a) $f_1(x) = \sqrt{x}$

normalizing $f_1(x)$:

$$\|f_1(x)\|^2 = \int_0^1 f_1^*(x)\, f_1(x)\, dx = \int_0^1 x\, dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$

but norm($f_1(x)$) is $\|f_1(x)\|$, so

normalizing $f_1(x)$ means: $\dfrac{f_1(x)}{\|f_1(x)\|} = \dfrac{\sqrt{x}}{1/\sqrt{2}} = \sqrt{2x}$

Rubric —

10 pts — everything correct

−3 pts — calculated the correct norm but did not normalize

−2 pts — did not calculate the norm properly or (algebra mistake)

−2 pts — normalized incorrectly but calculated the correct norm.

if little progress/conceptually wrong, points assigned on a case by case basis.

b) There are many orthogonal functions.

We can start arbitrarily with a random function like $f_2(x) = x$ or $f_2(x) = 1$ etc.

& then use the GS method to calculate an orthogonal function, eg. $f_2(x) = 1/\sqrt{x}$

then: $\int_0^1 f_1^*(x) f_2(x)\, dx = \int_0^1 \sqrt{2x}\, \frac{1}{\sqrt{x}}\, dx = \sqrt{2}$

so $f_2'(x) = f_2(x) - \langle f_1, f_2 \rangle f_1$

$= 1/\sqrt{x} - \sqrt{2}\sqrt{2x} = \frac{1}{\sqrt{x}} - 2\sqrt{x}$

again, there are many possible orthogonal functions, depending on which one you start with.

The other approach is to design a function that automatically gives a trivial integral.

eg. $f(x) = \frac{\sin(2\pi x)}{\sqrt{x}}$ etc.

* note: $\delta(x)$ is NOT a function so don't use that. Also, $f(x)=0$ is not an orthogonal function to $f(x) = \sqrt{x}$ or any other f^n bc. the angle is not unique.

rubric —

10 pts — correct

-3 pts — if GS method was not correctly calculated

-7 pts — $f_2(x) = 0$

-1 pts — for $f_2(x) = \delta(x)$

points for little progress assigned on a case by case basis.