Optimistic Policy Optimization via Multiple Importance Sampling

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Lorenzo Lupo Marcello Restelli

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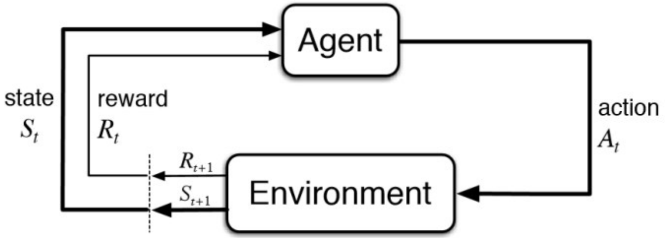
Reinforcement Learning [Sutton and Barto, 2018]



- Policy $\pi : \mathcal{S} \rightarrow \Delta(\mathcal{A})$
- Trajectories $\tau = (s_0, a_0, r_1, s_1, \dots)$
- Return $R(\tau) = \sum_{t=0}^{T-1} \gamma^t r_{t+1}$
- Goal: $\max_{\pi} \mathbb{E}_{\pi} [R(\tau)]$

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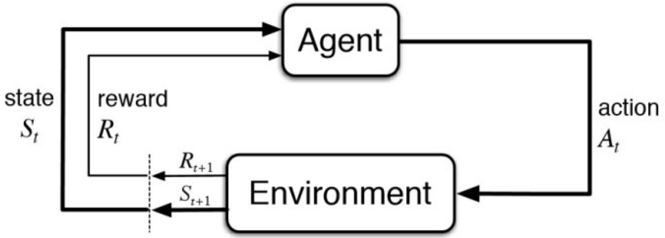

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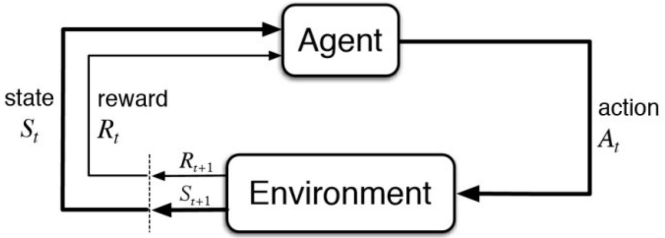

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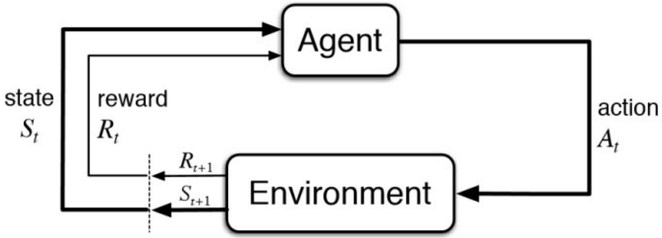

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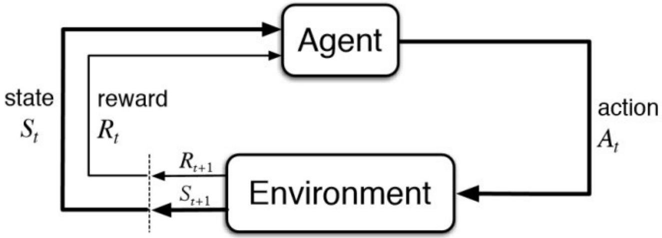

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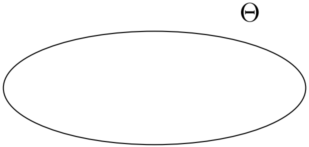
Exploration vs Exploitation

Continuous Reinforcement Learning

Policy Optimization

- **Parameter space** $\Theta \subseteq \mathbb{R}^d$



- A **parametric policy** $\pi_{\boldsymbol{\theta}}$ for each $\boldsymbol{\theta} \in \Theta$
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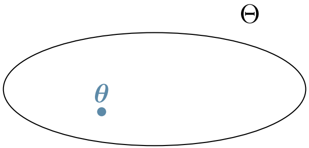

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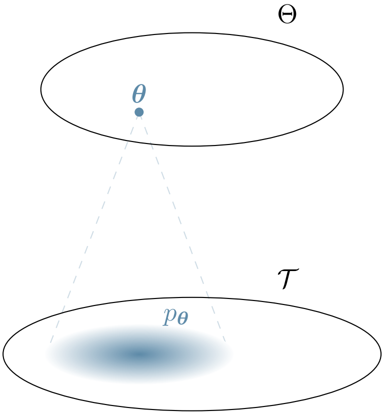

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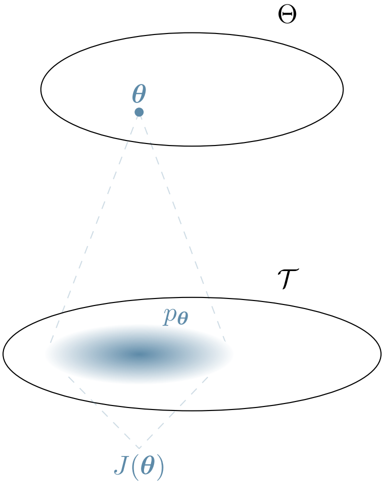

Policy Gradient Methods

- **Gradient ascent** on $J(\boldsymbol{\theta})$

- Popular algorithms: **REINFORCE** [Williams, 1992], **PGPE** [Sehnke et al., 2008], **TRPO** [Schulman et al., 2015], **PPO** [Schulman et al., 2017]

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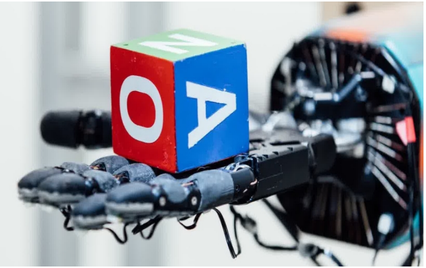
Dota 2 [OpenAI, 2018]



Manipulation [Andrychowicz et al., 2018]

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- Arms $a \in \mathcal{A}$
- Expected payoff $\mu(a)$
- Goal: $\min \mathit{Regret}(T) = \sum_{t=1}^{T} [\mu(a^*) - \mu(a_t)]$
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Optimism in Face of Uncertainty

- OFU strategy (e.g., UCB [Lai and Robbins, 1985]):

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- **Arms:** parameters $\boldsymbol{\theta}$
- **Payoff:** expected return $J(\boldsymbol{\theta})$
- **Continuous MAB**: we *need* structure [Kleinberg et al., 2013]

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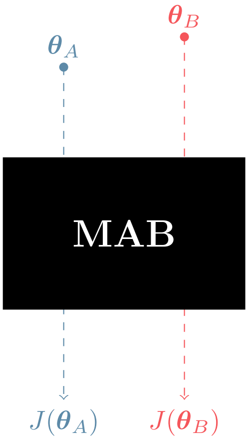

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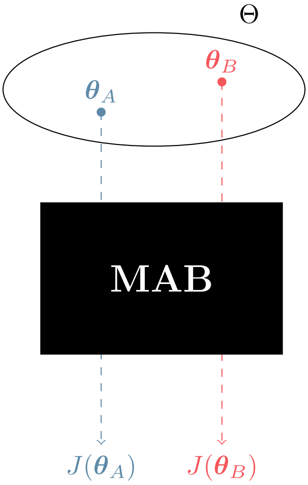
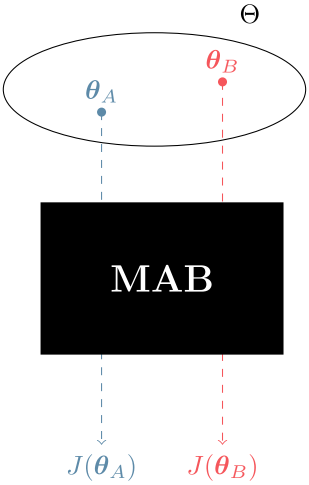


Exploiting Arm Correlation



- Arms correlate through overlapping trajectory distributions
- Use **Importance Sampling (IS)** to transfer information

$$J(\boldsymbol{\theta}_B) = \mathop{\mathbb{E}}_{\tau \sim p_{\boldsymbol{\theta}_A}} \left[\frac{p_{\boldsymbol{\theta}_B}(\tau)}{p_{\boldsymbol{\theta}_A}(\tau)} R(\tau) \right]$$

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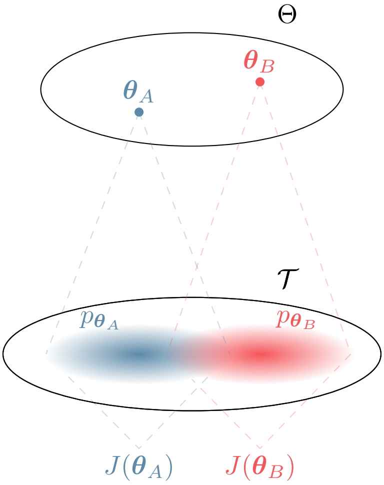

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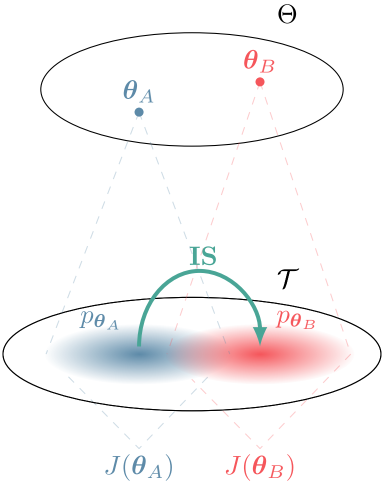

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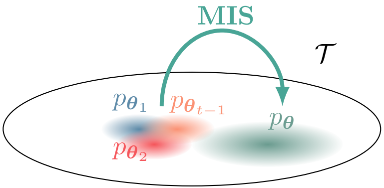
The OPTIMIST index [Papini et al., 2019]

- A **UCB-like** index:

$$\boldsymbol{\theta}_t = \arg\max_{\boldsymbol{\theta}\in\Theta} \underbrace{\check{J}_t(\boldsymbol{\theta})}_{\textbf{ESTIMATE}}$$

a **robust multiple**
importance sampling estimator



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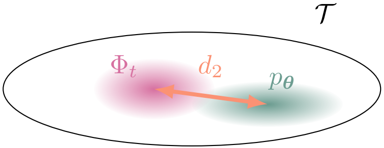
a **robust multiple** importance sampling estimator

distributional distance from previous solutions



Robust Multiple Importance Sampling Estimator

- Use **Multiple** Importance Sampling (MIS) [Veach and Guibas, 1995] to reuse *all* past experience

- Use **dynamic truncation** to prevent **heavy-tails** [Bubeck et al., 2013, Metelli et al., 2018]

$$\widehat{J}_t(\boldsymbol{\theta}) = \frac{1}{t}\sum_{k=0}^{t-1} \underbrace{\frac{p_{\boldsymbol{\theta}}(\tau_k)}{\Phi_t(\tau_k)}}_{\textbf{MIS weight}} R(\tau_k), \qquad \underbrace{\Phi_t(\tau) = \frac{1}{t}\sum_{k=0}^{t-1} p_{\boldsymbol{\theta}_k}(\tau)}_{\textbf{mixture}}$$

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Exploration Bonus

- Measure novelty with the *exponentiated* **Rényi divergence** [Cortes et al., 2010, Metelli et al., 2018]

$$d_2(p_{\boldsymbol{\theta}} \| \Phi_t) = \int \left(\frac{\mathrm{d}p_{\boldsymbol{\theta}}}{\mathrm{d}\Phi_t} \right)^2 \mathrm{d}\Phi_t$$

- Used to **upper bound** the true value (OFU):

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Sublinear Regret

- $Regret(T) = \sum_{t=0}^{T} J(\boldsymbol{\theta}^{*}) - J(\boldsymbol{\theta}_t)$
- **Compact**, d-dimensional parameter space Θ
- Under **mild assumptions** on the policy class, with high probability:

$$Regret(T) = \widetilde{\mathcal{O}}\left(\sqrt{dT}\right)$$

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- Easy implementation only for *parameter-based exploration* Sehnke et al. [2008]
- Difficult index optimization $\implies$ **discretization**
- Computational time can be traded-off with regret

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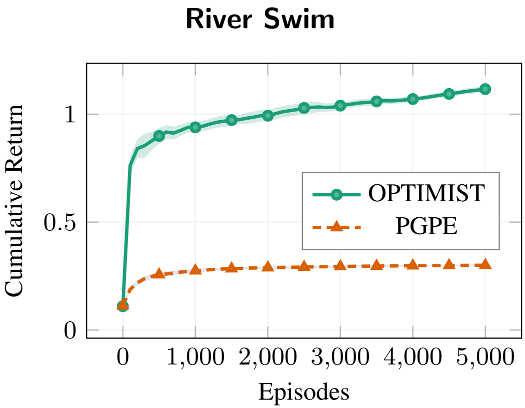

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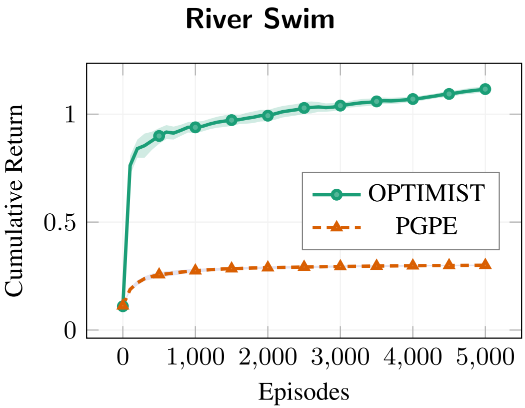

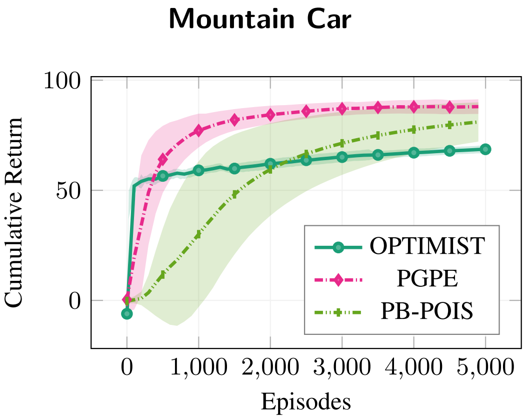

Future Work

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- Improve index optimization
- Posterior sampling [Thompson, 1933]

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- Outcome space $\mathcal{Z}$
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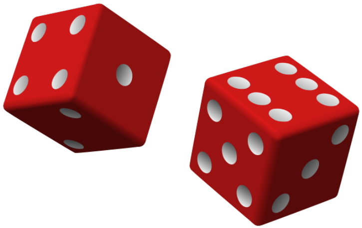

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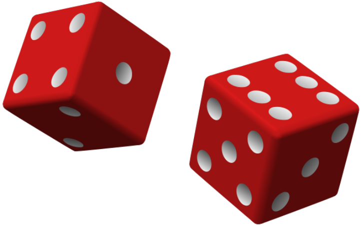

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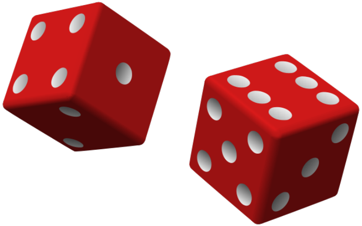

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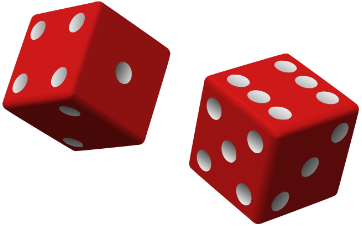

Thank you for your attention!

Papini, Matteo, Alberto Maria Metelli, Lorenzo Lupo, and Marcello Restelli. "Optimistic Policy Optimization via Multiple Importance Sampling." In International Conference on Machine Learning, pp. 4989-4999. 2019.

Code: `github.com/WolfLo/optimist`

Contact: matteo.papini@polimi.it

Web page: `t3p.github.io/icml19`

References

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