# Task3

November 21, 2021

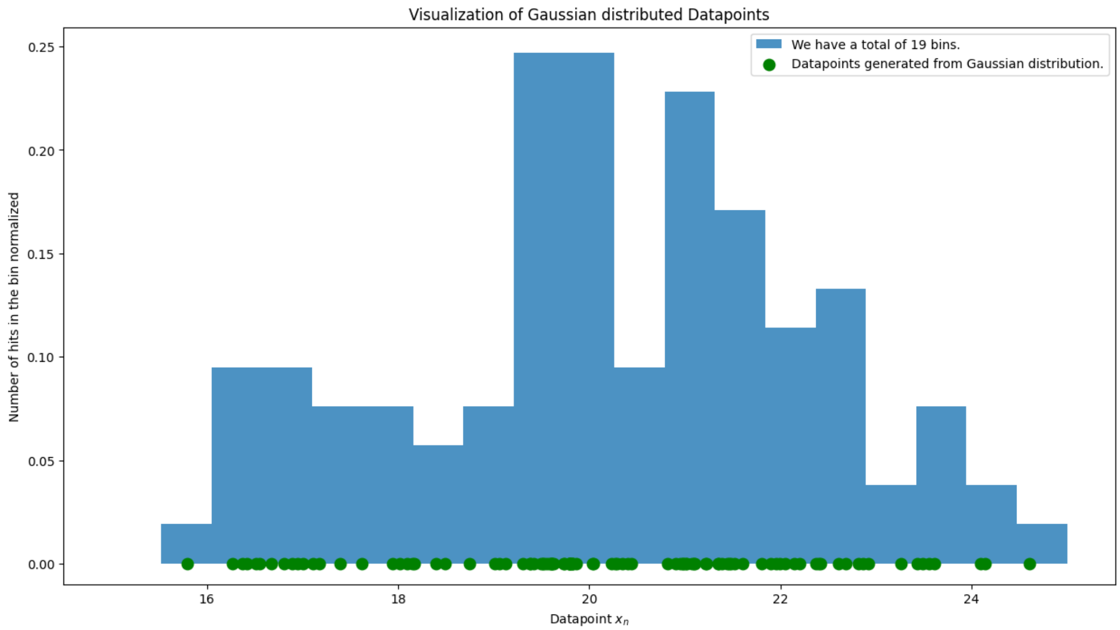
## 1 Exercise 1 Task 3

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display, Markdown, Math
import pandas as pd

plt.style.use("default")
```

## 2 Task A

[ ]:
```
fig, ax = plt.subplots(1,1, figsize = (15,8))

gaussian_samples = np.random.normal(loc=20, scale=2, size=100)
y=np.zeros(len(gaussian_samples))

# Create histogram of the distributed datapoints
# Round to nearest integer
hist_left = np.floor(np.min(gaussian_samples))
hist_right = np.ceil(np.max(gaussian_samples))
num_hist = int((hist_right - hist_left)*2)
bins = np.linspace(hist_left, hist_right, num_hist)

# Creates the histogram of gaussian_samples according to "bins"
ax.scatter(gaussian_samples, y, c="g", zorder=10, s=80, label="Datapoints␣
↪generated from Gaussian distribution.")
ax.hist(gaussian_samples, bins, alpha = 0.8, density=True, label=f"We have a␣
↪total of {len(bins)-1} bins.")

#
ax.set_xlabel("Datapoint $x_n$")
ax.set_ylabel("Number of hits in the bin normalized")
ax.set_title("Visualization of Gaussian distributed Datapoints")
ax.legend()
ax.set_ylim(bottom=-0.01)
```

[ ]: (-0.01, 0.25934999999999964)




# 3 Task B

[ ]:
```
# Compute Maximum Log Likelihood parameters for the datapoints
mu = 1/len(gaussian_samples)*np.sum(gaussian_samples)
sigma_squared = 1/len(gaussian_samples)*np.sum((gaussian_samples-mu)**2)
display(Math(r'Correct\ parameters:\ \mu={:.3f}\ and\ \sigma^2={:.3f}'.
 ↪format(20, 4)))
display(Math(r'Computed\ parameters\ at\ N=100:\ \mu={:.3f}\ and\ \sigma^2={:.
 ↪3f}'.format(mu, sigma_squared)))
```

$Correct\ parameters:\ \mu=20.000\ and\ \sigma^2=4.000$

$Computed\ parameters\ at\ N=100:\ \mu=20.178\ and\ \sigma^2=4.179$

# 4 Task C

### 4.1 We do not get the exact results because the computed parameters with the maximum log likelihood function is actually a computation with the sampling formula. Both approaches result in the same formula and the sampling formula is only an approximation with N datapoints.

# 5 Task D

### 5.1 On average, the more datapoints you have, the better the estimated parameters. Certainly it is possible that a simulation with fewer datapoints results in a better parameter approximation but on average more datapoints results in better estimations.

### 5.2 It seems that espectation value $\mu$ is not affected by the number of samples as much as the standard deviation.

[ ]:
```
results = []
for n_samples in [3, 5, 10, 50, 70, 100, 150, 250, 500, 1000, 5000, 10000,␣
 ↪50000, 100000]:
    gaussian_samples = np.random.normal(loc=20, scale=2, size=n_samples)
    mu = 1/len(gaussian_samples)*np.sum(gaussian_samples)
    sigma_squared = 1/len(gaussian_samples)*np.sum((gaussian_samples-mu)**2)

    results.append({"N_samples": n_samples, "mean": mu,
                    "std squared": sigma_squared})
df_results = pd.DataFrame(results)
df_results = df_results.set_index("N_samples")
df_results
```

[ ]:

| N_samples | mean | std squared |
|---|---|---|
| 3 | 21.173022 | 3.775418 |
| 5 | 19.424326 | 5.616439 |
| 10 | 20.212347 | 4.174615 |
| 50 | 19.720706 | 2.759251 |
| 70 | 19.891633 | 3.773514 |
| 100 | 20.154689 | 2.653187 |
| 150 | 20.086728 | 3.521250 |
| 250 | 19.995986 | 3.650498 |
| 500 | 19.949967 | 3.829200 |
| 1000 | 20.021011 | 3.939625 |
| 5000 | 19.991887 | 3.950861 |
| 10000 | 19.997612 | 3.968119 |
| 50000 | 19.997800 | 3.986841 |
| 100000 | 19.998166 | 3.988823 |