Q1 a) $\frac{3-2i}{2+i}$ in different forms

$x+iy$ form:

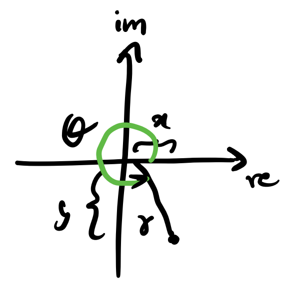
We multiply: $\frac{3-2i}{2+i} \times \left(\frac{2-i}{2-i}\right) = \frac{(3-2i)(2-i)}{4+1}$

$= \frac{6-3i-4i-2}{5} = \frac{4-7i}{5} = \underbrace{\frac{4}{5} - \frac{7}{5}i}_{x+iy \text{ form}}$

For the $re^{i\theta}$ form:

$r = \sqrt{x^2+y^2}$ $\theta = \tan^{-1} y/x$

$\Rightarrow r = \frac{\sqrt{65}}{5}$ $\theta = \tan^{-1}(-7/4)$



b) $(1+i)^{29} = re^{i\theta}$

$\Rightarrow r^{1/29} e^{i\theta/29} = 1+i = \sqrt{2}\, e^{i\pi/4}$

$\Rightarrow r = 2^{29/2}$ $\theta = 29\pi/4 = \pi/4$ (principal value)

$\Rightarrow x = r\cos\theta$ $y = r\sin\theta$ (converting b/w $x+iy$ & $re^{i\theta}$)

c) Similar solution.

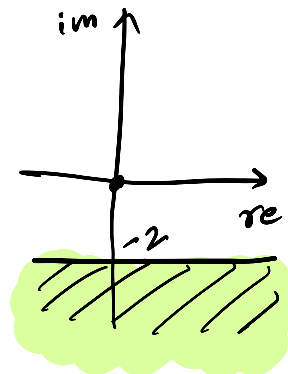
tanh $\longrightarrow$ write in terms of exps, convert exps to $x+iy$ form individually, add them!

d) $Re(e^{i\pi/2} z) > 2$

$\Rightarrow Re(iz) > 2$ let $z = x+iy$

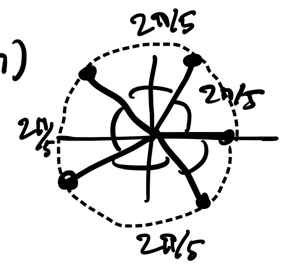
$\Rightarrow -y > 2$

$\Rightarrow y < -2$



Q2) a) $z^5 = 1 \Rightarrow z^5 = e^{i(0+2n\pi)}$

$\Rightarrow z = e^{i\,2\pi n/5}$



b) $z^4 = i$

$\Rightarrow z^4 = e^{i(\pi/2 + 2n\pi)}$

$\Rightarrow z = e^{i(\pi/8 + 2n\pi/4)}$

so $r = 1$ $\theta = \pi/8,\ 5\pi/8,\ 9\pi/4,\ 13\pi/8$

c) is similar, just write $2+i$ in the form $re^{i\theta}$

d) also similar! Replace $z-2 \equiv \alpha$ and solve normally

Q3) a) $I_{n,m} = \int_0^{2\pi} e^{-imx} e^{inx}\, dx = \int_0^{2\pi} e^{i(n-m)x}\, dx$

case I: $n=m$ $I_{n,n} = \int_0^{2\pi} e^{i0x}\, dx = \int_0^{2\pi} 1\, dx = 2\pi$

case II: $n \neq m$ let $n-m=\ell$

$I_{n,n-\ell} = \int_0^{2\pi} e^{i2\pi\ell x}\, dx = \int_0^{2\pi} \cos(2\pi\ell x)\, dx + i\int_0^{2\pi} \sin(2\pi\ell x)\, dx$

$= \left[\frac{\sin(2\pi\ell x)}{2\pi\ell} - i\,\frac{\cos(2\pi\ell x)}{2\pi\ell}\right]_0^{2\pi} = 0$

b) $\int_0^{2\pi} \sin mx \cos nx \, dx$

$\cos nx \rightarrow \left(\frac{e^{inx} + e^{-inx}}{2}\right)$

$\sin mx \rightarrow \left(\frac{e^{imx} - e^{-imx}}{2i}\right)$

answers:
a) $= \pi \;\; \forall \;\; m = n \neq 0$
$\;\;\; = 0 \;\; \forall \;\; m \neq n$ or $m = n = 0$
b) $= 0 \;\; \forall \;\; m, n$

& then you use a) to solve it!

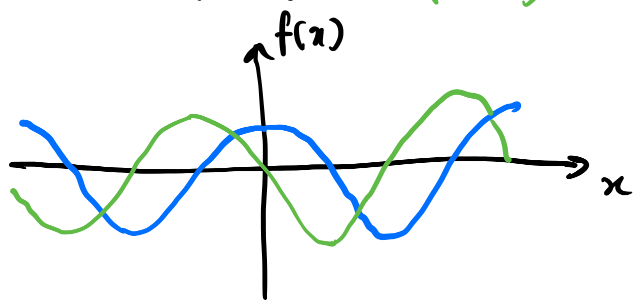
Q4) $f(z) = e^{-3iz}$ it's a complex function

a) let $z = x$ (real line)

$f(x) = e^{-3ix}$

$Re(f(x)) = \cos(-3x)$

$Im(f(x)) = \sin(-3x)$

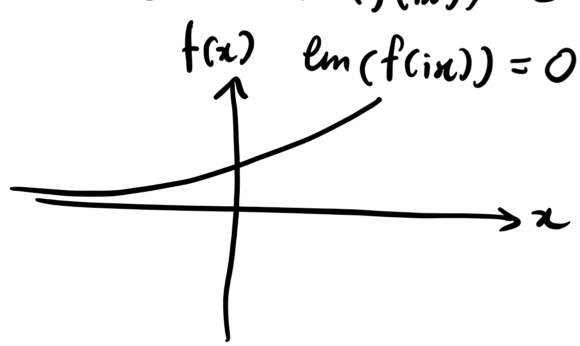


b) let $z = ix$ (imaginary line)

$f(ix) = e^{-3i(ix)} = e^{3x}$

$Re(f(ix)) = e^{3x}$

$Im(f(ix)) = 0$



c) Let's breakdown the question!

$f : \mathbb{C} \longrightarrow \mathbb{C}$ ← range

↗ function   ↖ domain

a complex function takes a complex number in the domain & spits out a complex number in the range.

if you input a point, then you get a point.

if you input a line, then you will get a curve!

So $\underbrace{f(\text{line})}_{\text{this is the input}} = \underbrace{\text{curve}}_{\text{this is the image}}$

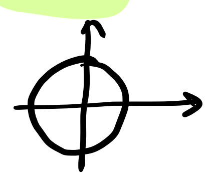
So if I input the real line, then the curve the function spits out for this line is the image!

How do I input a line into a function? Easy!
Just input it point by point!

So the real line is the collection of pts $z = x + 0i$ on the complex plane. So $f(z) = f(x+0i)$ will give me the image of the real line.

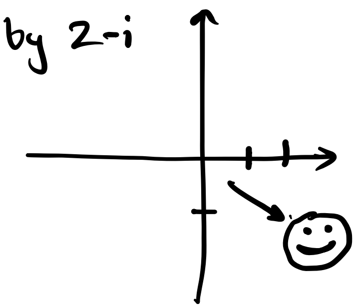
$f(x+0i) = e^{-3ix}$ ← This is a circle equation!

So the image of the real line is a circle under the function $f(z)$.



Q5) a) $f(z) = z + \underbrace{(2-i)}_{\text{linear displacement}}$

So the face will be displaced by 2-i



b) $g(z) = (1+i)z$

This one's more of an involved problem.

Take the eq^n of the circle:

$$z = \frac{1}{2}e^{i\theta} + \left(\frac{1}{2} - \frac{i}{2}\right)$$

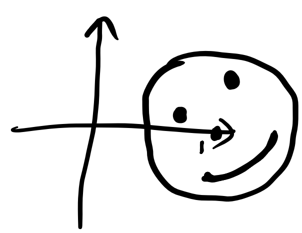
$$g(z) = (1+i)\left[\frac{1}{2}e^{i\theta} + \left(\frac{1}{2} - \frac{i}{2}\right)\right]$$

$$= \left(\frac{1+i}{2}\right)e^{i\theta} + \underbrace{\frac{1}{2}(1+i)(1-i)}_{\text{linear displacement}}$$

$$= \frac{1}{\sqrt{2}}e^{i(\theta+\pi/4)} + 1$$

also an eq^n of a circle!

It's turned 45°!



imo this looks better, & has a good nose too :)