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OAP – TP2

1) We model this problem using two decision variables:

X_{ij} = beer shipped from brewery i to distribution center j (ML)
Y_{pi} = malt shipped from plant p to brewery i (MT)

The objective function is the sum of the cost of shipping malt and the cost of shipping beer (known data which depends on the plant, brewery and distribution center). It should be minimized.

We have to consider four types of constraints:

- Every distribution center should be shipped more beer than the demand for that distribution center (the demand for each distribution center is known)
- Every brewery should ship less beer than its total production capacity
- Every brewery should ship less beer than what it can produce depending on the malt it was shipped
- Every production plant should ship less malt than its total production capacity

The current shipping cost is $ 11.244 million. Using CPLEX, here is the optimal solution with a cost of **$ 9.908 million**:

- Brewery 1 to DC 1: 103 ML of beer to ship
- Brewery 1 to DC 2: 39 ML of beer to ship
- Brewery 1 to DC 3: 0 ML of beer to ship
- Brewery 1 to DC 4: 60 ML of beer to ship
- Brewery 1 to DC 5: 0 ML of beer to ship
- Brewery 1 to DC 6: 0 ML of beer to ship

- Brewery 2 to DC 1: 0 ML of beer to ship
- Brewery 2 to DC 2: 35 ML of beer to ship
- Brewery 2 to DC 3: 50 ML of beer to ship
- Brewery 2 to DC 4: 0 ML of beer to ship
- Brewery 2 to DC 5: 102 ML of beer to ship
- Brewery 2 to DC 6: 13 ML of beer to ship

- Plant 1 to brewery 1: 24.240969639 MT of malt to ship
- Plant 1 to brewery 2: 5.759030361 MT of malt to ship

- Plant 2 to brewery 1: 0 MT of malt to ship
- Plant 2 to brewery 2: 18.241929677 MT of malt to ship

- Plant 3 to brewery 1: 0 MT of malt to ship
- Plant 3 to brewery 2: 0 MT of malt to ship

Legend:

- Brewery 1 = Istanbul
- Brewery 2 = Ankara

- DC 1 = Istanbul
- DC 2 = Izmir
- DC 3 = Antalya
- DC 4 = Bursa
- DC 5 = Kayseri
- DC 6 = Export (Izmir)

- Plant 1 = Afyon
- Plant 2 = Konya
- Plant 3 = Import (Izmir)

The savings amount to **$ 1.336 million**. It seems to be well worth it to stop the established relationship between Ankara and Bursa and between Istanbul and Antalya as the model does not propose to ship **any** beer between the two pairs of locations.

2) The reduced cost of $X_{1,3}$ (the amount of beer shipped from Istanbul to Antalya) is $ 0.014.

Therefore, the maximum cost of shipping 1 million liters of beer from Istanbul to Antalya should be 0.052 – 0.014 = **0.038 $** so that it becomes optimal to ship beer between the Istanbul brewery and the Antalya distribution center.

3) It is currently not cost effective for Efes to import malt in the first year.

Indeed, here are the reduced cost of importing malt:

- Reduced cost of importing malt to Istanbul: 0.003634945 $
- Reduced cost of importing malt to Ankara: 0.014453618 $

Those are both positive and we're working on a minimization problem, so importing malt would make Efes lose money compared to the solution presented in question 1.

For constraints, the dual variable measures the rate of change in the objective as the right-hand side of the constraint changes. Here, for a capacity constraint, the dual variable measures the improvement in the objective function (total shipping cost) per unit of additional capacity.

After computing the dual variables, we find that the dual variable of the **Ankara brewery capacity** is negative. Therefore, increasing this brewery capacity would reduce the objective function.

Dual var: **Brewery 2 capacity: -0.000080043 $**

4) Sensitivity analysis:

An increase in demand leads to an increase in transportation costs because the dual price of each demand constraint is positive and this is a minimization problem.

Sensitivity intervals table

Constraint Name	Dual Price	Down	Current	Up
demandDC(1)	0.0031	zero	103.0000	121.0000
demandDC(2)	0.0431	35.0000	74.0000	92.0000
demandDC(3)	0.0411	11.0000	50.0000	68.0000
demandDC(4)	0.0201	zero	60.0000	78.0000
demandDC(5)	0.0251	63.0000	102.0000	120.0000
demandDC(6)	0.0451	zero	13.0000	31.0000
capacityBrew(1)	zero	202.0000	220.0000	infinity
capacityBrew(2)	-0.0001	182.0000	200.0000	239.0000
beerYield(1)	-0.0031	-47.9900	zero	152.0100
beerYield(2)	-0.0020	-414.6340	zero	152.0100
capacityMalt(1)	zero	24.2410	30.0000	48.2419
capacityMalt(2)	zero	18.2419	68.0000	infinity
capacityMalt(3)	zero	zero	20.0000	infinity

Annual demand data

Annual demand at distribution centers

		(Million litres/year)	
Dist. Centers	**Year 1**	**Year 2**	**Year 3**
Istanbul	103	110	125
Izmir	74	80	90
Antalya	50	53	60
Bursa	60	75	85
Kayseri	102	110	125
Export (Izmir)	13	13	15
Total	402	441	500

In year 2, the current delivery network will still be the cheapest solution but the total production capacity of the two breweries will be less than the total demand by 1 ML of beer. The total production capacity of the breweries should be increased.

The demand in year 3 will exceed the maximum capacity of the current network to deliver beer to the Istanbul distribution center by 4 ML of beer, Bursa by 7 ML of beer and Kayseri by 5 ML of beer. The brewing capacity will have to be increased for Efes to be able to deliver in full.
On top of that, the 3 DC which wouldn't be able to be supplied in full are the 3 cheapest DCs to ship more beer to (their dual variables are the 3 lowest), which makes upgrading the total brewing capacity even more enticing.