ECEn 671: Mathematics of Signals and Systems

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Necessary and sufficient conditions:

- The following are equivalent:

  $P \iff Q$

  P if and only if Q

  Q is necessary and sufficient for P

- The following are equivalent:

  $P \Rightarrow Q$

  P only if Q

  Q is a necessary condition for P

- The following are equivalent:

  $P \Leftarrow Q$

  P if Q

  Q is a sufficient condition for P

Necessary and sufficient conditions (Examples)

Theorem
If $y(t)$ is continuously
differentiable, then
[$y(t')$ is a local maximum of y]
        only if
[$\frac{dy}{dt}(t') = 0$].

What are alternative statements
to this theorem?

Necessary and Sufficient Conditions

If a theorem is written as
$P \iff Q$, then the proof must
show:

- $P \Rightarrow Q$ and
- $P \Leftarrow Q$.

In other words, we need to show
that

- Q is a necessary condition
  for P $(P \Rightarrow Q)$, and

- Q is a sufficient condition
  for P $(Q \Rightarrow P)$.

Types of Proof

- Direct argument
- Proof by contradiction
- Proof by induction

Example of Direct Proof

Define

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}.$$

Theorem
If $a \geq 0$, then $|x| \leq a$ if and only if $-a \leq x \leq a$.