

Maths Year 11 Notes

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Contents

0.1	Algebraic Techniques	2
	Simplifying Algebraic expressions — 2 • Substitution in Formulae — 2 • Basic Polynomials — 3 • Fatorising The Sum/Difference of Two Cubes — 3 • Simplifying Algebraic Fractions — 4 • Adding & Subtracting Algebraic Fractions — 5 • Surds — 5	

0.1 Algebraic Techniques

0.1.1 Simplifying Algebraic expressions

Theorem 0.1.1 Simplifying Algebraic expressions

When you add & subtract in algebra you can only combine like terms
Questions in Fitzgeralds 1.1

Question 1

$$5x + 2y - 3 - (x - 7y + 9)$$

$$= 5x + 2y - 3 - x + 7y - 9$$

$$= 4x + 9y - 12$$

Question 2

$$3x(x + 2) - 4(x - 1)$$

$$= 3x^2 + 6x - 4x + 5$$

$$= 3x^2 - 2x + 5$$

0.1.2 Substitution in Formulae

Theorem 0.1.2 Substitution in Formulae

Substitution occurs when you substitute values into an algebraic equation and/or rearrange the equations to make a variable the subject
More Questions in 1.2 Fitzgeralds textbook

Question 3

$$\text{If } S = \frac{a(r^3 - 1)}{r - 1} \text{ find } S \text{ when } a = 5, r = 3$$

$$= \frac{5(3^3 - 1)}{3 - 1}$$

$$= \frac{5 \times 26}{2}$$

$$= 5 \times 13$$

$$= 65$$

Question 4

$$\text{If } A = P\left(1 + \frac{r}{100}\right)^n, \text{ find } A \text{ when } P = 1000, r = 10, n = 2$$

$$= 1000\left(1 + \frac{10}{100}\right)^2$$

$$= 1000 \times 1.21$$

$$= 1210$$

0.1.3 Basic Polynomials

Theorem 0.1.3 Basic Polynomials

There are different types of polynomials include monomial(one term), binomial(two terms) and trinomial(three terms)

Rules for expanding polynomials:

Expanding Perfect/Difference squares $((y + 4)^2)$, square first and last terms and multiply the first and last terms together. It should be for $a^2 + 2ab + b^2$ unless there is a negative between the two expressions in which case $-2ab$

Question 5

$$(2y + 5)^2$$

$$\begin{aligned} & a^2 + 2ab + b^2 \\ & = 2y^2 + 25 + 20y \end{aligned}$$

Question 6

$$(x + 2)(x^2 - 5x + 6)$$

$$\begin{aligned} & = -x^3 - 5x^2 + 6x + 2x^2 - 10x + 12 \\ & = x^3 - 3x^2 \\ & = x^3 - 3x^2 - 4x + 12 \end{aligned}$$

0.1.4 Factorising The Sum/Difference of Two Cubes

Theorem 0.1.4 Factorising The Sum/Difference of Two Cubes

When factoring Two cubes there are two rules to remember

Rule 1: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Rule 2: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

To remember the signs used in the factorisation an acronym is SOAP(SAME, OPPOSITE, ALWAYS, POSITIVE)

Question 7

$$a^3b - ab^3$$

$$= ab(a - b)(a + b)$$

Question 8

$$x^3 - x^2y - 9x + 9y$$

$$\begin{aligned} & = x^2(x - y) - 9(x - y) \\ & = (x - y)(x^2 - 9) \\ & = (x - y)(x + 3)(x - 3) \end{aligned}$$

Question 9

$$(x + 5)^3 + (x - 2)^3$$

$$\begin{aligned} &= (2x + 3)((x + 5)^2 - ((x + 5)(x - 2)) + (x - 2)^2) \\ &= (2x + 3)(x^2 + 10x + 25 - x^2 + 2x - 5x + 10 + x^2 - 2x - 2x + 4) \\ &= (2x + 3)(x^2 + 10x + 35 - x^2 + 2x - 5x + x^2 - 2x - 2x + 4) \\ &= (2x + 3)(x^2 + 10x + 35 + 2x - 5x - 2x - 2x + 4) \\ &= (2x + 3)(x^2 + 3x + 35 + 4) \\ &= (2x + 3)(x^2 + 3x + 39) \end{aligned}$$

Note:-

Remember to use FOIL(First, Outside, Inside Last) to expand brackets

0.1.5 Simplifying Algebraic Fractions

Theorem 0.1.5 Simplifying Algebraic Fractions

When simplifying algebraic fractions it is important to use these two steps:

1. Factorise the numerator and denominator
2. After factorising you can cancel any common factors

Question 10

$$\frac{8x^2 + 4x + 2}{8x^3 - 1}$$

$$\begin{aligned} &= \frac{2(4x^2 + 2x + 1)}{(2x - 1)((2x)^2 + (2x \times 1) + 1^2)} \\ &= \frac{2(4x^2 + 2x + 1)}{(2x - 1)(4x^2 + 2x + 1)} \\ &= \frac{2}{2x - 1} \end{aligned}$$

Question 11

$$\frac{(x + h)^3 - x^3}{h}$$

$$\begin{aligned} &= \frac{(x + h - x)((x + h)^2 + x(x + h) + x^2)}{h} \\ &= \frac{h(x^2 + 2xh + h^2 + x^2 + xh + x^2)}{h} \\ &= \frac{h(3x^2 + 3xh + h^2)}{h} \\ &= 3x^2 + 3xh + h^2 \end{aligned}$$

0.1.6 Adding & Subtracting Algebraic Fractions

Theorem 0.1.6 Adding & Subtracting Algebraic Fractions

To Add or Subtract Algebraic fractions there are three important steps you need to follow

Rule 1: Factorise all fractions on the numerator & denominator Rule 2: Find and create a common denominator for all fractions (remember to not repeat the same expression more than once)

Rule 3: Simplifying the fraction using like terms

Question 12

$$\frac{5}{2a+6} + \frac{a}{a^2-9}$$

$$\begin{aligned} &= \frac{5}{2(a+3)} + \frac{a}{(a+3)(a-3)} \\ &= \frac{5(a-3) + 2a}{2(a+3)(a-3)} \\ &= \frac{5a - 15 + 2a}{2(a+3)(a-3)} \\ &= \frac{7a - 15}{2(a+3)(a-3)} \end{aligned}$$

Question 13

$$\frac{6}{3x-2} - \frac{8}{4x+1}$$

$$\begin{aligned} &= \frac{6(4x+1) - 8(3x-2)}{(4x+1)(3x-2)} \\ &= \frac{24x+6 - 24x+16}{(4x+1)(3x-2)} \\ &= \frac{22}{(4x+1)(3x-2)} \end{aligned}$$

0.1.7 Surds

Theorem 0.1.7 Rationalising the denominator

Rationalising the denominator involves multiplying the entire fraction by the surd, denominator to rationalise it to a whole number

If the denominator is a binomial and has both a rational and irrational portion you will need to use the conjugate, the conjugate is the denominator with opposite signs.

if $\frac{1}{3+\sqrt{2}}$ is the fraction, the conjugate is $3 - \sqrt{2}$ as this results in the difference of squares

Question 14

$$\frac{2\sqrt{6}}{5\sqrt{2}}$$

$$\frac{2\sqrt{6}}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{2\sqrt{12}}{10}$$

$$\frac{4\sqrt{3}}{10}$$

$$\frac{2\sqrt{3}}{5}$$

Question 15

$$\frac{1}{3\sqrt{3} + 4}$$

$$\frac{1}{\sqrt{3} + 4} \times \frac{\sqrt{3} - 4}{\sqrt{3} - 4}$$

$$\frac{\sqrt{3} - 4}{3 - 16}$$

$$-\frac{\sqrt{3} - 4}{13}$$