

# MIDTERM P1 Sol

Friday, October 29, 2021 4:57 PM

## 1a) Rubric

10 pts - correct

- 5 pts - incorrect  $\theta$ 's (-1 for each wrong  $\theta$ )
- 3 pts - incorrect plot
- 2 pts - incorrect  $r$
- 2 pts - math error
- 1 pts - minor math error

## Solution

$$\text{solve } z^4 = \frac{4-4i}{1+i} = 8i-4$$

$$z^4 = \frac{4-4i}{1+i} \cdot \frac{1-i}{1-i} = \frac{-8i}{1+1} = \frac{-8i}{2} = -4i$$

Using  $z = re^{i\theta}$  we can write:

$$(re^{i\theta})^4 = r^4 e^{i4\theta} = r^4 (\cos 4\theta + i \sin 4\theta) = -4i$$

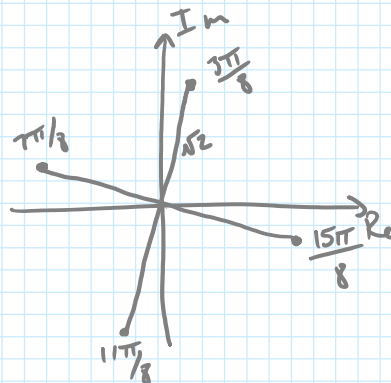
$$\text{Thus } r^4 = 4 \rightarrow r = \sqrt{2}$$

and

$$\cos 4\theta = 0, \sin 4\theta = -1$$

$$\rightarrow 4\theta = \frac{3\pi}{2} + n2\pi \rightarrow \theta = \frac{\pi}{2} (3 + 4n)$$

$$\text{So 4 solutions are } \theta = \left\{ \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8} \right\}$$



$$z_n = \sqrt{2} e^{i\theta_n}$$

## b) Rubric

10 pts - correct

- 4 pts - incorrect roots (2 pts each)
- 3 pts - incorrect  $\delta'$  handling
- 2 pts - math error
- 1 pts - minor math error

## Solution

$$I = \int_0^{2\pi} e^{3x} \delta'(\cos x) dx$$

$$= - \int_0^{2\pi} \frac{d}{dx} e^{3x} \delta(\cos x) dx$$

$$= -3 \int_0^{2\pi} e^{3x} \delta(\cos x) dx \quad \cos x \Big|_{\pi/2, 3\pi/2} = 0$$

$$= -3 \int_0^{2\pi} e^{3x} \left[ \frac{\delta(x - \pi/2)}{|\sin \pi/2|} + \frac{\delta(x - 3\pi/2)}{|\sin 3\pi/2|} \right] dx$$

$$= -3 \left[ e^{3\pi/2} + e^{a\pi/2} \right]$$