Maths
Year 11 Notes

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Contents

0.1 Algebraic Techniques

0.1.1 Simplifying Algebraic expressions

Theorem 0.1.1 Simplifying Algebraic expressions

When you add & subtract in algebra you can only combine like terms
Questions in Fitzgeralds 1.1

Question 1

$$5x + 2y - 3 - (x - 7y + 9)$$

$$\begin{aligned} &= 5x + 2y - 3 - x + 7y - 9 \\ &= 4x + 9y - 12 \end{aligned}$$

Question 2

$$3x(x + 2) - 4(x - 1)$$

$$\begin{aligned} &= 3x^2 + 6x - 4x + 5 \\ &= 3x^2 - 2x + 5 \end{aligned}$$

0.1.2 Substitution in Formulae

Theorem 0.1.2 Substituition in Formulae

Substituition occurs when you substitute values into an algebraic equation and/or rearrange the equations to make a variable the subject
More Questions in 1.2 Fitzgeralds textbook

Question 3

If $S = \frac{a(r^3-1)}{r-1}$ find S when $a = 5$, $r = 3$

$$\begin{aligned} &= \frac{5(3^3 - 1)}{3 - 1} \\ &= \frac{5 \times 26}{2} \\ &= 5 \times 13 \\ &= 65 \end{aligned}$$

Question 4

If $A = P(1 + \frac{r}{100})^n$, find A when $P = 1000$, $r = 10$, $n = 2$

$$\begin{aligned} &= 1000(1 + \frac{10}{100})^2 \\ &= 1000 \times 1.21 \\ &= 1210 \end{aligned}$$

0.1.3 Basic Polynomials

Theorem 0.1.3 Basic Polynomials

There are different types of polynomials include monomial(one term), binomial(two terms) and trinomial(three terms)
Rules for expanding polynomials:
Expanding Perfect/Difference squares ($(y+4)^2$), square first and last terms and multiply the first and last terms together. It should for $a^2+2ab+b^2$ unless there is a negative between the two expressions in hich case $-2ab$

Question 5

$$(2y+5)^2$$

$$a^2+2ab+b^2$$
$$= 2y^2+25+20y$$

Question 6

$$(x+2)(x^2-5x+6)$$

$$= -x^3-5x^2+6x+2x^2-10x+12$$
$$= x^3-3x^2$$
$$= x^3-3x^2-4x+12$$

0.1.4 Fatorising The Sum/Difference of Two Cubes

Theorem 0.1.4 Fatorising The Sum/Difference of Two Cubes

When factoring Two cubes there are two rules to remember
Rule 1: $a^3+b^3=(a+b)(a^2-ab+b^2)$
Rule 2: $a^3-b^3=(a-b)(a^2+ab+b^2)$
To remember the sings used in the factorisation an acronym is SOAP(SAME, OPPOSITE, ALWAY, POSITIVE)

Question 7

$$a^3b-ab^3$$

$$= ab(a-b)(a+b)$$

Question 8

$$x^3-x^2y-9x+9y$$

$$= x^2(x-y)-9(x-y)$$
$$= (x-y)(x^2-9)$$
$$= (x-y)(x+3)(x-3)$$

Question 9

$$(x+5)^3+(x-2)^3$$

$$\begin{aligned}
&= (2x+3)((x+5)^2-((x+5)(x-2))+(x-2)^2) \\
&= (2x+3)(x^2+10x+25-x^2+2x-5x+10+x^2-2x-2x+4) \\
&= (2x+3)(x^2+10x+35-x^2+2x-5x+x^2-2x-2x+4) \\
&= (2x+3)(x^2+10x+35+2x-5x-2x-2x+4) \\
&= (2x+3)(x^2+3x+35+4) \\
&= (2x+3)(x^2+3x+39)
\end{aligned}$$

Note:-

Remember to use FOIL(First, Outside,.Inside Last) to expand brackets

0.1.5 Simplifying Algebraic Fractions

Theorem 0.1.5 Simplifying Algebraic Fractions

When simplifying algebraic fractions it is important to use these two steps:
1. Factorise the numerator and denominator
2. After factorising you can cancel any common factors

Question 10

$$\frac{8x^2+4x+2}{8x^3-1}$$

$$\begin{aligned}
&= \frac{2(4x^2+2x+1)}{(2x-1)((2x)^2+(2x\times 1)+1^2)} \\
&= \frac{2(4x^2+2x+1)}{(2x-1)(4x^2+2x+1)} \\
&= \frac{2}{2x-1}
\end{aligned}$$

Question 11

$$\frac{(x+h)^3-x3}{h}$$

$$\begin{aligned}
&= \frac{(x+h-x)((x+h)^2+x(x+h)+x^2)}{h} \\
&= \frac{h(x^2+2xh+h^2+x^2+xh+x^2)}{h} \\
&= \frac{h(3x^2+3xh+h^2)}{h} \\
&= 3x^2+3xh+h^2
\end{aligned}$$

0.1.6 Adding & Subtracting Algebraic Fractions

Theorem 0.1.6 Adding & Subtracting Algebraic Fractions

To Add or Subtract Algebraic fractions there are three important steps you need to follow
Rule 1: Factorise all fractions on the numerator & denominator Rule 2: Find and create a common denomitor for all fractions(remember to not repeat the same expression more than once)
Rule 3:Simplifying the fraction using like terms

Question 12

$$\frac{5}{2a+6} + \frac{a}{a^2-9}$$

$$= \frac{5}{2(a+3)} + \frac{a}{(a+3)(a-3)}$$
$$= \frac{5(a-3)+2a}{2(a+3)(a-3)}$$
$$= \frac{5a-15+2a}{2(a+3)(a-3)}$$
$$= \frac{7a-15}{2(a+3)(a-3)}$$

Question 13

$$\frac{6}{3x-2} - \frac{8}{4x+1}$$

$$= \frac{6(4x+1)-8(3x-2)}{(4x+1)(3x-2)}$$
$$= \frac{24x+6-24x+16}{(4x+1)(3x-2)}$$
$$= \frac{22}{(4x+1)(3x-2)}$$

0.1.7 Surds

Theorem 0.1.7 Rationalising the denominator

Rationalising the denominator involves multiplying the entire fraction by the surd, denominator to rationalise it to a whole number
If the denominator is a binomial and has both a rational and rational ;portion you will need to use the conjugate,the conjugate is the denominator with opposite signs.
if $\frac{1}{3+\sqrt{2}}$ is the fraction, the conjugate is $3-\sqrt{2}$ as this results in the difference of squares

Question 14

$$\frac{2\sqrt{6}}{5\sqrt{2}}$$

$$\frac{2\sqrt{6}}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{2\sqrt{12}}{10}$$
$$\frac{4\sqrt{3}}{10}$$
$$\frac{2\sqrt{3}}{5}$$

Question 15

$$\frac{1}{3\sqrt{3}+4}$$

$$\frac{1}{\sqrt{3}+4} \times \frac{\sqrt{3}-4}{\sqrt{3}-4}$$
$$\frac{\sqrt{3}-4}{3-16}$$
$$-\frac{\sqrt{3}-4}{13}$$

0.1.8 Completing the square

Theorem 0.1.8 Completing the Square

To complete the square with monic quadratics $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$ to both sides of the equation $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2$ then solve for x When wanting to complete the square for non-monic quadratics you first must make the equation monic by diving the equation by a $ax^2 + bx + c = 0$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
Then solve like a normal completing the square, the non monic completing the square formula is also how the quadratic formula is derived

Question 16

$$2x^2 + 6x - 5 = 0$$

$$x^2 + 3x + \frac{9}{4} = \frac{5}{2} + \frac{9}{4}$$
$$(x + \frac{3}{2})^2 = \frac{19}{4}$$
$$x + \frac{3}{2} = \frac{\pm\sqrt{19}}{2}$$
$$x = \frac{-3 \pm \sqrt{19}}{2}$$

Question 17

$$3x^2 - 5x - 1 = 0$$

$$x^2 - \frac{5}{3}x + \left(-\frac{5}{6}\right)^2 = \frac{1}{3} + \left(-\frac{5}{6}\right)^2$$
$$\left(x - \frac{5}{6}\right)^2 = \frac{37}{36}$$
$$x - \frac{5}{6} = \frac{\pm\sqrt{37}}{6}$$
$$x = \frac{5 \pm \sqrt{37}}{6}$$

0.1.9 Indices

Theorem 0.1.9 Indices

Index Laws:
$a^m \times a^n = a^{n+m}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{nm}$
$(ab)^n = a^n b^n$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
NEGATIVE INDICES:
$x^{-n} = \frac{1}{x^n}$
Fractional Indices:
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Question 18

$$\frac{1}{\sqrt[3]{(4x^2-1)^2}}$$

$$(4x^2-1)^{-\frac{2}{3}}$$

Question 19

$$\frac{x-5+6x^{-1}}{1-2x^{-1}}$$

$$\frac{x-5+6x^{-1}}{1-2x^{-1}} \times \frac{x}{x}$$
$$\frac{x^2-5x+6x}{x-2x}$$
$$\frac{(x-3)(x-2)}{x(1-2)}$$

0.2 Functions

0.2.1 Funtions and Relations

Theorem 0.2.1 Functrions and Relations

A relation is a set of ordered pairs where variables are related to each other according to a rules
A set is a list of numbers, ordered pairs etc

Types of Relations:
One-to-One - every element corresponds to on element in the other set
One-to-Many - where a element in Set A corresponds to 2 or more elemnts in Set B
Many-to-One - 2 or more elemnts of Set A correspond with 2 or more elements in Set B

Functions: Functions are a special typoe of relation where every elemnt of Set A corresponds with a unique element of Set B. In a function the domain is the set of all x values that the function could input, the range in the function is the set of all y values that can be potentialy outputted by the function.

Vertical Line Test: To determine whether something is a function vs a relation we can use the vertical line test which states that if a line onlycuts the y axis at one point it must represent a function.
Horizontal line test: We can use the horizontal line test to determine if a relation is one-one or not, if multiple points lie on the same y coordinate then the function cannot be one-one.

Question 20

Find the Domain and range of the equation $\sqrt{x}$

Domain: $x \geqslant 0$
Range: $y \geqslant 0$

Question 21

Find the domain and range of the equation $2 + x^2$

Domain: $\mathbb{R}$
Range: $y \geqslant 2$

0.2.2 Function & Interval notation

Theorem 0.2.2 Function and Interval notation

Function notaion:
With function notation like $f(x)$, f is the name of our function and x inside the brackets is the input of the function
So when $f(x) = 2x$ then $f(3) = 6$

Interval notation:
A closed interval is when the interval contains all endpoints within it.
Example: $y \geqslant x \geqslant b$ or in bracket notation $[y, b]$

The open interval:
The open interval occurs when the interval does not contain its endpoints.
Example: $y < x < b$ or in bracket notation (y, b)

The closed ray:
The closed ray occurs when x is unbounded inn one direction and contains its endpoint.
Example: $x \geqslant y$ or in bracket notation $[y, \infty)$

The open ray:
The open ray occurs when x is unbounded and does not contain its endpoint.
Example: $x < y$ or or in bracket notation $(-\infty, y)$

0.2.3 Absolute values

Theorem 0.2.3 Absolute values

Absolute values:
Absolute values are a way or measuring the distance a number is from its origin(0), trhis means that an absolute value will always be positive. To denote an absolute values we use the symbols $|x|$
When you are trying to solve an equation with absolute values, it can be positive or negative. Hint, if there is an equatio in an absolute value do not solve until you get rid of the absolute value.
For an example if $|x - b| = a$ then $x - b = \pm a$

Question 22

$$Solve : |x - 2| = 3$$

$$x - 2 = \pm 3$$
$$x = 2 \pm 3$$
$$x = 5, -1$$

Question 23

$$|m - 5| \geqslant 0$$

$$0 \geqslant m - 5 \geqslant 0$$
$$5 \geqslant m \geqslant 5$$
$$m = 5$$

Odd and Even functions

Theorem 0.2.4 Odd and Even functions

A function/relation is even if when graphed it has line of symmetry from the y axis
To determine whether a function is even $f(x) = f(-x)$
A function/relation is called odd if the point of symmetry in the origin, this means that if rotated 180 deg trhe graph remains unchanged
To determine whether a function is off $f(x) = -f(x)$
If a function is neither odd nor even you just use "neither"

Question 24

Determine whether the function is odd,even or neither $f(x) = \frac{3}{x^2-4}$

$$f(-x) = \frac{3}{(-x)^2 - 4}$$
$$= \frac{3}{x^2 - 4}$$
$$\therefore f(x) \text{ is even function}$$

Question 25

Determine whether the function is odd,even or neither

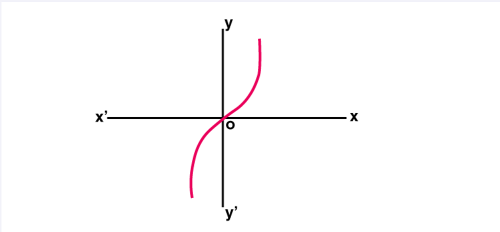
$$f(x) = \frac{x^3}{x^4 - x^2}$$

$$f(-x) = \frac{(-x)^3}{(-x)^4 - (-x)^2}$$
$$= -\frac{x^3}{x^4 - x^2}$$
$$-f(x) = \frac{(-x)^3}{(-x)^4 - (-x)^2}$$
$$\therefore f(x) \text{ is odd function}$$

0.2.4 Cubic Polynomials

Theorem 0.2.5 Cubic Polynomials

Cubic polymials are polynomials of the third degree, a cubic function has 1 y intercept but upto 3 x intercepts



- in $f(x) = x^3 + 1$ the constant 1 is the y intercept
If ther form is kx^3 when $k > 0$ it is a increasing function and when $k < 0$ it is a decreassing function. The point of inflection in the x^3 function is wear the gradient of the line changes
To find the point of inflexion we can get the equation in the form $f(x) = k(x - b)^3 + 6$ which is the same as kx^3 however the inflexion point is (b, c)
The final way a cubic function can be displayed is of $f(x) = k(x - a)(x - b)(x - c)$

Question 26

Find inflexion point of the equation $2(x - 1)^3 - 16$

$$= k(x - b)^3 + c$$
$$POI = (b, c)$$
$$POI = (1, 16)$$

Question 27

Find inflexion point of the equation $(x + 2)^3 + 8 = 0$

$$= k(x - b)^3 + c$$
$$POI = (b, c)$$
$$POI = (-2, 8)$$

Note:-

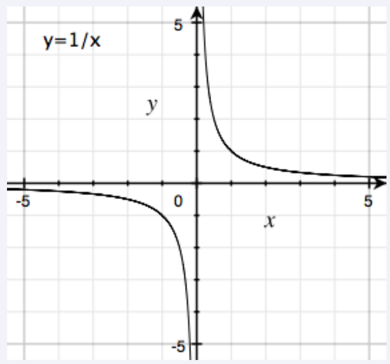
As the form is -b, the b value for POI is negative rather than positive. If the b vlue was negative it the POI b would be positive.

0.2.5 Hyperbolic Function(Inverse variation)

Theorem 0.2.6 Hyperbolic Function

A hyperbola occurs in the form of $\frac{k}{x}$ where k is a constant. This equation tells us that when oine variable increases the other decreases..

A hyperbola is a discontinuous function meaning there are gaps. The are two asymptotes along the x and y axis. This means that x cannot equal 0 and y cannot equal 0.



To find the y intercept set x = 0 and solve for y

To find the x asymptote solve expression on the denominator for x.

To find the y asymptote substitute the x asymtote into the denominator expression and solve for y

Question 28

Question 29