Exercise 3: Least Squares Principle

Lecture Information Processing and Communication

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Submit solutions until Tuesday 2022-05-10, 23:59h, by uploading to your group's exercise folder on cs.uol.de. You may submit your solutions in groups of at most two students.

1. Linear separation with the least squares principle

Assume that you are given training vectors $\mathbf{x}^{(n)}$ with corresponding class-labels $y^{(n)} \in \{-1, 1\}$ for training examples $n = 1, \ldots, N$. The decision value of a linear classifier is computed as $\hat{y} = \mathbf{w}^T\mathbf{x}$, with a positiv sign of $\hat{y}$ indicating one class and a negative sign indicating the other class as being the most likely class. Note that we assume that the bias term w_0 can be set to zero in this exercise, i.e., it is omitted from the calculations.

According to the least squares principle, the cost-function is defined as

$$E(\mathbf{w}) = \sum_{n=1}^{N} \left(y^{(n)} - \mathbf{w}^T\mathbf{x}^{(n)} \right)^2 .$$

1. Explain how all training vectors and lables can be subsumed into one matrix $\mathbf{X}$ and one vector $\mathbf{y}$, such that the cost-function (or "loss-function") can be written as $E(\mathbf{w}) = (\mathbf{y} - \mathbf{w}^T\mathbf{X})\,(\mathbf{y} - \mathbf{w}^T\mathbf{X})^T$. Of which size are $\mathbf{X}$ and $\mathbf{y}$?

2. Show that the opimal separation plane is defined by its normal vector

$$\hat{\mathbf{w}} = \left(\mathbf{X}\mathbf{X}^T\right)^{-1}\mathbf{X}\mathbf{y}^T .$$

2. Linear receptive fields

Write a small matlab script that reads in an image, converts it to a single (black&white) color channel, and filters it with a two-dimensional receptive field filter. You can design a receptive field filter yourself, or you can use a two-dimensional Gabor function (e.g., https://de.mathworks.com/matlabcentral/fileexchange/37471-gabor-function). Investigate and describe which properties of the images are reflected in the output of the receptive field filter.