Exercise 3
Fuid Dynamics II SS 2022

4.5.2022

1 Exercise: Averages

Consider the velocity field of a so-called Burgers vortex, which is given in cylindrical coordinates according to

$$u_r(r,z,t) = -\frac{a}{2}r \ , \quad u_\varphi(r,z,t) = \frac{\Gamma}{2\pi r}\left(1 - e^{-\frac{r^2}{r_B^2}}\right) \ , \quad u_z(r,z,t) = az \ . \tag{1}$$

Here, $r_B^2 = \frac{4\nu}{a}$, and the strain parameter a is assumed to be constant.

1. Sketch the resulting velocity field.
2. Show that the velocity field is incompressible.
3. Show that a Lagrangian tracer particle $\dot{\mathbf{X}}(\mathbf{x}_0, t) = \mathbf{u}(\mathbf{X}(\mathbf{x}_0, t), t)$ with the initial condition $\mathbf{X}(\mathbf{x}_0, t = 0) = \mathbf{x}_0$, obeys the following system of equations

$$\dot{r}(r_0, t) = -\frac{a}{2}r(r_0, t) \ , \tag{2}$$

$$\dot{\varphi}(r_0, t) = \frac{\Gamma}{2\pi r^2(r_0, t)}\left(1 - e^{-\frac{r^2(r_0,t)}{r_B^2}}\right) \ , \tag{3}$$

$$\dot{z}(z_0, t) = az(z_0, t) \ . \tag{4}$$

4. Solve Eqs. (2) and (4) and determine the correlation functions for the radial and vertical correlation functions in terms of temporal averages

$$\langle r(r_0, t)r(r_0, t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_t^{t+T} \mathrm{d}t' r(r_0, t')r(r_0, t'+\tau) \ , \tag{5}$$

$$\langle z(z_0, t)z(z_0, t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_t^{t+T} \mathrm{d}t' z(z_0, t')z(z_0, t'+\tau) \ . \tag{6}$$

5. Discuss whether the ergodicity hypothesis holds or not.

2 Exercise: Gaussian distribution

Draw random numbers N from a standard normal distributions with the help of a computer program (in python, with the function np.random.normal() and in matlab with the function normrnd). Consider the quantity

$$\mu + \sigma N \tag{7}$$

and sort the resulting numbers in a histogram (for different values of μ and σ). If you have difficulties with the programming task, take a look at the jupyter notebook in: src/randomgauss.ipynb of the lecture material.