primary topic: Dirac delta function

1. Evaluate the following integrals or explain why they diverge.

- a) $\int_0^\infty f(x)\delta(2x-3)dx$ e) $\int_{-\infty}^\infty e^x\delta(x^3)dx$
- b) $\int_0^{2\pi} \frac{\sin z}{\cos^2 2z + \tan^2 z/2} \delta(\pi-z)dz$ f) $\int_0^{2\pi} \sin(\theta)\delta(e^\theta \cos \theta)d\theta$
- c) $\int_0^3 f(x)\delta(2x^2-x-1)dx$ g) $\int_{-\infty}^\infty f(x)\delta(2x^2+1)dx$
- d) $\int_{-\infty}^\infty \cos x\delta(x^3-\pi^2x)dx$ h) $\int_0^r f(x)\delta(\sqrt{r^2-x^2}-2)dx$, for $r > 0$
(Take two cases, $r > 2$ and $r < 2$.)

2. Evaluate the following integrals or explain why they diverge.

- a) $\int_{-\infty}^\infty f(x)\delta'(x-1)dx$
- b) $\int_{-5\pi/2}^{5\pi/2} e^{-2x}\delta'(\sin x)dx$
- c) $\int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty dx dy dz f(x,y,z)\delta(\frac{x}{2}+1)\delta(y-3)\delta(\sqrt{6}z+1)$
- d) $\int_{-\infty}^\infty \int_{-\infty}^\infty dx dy (x^2+xy+1)\delta(3x-5)\delta(y)$
- e) $\int_0^\infty dx \int_0^\infty dy \frac{\delta(x-y)}{(4x^2+5y^2+1)^{3/2}}$
Also integrate the same integrand over the upper half-plane (i.e., $y > 0$, $-\infty < x < \infty$)
and over the whole plane.
- f) $\int_0^{\pi/4} \int_0^{\pi/4} dx dy \delta(x-2y) \cos \sqrt{2xy}$
Do this one in both orders. (That is, x-integral followed by y-integral and vice versa.) If
you don't get the same answer both ways, think carefully about your integration limits.

3. Evaluate the following integrals.

- a) $\int_{-\pi/2}^{\pi/2} (\cos \theta)\delta((\theta - \frac{\pi}{4}) \sin \theta)d\theta$ b) $\int_0^\infty \left[\int_0^5 \delta(x-y)dx \right] dy$