Exercise 4
Fuid Dynamics II SS 2022

11.5.2022

1   Exercise: Reynolds-averaged Navier-Stokes equation

Repeat the derivation of the Reynolds-averaged Navier-Stokes equation

$$\frac{\partial}{\partial t}\bar{\mathbf{u}}(\mathbf{x},t) + \bar{\mathbf{u}}(\mathbf{x},t) \cdot \nabla\bar{\mathbf{u}}(\mathbf{x},t) + \nabla \cdot \langle \mathbf{u}'(\mathbf{x},t)\mathbf{u}'(\mathbf{x},t)\rangle = -\nabla\bar{p}(\mathbf{x},t) + \nu\Delta\bar{\mathbf{u}}(\mathbf{x},t\ .) \tag{1}$$

Derive an evolution equation for the kinetic energy

$$E_{kin}(t) = \frac{1}{2}\int \mathrm{d}\mathbf{x}\ \bar{u}^2(\mathbf{x},t) \tag{2}$$

Hint: Repeat the derivation for the ordinary equation of energy balance from the first lecture and take care of the nonlinear terms.