

# Convex Spectra Set of Dissected Regular Polygons

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The convex spectrum of a shape is a set of positive integers for the numbers of the congruent shapes to form some convex shapes.

The idea of convex spectra was introduced by Erich Friedman in 1999.

This document provides sets of convex spectra (hereinafter convex spectra set) of generalized shapes those convex spectra contributed on the Erich's web site by authors.

A convex spectra set consists of convex spectra of distinct shapes determined by positive integers  $j$  and/or  $k$ .

Positive integer  $r$  and/or  $s$  are parameters to form convex shapes. The  $r$  makes a convex shape large by surrounding it. The  $s$  makes a convex shape longer.

The  $N$  is the set of all positive integers.

Erich's web site <https://erich-friedman.github.io/mathmagic/0499.html>

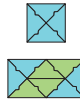
## Type 1-even

Convex spectra set  
and its sample shape

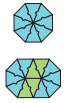


$$4N+4(k-1)$$

Some elements of convex spectra set  
and their convex shapes



$$4N$$



$$4N+4$$



$$4N+12$$

## Type 1-odd



$$\{ (2k+3)r^2 \}$$



$$\{ 5r^2 \}$$



$$\{ 7r^2 \}$$

## Type 2



$$2N+2k$$

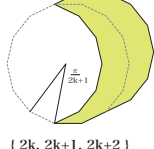


$$2N+2$$



$$2N+4$$

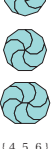
## Type 2-double



$$\{ 2k, 2k+1, 2k+2 \}$$



$$\{ 2, 3, 4 \}$$

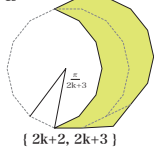


$$\{ 4, 5, 6 \}$$



$$\{ 6, 7, 8 \}$$

## Type 2-double-mod



$$\{ 2k+2, 2k+3 \}$$

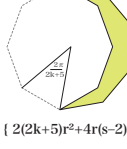


$$\{ 4, 5 \}$$



$$\{ 6, 7 \}$$

## Type 3



$$\{ 2(2k+5)r^2+4r(s-2) \}$$

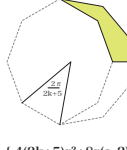


$$\{ 14r^2+4r(s-2) \}$$



$$\{ 18r^2+4r(s-2) \}$$

## Type 3-half



$$\{ 4(2k+5)r^2+8r(s-2) \}$$

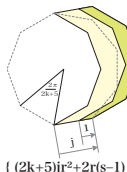


$$\{ 28r^2+8r(s-2) \}$$



$$\{ 36r^2+8r(s-2) \}$$

## Type 3-double-sliced



$$\{ (2k+5)jr^2+2r(s-1) \}$$



$$\{ 28r^2+2r(s-1) \}$$

$$j=4, k=1$$

## Type 3-triple



$$\{ 4k+2, 4(4k+2) \}$$



$$\{ 6, 24 \}$$

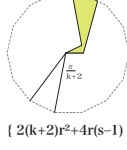


$$\{ 10, 40 \}$$

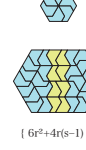


$$\{ 14, 56 \}$$

## Type 4



$$\{ 2(k+2)r^2+4r(s-1) \}$$



$$\{ 6r^2+4r(s-1) \}$$



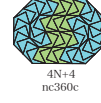
$$\{ 8r^2+4r(s-1) \}$$



$$\{ 10r^2+4r(s-1) \}$$

## Livio's Equilateral Pentagons

<https://www.iread.it/lz/pentagons.html>



$$4N+4$$

$$nc360c$$



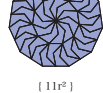
$$\{ 7r^2 \}$$

$$nc360b$$



$$\{ 9r^2 \}$$

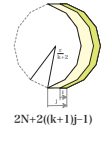
$$nc360d$$



$$\{ 11r^2 \}$$

$$nc360f$$

## Type 2-sliced

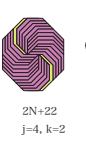


$$2N+2((k+1)j-1)$$



$$2N+14$$

$$j=4, k=1$$



$$2N+22$$

$$j=4, k=2$$

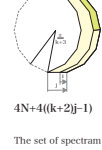


$$2N+46$$

$$j=4, k=5$$

The set of spectrum of this kind is a subset of to Type 2,  $2N+2k$  because the last term of  $2N+2((k+1)j-1)$  is always a positive even number regardless of the value of  $j$  and  $k$ .  
Note that each convex spectrum in the set is equivalent to some of convex spectrum in set of Type 2.

## Type 2-half

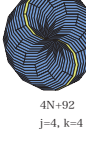


$$4N+4((k+2)j-1)$$



$$4N+44$$

$$j=4, k=1$$

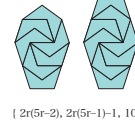


$$4N+92$$

$$j=4, k=4$$

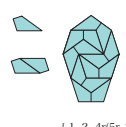
The set of spectrum of this kind is a subset of Type 1-even,  $4N+4(k-1)$ , because the last term of  $4N+4((k+2)j-1)$  is always a multiple of four regardless of the value of  $j$  and  $k$ .  
Note that each convex spectrum in the set is equivalent to some of convex spectrum in set of Type 1-even.

## Type 3-special



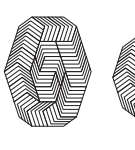
$$\{ 2r(5r-2), 2r(5r-1)-1, 10r^2+4r(s-2) \}$$

## Type 3-half-special



$$\{ 1, 2, 4r(5r-2), 4r(5r-1)-1, 20r^2+8r(s-2) \}$$

## Type 3-double-sliced-special



$$\{ 5jr^2-j, 5jr^2+2r(s-1) \}$$