

max(0.1x, x)

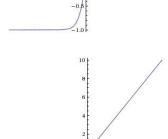
Leaky ReLU



$$\sigma(x) = 1/(1+e^{-x})$$

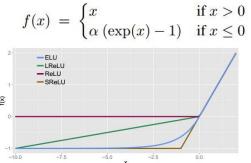
 $\max(w_1^T x + b_1, w_2^T x + b_2)$

tanh(x) tanh



ELU

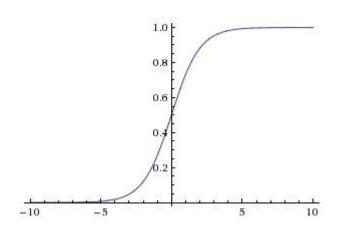
Maxout



ReLU max(0,x)Fei-Fei Li & Andrej Karpathy & Justin Johnson

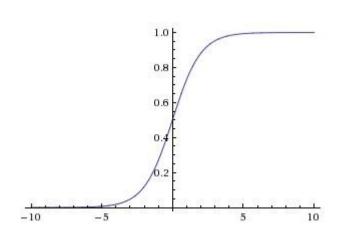
Lecture 5 - 29

20 Jan 2016



$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron



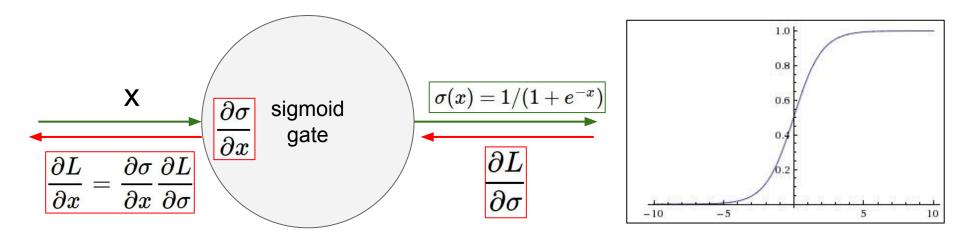
Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
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3 problems:

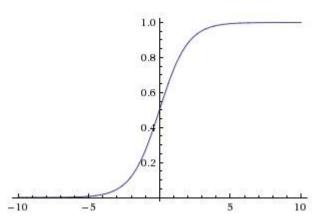
 Saturated neurons "kill" the gradients



What happens when x = -10?

What happens when x = 0?

What happens when x = 10?



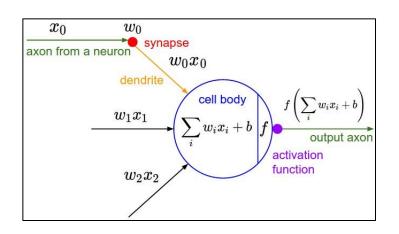
$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

3 problems:

- Saturated neurons "kill" the gradients
- Sigmoid outputs are not zerocentered

Consider what happens when the input to a neuron (x) is always positive:



$$f\left(\sum_{\pmb{i}} w_{\pmb{i}} x_{\pmb{i}} + b
ight)$$

What can we say about the gradients on w?

Consider what happens when the input to a neuron is

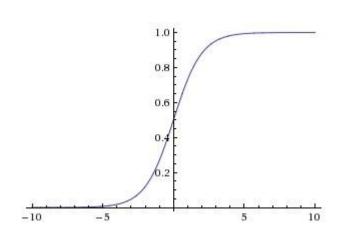
always positive...

$$f\left(\sum_{\pmb{i}} w_{\pmb{i}} x_{\pmb{i}} + b
ight)$$

What can we say about the gradients on **w**? Always all positive or all negative :((this is also why you want zero-mean data!)

gradient update directions zig zag path allowed gradient update directions hypothetical optimal w vector

allowed



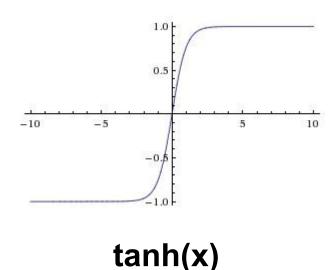
Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

3 problems:

- Saturated neurons "kill" the gradients
- Sigmoid outputs are not zerocentered
- 3. exp() is a bit compute expensive



- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

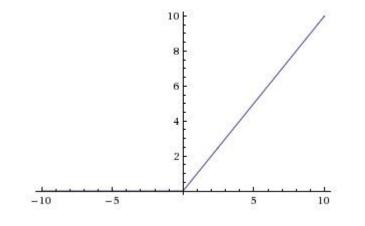
[LeCun et al., 1991]

Computes f(x) = max(0,x)



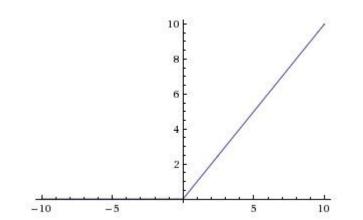
- Does not saturate (in +region)

 - Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)



ReLU (Rectified Linear Unit)

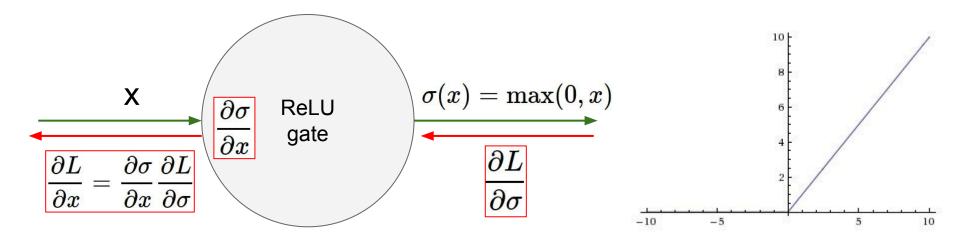
[Krizhevsky et al., 2012]



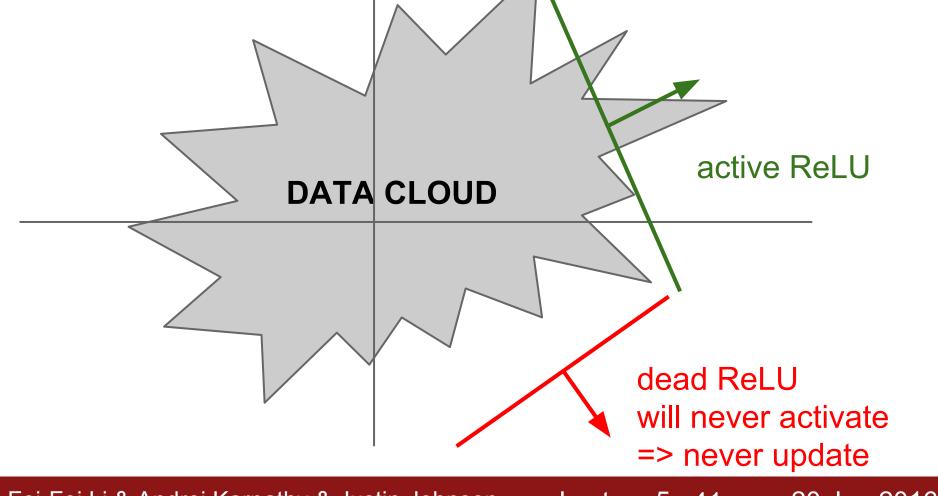
ReLU (Rectified Linear Unit)

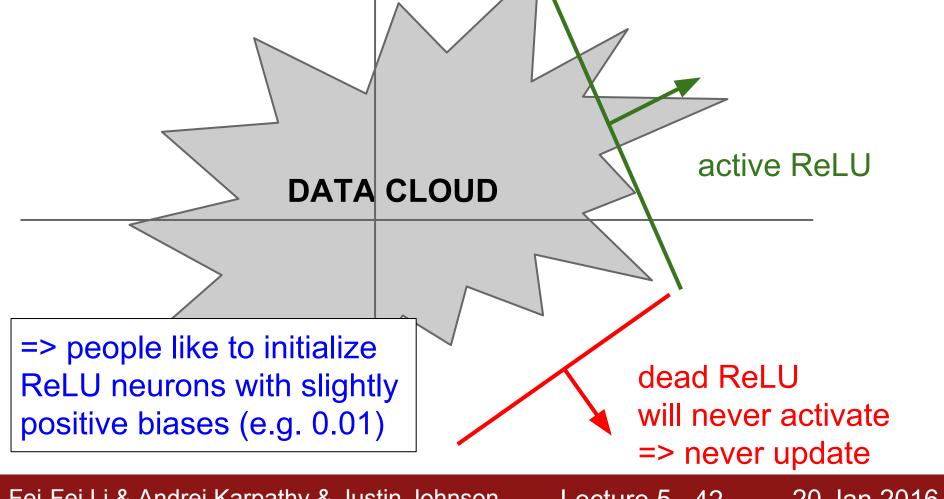
- Computes f(x) = max(0,x)
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Not zero-centered output
- An annoyance:

hint: what is the gradient when x < 0?

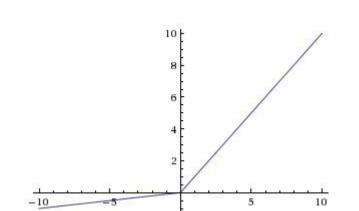


What happens when x = -10? What happens when x = 0? What happens when x = 10?





[Mass et al., 2013] [He et al., 2015]

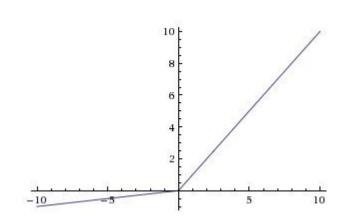


- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not "die".

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

[Mass et al., 2013] [He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not "die".

Leaky ReLU

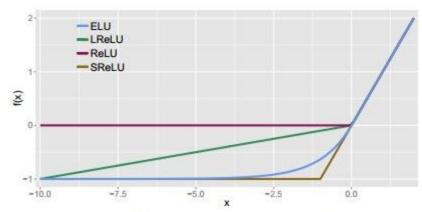
$$f(x) = \max(0.01x, x)$$

Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into \alpha / (parameter)

Exponential Linear Units (ELU)



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

- All benefits of ReLU
- Does not die
- Closer to zero mean outputs
- Computation requires exp()

Maxout "Neuron"

- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

Problem: doubles the number of parameters/neuron:(

TLDR: In practice:

- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / Maxout / ELU
- Try out tanh but don't expect much
- Don't use sigmoid

Data Preprocessing

TLDR: In practice for Images: center only

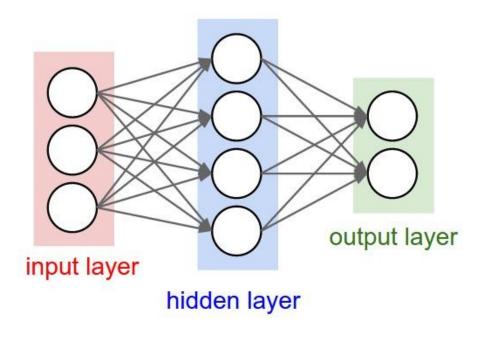
e.g. consider CIFAR-10 example with [32,32,3] images

- Subtract the mean image (e.g. AlexNet) (mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet)
 (mean along each channel = 3 numbers)

Not common to normalize variance, to do PCA or whitening

Weight Initialization

Q: what happens when W=0 init is used?



- First idea: **Small random numbers** (gaussian with zero mean and 1e-2 standard deviation)

$$W = 0.01* np.random.randn(D,H)$$

- First idea: **Small random numbers** (gaussian with zero mean and 1e-2 standard deviation)

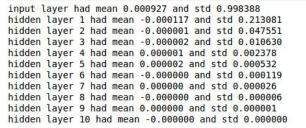
$$W = 0.01* np.random.randn(D,H)$$

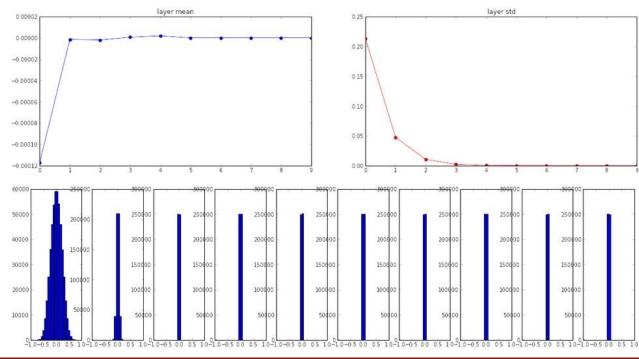
Works ~okay for small networks, but can lead to non-homogeneous distributions of activations across the layers of a network.

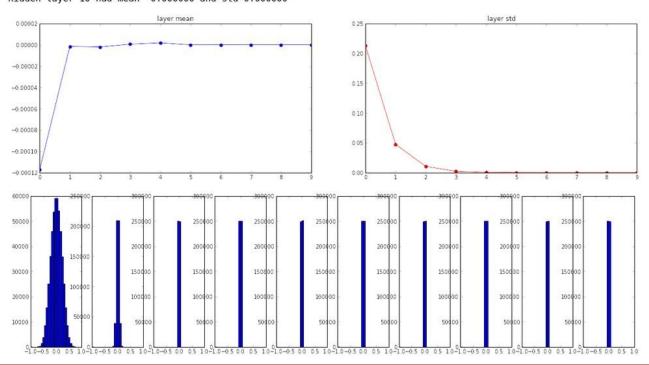
Lets look at some activation statistics

E.g. 10-layer net with 500 neurons on each layer, using tanh non-linearities, and initializing as described in last slide.

```
# assume some unit gaussian 10-D input data
D = np.random.randn(1000, 500)
hidden layer sizes = [500]*10
nonlinearities = ['tanh']*len(hidden layer sizes)
act = {'relu':lambda x:np.maximum(0,x), 'tanh':lambda x:np.tanh(x)}
Hs = \{\}
for i in xrange(len(hidden layer sizes)):
    X = D if i == 0 else Hs[i-1] # input at this layer
    fan in = X.shape[1]
    fan out = hidden laver sizes[i]
    W = np.random.randn(fan in, fan out) * 0.01 # layer initialization
    H = np.dot(X, W) # matrix multiply
    H = act[nonlinearities[i]](H) # nonlinearity
    Hs[i] = H # cache result on this layer
# look at distributions at each layer
print 'input layer had mean %f and std %f' % (np.mean(D), np.std(D))
layer means = [np.mean(H) for i,H in Hs.iteritems()]
layer stds = [np.std(H) for i,H in Hs.iteritems()]
for i,H in Hs.iteritems():
    print 'hidden layer %d had mean %f and std %f' % (i+1, layer means[i], layer stds[i])
# plot the means and standard deviations
plt.figure()
plt.subplot(121)
plt.plot(Hs.keys(), layer means, 'ob-')
plt.title('layer mean')
plt.subplot(122)
plt.plot(Hs.keys(), layer stds, 'or-')
plt.title('layer std')
# plot the raw distributions
plt.figure()
for i,H in Hs.iteritems():
    plt.subplot(1.len(Hs).i+1)
    plt.hist(H.ravel(), 30, range=(-1,1))
```

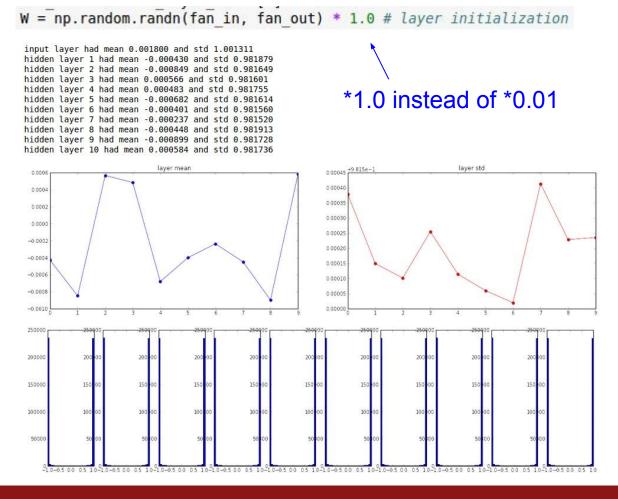




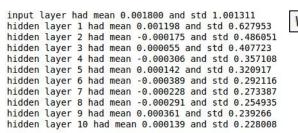


All activations become zero!

Q: think about the backward pass.
What do the gradients look like?

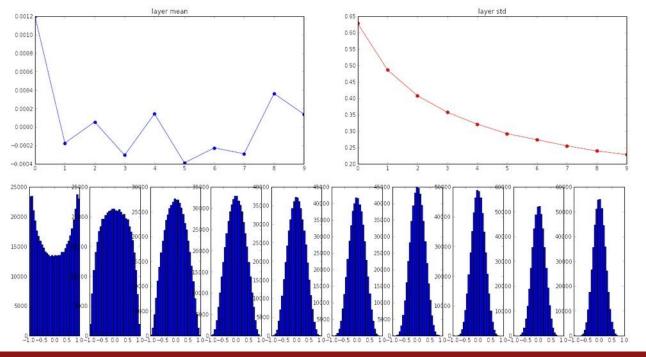


Almost all neurons completely saturated, either -1 and 1. Gradients will be all zero.

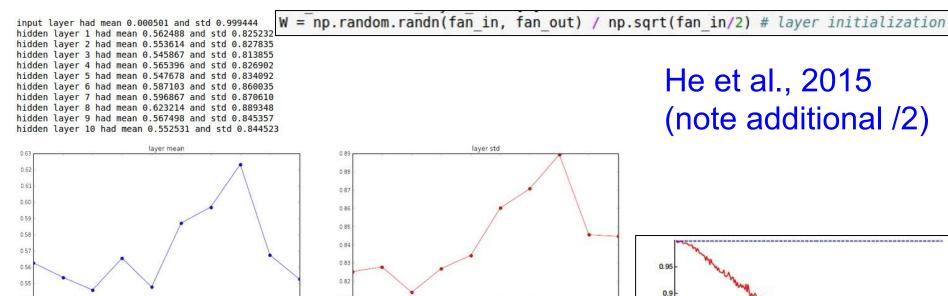




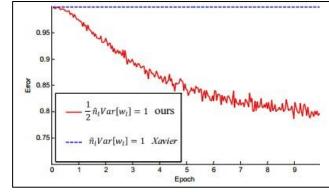
"Xavier initialization" [Glorot et al., 2010]



Reasonable initialization. (Mathematical derivation assumes linear activations)







Batch Normalization

"you want unit gaussian activations? just make them so."

consider a batch of activations at some layer. To make each dimension unit gaussian, apply:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

Batch Normalization

"you want unit gaussian activations? just make them so."

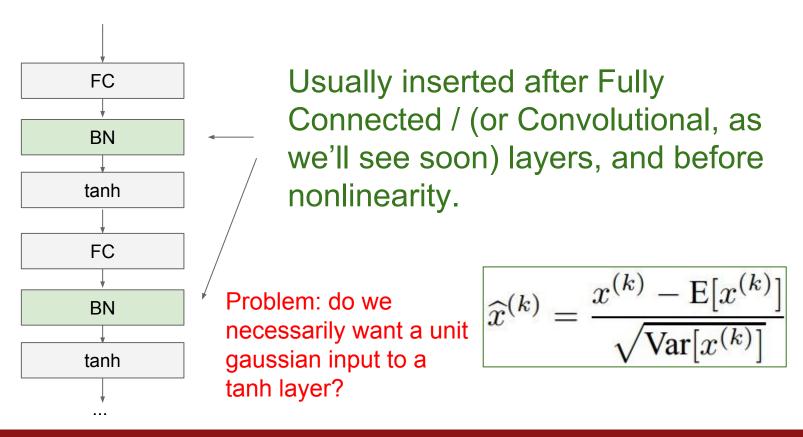
N

1. compute the empirical mean and variance independently for each dimension.

2. Normalize

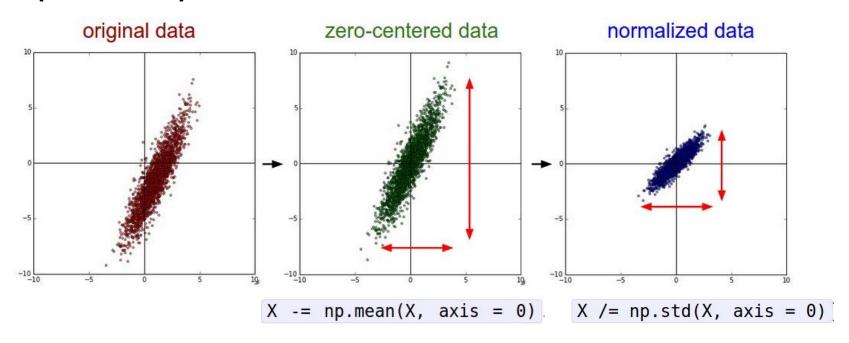
$$\widehat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

Batch Normalization



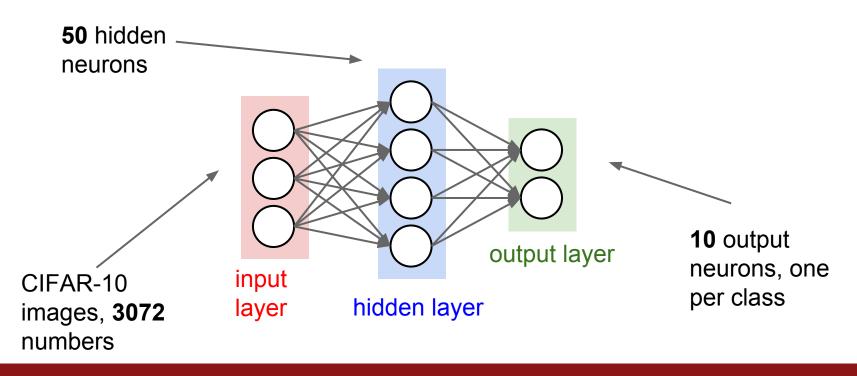
Babysitting the Learning Process

Step 1: Preprocess the data



(Assume X [NxD] is data matrix, each example in a row)

Step 2: Choose the architecture: say we start with one hidden layer of 50 neurons:



Double check that the loss is reasonable:

```
def init_two_layer_model(input_size, hidden_size, output_size):
    # initialize a model
    model = {}
    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)
    model['b1'] = np.zeros(hidden_size)
    model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)
    model['b2'] = np.zeros(output_size)
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes loss, grad = two_layer_net(X_train, model, y_train 0.0) disable regularization

2.30261216167 loss ~2.3.

"correct " for returns the loss and the gradient for all parameters
```

Double check that the loss is reasonable:

```
def init_two_layer_model(input_size, hidden_size, output_size):
    # initialize a model
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    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)
    model['b1'] = np.zeros(hidden_size)
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    model['b2'] = np.zeros(output_size)
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input_size, hidden size, number of classes loss, grad = two_layer_net(X_train, model, y_train, le3) crank up regularization

3.06859716482 loss went up, good. (sanity check)
```

Tip: Make sure that you can overfit very small portion of the training data

Very small loss, train accuracy 1.00, nice!

```
model = init two layer model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X tiny = X train[:20] # take 20 examples
y tiny = y train[:20]
best model, stats = trainer.train(X tiny, y tiny, X tiny, y tiny,
                                  model, two layer net,
                                  num epochs=200, reg=0.0,
                                  update='sgd', learning rate decay=1,
                                  sample batches = False.
                                  learning rate=1e-3, verbose=True)
Finished epoch 1 / 200: cost 2.302603, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 2 / 200: cost 2.302258, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 3 / 200: cost 2.301849, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 4 / 200: cost 2.301196, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 5 / 200: cost 2.300044, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 6 / 200: cost 2.297864, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 7 / 200: cost 2.293595, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 8 / 200: cost 2.285096, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 9 / 200: cost 2.268094, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 10 / 200: cost 2.234787, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 11 / 200: cost 2.173187, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 12 / 200: cost 2.076862, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 13 / 200: cost 1.974090, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 14 / 200: cost 1.895885, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 15 / 200: cost 1.820876, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 16 / 200: cost 1.737430, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 17 / 200: cost 1.642356, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 18 / 200: cost 1.535239, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 19 / 200: cost 1.421527, train: 0.600000, val 0.600000, lr 1.000000e-03
      Finished epoch 195 / 200: cost 0.002694, train: 1.000000, val 1.000000, lr 1.000000e-03
      Finished epoch 196 / 200: cost 0.002674, train: 1.000000, val 1.000000, lr 1.000000e-03
      Finished epoch 197 / 200: cost 0.002655, train: 1.000000, val 1.000000, lr 1.000000e-03
      Finished epoch 198 / 200: cost 0.002635, train: 1.000000, val 1.000000, lr 1.000000e-03
      Finished epoch 199 / 200: cost 0.002617, train: 1.000000, val 1.000000, lr 1.000000e-03
      Finished epoch 200 / 200: cost 0.002597, train: 1.000000, val 1.000000, lr 1.000000e-03
      finished optimization. best validation accuracy: 1.000000
```

I like to start with small regularization and find learning rate that makes the loss go down.

I like to start with small regularization and find learning rate that makes the loss go down.

```
model = init two layer model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best model, stats = trainer.train(X train, y train, X val, y val,
                                  model, two layer net,
                                  num epochs=10, reg=0.000001,
                                  update='sqd', learning rate decay=1,
                                  sample batches = True,
                                  learning rate=le-6, verbose=True)
Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10 cost 2.302420 train: 0.190000, val 0.192000, lr 1.000000e-06
```

Loss barely changing

finished optimization, best validation accuracy: 0.192000

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down: learning rate too low

```
model = init two layer model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best model, stats = trainer.train(X train, y train, X val, y val,
                                  model, two layer net.
                                  num epochs=10, reg=0.000001,
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Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
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Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10 cost 2.302420 train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization, best validation accuracy: 0.192000
```

Loss barely changing: Learning rate is probably too low

I like to start with small regularization and find learning rate that makes the loss go down.

```
model = init two layer model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best model, stats = trainer.train(X train, y train, X val, y val,
                                  model, two layer net,
                                  num epochs=10, reg=0.000001.
                                  update='sgd', learning rate decay=1,
                                  sample batches = True,
                                  learning rate=1e6, verbose=True)
/home/karpathy/cs231n/code/cs231n/classifiers/neural net.py:50: RuntimeWarning: divide by zero en
countered in log
 data loss = -np.sum(np.log(probs[range(N), y])) / N
/home/karpathy/cs231n/code/cs231n/classifiers/neural net.py:48: RuntimeWarning: invalid value enc
ountered in subtract
  probs = np.exp(scores - np.max(scores, axis=1, keepdims=True))
Finished epoch 1 / 10: cost nan, train: 0.091000, val 0.087000, lr 1.000000e+06
Finished epoch 2 / 10: cost nan, train: 0.095000, val 0.087000, lr 1.000000e+06
Finished epoch 3 / 10: cost nan, train: 0.100000, val 0.087000, lr 1.000000e+06
```

loss not going down: learning rate too low loss exploding: learning rate too high cost: NaN almost always means high learning rate...

I like to start with small regularization and find learning rate that makes the loss go down.

Finished epoch 1 / 10: cost 2.186654, train: 0.308000, val 0.306000, lr 3.000000e-03
Finished epoch 2 / 10: cost 2.176230, train: 0.330000, val 0.350000, lr 3.000000e-03
Finished epoch 3 / 10: cost 1.942257, train: 0.376000, val 0.352000, lr 3.000000e-03
Finished epoch 4 / 10: cost 1.827868, train: 0.329000, val 0.310000, lr 3.000000e-03
Finished epoch 5 / 10: cost inf, train: 0.128000, val 0.128000, lr 3.000000e-03
Finished epoch 6 / 10: cost inf, train: 0.144000, val 0.147000, lr 3.000000e-03

3e-3 is still too high. Cost explodes....

loss not going down: learning rate too low loss exploding: learning rate too high

=> Rough range for learning rate we should be cross-validating is somewhere [1e-3 ... 1e-5]

Hyperparameter Optimization

TODO

Look at:

- Parameter update schemes
- Learning rate schedules
- Gradient Checking
- Regularization (Dropout etc)
- Evaluation (Ensembles etc)