

① CPS 803 Fall 2021 Assignment - 1

Name: TUSAIF AZMAT

Student #: 500660278

Q1: Linear Regression

Problem: 1

Solution: ^{same} As in Example discussed in class.

$$① - h_{\theta}(\hat{x}^{(i)}) = \theta_3 \hat{x}_3 + \theta_2 \hat{x}_2 + \theta_1 \hat{x}_1 + \theta_0 \hat{x}_0$$

$$J(\theta) = \frac{1}{2} \sum_{i=1}^n (h_{\theta}(\hat{x}^{(i)}) - y^{(i)})^2 \rightarrow ①$$

objective function $J(\theta)$ of the linear regression problem on the new dataset $\{(\hat{x}^{(i)}, y^{(i)})\}_{i=1}^n$

② Gradient descent
we start with some θ and keep changing θ to reduce $J(\theta)$.

we use the gradient $\theta_j^* := \theta_j - \alpha \frac{2}{2\theta_j} J(\theta)$ ②
putting the value ^{eq (1)} in ^{eq (2)}

we get

$$\theta_j := \theta_j - \alpha \sum_{i=1}^n [(h_{\theta}(\hat{x}^{(i)}) - y^{(i)}) \hat{x}_j^{(i)}] \checkmark$$

Q2: Calculations

given

$$\forall x, P(y^{(i)} = 1 | t^{(i)} = 1, x^{(i)} = x) = \alpha$$

$$\forall x, P(y^{(i)} = 0 | t^{(i)} = 1, x^{(i)} = x) = 1 - \alpha$$

$$\forall x, P(y^{(i)} = 1 | t^{(i)} = 0, x^{(i)} = x) = 0$$

$$\forall x, P(y^{(i)} = 0 | t^{(i)} = 0, x^{(i)} = x) = 1$$

$$P(t^{(i)} = 1 | y^{(i)} = 1, x^{(i)}) = 1$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(t^{(i)} = 1 | y^{(i)} = 1, x^{(i)}) = \frac{P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) P(t^{(i)} = 1)}{P(y^{(i)} = 1, x^{(i)})}$$

$$P(t^{(i)} = 1 | y^{(i)} = 1, x^{(i)}) = \frac{P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) P(t^{(i)} = 1)}{P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 0) P(t^{(i)} = 0) + P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) P(t^{(i)} = 1)}$$

$$P(t^{(i)} = 1 | y^{(i)} = 1, x^{(i)}) = \frac{P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) (t^{(i)} = 1)}{(y^{(i)} = 1, x^{(i)} | t^{(i)} = 0) (t^{(i)} = 0) + P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) (t^{(i)} = 1)}$$

Since, $\forall x, P(y^{(i)} = 1 | t^{(i)} = 0, x^{(i)} = 0) = 0 \leftarrow$ use this in equation

$$P(t^{(i)} = 1 | y^{(i)} = 1, x^{(i)}) = \frac{P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) (t^{(i)} = 1)}{(0)(0.5) + P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) (t^{(i)} = 1)}$$

$$P(t^{(i)} = 1 | y^{(i)} = 1, x^{(i)}) = \frac{P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) (t^{(i)} = 1)}{P(y^{(i)} = 1, x^{(i)} | t^{(i)} = 1) (t^{(i)} = 1)} = \underline{\underline{1}} \quad \checkmark$$