Maths
Year 11 Notes

Jackson Love

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Contents

0.1 Algebraic Techniques

0.1.1 Simplifying Algebraic expressions

Theorem 0.1.1 Simplifying Algebraic expressions

When you add & subtract in algebra you can only combine like terms
Questions in Fitzgeralds 1.1

Question 1

$$5x + 2y - 3 - (x - 7y + 9)$$

$$= 5x + 2y - 3 - x + 7y - 9$$
$$= 4x + 9y - 12$$

Question 2

$$3x(x + 2) - 4(x - 1)$$

$$= 3x^2 + 6x - 4x + 5$$
$$= 3x^2 - 2x + 5$$

0.1.2 Substitution in Formulae

Theorem 0.1.2 Substituition in Formulae

Substituition occurs when you substitute values into an algebraic equation and/or rearrange the equations to make a variable the subject
More Questions in 1.2 Fitzgeralds textbook

Question 3

If $S = \frac{a(r^3-1)}{r-1}$ find S when $a = 5$, $r = 3$

$$= \frac{5(3^3 - 1)}{3 - 1}$$
$$= \frac{5 \times 26}{2}$$
$$= 5 \times 13$$
$$= 65$$

Question 4

If $A = P(1 + \frac{r}{100})^n$, find A when $P = 1000$, $r = 10$, $n = 2$

$$= 1000(1 + \frac{10}{100})^2$$
$$= 1000 \times 1.21$$
$$= 1210$$

0.1.3 Basic Polynomials

Theorem 0.1.3 Basic Polynomials

There are different types of polynomials include monomial(one term), binomial(two terms) and trinomial(three terms)
Rules for expanding polynomials:
Expanding Perfect/Difference squares ($(y+4)^2$), square first and last terms and multiply the first and last terms together. It should for $a^2+2ab+b^2$ unless there is a negative between the two expressions in hich case $-2ab$

Question 5

$$(2y+5)^2$$

$$a^2+2ab+b^2$$
$$=2y^2+25+20y$$

Question 6

$$(x+2)(x^2-5x+6)$$

$$=-x^3-5x^2+6x+2x^2-10x+12$$
$$=x^3-3x^2$$
$$=x^3-3x^2-4x+12$$

0.1.4 Fatorising The Sum/Difference of Two Cubes

Theorem 0.1.4 Fatorising The Sum/Difference of Two Cubes

When factoring Two cubes there are two rules to remember
Rule 1: $a^3+b^3=(a+b)(a^2-ab+b^2)$
Rule 2: $a^3-b^3=(a-b)(a^2+ab+b^2)$
To remember the sings used in the factorisation an acronym is SOAP(SAME, OPPOSITE, ALWAY, POSITIVE)

Question 7

$$a^3b-ab^3$$

$$=ab(a-b)(a+b)$$

Question 8

$$x^3-x^2y-9x+9y$$

$$=x^2(x-y)-9(x-y)$$
$$=(x-y)(x^2-9)$$
$$=(x-y)(x+3)(x-3)$$

Question 9

$$(x+5)^3+(x-2)^3$$

$$\begin{aligned}
&= (2x+3)((x+5)^2-((x+5)(x-2))+(x-2)^2) \\
&= (2x+3)(x^2+10x+25-x^2+2x-5x+10+x^2-2x-2x+4) \\
&= (2x+3)(x^2+10x+35-x^2+2x-5x+x^2-2x-2x+4) \\
&= (2x+3)(x^2+10x+35+2x-5x-2x-2x+4) \\
&= (2x+3)(x^2+3x+35+4) \\
&= (2x+3)(x^2+3x+39)
\end{aligned}$$

Note:-

Remember to use FOIL(First, Outside,.Inside Last) to expand brackets

0.1.5 Simplifying Algebraic Fractions

Theorem 0.1.5 Simplifying Algebraic Fractions

When simplifying algebraic fractions it is important to use these two steps:

1. Factorise the numerator and denominator
2. After factorising you can cancel any common factors

Question 10

$$\frac{8x^2+4x+2}{8x^3-1}$$

$$\begin{aligned}
&= \frac{2(4x^2+2x+1)}{(2x-1)((2x)^2+(2x\times 1)+1^2)} \\
&= \frac{2(4x^2+2x+1)}{(2x-1)(4x^2+2x+1)} \\
&= \frac{2}{2x-1}
\end{aligned}$$

Question 11

$$\frac{(x+h)^3-x3}{h}$$

$$\begin{aligned}
&= \frac{(x+h-x)((x+h)^2+x(x+h)+x^2)}{h} \\
&= \frac{h(x^2+2xh+h^2+x^2+xh+x^2)}{h} \\
&= \frac{h(3x^2+3xh+h^2)}{h} \\
&= 3x^2+3xh+h^2
\end{aligned}$$

0.1.6 Adding & Subtracting Algebraic Fractions

Theorem 0.1.6 Adding & Subtracting Algebraic Fractions

To Add or Subtract Algebraic fractions there are three important steps you need to follow
Rule 1: Factorise all fractions on the numerator & denominator Rule 2: Find and create a common denomitor for all fractions(remember to not repeat the same expression more than once)
Rule 3:Simplifying the fraction using like terms

Question 12

$$\frac{5}{2a+6}+\frac{a}{a^2-9}$$

$$\begin{aligned}
&= \frac{5}{2(a+3)}+\frac{a}{(a+3)(a-3)}\\
&= \frac{5(a-3)+2a}{2(a+3)(a-3)}\\
&= \frac{5a-15+2a}{2(a+3)(a-3)}\\
&= \frac{7a-15}{2(a+3)(a-3)}
\end{aligned}$$

Question 13

$$\frac{6}{3x-2}-\frac{8}{4x+1}$$

$$\begin{aligned}
&= \frac{6(4x+1)-8(3x-2)}{(4x+1)(3x-2)}\\
&= \frac{24x+6-24x+16}{(4x+1)(3x-2)}\\
&= \frac{22}{(4x+1)(3x-2)}
\end{aligned}$$

0.1.7 Surds

Theorem 0.1.7 Rationalising the denominator

Rationalising the denominator involves multiplying the entire fraction by the surd, denominator to rationalise it to a whole number
If the denominator is a binomial and has both a rational and rational ;portion you will need to use the conjugate,the conjugate is the denominator with opposite signs.
if $\frac{1}{3+\sqrt{2}}$ is the fraction, the conjugate is $3-\sqrt{2}$ as this results in the difference of squares

Question 14

$$\frac{2\sqrt{6}}{5\sqrt{2}}$$

$$\frac{2\sqrt{6}}{5\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{2\sqrt{12}}{10}$$

$$\frac{4\sqrt{3}}{10}$$

$$\frac{2\sqrt{3}}{5}$$

Question 15

$$\frac{1}{3\sqrt{3}+4}$$

$$\frac{1}{\sqrt{3}+4} \times \frac{\sqrt{3}-4}{\sqrt{3}-4}$$

$$\frac{\sqrt{3}-4}{3-16}$$

$$-\frac{\sqrt{3}-4}{13}$$

0.1.8 Completing the square

Theorem 0.1.8 Completing the Square

To complete the square with monic quadratics $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$ to both sides of the equation $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2$ then solve for x When wanting to complete the square for non-monic quadratics you first must make the equation monic by diving the equation by a $ax^2 + bx + c = 0$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
Then solve like a normal completing the square, the non monic completing the square formula is also how the quadratic formula is derived

Question 16

$$2x^2 + 6x - 5 = 0$$

$$x^2 + 3x + \frac{9}{4} = \frac{5}{2} + \frac{9}{4}$$

$$(x + \frac{3}{2})^2 = \frac{19}{4}$$

$$x + \frac{3}{2} = \frac{\pm\sqrt{19}}{2}$$

$$x = \frac{-3 \pm \sqrt{19}}{2}$$

Question 17

$$3x^2 - 5x - 1 = 0$$

$$x^2 - \frac{5}{3}x + \left(-\frac{5}{6}\right)^2 = \frac{1}{3} + \left(-\frac{5}{6}\right)^2$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{37}{36}$$

$$x - \frac{5}{6} = \frac{\pm\sqrt{37}}{6}$$

$$x = \frac{5 \pm \sqrt{37}}{6}$$

0.1.9 Indices

Theorem 0.1.9 Indices

Index Laws:
$a^m \times a^n = a^{n+m}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{nm}$
$(ab)^n = a^n b^n$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
NEGATIVE INDICES:
$x^{-n} = \frac{1}{x^n}$
Fractional Indices:
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Question 18

$$\frac{1}{\sqrt[3]{(4x^2-1)^2}}$$

$$(4x^2 - 1)^{-\frac{2}{3}}$$

Question 19

$$\frac{x - 5 + 6x^{-1}}{1 - 2x^{-1}}$$

$$\frac{x - 5 + 6x^{-1}}{1 - 2x^{-1}} \times \frac{x}{x}$$

$$\frac{x^2 - 5x + 6x}{x - 2x}$$

$$\frac{(x-3)(x-2)}{x(1-2)}$$

0.2 Functions

0.2.1 Funtions and Relations

Theorem 0.2.1 Functrions and Relations

A relation is a set of ordered pairs where variables are related to each other according to a rules
A set is a list of numbers, ordered pairs etc

Types of Relations:
One-to-One - every element corresponds to on element in the other set
One-to-Many - where a element in Set A corresponds to 2 or more elemnts in Set B
Many-to-One - 2 or more elemnts of Set A correspond with 2 or more elements in Set B

Functions: Functions are a special typoe of relation where every elemnt of Set A corresponds with a unique element of Set B. In a function the domain is the set of all x values that the function could input, the range in the function is the set of all y values that can be potentialy outputted by the function.

Vertical Line Test: To determine whether something is a function vs a relation we can use the vertical line test which states that if a line onlycuts the y axis at one point it must represent a function.
Horizontal line test: We can use the horizontal line test to determine if a relation is one-one or not, if multiple points lie on the same y coordinate then the function cannot be one-one.

Question 20

Find the Domain and range of the equation $\sqrt{x}$

Domain: $x \geqslant 0$
Range: $y \geqslant 0$

Question 21

Find the domain and range of the equation $2 + x^2$

Domain: $\mathbb{R}$
Range: $y \geqslant 2$

0.2.2 Function & Interval notation