Exercise 2: Normal Distribution Recap and 2-dimensional Fourier Transformation

Lecture Information Processing and Communication

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Submit solutions until Tuesday 2022-05-03, 12:00 noon, by uploading to your group's exercise folder on cs.uol.de. You may submit your solutions in groups of at most two students.

1. 1-dimensional normal distribution

(a) Give the definition of the 1-dimensional normal distribution.

(b) Given some measurement data $x^{(1)}, x^{(2)}, \ldots, x^{(N)}$ that are known to be drawn from a normal distribution: How can you estimate the parameters of the normal distribution from these data?

2. z-scoring

Measurement data are often z-scored prior to further processing or classification. Define how z-scored data is obtained from measurement data $x^{(1)}, x^{(2)}, \ldots, x^{(N)}$.

3. N-dimensional normal distribution

(a) Give the definition of the N-dimensional normal distribution.

(b) Given some measurement data $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(N)}$ that are known to be drawn from an N-dimensional normal distribution: How can you estimate the parameters of the normal distribution from these data?

(c) Characterize (qualitatively) the geometric shape of an N-dimensional normal distribution, in particular in relation to the (orthogonal) basis vectors of the Euclidian coordinate system. How does the geometric shape relate to the normal distribution's parameters?

4. Mahalanobis distance

Define the Mahalanobis distance and discuss its relation to z-scoring from question 2.

5. Linear receptive fields

Write a small matlab script that reads in an image and converts it to a single (black&white) color channel. Then use matlab's fft2 function for the two-dimensional Fourier transformation in order to analyze the spatial characteristics of the image. You may consider the magnitude (absolute) value of the fft2 output, disregarding the phase. Interpret the resulting two-dimensional image: which properties of the original image are reflected in which properties of the 2D Fourier transformation? Try this with about three images. Among those, use also images with a dominant spatial characteristics such as vertical oder horizontal structures, or with a dominant diagonal structure. How are these structures reflected in the 2D Fourier transformation result?