Homework 4

1)
$$\int_0^{V_{LSB}} \frac{\left(x - \frac{0+V_{LSB}}{2}\right)^2}{V_{LSB} - 0}\,dx = \left.\frac{\left(x - \frac{V_{LSB}}{2}\right)^3}{3\,V_{LSB}}\right]_0^{V_{LSB}}$$

$$= \frac{\left(V_{LSB} - \frac{V_{LSB}}{2}\right)^3}{3\,V} - \frac{\left(0 - \frac{V_{LSB}}{2}\right)^3}{3\,V}$$

$$= \frac{\left(V_{LSB} - \frac{V_{LSB}}{2}\right)^3}{3\,V} + \frac{\left(\frac{V_{LSB}}{2}\right)^3}{3\,V}$$

$$= \frac{\left(V_{LSB} - \cancel{\frac{V_{LSB}}{2}} + \cancel{\frac{V_{LSB}}{2}}\right)\left(\cancel{V_{LSB}^2} - \cancel{V_{LSB}^2} + \cancel{\frac{V_{LSB}^2}{4}} - \cancel{\frac{V_{LSB}^2}{2}} + \cancel{\frac{V_{LSB}^2}{4}} + \frac{V_{LSB}^2}{4}\right)}{3\,V_{LSB}}$$

$$= \frac{\cancel{V_{LSB}}\,\frac{V_{LSB}^2}{4}}{3\,\cancel{V_{LSB}}} = \frac{V_{LSB}^2}{12}$$

$$V_{LSB} = \frac{V_{ref}}{2^n}$$

$$\sigma = \sqrt{\frac{V_{LSB}^2}{12}} = \sqrt{\frac{1}{12}\left(\frac{V_{ref}}{2^n}\right)^2} = \frac{1}{\sqrt{12}}\,\frac{V_{ref}}{2^n}$$

2) a) $\frac{2}{38} = \frac{1}{19}$ chance that the House will win

b) $\lambda = pN$

$\Rightarrow N = \frac{\lambda}{p} = \frac{2}{\frac{1}{19}} = 38$ (times)

c) the probability that the House wins mothing:

$$\binom{38}{0} \left(\frac{2}{38}\right)^{0} \left(\frac{36}{38}\right)^{38} = 0.128$$

d) $1000 \times \frac{2}{38} \times \$100 = \$5263.16$

e) $\sigma = \sqrt{1000\left(\frac{2}{38}\right)\left(1 - \frac{2}{38}\right)} = 7.06$

f) $\int_{-\infty}^{26.5} \frac{1}{7.06\sqrt{2\pi}} e^{\frac{-(k-53)^2}{2(7.06)^2}} dk = 0.00008$

5) a) $e^{-0.37} = 0.69$

probability: $1 - 0.69 = 0.31$

this is the probability that there will be at least one event

b) $P(k) = \frac{1}{k!} \lambda^k e^{-\lambda} < 0.000001$

$k = 6$, $P(6) = 2.4 \times 10^{-6} > 0.000001$

$k = 7$, $P(7) = 1.3 \times 10^{-7}$

$\Rightarrow k = 7$