ECEn 671: Mathematics of Signals and Systems

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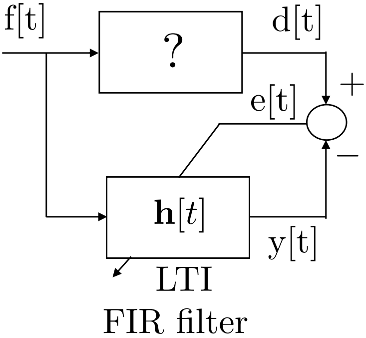
Section 1

Application: LMS Adaptive Filtering

LMS Adaptive Filtering



Recall the RLS adaptive filter algorithm.
The objective is to minimize the error

$$J(\mathbf{h}) = (d[t] - y[t])^2.$$

- The RLS minimizes the squared error of all past outputs, but LMS only minimizes the squared error of the current output.
- The RLS algorithm was derived using the projection theorem.
- LMS is derived using gradient descent.

LMS Adaptive Filtering

Assume that the output of the adaptive filter is

$$y[t] = \sum_{\ell=0}^{m-1} h[\ell] f[t-\ell] = \mathbf{f}^\top[t]\mathbf{h}$$

where

$$\mathbf{f}[t] = \begin{pmatrix} f[t] \\ f[t-1] \\ \vdots \\ f[t-m+1] \end{pmatrix} \text{ and } \mathbf{h} = \begin{pmatrix} h[0] \\ h[1] \\ \vdots \\ h[m-1] \end{pmatrix}$$

LMS Adaptive Filtering

Then

$$\begin{aligned} J(\mathbf{h}) &= (d[t] - y[t])^2 \\ &= (d[t] - \mathbf{f}^\top[t]\mathbf{h})^2 \\ &= d^2[t] - d[t]\mathbf{f}^\top[t]\mathbf{h} - d[t]\mathbf{h}^\top f[t] + \mathbf{h}\mathbf{f}[t]\mathbf{f}^\top[t]\mathbf{h} \end{aligned}$$

where

$$\frac{\partial J}{\partial \mathbf{h}} = 2\mathbf{f}[t]\mathbf{f}^\top[t]\mathbf{h} - 2d[t]\mathbf{f}[t]$$

LMS Adaptive Filtering

So let

$$\mathbf{h}[t+1] = \mathbf{h}[t] - \alpha \frac{\partial J}{\partial \mathbf{h}}(\mathbf{h}[t])$$

gives

$$\begin{aligned}\mathbf{h}[t+1] &= \mathbf{h}[t] - 2\alpha(\mathbf{f}[t]\mathbf{f}^\top[t]\mathbf{h}[t] - d[t]\mathbf{f}[t]) \\ &= \mathbf{h}[t] + \mu\mathbf{f}[t](d[t] - \mathbf{f}^\top[t]\mathbf{h}[t])\end{aligned}$$

$$\boxed{\mathbf{h}[t+1] = \mathbf{h}[t] + \mu\mathbf{f}[t]e[t]}$$

This is known as the LMS adaptive filter.

Compare to RLS...

For discussion on convergence, consult Moon Chap 14...