Maths
Year 10 Notes

Jackson

2022

Contents

0.1 Factoring Quadratics

0.1.1 Difference of squares

Theorem 0.1.1 Completing the Square

This factorisation involves taking the square root of the numbers involved and factorising.
See Q1 and Q2

Question 1

$$-x^2 - 9$$

$$(x + 3)(x - 3)$$
$$x = \pm 3$$

Question 2

$$64x^2 - 25y^2$$

$$(8x - 5y^{)}(8x + 5y^{)}$$

Note:-
Cannot gain intercepts from this(too many variables)

0.1.2 Common factorising

Theorem 0.1.2 Common factorising

When we have a common factor, we take the GCF out of both numbers by dividing
and then factor the smaller numbers. SEE examples below.

Question 3

$$3x^2 - 75y^2$$

$$= 3(x^2 + 25y^2)$$
$$= 3(x - 5y)(x + 5y)$$

Note:-
Remember to factor out the coefficient first

Question 4

$$6x^2 + 12x$$

$$= 6x(x + 2)$$

0.1.3 Factoring by Grouping Pairs(IN Progress)

Theorem 0.1.3 Grouping pairs

Grouping pairs involves four-term expressions and factorising them by grouping them into like terms and then obtaining the GCF. SEE Q5 and Q6 for examples

Question 5

$$x^2 + 4x + ax + 4a$$

$$= (4x + 4a)(x^2 + ax)$$
$$= 4(x + a)x(x + a)$$
$$= (x + a)(4 + x)$$

Note:-

Remember that if the same expression is inside the brackets
you can factorise the coefficient of the brackets into one of the sets

Question 6

$$x^2 + 7x + bx + 7b$$

$$= (7x + 7b)(x^2 + bx)$$
$$= 7(x + b)x(x + b)$$
$$= (x + b)(7 + x)$$

0.1.4 Trinomials

Trinomial A

Theorem 0.1.4 Trinomial A

In monic quadratics the coefficient of x^2 is 1
Monic quadratics of the form $x^2 + bx$ + c can be factorised by finding the two numbers that multiply to give the constant term (c) and add to give the coefficient of $x(b)$

Question 7

$$x^2 + 4 - 12$$

$$(x + 6)(x - 2)$$
$$x = -6, 2$$

Question 8

$$z^2 + 5x + 6$$

$$(x + 3)(x + 2)$$
$$x = -2, -3$$

Note:-

When factorising trinomials, ask what the factors of c add to bx

Trinomial B

Theorem 0.1.5 Trinomial B

One method that can be used to factorise a non-monic trinomial of the form $ax^2 + bx + c$
Find two numbers that multiply to give a×c and add to give b

Question 9

$$2h^2 + 4h + 2 = 0$$

$$(2h + 2)^2 = 0$$
$$2(h + 1)^2 = 0$$
$$h = -1$$

Question 10

$$10s^2 - 21s + 9 = 0$$

$$(10s - 15)(10s - 6)$$
$$5(2s - 3)2(5s - 3)$$

Note:-

Always remember to simplify numbners in brackets like $\frac{15}{10}$ is equivalent $\frac{3}{2}$

0.1.5 Completing the Square

Theorem 0.1.6 Completing the Square

To complete the square for $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$

$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Question 11

$$x^2 + 8x + 7 = 0$$

$$x^2 = 8x + 16 = 9$$
$$(x + 4)^2 = 9$$
$$x + 4 = \sqrt{9}$$
$$x = -4 \pm 3$$

Question 12

$$x^2 + 20x - 13$$

$$x^2 + 20x + 100 = 87$$
$$(x + 10)^2$$
$$x + 10 = \pm\sqrt{87}$$
$$x = -10 \pm \sqrt{87}$$

Note:-

Remember to simplify your surds and when you introduce a surd can have a $\pm$ value

0.1.6 The Quadratic Formula

Theorem 0.1.7 The quadratic formula

The quadratic formula is very useful when $ax^2 + bx + c$ is difficult to factorise.
The discriminant $\Delta = b^2 - 4ac$ is used to find the number of solutions to an Equations
If $\Delta < 0$, there are no real solutions due to it being an undefined number
If $\Delta = 0$, there is only one real solution
If $\Delta > 0$, there are only two real solutions. The Quadratic formula is:

$$x = \left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

Question 13

$$x^2 + 5x + 3$$

$$x = \left(\frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 3}}{2 \times 1}\right)$$
$$= \left(\frac{-5 \pm \sqrt{25 - 12}}{2}\right)$$
$$= \left(\frac{-5 \pm \sqrt{13}}{2}\right)$$

Question 14

$$2x^2 - 2x - 1$$

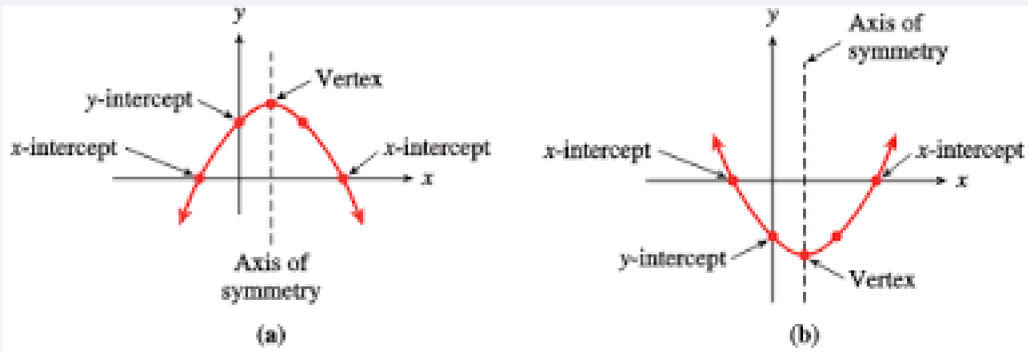
$$x = \left(\frac{2 \pm \sqrt{(-2)^2 - 4 \times 2 \times -1}}{2 \times 2}\right)$$
$$x = \left(\frac{2 \pm \sqrt{12}}{4}\right)$$
$$x = \left(\frac{2 \pm 2\sqrt{3}}{4}\right)$$
$$x = \left(\frac{1 \pm \sqrt{3}}{2}\right)$$

0.2 Parabolas

0.2.1 Features of a Parabola

Theorem 0.2.1 Parabola

Terms:
Y-intercept: where the parabola cuts the Y axis(found by substituting x = 0 into the equation)
X intercept: where the parabola cuts the X axis one or two times (find x int using factorisation)
Axis of symmetry: vertical line that splits the parabola into two halves $\left(\frac{-b}{2a}\right)$
Turning Point/ Vertex: Where the parabola cuts the axis of symmetry (found by substituting the axis of symmetry value into the quadratic equation)



0.2.2 Parabolic Transformations

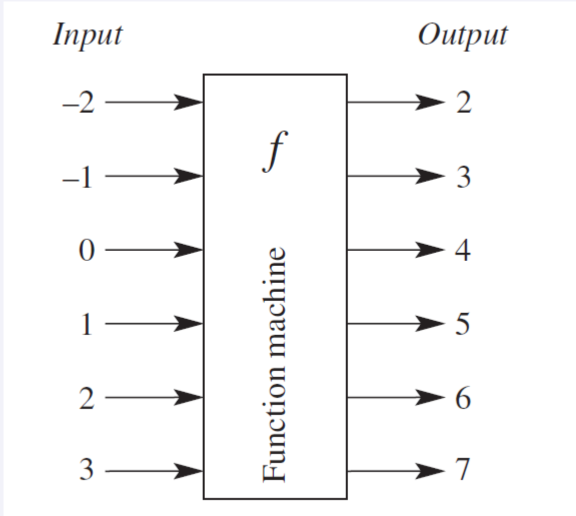
Theorem 0.2.2 Parabolic Transformations

Parabola Transformations:
if $a < 0$, the parabola is concave down
if $a > 0$, the parabola is concave up
if $c < 0$ move parabola down y axis
if $c > 0$ move parabola up y axis
if $b < 0$ move parabola left along x axis
if $b > 0$, the move parabola right along the x-axis

0.3 Functions and their notation

Theorem 0.3.1 Functions and their notation

Functions are another way of writing equations for graphs, $y = x^2$ can be written as $f(x) = x^2$
Any set of ordered pairs is called a relation when referring to a function, a relation where each x value produces only one y value is called a function.
Any relation that also passes the vertical line test can be referred to as a Function.
The set of permissible x values in a relation is referred to as the domain
The range is the set of y coordinates the function can output into a relation



Question 15

$$f(x) = 3x + 4 \text{ find } f(\tfrac{1}{2})$$

$$\begin{aligned} f(\frac{1}{2}) &= 3(\frac{1}{2}) + 4 \\ &= \frac{3}{2} + 4 \\ &= \frac{11}{2} \end{aligned}$$

Question 16

Find the domain of the following equation $f(x) = \frac{2}{x}$

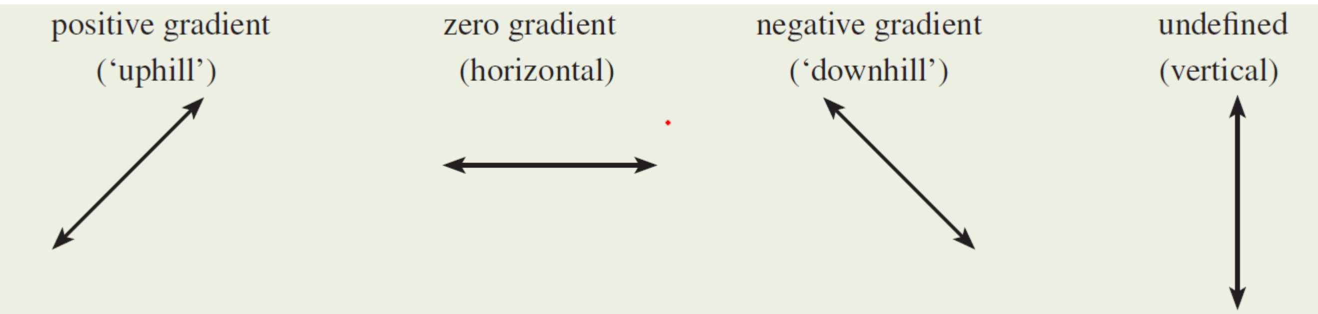
The domain for the equation is $x \neq 0$

0.4 Linear Relationships

0.4.1 Straight lines

Straight line formula

Theorem 0.4.1 Straight line formula
Gradient states the slope of a line abbreviated to m in equations To find the gradient the equation is $\frac{y2-y1}{x2-x1}$ If the gradient is positive, the slope is uphill If the gradient is negative, the slope is downhill No gradient = horizontal line Vertical line = Undefined line
The straight line equation is $y = mx + c$ where m is the gradient and c is the y-intercept



Question 17: Find Y intercept

$$y = 2x - 8$$

$$y = 2 \times 0 - 8$$
$$y = -8$$

Question 18: Find equation of the line using the points below

$$(3, 11)(6, 7)$$

$$\frac{11-7}{3-6}$$
$$-\frac{4}{3}$$
$$y = -\frac{4}{3}x + c$$
$$11 = -\frac{4}{3} \times 3 + c$$
$$11 = 4 + c$$
$$c = 7$$

Point Gradient Formula

Theorem 0.4.2 Point Gradient Formula

If you are given one point and gradient, you can use $y - y1 = m(x - x1)$ where (y1, x1,) are the points on the line and m is the gradient

Question 19: Find the equation of the line

$$(3, 2) and m = 4$$

$$y - 2 = 4(x - 3)$$
$$y - 2 = 4x - 12$$
$$y = 4x - 10$$

Question 20: Find the equation of the line

$$(-3, -4) m = -1$$

$$y + 4 = -1(x + 3)$$
$$y + 4 = -x - 3$$
$$y = -x - 7$$

Additional formulas

Theorem 0.4.3 Additional formulas

Midpoint Formula:
The midpoint is between two points of a line segment; the formula is $M = (\frac{x1+x2}{2}, \frac{y1+y2}{2})$
Line Segment Length:
The length of a line segment is the distance between two points; the formula is $d = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$

Question 21: Find the midpoint

$$(-3, -5) and (2, 8)$$

$$M = (\frac{-3 + 2}{2}, \frac{8 - 5}{2})$$
$$M = (-\frac{1}{2}, \frac{3}{2})$$

Question 22: Find the distance between the points

$$(-3, 8) and (4, -1)$$

$$d = \sqrt{(4 + 3)^2 + (-1 - 8)^2}$$
$$d = \sqrt{7^2 + 9^2}$$
$$d = \sqrt{130}$$

Parallel and perpendicular lines

Theorem 0.4.4 Parallel and Perpendicular lines

Parallel Lines:
All parallel lines have the same gradient
Perpendicular lines:
To find perpendicular lines $m1 \times m2 = -1$ or $m2 = -\frac{1}{m1}$, m2 is the negative reciprocal of m1

Question 23: Are these lines perpendicular or parallel

$$y = \frac{1}{2}x + 2$$

and

$$2y - x = 5$$

$$2y = x + 5$$
$$y = \frac{1}{2}x + \frac{5}{2}$$
$$These lines are parallel to each other$$

Question 24: Are these lines perpendicular or parallel

$$y = -3x - 8$$

and

$$y = \frac{1}{3}x + 1$$

$$-3 \times \frac{1}{3} = -1$$
$$Thes lines are perpendicular$$

0.4.2 Simultaneous Equations

Simultaneous Equations Substitution

Theorem 0.4.5 Simultaneous Equations Substitution
The substitution method is used to solve simultaneous equations when one pronumeral is isolated and can be substituted into the other equation.

Question 25

$$1. 2x - 3y = -8$$
$$2. y = x + 3$$

Sub 2 into 1

$$2x - 3(x + 3) = -8$$
$$2x - 3x - 9 = -8$$
$$x = -1$$
$$y = -1 + 3$$
$$y = 2$$
$$(-1, 2)$$

Note:-
Always remember to isolate pronumerals before substituting

Question 26

$$1. y = -3x + 2$$
$$2. y = 7x - 8$$

Sub 1 into 2

$$7x - 8 = -3x + 2$$
$$10x = 10$$
$$x = 1$$
$$y = 7 - 8$$
$$y = -1$$
$$(1, -1)$$

Simultaneous Equations Elimination

Theorem 0.4.6 Simultaneous Equations Elimination
The elimination method is used when you cannot isolate one variable, and the equation is in the form of $ax + by = d$
The method used to complete the elimination method is multiplying both equations by a chosen factor and then adding or subtracting one of the equations from the other to eliminate a variable.

Question 27

$$1.x + y = 6$$
$$2.3x - y = 10$$

add 1 to 2 2a.

$$4x = 16$$
$$x = 4$$
$$4 + y = 6$$
$$y = 2$$

Note:-
Once you find one variable, sub into an equation to find the other

Question 28

$$1.3x + 2y = 6$$
and
$$2.5x + 3y = 11$$

1. times 3 2. times 2

$$1a.9x + 6y = 18$$
$$2a.10x + 6y = 22$$

2a - 1a

$$x = 4$$
$$12 + 2y = 6$$
$$y = -3$$

Note:-
When multiplying the equations by the chosen factor, remember that one coefficient and variable in each equation should be the same

0.4.3 Linear Inequalities

Theorem 0.4.7 Linear inequalities

We solve Linear inequalities the same way we solve linear equations, except we flip the sign if we multiply or divide by a negative.
When drawing linear inequalities, we use an open circle if the greater or less than signs are used. We use a closed circle if the greater/less than or equal symbols are used.

Question 29

$$3x + 4 > 13$$

$$3x > 9$$
$$x > 3$$

Question 30

$$4 - \frac{x}{3} \leqslant 6$$

$$-\frac{x}{3} \leqslant 2$$
$$x \geqslant -6$$

0.5 Indices and Surds

0.5.1 Surds

0.5.2 Surd Rules and Simplification

Theorem 0.5.1 Surds Rules and Simplification

Surds are irrational numbers that don't have a terminating decimal; surds use $\sqrt{}$
Surd Rules:
$\sqrt{x^2} = x$
$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$
$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$
When a factor of a number is a perfect square, its called a square factor
Simplifying Surds:
To simplify a surd, look for square factors of that number $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$

Question 31: Simplify the following Surd

$$\sqrt{32}$$

$$\sqrt{16 \times 2}$$
$$4\sqrt{2}$$

Question 32: Simplify the following Surd

$$\sqrt{\frac{75}{9}}$$

$$\frac{\sqrt{25 \times 3}}{3}$$
$$\frac{5\sqrt{3}}{3}$$

Adding and Subtracting Surds

Theorem 0.5.2 Adding and Subtracting Surds

Like surds are multiples of the same surd, only like surds can be added or subtracted.
Remember to simplify surds before adding or subtracting them.

Question 33

$$2\sqrt{3} + 4\sqrt{3}$$

$$= 6\sqrt{3}$$

Question 34

$$5\sqrt{2} - \sqrt{8}$$

$$= 5\sqrt{2} - \sqrt{4 \times 2}$$
$$= 5\sqrt{2} - 2\sqrt{\times 2}$$
$$3\sqrt{2}$$

Multiplying and Dividing Surds

Theorem 0.5.3 Multiplying and Dividing

When multiply surds use $a\sqrt{x} \times b\sqrt{y} = ab\sqrt{xy}$
When dividing surds use $\frac{a\sqrt{x}}{b\sqrt{y}} = \frac{a}{b}\sqrt{\frac{x}{y}}$
Also, remember to use the distributive law to expand brackets.

Question 35

$$2\sqrt{3} \times 3\sqrt{15}$$

$$6\sqrt{45}$$
$$6\sqrt{9 \times 5}$$
$$18\sqrt{5}$$

Note:-
Remember to times all square numbers by the number multiply the surd as seen above

Question 36

$$\frac{12\sqrt{18}}{3\sqrt{3}}$$

$$\frac{12}{3}\sqrt{\frac{18}{3}}$$

$$4\sqrt{6}$$

Note:-
Always separate the fraction before simplifying when dividing Surds

Rationalising the Denominator

Theorem 0.5.4 Rationalising the denominator
Rationalising the denominator involves multiplying the entire fraction by the surd, denominator to rationalise it to a whole number

Question 37

$$\frac{2\sqrt{6}}{5\sqrt{2}}$$

$$\frac{2\sqrt{6}}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{2\sqrt{12}}{10}$$

$$\frac{4\sqrt{3}}{10}$$

$$\frac{2\sqrt{3}}{5}$$

Question 38

$$\frac{1-\sqrt{3}}{\sqrt{3}}$$

$$\frac{1-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{\sqrt{3}-3}{3}$$

0.5.3 Indices

Fractional Indices

Theorem 0.5.5 Fractional Indices

$a^{\frac{1}{n}} = \sqrt[n]{a}$ this is the nth root of a

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

Question 39: Write in index form

$$\sqrt[4]{x^7}$$

$$= x^{\frac{7}{4}}$$

Question 40: Write in surd form

$$5^{\frac{2}{3}}$$

$$= \sqrt[3]{5^2}$$

Exponential Equations

Theorem 0.5.6 Exponential Equations

An exponential equation looks like $a^x = b$. There is only one solution to an exponential equation like the above. Many of these equations can be solved by using the same base for each exponent.

Question 41

$$25^x = 125$$

$$(5^2)^x = 5^3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

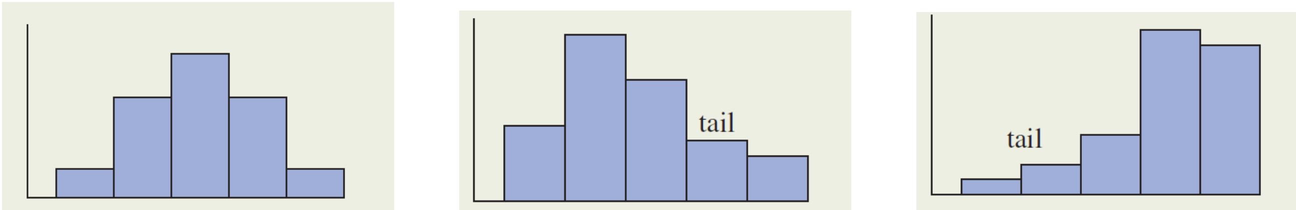
Question 42

$$3^{2x-1} = 27^x$$

$$3^{2x-1} = 3^{3x}$$

$$2x - 1 = 3x$$

$$x = -1$$



(a) The median and mean will be equal

(b) The median will be less than the mean

(c) The median will be greater than the mean

Figure 1: SYMMETRICAL AND SKEWED DATA

0.6 Statistics

0.6.1 Types of Statistics and Data Displays

Theorem 0.6.1 Types of Statistics and Data Displays

Types of Statistics:
There are two types of Statistics Categorical and Numerical. Within these types, there are subsections or statistics.
For Categorical Data, there are nominal and Ordinal data. Nominal data has no order, i.e. red, green, blue. Ordinal data can be ordered, i.e. low, medium, and high.
For Numerical, there are Discrete and Continuous data. Discrete data can only have a limited amount of values, i.e. the number of children in a family. Continuous Data can take any value of numbers in a range, i.e. time taken to run a race.
Types of Data sets:
There are three types of data sets, Symmetrical where the mean and median will be equal,
Positively Skewed where the median will be less than the mean and
Negatively skewed where the median will be greater than the mean. See the Figures Above for a visual representation.

0.6.2 Summary Statistics

Theorem 0.6.2 Summary Statistics

Summary statistics highlight important aspects of a data set
The median is the middle value of a data set. When the data set is even, you should divide the middle two numbers by 2
The mode is the most frequently occurring value in a dataset

The frequency and the percentage frequency are the number of times a favourable outcome occurred. To get percentage frequency, you divide the number of favourable outcomes by the total outcomes.

Five-figure summary:
Min/max value, the smallest/largest value in a dataset
Lower Quartile(Q_1), the value above 25% of the ordered data. The formula to obtain Q_1 is $0.25(n+1)$, with n being the number of data points given.

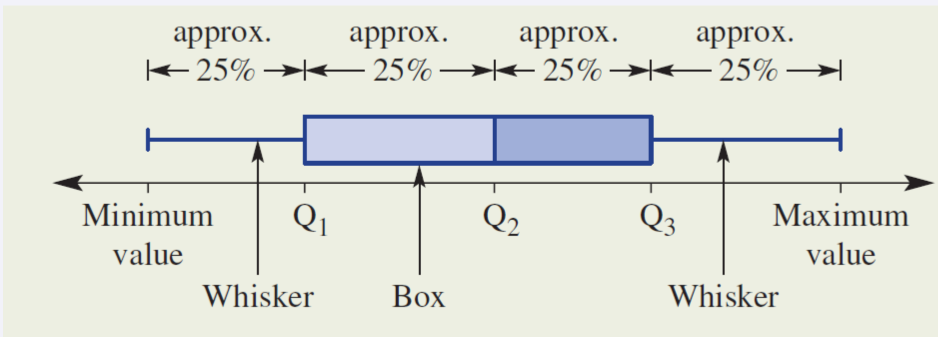
The median(Q_2), the value above 50% of the ordered data.

Upper Quartile(Q_3), the value above 75% of the ordered data. The formula to obtain Q_1 is $0.25(n+1)$, with n being the number of data points given.

A measure of spread:
Range = max value - min value
Interquartile Range(IQR). The formula for this is Q_3Q_1
Standard deviation(SD) Outliers are outside the parameters o a data set. To detect if a data point is an outlier, the formula is $if < Q_1 - 1.5 \times IQR$ or $> Q_3 + 1.5 \times IQR$

0.6.3 Box Plots

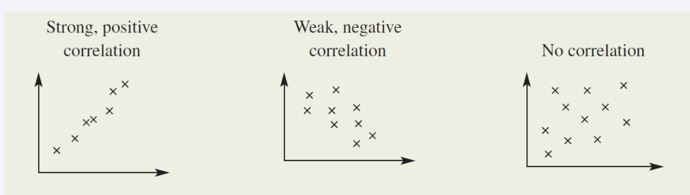
Theorem 0.6.3 Box Plots
Box Plots are used to summarise data sets. The dataset is divided into four groups, as seen in the figure below,



An outlier is marked with an x on the plot

0.6.4 BiVariate Data

Theorem 0.6.4 Bivariate Data
Types of correlation with bivariate data:



0.6.5 Line of Best fit

Theorem 0.6.5 Line of best fit
What is a line of best fit?
A line of best fit is placed on a graph with an even amount of points across each side. This shows the trend of the data.
To find the line of best fit, you can use the straight line formula or Point gradient Formula.
This line can be used for extrapolation or interpolation.

0.7 Probability

0.7.1 Arrays

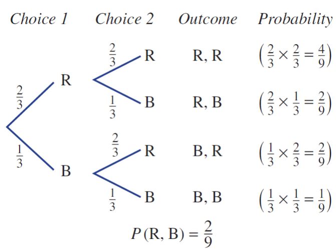
Theorem 0.7.1 Arrays

An array is used to list the data of a two-step experiment. If replacement is allowed in the experiment, then the outcome can occur in both steps of the experiment. However, if replacement is not allowed, the outcome cannot occur in both steps of the experiment; w highlight these possibilities with an X in the array table.

0.7.2 Tree Diagrams

Theorem 0.7.2 Tree Diagrams

Tree diagrams are used to list the probability of outcomes in an experiment with two or more steps
The branches on the tree describe the probability of an outcome at each stage. To gain the probability of an outcome occurring, you multiply the probability of each branch by each other. See the diagram below for example:



0.8 Measurement

0.8.1 Area of quadrilaterals

Theorem 0.8.1 Area of quadrilaterals

Area formulas:
Rhombus : $\frac{1}{2}xy$
Parallelogram: bh
Trapezium: $h\left(\frac{a+b}{2}\right)$
Kite: $\frac{1}{2}xy$

0.8.2 Cylinders and Prisms

SA of Cylinders and Prisms

Theorem 0.8.2 SA of Cylinders and Prisms

To find the Surface area of objects, you need to find the area of each 2d shape that make up the 3d object
To find the SA of a cylinder, the formula is $2\pi r^2 + \pi dh$ with (d) being the diameter and (h) being the height
To find the SA of a prism, the formula is $2(w \times h) + 2(h \times l) + 2(l \times w)$

Question 43

If the radius of a cylinder is three and the height of the cylinder is 3, what is the SA, leave your answer in exact form

$$SA = 2\pi 3^2 + \pi 6 \times 3$$

$$= 18\pi + 18\pi$$
$$= 36\pi$$

Question 44

If the height of a prism is five and the length of the prism is seven, and the width of a prism is 2, find the SA

$$SA = 2(2 \times 5) + 2(7 \times 5) + 2(2 \times 7)$$
$$= 40 + 70 + 28$$
$$= 138$$

Volume of Cylinders and Prisms

Theorem 0.8.3 Volume of Cylinders and Prisms

To calculate the volume of a cylinder and prism, you first need to find the area of the cross-section that makes up that shape. The volume formula for a cylinder is $\pi r^2 \times h$
The volume formula for a prism is ah, where a is the area of the cross-section of the shape.

Question 45

Find the volume of a prism where the width is 2m, length is 3m, and height is 4 m

$$\begin{aligned} V &= 4(2 \times 3) \\ &= 24 \end{aligned}$$

Question 46

Find the volume of a cylinder where the radius is 2m and the height is 6m in exact form

$$\begin{aligned} V &= \pi 2^2 \times 6 \\ &= 24\pi \end{aligned}$$

0.8.3 Pyramids and cones

SA of Pyramids and Cones

Theorem 0.8.4 SA of Pyramids and Cones
The formula for the surface area of a right cone is $\pi rl + \pi r^2$ where l is the slant height. If you need to find the slant height or any measurement in a cone, use the Pythagorean theorem, the Sine rule or the Cosine Rule
The surface area for a right pyramid is $4(b \times h) + b^2$

Question 47

The slant height of a cone is four and the radius of a cone is 6, what is the SA in exact form

$$\begin{aligned} SA &= \pi 6 \times 7 + \pi 6^2 \\ &= 42\pi + 36\pi \\ &= 78\pi \end{aligned}$$

Question 48

The base of a pyramid is 4 and the height is 6 find the SA

$$\begin{aligned} SA &= 2(4 \times 6) + 4^2 \\ &= 48 + 16 \\ &= 64 \end{aligned}$$

Volume of Pyramids and Cones

Theorem 0.8.5 Volume of Pyramids and Cones
To find the volume of a right pyramid or cone the formula $\frac{1}{3}AH$ where A is the area of the pyramids or cone cross-section, x^2 for pyramid orπr^2 for cone.

Question 49

Find the volume of this pyramid if the length is 4 and the height is 6

$$\begin{aligned} V &= \frac{1}{3}4^2 \times 6 \\ &= \frac{16}{3} \times 6 \\ &= \frac{96}{3} \\ &= 32 \end{aligned}$$

Question 50

Find the volume of a cone in the exact form if the radius is 10 and the height is 15

$$\begin{aligned} V &= \frac{1}{3}(\pi 10^2) \times 15 \\ &= \frac{100\pi}{3} \times 15 \\ &= \frac{1500\pi}{3} \\ &= 500\pi \end{aligned}$$

0.8.4 Spheres

SA of Spheres

Theorem 0.8.6 SA of Spheres

The formula for a Spheres SA is $4\pi r^2$

Question 51

If the radius of the sphere is 3 find the SA in exact form

$$\begin{aligned} SA &= 4\pi 3^2 \\ &= 4\pi 9 \\ &= 36\pi \end{aligned}$$

Question 52

If the radius of a sphere is 5 find the SA in exact form

$$\begin{aligned} SA &= 4\pi 5^2 \\ &= 100\pi \end{aligned}$$

Volume of a Sphere

Theorem 0.8.7 Volume of a Sphere

The formula for the Volume of a Sphere is $\frac{4}{3}\pi r^3$

Question 53

If the Radius of the sphere is 7 find the volume in exact form

$$\frac{4}{3}\pi 7^3$$

$$\frac{4}{3}343\pi$$

$$\frac{1372\pi}{3}$$

Question 54

If the radius of a sphere is 3 find the volume in exact form

$$\frac{4}{3}\pi 2^3$$
$$\frac{4}{3}8\pi$$
$$\frac{32\pi}{3}$$

0.9 Trigonometry

0.9.1 Trigonometric Ratios

Theorem 0.9.1 Trigonometric Ratios

Trigonometric ratios help us find the angles inside a RAT
The three trigonometric Ratios:
$Sin\theta = \frac{opp}{hyp}$
$Cos\theta = \frac{adj}{hyp}$
$Tan\theta = \frac{opp}{adj}$
A mnemonic to remember these ratios is SOHCAHTOA
To find the unknown length of a side you can find a ratio that links the angle and the two sides together and solve for x See the second example below

Question 55

Find the angle if the adjacent side is 5 and the hypotenuse 7

$$\cos\theta = \cos^{-1}\frac{5}{7}$$
$$\cos\theta = 44.4$$

Question 56

Find the unknown side if the opp is 5cm, hyp = unknown and the angle is 50

$$\sin 50 = \frac{5}{x}$$
$$x = \frac{5}{\sin 50}$$
$$x = 6.53$$

Note:-

Remember if the variable is the numerator multiply the ratio and the length, if the variable is the denominator remember to divide the length by the variable

0.9.2 Bearings

Theorem 0.9.2 Bearings **Come back to this later**

Bearings are three-digit numbers that give us a more accurate direction

Question 57

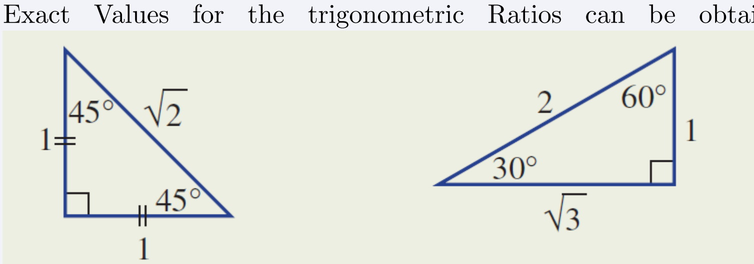
Question 58

0.9.3 Obtuse Angles and Unit Circle

Theorem 0.9.3 Obtuse angles and Unit Circle

The unit circle has a radius of 1. It is used to define the values of $Cos\theta$ $Sin\theta$ and $Tan\theta$
Obtuse Angles:
To find the angle supplementary to an obtuse angle in the Q2 use the below
$cos(180 - \theta) = -cos\theta$
$sin(180 - \theta) = sin\theta$
$tan(180 - \theta) = -tan\theta$
A mnemonic to remember the sign in each of the four Quadrants is All Stations To Central(ALL SIN TAN COS) in the first quadrant(acute) all values are positive and in the Second only Sin is positive. etc.

Exact Values for the trigonometric Ratios can be obtained using two triangles seen below



Here are the values in a table:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

Question 59

Find the supplementary angle that goes with $\sin 150$

$$\theta = \sin 180 - 150$$
$$= 30$$
$$\sin 30 = \frac{1}{2}$$

Question 60

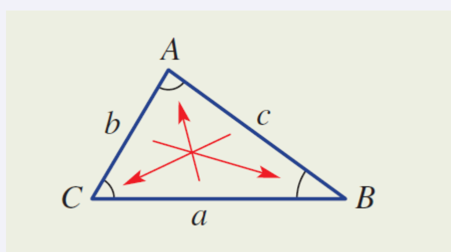
Find the supplementary angle that goes with $\tan 135$

$$\theta = \tan 180 - 135$$
$$= \tan 45$$
$$\tan 45 = 1$$

0.9.4 Sine Rule

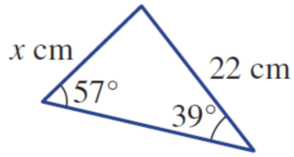
Theorem 0.9.4 The Sine Rule

The Sine rule describes the relationship between the angles in a triangle and their opposite sides, being that they're equal.
the Sine Rule is $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$, this rule can be manipulated to find either the side length or angle size.
It is important to note that to use the sine rule you need to know the size of one angle, the length of the side opposite that angle, and another side or angle.



Question 61

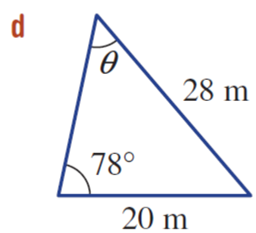
Find for x in the following:



$$\frac{x}{\sin 39} = \frac{22}{\sin 57}$$
$$x = \frac{22 \times \sin 39}{\sin 57}$$
$$x = 16.51cm$$

Question 62

Find θ in the following:



$$\frac{\sin\theta}{20} = \frac{\sin 78}{28}$$
$$\sin\theta = \frac{20 \times \sin 78}{28}$$
$$\sin\theta = 44.32$$

Note:-

Remember to use $\sin^{-1}($ to gain the size of the angle. Additionally when using the sine rule to find an angle, use the reciprocal of the rule stated in the theorem

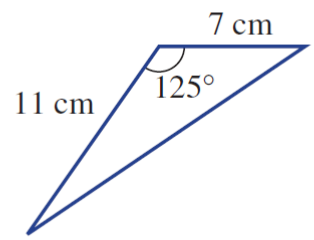
0.9.5 Cosine Rule

Theorem 0.9.5 Cosine Rule

The cosine rule describes the relationship between three sides and one angle in a triangle.
The rule to find a side is $a^2 = b^2 + c^2 - 2bc \times \cos(A)$
The rule to find the angle is $\cos(A) = \frac{b^2+c^2-a^2}{2bc}$
You can use the Cosine Rule when you are given two sides and angles and you are required to find the third side. You are required to find the angle when given three sides
Checking your answer, the longest side is opposite the largest $\angle$ $\therefore$ smallest angle is opp smallest $\angle$

Question 63

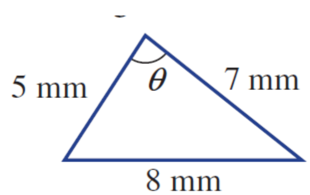
Solve for the missing side



$$a^2 = 7^2 + 11^2 - 2 \times 7 \times 11 \times \cos(125)$$
$$a = \sqrt{258.33}$$
$$a = 16.07cm$$

Question 64

Solve for θ



$$\cos\theta = \frac{5^2 + 7^2 - 8^2}{2 \times 5 \times 7}$$
$$= \frac{10}{70}$$
$$= \cos^{-1}\left(\frac{1}{7}\right)$$
$$\theta = 81.79$$

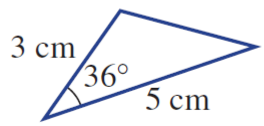
0.9.6 Area Rule

Theorem 0.9.6 Area Rule

The area rule is primarily used for NRATS but you can use it for RATS as well, it relates two sides of a triangle plus the including angle with the area.
The formula is $A = \frac{1}{2}ab\sin(C)$

Question 65

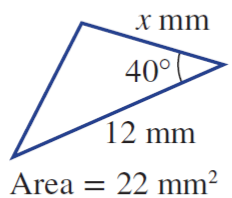
Find the area



$$A = \frac{1}{2} \times 3 \times 5 \times \sin(36)$$
$$A = 4.41cm^2$$

Question 66

Solve for x



$$22 = \frac{1}{2} \times 12 \times x \times \sin(40)$$
$$\frac{22}{6\sin(40)} = x$$
$$x = 5.7mm^2$$

0.9.7 The four Quadrants

Theorem 0.9.7 The four Quadrants

Dependent on the quadrant the angle is, the angle ratio could be positive or negative:
Q1(0-90): All ratios are positive
Q2(90-180): Only Sine Ratio is positive $180 - \theta$
Q3(180-270): Only the Tan ratio is positive, to get this angle the equation is $\theta - 180$
Q4(270-360): Only the Cosine ratio is positive the equation to find the cute angle is $360 - \theta$
A mnemonic to remember this is All Stations to Central

Question 67

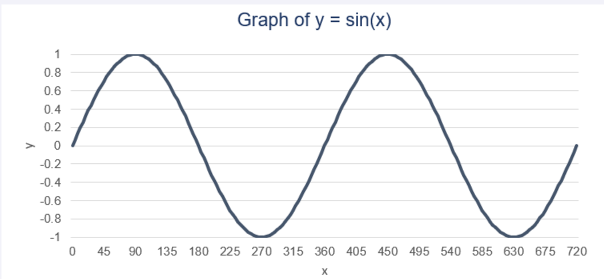
Find the acute angle if it is 280

$$\theta = 360 - 280$$
$$\theta = 80 = -tan(80), -sin(80), cos(80)$$

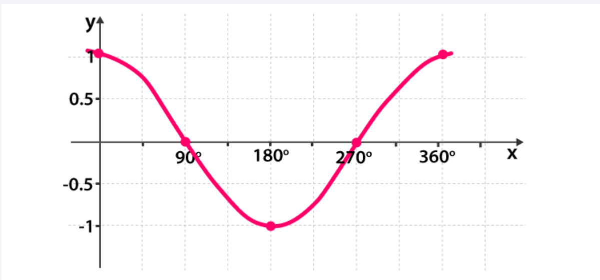
0.9.8 Trigonometric Function Graphs

Theorem 0.9.8 Trigonometric exact value graphs

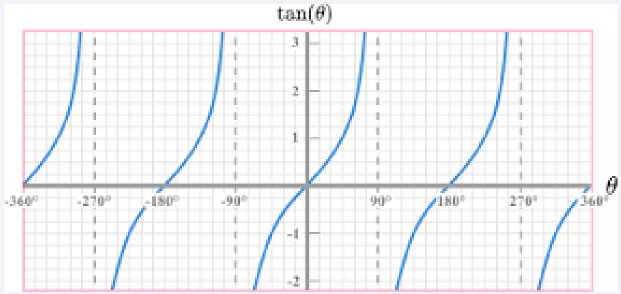
Here are the graphs that show the pattern of trigonometric exact values
Sine Wave:



Cosine Wave:



Tangent Wave:



0.10 Geometry

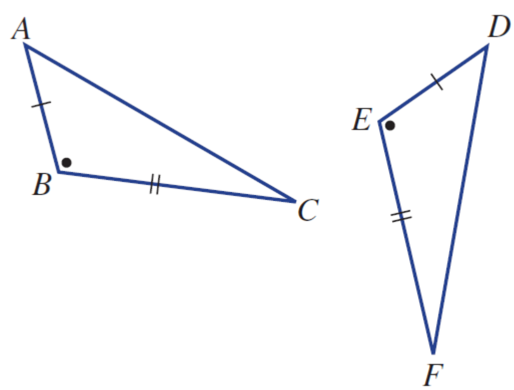
0.10.1 Triangle Congruency

Theorem 0.10.1 Triangle Congruency
Two triangles are considered to be congruent when they are the same shape and size
Triangle congruency can be proven using one of the tests below:

SSS(3 Sides): The three sides of each triangle are equal to each other
SAS(Two sides one angle): The two sides and the including angle of the triangles are equal to each other
AAS(Two angles and one side): The two angles and one side of the triangles are equal
RHS(right-angle, hypotenuse, side): One side, 90-degree angle and the hypotenuse of triangles are equal

Question 68

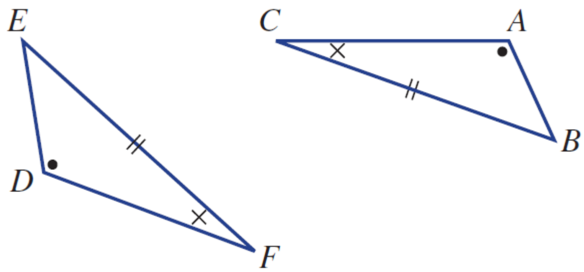
Prove this pair of triangles is congruent



$$BC = EF(given)$$
$$AB = DE(given)$$
$$\angle ABC = \angle DEF(given)$$
$$\therefore \triangle ABC = \triangle DEF(SAS)$$

Question 69

Prove this pair of triangles is congruent



$$EF = CB(given)$$
$$\angle EDF = \angle CAB(given)$$
$$\angle EFD = \angle ACB(given)$$
$$\therefore \triangle EDF = \triangle CAB(AAS)$$

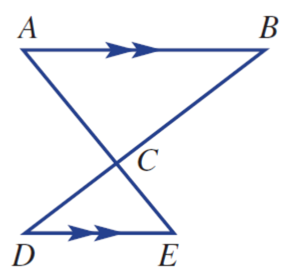
0.10.2 Similarity in Shapes

Theorem 0.10.2 Similarity in Shapes

For two figures to be considered similar, they need to be the same shape(proportional) but have different sizes.
The symbol used to describe similarity in shapes is
To prove two shapes are similar we can use four different tests:
PPP(three proportional sides): Three sides of each triangle are proportional to each other
PAP(two proportional sides and included angle): Two sides of each triangle are proportional and the included angle is equal
AA(Two Angles): Two angles in each triangle are equal to each other
RHS(right angle, hypotenuse, side): The right angle, hypotenuse and side are proportional to each triangle

Question 70

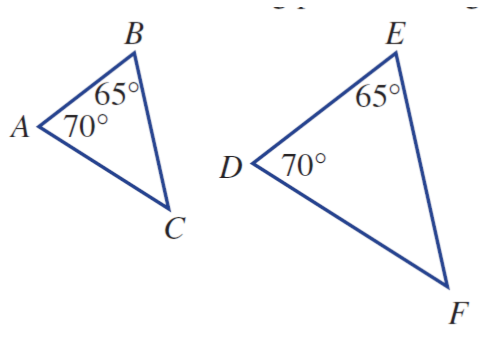
Prove these two triangles are similiar



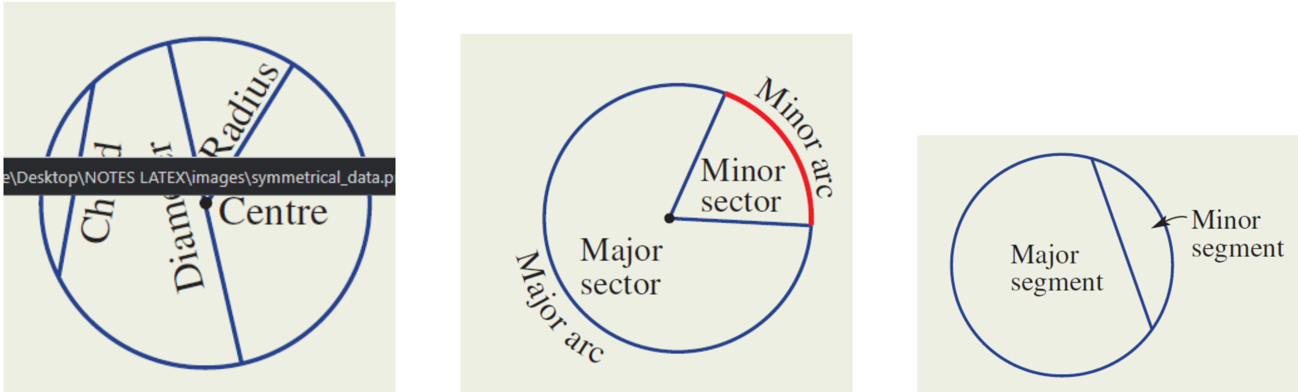
$$\angle DCE = \angle ACB(vertopp)$$
$$\angle CDE = \angle ABC(altangles)$$
$$\therefore \triangle DCE ||| \triangle ACB(AA)$$

Question 71

Prove these two triangles are similar



$$\angle BAC = \angle EDF(given)$$
$$\angle ABC = \angle DEF$$
$$\therefore \triangle BAC ||| \triangle EDF(AA)$$



(a) The Chord is a segment whose both points lie on a circular arc

(b) An arc is a smooth curve joining two endpoints

(c) A segment is part of the circle which is cut off by a chord or secant

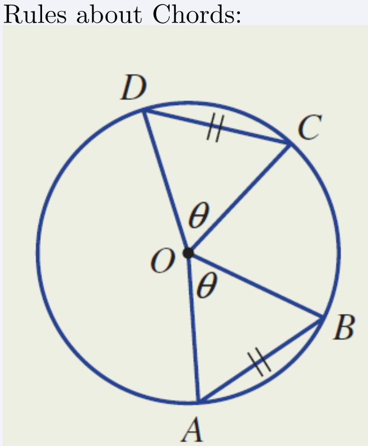
Figure 2: STypes of lines in Circles

0.10.3 Circle terminology and chord properties

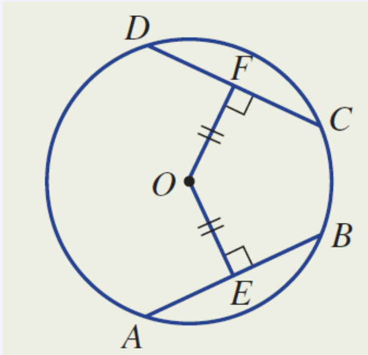
Theorem 0.10.3 Circle terminology and Chord properties

Look at the figures above to revise basic circle terms, an important term to remember is subtended which is a mathematical term for make/made.
An angle is subtended by an arc/chord when the arms of the angle arms meet the chord/arc's endpoint. See the example below.

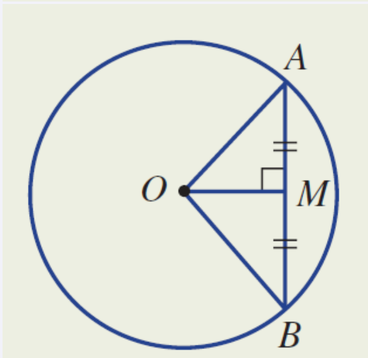
Rules about Chords:



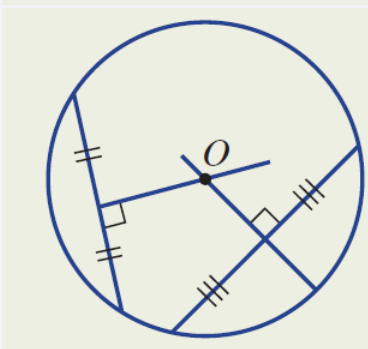
Chords of equal length subtend equal angles at the centre of a circle, and chords that subtend equal angles are equal length.



Chords which are equal distances from the centre are equal, and chords of equal length are equal distances from the centre.



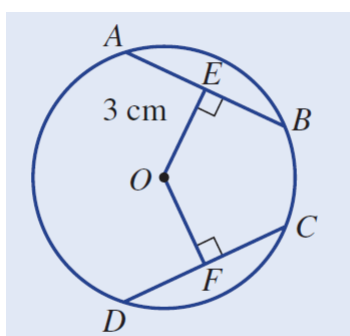
The perpendicular from the centre of a circle bisects the chord, and a line through the centre of a circle that bisects a chord is perpendicular to the chord.



Perpendicular bisectors of each chord intersect at the centre of a circle, you can use two perp bisectors to find the circle centre.

Question 72

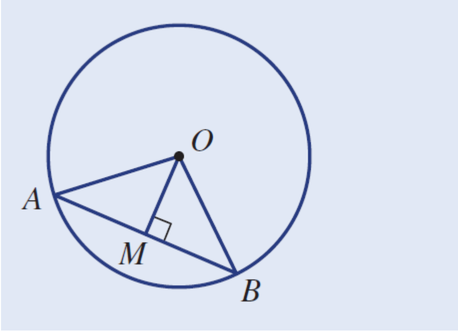
Given AB = CD and OE = 3 cm , find OF.



centre $OF = 3cm$ (chords of equal length are the same distance from centre)

Question 73

Given $OM \perp AB$, AB = 10 cm and $\angle AOB = 92$, find AM and $\angle AOM$



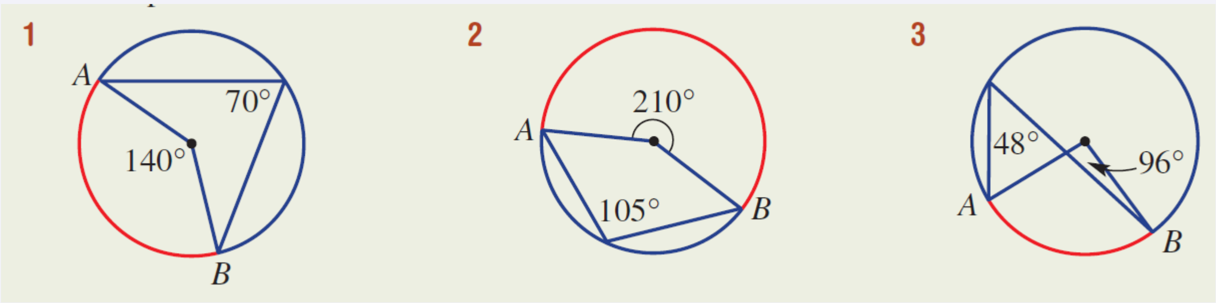
$$AM = 5cm, \text{ perp centre bisects chord}$$

$$\angle AOM = 92 \div 2 = 46$$

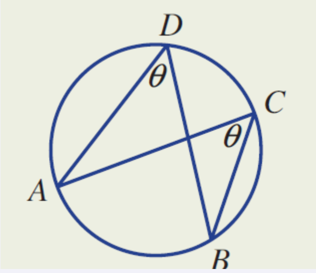
0.10.4 Circle Angle properties

Theorem 0.10.4 Circle angle properties

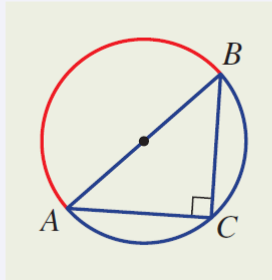
Here are the angle properties at the centre and circumference of a circle
The angle at the circumference of a circle is $\frac{1}{2}$ the size of the angle at the centre when standing on the same arc. The angle at the circumference is called an inscribed angle. See diagram.



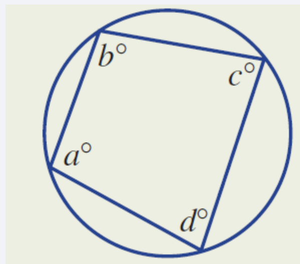
The inscribed angle theory states that Angles subtended by the same arc are equal. See diagram.



The angle formed by the diameter to a point on a semi-circle is 90

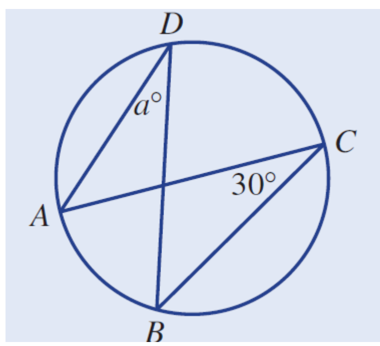


Opposite angles in a cyclic quadrilateral add to 180 degrees, as long as all vertices touch the circumference of the Circle. See diagram.



Question 74

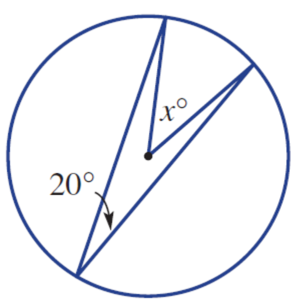
Find the value of a



$a = 30$ inscribed angles on the same arc are equal

Question 75

Find the value of x

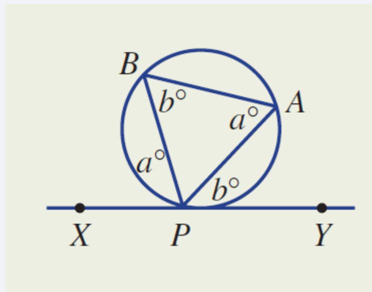


$x = 80$ centre angles are 2 times the size of an inscribed angle when standing on the same arc

0.10.5 Tangent Theorems

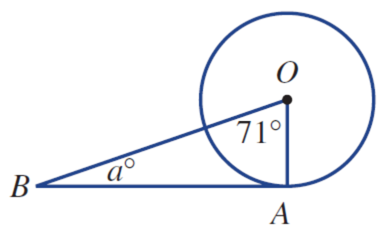
Theorem 0.10.5 Tangent Theorems

What is a tangent? A tangent is a line that has one point of contact with the circle/object.
In a circle, the tangent intersects the circle once and the tangent is perpendicular to the radius at the point of contact.
Two tangents that originate at the same point are equal.
Alternate angle theorem:
The angle formed between the tangent and the chord through the point of contact of the tangent is equal to the angle formed by the chord in the alternate segment. See diagram.



Question 76

Solve for a



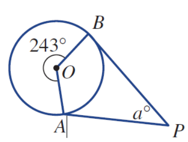
$$\angle BOA = 90 \text{ alternate angle theorem}$$

$$180 - (90 + 71) = 19$$

$$\therefore \angle OBA = 19$$

Question 77

Solve for a



$$\angle BOA = 360 - 243 = 117$$

$$\angle OBP = \angle OAP = 90 \text{ alternate angle theorem}$$

$$a = 360 - (117 + 90 + 90) = 63$$

$$a = 63$$

0.11 Logarithms and Polynomials

0.11.1 Logarithm Intro

Theorem 0.11.1 Logarithm Intro

Logarithms are is used to find the power a base was raised to get a value.
Example:
$\log_2(16) = 4$ since$2^4 = 16$
The way a log is written is the base is the subscript, and the value of the equation is inside the function which outputs the exponent ie $a^x = y = \log_a(y) = x$

Question 78

Write $10^2 = 100$ in log form

$$\log_{10}(100) = 2$$

Question 79

Write $\log_7(49) = 2$ in index form

$$7^2 = 49$$

0.11.2 Log Laws

Theorem 0.11.2 Log Laws

The Log laws Relate to index Laws, the log laws help us simplify and expand Logs when needed

Product Rule:
$log_a(xy) = log_a(x) + log_a(y)$ This law relates to the index law of $a^x \times a^y = a^{x+y}$
Quotient Rule:
$\log_a(\frac{x}{y}) = \log_a x - log_a y$ This relates to $a^m \div a^n = a^{m-n}$
Power Rule:
$\log_a(x^n) = n \log_a x$ This law relates to $(a^m)^n = a^{m \times n}$
Change of Base Rule:
$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$
$\log_a \frac{1}{x} = \log_a(x^{-1}) = -\log_a(x)$

Question 80

Simplify

$$\log_a(3) + \log_a(2)$$

$$\log_a(6)$$

Question 81

Simplify

$$2 \log_a(3)$$

$$\log_a(3^2)$$