ECEn 671: Mathematics of Signals and Systems

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Section 1

Underdetermined Problems

Section 3.15: Underdetermined Problems

Given $Ax = b$ where A is fat, i.e. fewer equations than unknowns, solve the following problem:

$$\begin{aligned} \min \quad & \|x\|_2 \\ \text{s.t.} \quad & Ax = b \end{aligned}$$

where $A = \begin{pmatrix} y_1^H \\ \vdots \\ y_m^H \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$,
and $y_i \in \mathbb{C}^n$ and $b \in \mathbb{C}^m$.

Section 3.15: Underdetermined Problems, cont.

$Ax = b$ is a set of inner product constraints

$$y_1^H x = b_1$$

$$\vdots$$

$$y_m^H x = b_m$$

Let $M = span\{y_1, \cdots, y_m\}$.
Theorem 3.4 implies that $x_0 = \arg\min \|x\| \in M$

$$\Rightarrow x_0 = \sum c_j y_j = A^H c$$

and that c satisfies

$$R\mathbf{c} = \mathbf{b} \text{ where } R = AA^H$$

if $\{y_1, \cdots, y_m\}$ are linearly independent then

$$\mathbf{c} = (AA^H)^{-1}\mathbf{b} \qquad \Rightarrow \qquad x_0 = \underbrace{A^H(AA^H)^{-1}}_{\text{pseudo-inverse}}\mathbf{b}$$