IOITC 2020 TST 1

Removing Leaves

For a rooted tree T, define the depth of a node to be the number of edges on the path from the root to it. Also, define $f(T)$ as the sum of depths of all the nodes in the tree.

You are given a rooted tree T with N nodes numbered $1, 2, \ldots, N$, rooted at node 1. For $i \geq 2$, the parent of node i is p_i. It is given that $p_i < i$ for all $i \geq 2$.

You want to remove all nodes from the tree one by one until only the root remains. In a single step, you can remove any leaf node (a non-root node which has exactly one adjacent node). Note that a node which is not a leaf node initially can become a leaf node later and become eligible for removal.

In this process, let T_0(the original tree), $T_1, T_2, \ldots T_{n-1}$(the tree with only the root remaining) be the sequence of trees formed. In other words, T_i is the tree formed after i nodes have been removed. What is the maximum possible value of $\sum_{i=0}^{n-1} f(T_i)$

Input

- The first line contains Q, the number of testcases. Each testcase contains two lines
- The first line of each testcase contains N, the number of nodes in the tree T.
- The second line of each testcase contains $N-1$ space separated integers, $p_2, p_3, \ldots p_N$.

Output

Print the maximum possible value of $\sum_{i=0}^{n-1} f(T_i)$

Test Data

In all inputs,

- $1 \leq N$
- The sum of N over all testcases doesn't exceed 40.
- For each valid i, $p_i < i$.

Subtask 1 (10 Points): $1 \leq N \leq 20$

Subtask 2 (20 Points): $1 \leq N \leq 30$ and the input tree is a rooted binary tree. (That is, each node has ≤ 2 children).

Subtask 2 (40 Points): The input tree is a rooted binary tree.

Subtask 2 (30 Points): No additional constraints

Sample Input

```
2
4
1 1 1
5
1 2 2 1
```

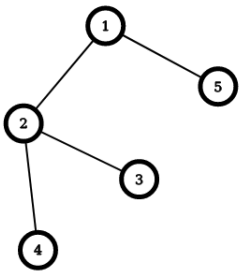
Sample Output

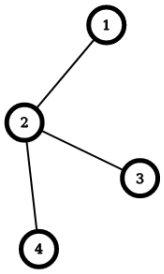
```
6
15
```

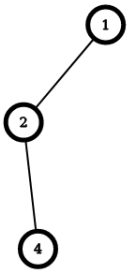
Explanation

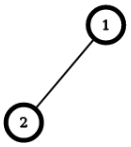
In the first testcase, the depth of node 1 is 0, and that of nodes $2, 3, 4$ are 1. If we remove the nodes in the order $2, 3, 4$, the objective value is $(0 + 1 + 1 + 1) + (0 + 1 + 1) + (0 + 1) + 0 = 6$. This is the maximum possible value, and so is the answer.

In the second testcase, the depths of nodes $1, 2, 3, 4, 5$ are $0, 1, 2, 2, 1$ respectively. We can remove nodes in the order $5, 3, 4, 2$ to get the objective value as $6 + 5 + 3 + 1 + 0 = 15$. The sequence $T_0, T_1, \ldots T_4$ is :









Limits

Time: 2 seconds

Memory: 256 MB