Maths
Year 11 Notes

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2022

Contents

0.1 Algebraic Techniques

0.1.1 Simplifying Algebraic expressions

Theorem 0.1.1 Simplifying Algebraic expressions

When you add & subtract in algebra you can only combine like terms
Questions in Fitzgeralds 1.1

Question 1

$$5x + 2y - 3 - (x - 7y + 9)$$

$$= 5x + 2y - 3 - x + 7y - 9$$
$$= 4x + 9y - 12$$

Question 2

$$3x(x + 2) - 4(x - 1)$$

$$= 3x^2 + 6x - 4x + 5$$
$$= 3x^2 - 2x + 5$$

0.1.2 Substitution in Formulae

Theorem 0.1.2 Substituition in Formulae

Substituition occurs when you substitute values into an algebraic equation and/or rearrange the equations to make a variable the subject
More Questions in 1.2 Fitzgeralds textbook

Question 3

If $S = \frac{a(r^3-1)}{r-1}$ find S when $a = 5$, $r = 3$

$$= \frac{5(3^3 - 1)}{3 - 1}$$
$$= \frac{5 \times 26}{2}$$
$$= 5 \times 13$$
$$= 65$$

Question 4

If $A = P(1 + \frac{r}{100})^n$, find A when $P = 1000$, $r = 10$, $n = 2$

$$= 1000(1 + \frac{10}{100})^2$$
$$= 1000 \times 1.21$$
$$= 1210$$

0.1.3 Basic Polynomials

Theorem 0.1.3 Basic Polynomials

There are different types of polynomials include monomial(one term), binomial(two terms) and trinomial(three terms)
Rules for expanding polynomials:
Expanding Perfect/Difference squares ($(y+4)^2$), square first and last terms and multiply the first and last terms together. It should for $a^2 + 2ab + b^2$ unless there is a negative between the two expressions in hich case $-2ab$

Question 5

$$(2y+5)^2$$

$$a^2 + 2ab + b^2$$
$$= 2y^2 + 25 + 20y$$

Question 6

$$(x+2)(x^2 - 5x + 6)$$

$$= -x^3 - 5x^2 + 6x + 2x^2 - 10x + 12$$
$$= x^3 - 3x^2$$
$$= x^3 - 3x^2 - 4x + 12$$

0.1.4 Fatorising The Sum/Difference of Two Cubes

Theorem 0.1.4 Fatorising The Sum/Difference of Two Cubes

When factoring Two cubes there are two rules to remember
Rule 1: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
Rule 2: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
To remember the sings used in the factorisation an acronym is SOAP(SAME, OPPOSITE, ALWAY, POSITIVE)

Question 7

$$a^3b - ab^3$$

$$= ab(a-b)(a+b)$$

Question 8

$$x^3 - x^2y - 9x + 9y$$

$$= x^2(x-y) - 9(x-y)$$
$$= (x-y)(x^2-9)$$
$$= (x-y)(x+3)(x-3)$$

Question 9

$$(x+5)^3 + (x-2)^3$$

$$\begin{aligned}
&= (2x+3)((x+5)^2 - ((x+5)(x-2)) + (x-2)^2) \\
&= (2x+3)(x^2 + 10x + 25 - x^2 + 2x - 5x + 10 + x^2 - 2x - 2x + 4) \\
&= (2x+3)(x^2 + 10x + 35 - x^2 + 2x - 5x + x^2 - 2x - 2x + 4) \\
&= (2x+3)(x^2 + 10x + 35 + 2x - 5x - 2x - 2x + 4) \\
&= (2x+3)(x^2 + 3x + 35 + 4) \\
&= (2x+3)(x^2 + 3x + 39)
\end{aligned}$$

Note:-
Remember to use FOIL(First, Outside,.Inside Last) to expand brackets

0.1.5 Simplifying Algebraic Fractions

Theorem 0.1.5 Simplifying Algebraic Fractions
When simplifying algebraic fractions it is important to use these two steps:
1. Factorise the numerator and denominator
2. After factorising you can cancel any common factors

Question 10

$$\frac{8x^2 + 4x + 2}{8x^3 - 1}$$

$$\begin{aligned}
&= \frac{2(4x^2 + 2x + 1)}{(2x-1)((2x)^2 + (2x \times 1) + 1^2)} \\
&= \frac{2(4x^2 + 2x + 1)}{(2x-1)(4x^2 + 2x + 1)} \\
&= \frac{2}{2x-1}
\end{aligned}$$

Question 11

$$\frac{(x+h)^3 - x3}{h}$$

$$\begin{aligned}
&= \frac{(x+h-x)((x+h)^2 + x(x+h) + x^2)}{h} \\
&= \frac{h(x^2 + 2xh + h^2 + x^2 + xh + x^2)}{h} \\
&= \frac{h(3x^2 + 3xh + h^2)}{h} \\
&= 3x^2 + 3xh + h^2
\end{aligned}$$

0.1.6 Adding & Subtracting Algebraic Fractions

Theorem 0.1.6 Adding & Subtracting Algebraic Fractions

To Add or Subtract Algebraic fractions there are three important steps you need to follow
Rule 1: Factorise all fractions on the numerator & denominator Rule 2: Find and create a common denomitor for all fractions(remember to not repeat the same expression more than once)
Rule 3:Simplifying the fraction using like terms

Question 12

$$\frac{5}{2a+6} + \frac{a}{a^2-9}$$

$$= \frac{5}{2(a+3)} + \frac{a}{(a+3)(a-3)}$$
$$= \frac{5(a-3)+2a}{2(a+3)(a-3)}$$
$$= \frac{5a-15+2a}{2(a+3)(a-3)}$$
$$= \frac{7a-15}{2(a+3)(a-3)}$$

Question 13

$$\frac{6}{3x-2} - \frac{8}{4x+1}$$

$$= \frac{6(4x+1)-8(3x-2)}{(4x+1)(3x-2)}$$
$$= \frac{24x+6-24x+16}{(4x+1)(3x-2)}$$
$$= \frac{22}{(4x+1)(3x-2)}$$

0.1.7 Surds

Theorem 0.1.7 Rationalising the denominator

Rationalising the denominator involves multiplying the entire fraction by the surd, denominator to rationalise it to a whole number
If the denominator is a binomial and has both a rational and rational ;portion you will need to use the conjugate,the conjugate is the denominator with opposite signs.
if $\frac{1}{3+\sqrt{2}}$ is the fraction, the conjugate is $3-\sqrt{2}$ as this results in the difference of squares

Question 14

$$\frac{2\sqrt{6}}{5\sqrt{2}}$$

$$\frac{2\sqrt{6}}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{2\sqrt{12}}{10}$$
$$\frac{4\sqrt{3}}{10}$$
$$\frac{2\sqrt{3}}{5}$$

Question 15

$$\frac{1}{3\sqrt{3}+4}$$

$$\frac{1}{\sqrt{3}+4} \times \frac{\sqrt{3}-4}{\sqrt{3}-4}$$
$$\frac{\sqrt{3}-4}{3-16}$$
$$-\frac{\sqrt{3}-4}{13}$$

0.1.8 Completing the square

Theorem 0.1.8 Completing the Square

To complete the square with monic quadratics $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$ to both sides of the equation $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2$ then solve for x When wanting to complete the square for non-monic quadratics you first must make the equation monic by diving the equation by a $ax^2 + bx + c = 0$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
Then solve like a normal completing the square, the non monic completing the square formula is also how the quadratic formula is derived

Question 16

$$2x^2 + 6x - 5 = 0$$

$$x^2 + 3x + \frac{9}{4} = \frac{5}{2} + \frac{9}{4}$$
$$(x + \frac{3}{2})^2 = \frac{19}{4}$$
$$x + \frac{3}{2} = \frac{\pm\sqrt{19}}{2}$$
$$x = \frac{-3 \pm \sqrt{19}}{2}$$

Question 17

$$3x^2 - 5x - 1 = 0$$

$$x^2 - \frac{5}{3}x + \left(-\frac{5}{6}\right)^2 = \frac{1}{3} + \left(-\frac{5}{6}\right)^2$$
$$\left(x - \frac{5}{6}\right)^2 = \frac{37}{36}$$
$$x - \frac{5}{6} = \frac{\pm\sqrt{37}}{6}$$
$$x = \frac{5 \pm \sqrt{37}}{6}$$

0.1.9 Indices

Theorem 0.1.9 Indices

Index Laws:
$a^m \times a^n = a^{n+m}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{nm}$
$(ab)^n = a^n b^n$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
NEGATIVE INDICES:
$x^{-n} = \frac{1}{x^n}$
Fractional Indices:
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Question 18

$$\frac{1}{\sqrt[3]{(4x^2-1)^2}}$$

$$(4x^2-1)^{-\frac{2}{3}}$$

Question 19

$$\frac{x - 5 + 6x^{-1}}{1 - 2x^{-1}}$$

$$\frac{x - 5 + 6x^{-1}}{1 - 2x^{-1}} \times \frac{x}{x}$$
$$\frac{x^2 - 5x + 6x}{x - 2x}$$
$$\frac{(x-3)(x-2)}{x(1-2)}$$

0.2 Functions

0.2.1 Funtions and Relations

Theorem 0.2.1 Functrions and Relations

A relation is a set of ordered pairs where variables are related to each other according to a rules
A set is a list of numbers, ordered pairs etc

Types of Relations:
One-to-One - every element corresponds to on element in the other set
One-to-Many - where a element in Set A corresponds to 2 or more elemnts in Set B
Many-to-One - 2 or more elemnts of Set A correspond with 2 or more elements in Set B

Functions: Functions are a special typoe of relation where every elemnt of Set A corresponds with a unique element of Set B. In a function the domain is the set of all x values that the function could input, the range in the function is the set of all y values that can be potentialy outputted by the function.

Vertical Line Test: To determine whether something is a function vs a relation we can use the vertical line test which states that if a line onlycuts the y axis at one point it must represent a function.
Horizontal line test: We can use the horizontal line test to determine if a relation is one-one or not, if multiple points lie on the same y coordinate then the function cannot be one-one.

Question 20

Find the Domain and range of the equation $\sqrt{x}$

Domain: $x \geqslant 0$
Range: $y \geqslant 0$

Question 21

Find the domain and range of the equation $2 + x^2$

Domain: $\mathbb{R}$
Range: $y \geqslant 2$

0.2.2 Function & Interval notation

Theorem 0.2.2 Function and Interval notation

Function notaion:
With function notation like $f(x)$, f is the name of our function and x inside the brackets is the input of the function
So when $f(x) = 2x$ then $f(3) = 6$

Interval notation:
A closed interval is when the interval contains all endpoints within it.
Example: $y \geqslant x \geqslant b$ or in bracket notation $[y, b]$

The open interval:
The open interval occurs when the interval does not contain its endpoints.
Example: $y < x < b$ or in bracket notation (y, b)

The closed ray:
The closed ray occurs when x is unbounded inn one direction and contains its endpoint.
Example: $x \geqslant y$ or in bracket notation $[y, \infty)$

The open ray:
The open ray occurs when x is unbounded and does not contain its endpoint.
Example: $x < y$ or or in bracket notation $(-\infty, y)$

0.2.3 Absolute values

Theorem 0.2.3 Absolute values

Absolute values:
Absolute values are a way or measuring the distance a number is from its origin(0), trhis means that an absolute value will always be positive. To denote an absolute values we use the symbols $|x|$
When you are trying to solve an equation with absolute values, it can be positive or negative. Hint, if there is an equatio in an absolute value do not solve until you get rid of the absolute value.
For an example if $|x - b| = a$ then $x - b = \pm a$

Question 22

$$Solve : |x - 2| = 3$$

$$x - 2 = \pm 3$$
$$x = 2 \pm 3$$
$$x = 5, -1$$

Question 23

$$|m - 5| \geqslant 0$$

$$0 \geqslant m - 5 \geqslant 0$$
$$5 \geqslant m \geqslant 5$$
$$m = 5$$

Odd and Even functions

Theorem 0.2.4 Odd and Even functions

A function/relation is even if when graphed it has line of symmetry from the y axis
To determine whether a function is even $f(x) = f(-x)$
A function/relation is called odd if the point of symmetry in the origin, this means that if rotated 180 deg trhe graph remains unchanged
To determine whether a function is odd $f(-x) = -f(x)$
If a function is neither odd nor even you just use "neither"

Question 24

Determine whether the function is odd,even or neither $f(x) = \frac{3}{x^2-4}$

$$f(-x) = \frac{3}{(-x)^2 - 4}$$
$$= \frac{3}{x^2 - 4}$$
$$\therefore f(x) \text{ is even function}$$

Question 25

Determine whether the function is odd,even or neither

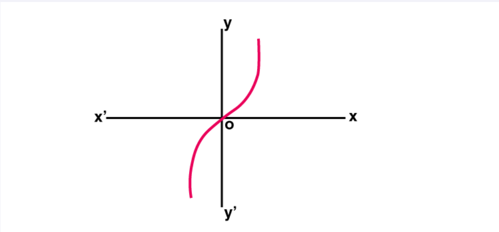
$$f(x) = \frac{x^3}{x^4 - x^2}$$

$$f(-x) = \frac{(-x)^3}{(-x)^4 - (-x)^2}$$
$$= -\frac{x^3}{x^4 - x^2}$$
$$-f(x) = \frac{(-x)^3}{(-x)^4 - (-x)^2}$$
$$\therefore f(x) \text{ is odd function}$$

0.2.4 Cubic Polynomials

Theorem 0.2.5 Cubic Polynomials

Cubic polymials are polynomials of the third degree, a cubic function has one y intercept but upto three x intercepts



- in $f(x) = x^3 + 1$ the constant 1 is the y intercept
If the form is kx^3 when $k > 0$ it is a increasing function and when $k < 0$ it is a decreassing function. The point of inflection in the x^3 function is wear the gradient of the line changes
To find the point of inflexion we can get the equation in the form $f(x) = k(x - b)^3 + c$ which is the same as kx^3 however the inflexion point is (b, c)
The final way a cubic function can be displayed is of $f(x) = k(x - a)(x - b)(x - c)$

Question 26

Find inflexion point of the equation $2(x - 1)^3 - 16$

$$= k(x - b)^3 + c$$
$$POI = (b, c)$$
$$POI = (1, 16)$$

Question 27

Find inflexion point of the equation $(x + 2)^3 + 8 = 0$

$$= k(x - b)^3 + c$$
$$POI = (b, c)$$
$$POI = (-2, 8)$$

Note:-

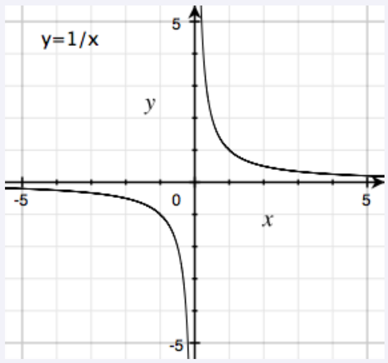
As the form is -b, the b value for POI is negative rather than positive. If the b vlue was negative it the POI b would be positive.

0.2.5 Hyperbolic Function(Inverse variation)

Theorem 0.2.6 Hyperbolic Function

A hyperbola occurs in the form of $\frac{k}{x}$ where k is a constant. This equation tells us that when one variable increases the other decreases..

A hyperbola is a discontinuous function meaning there are gaps. There are two asymptotes in a hyperbolic function one on the x and one on the y axis.



To find the y intercept set x = 0 and solve for y

To find the x asymptote solve expression on the denominator for x.

To find the y asymptote substitute the x asymtote into the denominator expression and solve for y

Question 28

Find the asymptotes and the Domain and Range of the following function $f(x) = \frac{3}{x-3}$

$$x - 3 \neq 0$$

$$x \neq 3$$

$$y \neq 0$$

$$\text{y int} \neq \frac{3}{0-3}$$

$$\text{y int} \neq -1$$

$$\therefore \text{ x asymptote} = 3, \text{ y asymptote} = 0, \text{ y int} = 0$$

Domain:
$(-\infty, 3) \cup (3, \infty)$
Range:

$$(-\infty, 0) \cup (0, \infty)$$

Question 29

Find the x and y asymptotes and the y intercept = for the following function $f(x) = -\frac{1}{2x+4}$

$$2x + 4 \neq 0$$
$$2x \neq -4$$
$$x \neq -2$$
$$\text{x asymp} = -2$$
$$\text{y int} = -\frac{1}{0+4}$$
$$\text{y int} = -\frac{1}{4}$$
$$\text{y asymp} = 0$$

0.2.6 Linear functions

Intersection of Two Lines

Theorem 0.2.7 Intersection of two lines

To find the point where two lines intersect we need to solve the two equations simultaneously

Question 30

Find the intersection points of the equations $4x + 2y + 2 = 0$ and $3x + 5y - 9 = 0$

$$4x + 2y + 2 = 0...1$$
$$3x + 5y - 9 = 0...2$$
$$...1 \times 3$$
$$12x + 6y + 6 = 0...3$$
$$...2 \times 4$$
$$12x + 20y - 36 = 0...4$$
$$...4 - ...3$$
$$14y - 42 = 0$$
$$y = 3$$
$$4x + 6 + 2 = 0$$
$$x = -2$$
$$\therefore \text{ the intersection point of the two lines is } (-2, 3)$$

Question 31

Find the equation of the line above if it passes through points $(4, -2)$

$$m = \frac{3+2}{-2-4}$$
$$m = -\frac{5}{6}$$
$$y - 3 = -\frac{5}{6}(x + 2)$$
$$y - 3 = -\frac{5}{6}x + -\frac{10}{6}$$
$$6y - 18 = -5x - 10$$
$$5x + 6y - 8 = 0$$

Note:-

Two straight lines have three possible variations with them intersecting at the same point, being parallel or coinciding(infinite solutions)

Solving Simultaneous equations using the k method

Theorem 0.2.8 K method

The k method works for when you need to find the equation of a line that passes through a point aswell as the intersection point of two lines. The formula is $y + k(x) = 0$ Where y is the first equation and x is the second equation, It is worth noting that each equations has to equal zero before using the k method. You then subsitute the point values in the equation and solve for k. Once you solve for k you subsitute into the first form of the k equation and simplify.

Question 32

Two equations $4x + 2y + 2 = 0$ and $3x + 5y - 9 = 0$ intersect at $(4, -2)$ Find the equation using the k method

$$\begin{gathered}
4x + 2y + 2 + k(3x + 5y - 9) = 0 \\
16 - 4 + 2 + k(12 - 10 - 9) = 0 \\
14 - 7k = 0 \\
7k = 14 \\
k = 2 \\
4x + 2y + 2 + 2(3x + 5y - 9) = 0 \\
4x + 2y + 2 + 6x + 10y - 18 = 0 \\
10x + 12y - 16 = 0 \\
5x + 6y - 8 = 0
\end{gathered}$$

0.2.7 Composite functions

Theorem 0.2.9 Composite functions

Composite functions are functons when the output of one is used as the input for another eg. $g(f(x))$. In other words you combine two functions to create a third function with a different output.

Question 33

$$f(x) = 3x \ g(x) = x + 4 find \ g(f(x))$$

$$g(f(x)) = 3x + 4$$

Question 34

$$f(x) = \frac{x}{3} \ g(x) = \frac{3}{x} \ find \ g(f(x))$$

$$g(f(x)) = x$$

0.2.8 Simultaneous equations of the 2nd degree

Theorem 0.2.10 quadratic simultaneous equations
When dealing with second degreee simultaneous equations generally you will have two sets of coordinates.
To solve we first either use the subsitute or elimination method, we then factor and solve as usual.
The reason why we solve these is to find where these two equations intersect eachother.

Question 35

Solve simultaneously $y = x - 1$ $y = x^2 + 4x + 1$

$$y = x - 1...1$$
$$y = x^2 + 4x + 1...2$$
$$\text{sub 1 in 2}$$
$$x - 1 = x^2 + 4x + 1$$
$$x^2 + 3x + 2 = 0$$
$$(x + 2)(x + 1)$$
$$x = -2, -1$$
$$y = -3, -2$$
$$\therefore POI : (-2, -3), (-1, -2)$$

Question 36

Solve simultaneously $y - 2x + 1 = 0$ $3y^2 - y - 2x^2 = 0$

$$y = 2x - 1...1$$
$$3y^2 - y - 2x^2 = 0...2$$
$$\text{Sub 1 in 2}$$
$$3(2x - 1)^2 - (2x - 1) - 2x^2 = 0$$
$$3(4x^2 - 4x + 1) - 2x + 1 - 2x^2 = 0$$
$$12x^2 - 12x + 3 - 2x + 1 - 2x^2$$
$$10x^2 - 14x + 4 = 0$$
$$(10x - 4)(10x - 10) = 0$$
$$(5x - 2)(x - 1) = 0$$
$$x = \frac{5}{2}, 1$$
$$y = 2 - 1$$
$$y = 1$$
$$y = 2 \times \frac{2}{5} - 1$$
$$y = -\frac{1}{5}$$
$$y = 1, -\frac{1}{5}$$
$$\therefore (\frac{2}{5}, \frac{2}{5}), (1, 1)$$

0.2.9 Non-Linear Simultaneous equations

Theorem 0.2.11 Non-Linear Simultaneous equations

To solve non-linear simultaneous equations, you first want to make a variable the subject of one equation to subsitute or eliminate one variable by another operation. After this is complete you can solve like a normal simultaneous equation

Question 37

Solve simultaneously $x^2 + y^2 = 25$ $x + y = 5$

$$x = 5 - y...1$$
$$x^2 - y^2 = 25...2$$
$$\text{Sub 1 in 2}$$
$$(5 - y)^2 + y^2 = 25$$
$$25 - 10y + 2y^2 = 25$$
$$-10y + 2y^2 = 0$$
$$2y(y - 5) = 0$$
$$y = 0, 5$$
$$x = 5, 0$$
$$\therefore (0, 5), (5, 0)$$

Question 38

Solve simultaneously $x + 2y = -8$ $xy = 8$

$$x = -2y - 8...1$$
$$xy = 8...2$$
$$\text{Sub 1 in 2}$$
$$y(-2y - 8) = 8$$
$$2y^2 + 8y + 8 = 0$$
$$(2y + 4)^2 = 0$$
$$(y + 2)^2 = 0$$
$$y = -2$$
$$x = -4$$
$$\therefore (-4, -2)$$

0.2.10 Quadratic functions

Theorem 0.2.12 Quadratic function

Quadratic functuion are polynomials of the 2nd degree $ax^2 + bx + c$, these equations form parabolas

Turning point

Theorem 0.2.13 Turning point

Minimum and maximum values:
The minimum or maximum value of a parabola defines where its turning point will be
If $a > 0$ we use call it the minimum value and if $a < 0$ we call it the maximum value
Turning point:
The turning point of a parabola lies on the axis of symmetry, the axis of sym is halfway between the x intercepts.
To find the axis of symmetry we use $x = \frac{-b}{2a}$
Once we know the value of the axis of symmetry we then subsitute its value as x. After the equation is solved the answer is our y coordinate.
Here is an equation for the value of the coordinate pair for the turning point $(\frac{-b}{2a}, a(\frac{-b}{2a})^2 + b(\frac{-b}{2a}) + c)$

Question 39

Find the turning point of the equation $f(x) = x^2 - 5x + 1$

$$x = \frac{5}{2}$$
$$y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 1$$
$$y = \frac{25}{4} - \frac{25}{2} + 1$$
$$y = -\frac{25}{4} + 1$$
$$y = -\frac{21}{4}$$
$$y = -5\frac{1}{4}$$
$$\therefore minval = (2\frac{1}{2}, -5\frac{1}{4})$$

Question 40

Find the turning point of the equation $f(x) = -3x^2 + x - 5$

$$x = \frac{-1}{-6}$$
$$x = \frac{1}{6}$$
$$y = -3(\frac{1}{6})^2 + \frac{1}{6} - 5$$
$$y = -\frac{1}{12} + \frac{2}{12} - 5$$

$$y = -\frac{59}{12}$$
$$y = -4\frac{11}{12}$$
$$\therefore maxval(\frac{1}{6}, -4\frac{11}{12})$$

The discriminant

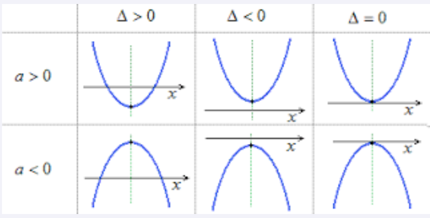
Theorem 0.2.14 The discriminant

The discriminant gives information about the roots of a quadratic equation. The discriminant is $\Delta = b^2 - 4ac$

If the $\Delta > 0$ then there are two real roots(two x intercepts), if the discriminant is a square number these roots are rational and if the discriminant is not a square then the roots are irrational.

If $\Delta = 0$ there are 2 equal rational roots(one x intercept)

If $\Delta < 0$ there are no real roots(no x intercept)



Question 41

Find k for which $5x^2 - 2x + k = 0$ has real roots and $\Delta \geqslant 0$

$$a > 0$$
$$4 - 4(5 \times k) \geqslant 0$$
$$4 - 20k \geqslant 0$$
$$-20k \geqslant -4$$
$$k \leqslant \frac{1}{5}$$
$$k = \frac{1}{5}$$

Note:-

if $a < 0$ we would use a $\leqslant$ symbol instead of $\geqslant$

Question 42

Show that $f(x) = x^2 - x - 2$ has 2 real roots

$$\Delta = 1 - 4(1 \times -2)$$
$$= 1 + 8$$
$$= 9$$

$\therefore \Delta > 0$ and is a perfect square so $x^2 - x - 2$ has two real rational roots

Finding a quadratic equation

Theorem 0.2.15 Finding a quadratic equation
To find a quadratic equation we use subsitute the coordinate pairs and solve like a simultaneous equation

Question 43

Find the equation that passes $(-1,-3)(0,3),(2,21)$

$$y = ax^2 + bx + c =$$
$$-3 = a - b + c...1$$
$$3 = c...2$$
$$21 = 4a + 2b + c...3$$

Sub 2 in 1

$$a - b = -6...4$$

Sub 2 in 3

$$4a + 2b + 3 = 21...5$$

From 5

$$4a + 2b = 18$$
$$2a + b = 9...6$$

Add 6 and 7

$$3a = 3$$
$$a = 1$$
$$1 - b = -6$$
$$b = 7$$
$$\therefore a = 1, b = 7, c = 3$$

Solution Set of simultaneous equations

Theorem 0.2.16 Solution Set of simultaneous equations

A solution set of simultaneous equations is where you are given two equations ad are to find the coordinates where the two lines touch, intersect, not intersect each other.
To do this we use the discriminant For an example, once we find the point of intersection line via subsituition we can find where the two lines touch by using the Δ and the values of the POI line.
We can find the intersection points by using $b^2 - 4ac > 0$ on the new POI line
We can also find where the line do not inteersect by using $b^2 - 4ac < 0$

Note:-
To find these areas, we use ineqwualities and factorisation. Before we complete the steps above we first need to factorise the POI line and use the x values to test the discriminant inequalities as seen below.

Question 44

For what values of m does line $y = mx - 6$ algebraic) touch b)intersect c) not intersect the parabola y+x$^2 - 2x + 3$

Point of intersection:

$$4x^2 - 2x + 3 = mx - 6$$
$$x^2 - 2x - mx + 9$$
$$x^2 - (m + 2)x + 9$$

Touch when $\Delta = 0$

$$(m + 2)^2 - 4(1 \times 9) = 0$$
$$m^2 - 4m + 32 = 0$$
$$(m + 8)(m - 4) = 0$$

$\therefore m = 4, -8$ when the two lines touch

Note:-
Remember to use the point of interesection equation when subsituting values for the discriminant

Intersect when $\Delta > 0$

$$(m + 8)(m - 4) > 0$$

We now need to look at for what numbers satisfy m in this inequality. So we plot -8 and 4 on a number line and see if a value less than -8 can satify the inequality then inbetweeen -8 and 4 and then greter than 4

$$(-9 + 8)(-9 - 4) > 0$$

This is true

$$(-2 + 8)(2 - 4) > 0$$

This is false

$$(5 + 8)(5 - 4) > 0$$

This is true

$$\therefore m < -8, m > 4$$

Not intersect $\Delta < 0$

$$(m+8)(m-4) < 0$$

Like before we neeed to plot -8 and 4 ona number line and look for values which satify the inequality

$$(-9+8)(-9-4) < 0$$

False

$$(-2+8)(-2-4)$$

True

$$(5+8)(5-4)$$

False

$$\therefore -8 < m < 4$$

Simultaneous equations with three unknowns

Theorem 0.2.17 Simultaneous equations with three unknowns

When dealing with three unknown variables we first have to eliminate a variable and tranistion into a two variable simultaneous equation. Once two variables have been found you can easily find the third.
You must note that there are multiple ways to solve these equations correctly aswell as the importance of numbering the equations

Question 45

Solve:

$$a - b + c = 7...1$$
$$a + 2b - c = -4...2$$
$$3a - b - c = 3...3$$

We can first add 1 and 2 as we notice this will eliminate the c variable

$$2a + b = 3...4$$

We can also add 1 and 3 as we notice thisa will also eliminate the c variable

$$4a - 2b = 10$$
$$2a - b = 5...5$$

We can also notice that if we add 4 and 5 we eliminate the b variable

$$4a = 8$$
$$a = 2$$

Sub a back into 4

$$4 + b = 3$$
$$b = -1$$

We can then subsitute both a and b back into equation 1

$$2 + 1 + c = 7$$
$$c = 4$$
$$\therefore a = 2, b = -1, c = 4$$

0.3 Further work with Functions (Extension 1)

0.3.1 Quadratic inequalities

Theorem 0.3.1 Quadratic inequalities

A quadratic inequality is where there is a polynomial of the second degreee in an inequality.
To solve a quadratic inequality you first want to move everything to the lefthand side of the inequality then factor the quadratic equation.
After factoring, you use the solution to find the bounds of the inequality. To do this you plot them on a numberline and check each area to see if it fits the inequality. For an example if the solutions after factorsing were 3 and 7, you would check a number < 3 in between 3 and 7 and a number > 7.

Question 46

Solve:

$$x^2 + x - 6 > 0$$

$$(x + 3)(x - 2) > 0$$

We now check each are to see what values fit the inequality of < -3 between -3 and 2 and > 2

$$(-4 + 3)(-4 - 2) > 0$$

True

$$(-1 + 3)(-1 - 2) > 0$$

False

$$(3 + 3)(3 - 2) > 0$$

True

$$\therefore x < -3 or x > 2$$

Question 47

Solve:

$$(2 - x)(x - 5)(x + 1) > 0$$

The roots are 2, 5, -1. We now need to check numbers both less than or greater than each of these roots

$$(2 + 2)(-2 - 5)(-2 + 1) > 0$$

True

$$(2 + 0)(0 + -5)(0 + 1) > 0$$

False

$$(2 - 4)(4 - 5)(4 + 1) > 0$$

True

$$(2 - 6)(6 - 5)(6 + 1) > 0$$

False

$$\therefore x < -1, 2 < x < 5$$

Question 48

Solve:

$$2^{2x} - 5(2^x) + 4 \leqslant 0$$

First we can subsitute 2^x as m

$$m^2 - 5m + 4 \leqslant 0$$

$$(m-4)(m-1)$$

$$1 \leqslant 4$$

$$2^x = 4$$

$$x = 2$$

$$2^x = 1$$

$$x = 0$$

$$\therefore 0 \leqslant x \leqslant 2$$

0.3.2 Inequalities with unknown denominator

Theorem 0.3.2 Inequalities with unknown denominator

When solving inequalities with an unknown denominator there are two methods, the critical values method and the multily by the square denominator method.
However they both start by finding what x cannot be
Critical Values method:
After finding out what x cannot be you solve the inequality by changing the sign to an =. Following this step you plot what x cannot be and the solved value for x and test each area/range(like with quadratic inequalities) to find what bounds work.
After this is completre you can format the inequality
Multiply by Square denominator method:
After finding what value x cannot be you muliply each side by the denominator squared, you then move all componnts of the inequality to make one side equal to zero. Following this you factorise this quadratic equation and find the solution for x. You then complete the numberline method as you do with quadratic inequalities

Question 49

Solve

$$\frac{y^2-6}{y} \leqslant 1$$

$$y \neq 0$$

$$\frac{y^2-6}{y} \times y^2 \leqslant 1 \times y^2$$

$$y^3 - 6y \leqslant y^2$$

$$y(y^2-6) \leqslant y^2$$

$$y(y^2-6) + y^2 \leqslant 0$$

$$y(y^2-y-6) \leqslant 0$$

$$y(y-3)(y+2) \leqslant 0$$

The roots are 3 and -2
We now use the numberline method and test the numbers in original equation

$$\frac{-32-6}{-3} \leqslant 1$$

True

$$\frac{-1^2-6}{-1} \leqslant 1$$

False

$$\frac{1^2-6}{1} \leqslant 1$$

True

$$\frac{4^2-6}{4} \leqslant 1$$

False

$$\therefore y \leqslant 2, 0 < y \leqslant 3$$

Question 50

Solve

$$\frac{6}{x+3} \geqslant 1$$

$$x \neq -3$$

$$\frac{6}{x+3} \times (x+3)^2 \geqslant (x+3)^2$$

$$6x + 18 \geqslant x^2 + 6x + 9$$

$$x^2 - 9 \leqslant 0$$

$$(x-3)(x+3)$$

$$\therefore -3 < x \leqslant 3$$

Note:-

We do not need to use the numberline method as we know that because of the < sign, we are looking for all x values below the x axis.
If the sign was > we would be looking for all coordinates above the x axis

0.3.3 Inequalities Involving Absolute values

Theorem 0.3.3 Inequalities involving Absolute values

As absolute values can have two inputs(+ or -) for the same one output, we split them up into two simultaneous inequalities to solve.
Because of this, the negative potential input flips the inequality sign. After splitting up the absolute value into to inequalities you should either have a < and a > or a ⩽ and *geq*

Question 51

Solve :

$$|5b - 7| \geqslant 3$$

$$5b - 7 \geqslant 3$$

$$5b - 7 \leqslant -3$$

$$5b \geqslant 10$$

$$b \geqslant 2$$

$$5b \leqslant 4$$

$$b \leqslant \frac{4}{5}$$

$$\therefore b \geqslant 2, b \leqslant \frac{4}{5}$$

Question 52

$$|2y - 1| < 5$$

$$2y - 1 < 5$$

$$2y < 6$$
$$y < 3$$
$$2y - 1 > -5$$
$$2y > -4$$
$$y > -2$$
$$\therefore y > -2, y < 3$$

0.3.4 Inequalities Involving Square roots

Theorem 0.3.4 Inequalities involving Square roots

Question 53

Question 54