Optimistic Policy Optimization via Multiple Importance Sampling

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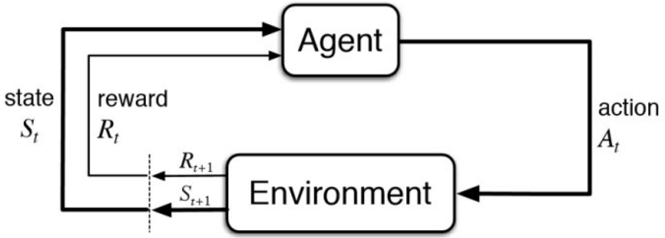
Reinforcement Learning [Sutton and Barto, 2018]



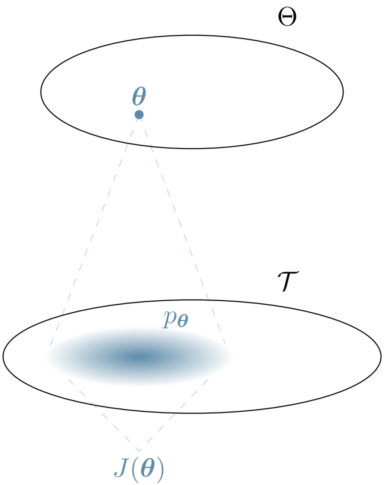
- Policy $\pi : \mathcal{S} \rightarrow \Delta(\mathcal{A})$
- Trajectories $\tau = (s_0, a_0, r_1, s_1, \dots)$
- Return $R(\tau) = \sum_{t=0}^{T-1} \gamma^t r_{t+1}$
- Goal: $\max_{\pi} \mathbb{E}_{\pi} [R(\tau)]$

Exploration vs Exploitation

Continuous Reinforcement Learning

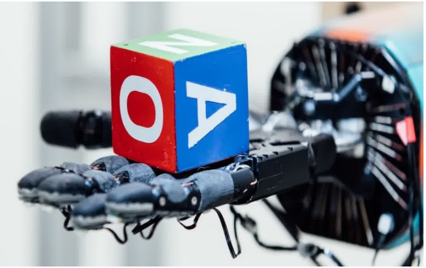
Policy Optimization

- **Parameter space** $\Theta \subseteq \mathbb{R}^d$
- A **parametric policy** $\pi_{\boldsymbol{\theta}}$ for each $\boldsymbol{\theta} \in \Theta$
- Each inducing a distribution $p_{\boldsymbol{\theta}}$ over **trajectories**
- Goal: $\max_{\boldsymbol{\theta} \in \Theta} J(\boldsymbol{\theta}) = \mathbb{E}_{\tau \sim p_{\boldsymbol{\theta}}} [R(\tau)]$



- **Gradient ascent** on $J(\boldsymbol{\theta})$
- Popular algorithms: **REINFORCE** [Williams, 1992], **PGPE** [Sehnke et al., 2008], **TRPO** [Schulman et al., 2015], **PPO** [Schulman et al., 2017]

Dota 2 [OpenAI, 2018]



Manipulation [Andrychowicz et al., 2018]

Exploration in Policy Optimization

- Policy Gradient fails with **sparse rewards** [Kakade and Langford, 2002]

- Non-convex objective $\implies$ **local minima**

Entropy bonus [Haarnoja et al., 2018]:

- *Undirected*
- **Unsafe**
- Little theoretical understanding [Ahmed et al., 2018]

Multi Armed Bandit (MAB)

- Arms $a \in \mathcal{A}$
- Expected payoff $\mu(a)$
- Goal: $\min \textit{Regret}(T) = \sum_{t=1}^{T} [\mu(a^*) - \mu(a_t)]$
- Wide literature on **directed exploration** [Bubeck et al., 2012, Lattimore and Szepesvári, 2019]

Optimism in Face of Uncertainty

- OFU strategy (e.g., UCB [Lai and Robbins, 1985]):

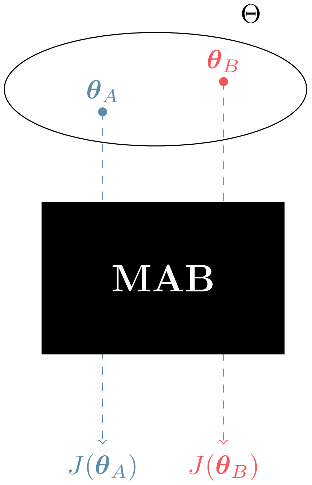
$$a_t = \arg\max_{a \in \mathcal{A}} \underbrace{\widehat{\mu}(a)}_{\textbf{ESTIMATE}} + \underbrace{C\sqrt{\frac{\log(\frac{1}{\delta})}{\#a}}}_{\textbf{EXPLORATION BONUS}}$$

- Idea: be **optimistic** about unknown arms
- Can be applied to RL (e.g., Jaksch et al. [2010])

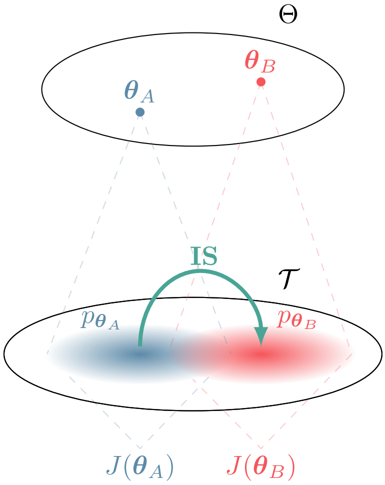
Policy Optimization as a MAB

- **Arms:** parameters $\boldsymbol{\theta}$
- **Payoff:** expected return $J(\boldsymbol{\theta})$
- **Continuous MAB**: we *need* structure [Kleinberg et al., 2013]

$$\boldsymbol{\theta}_t = \arg\max_{\boldsymbol{\theta}\in\Theta} \widehat{J}(\boldsymbol{\theta}_t) + C\sqrt{\frac{\log(\frac{1}{\delta})}{\#\boldsymbol{\theta}}}$$



Exploiting Arm Correlation



- Arms correlate through overlapping trajectory distributions
- Use **Importance Sampling (IS)** to transfer information

$$J(\boldsymbol{\theta}_B) = \mathop{\mathbb{E}}_{\tau \sim p_{\boldsymbol{\theta}_A}} \left[\frac{p_{\boldsymbol{\theta}_B}(\tau)}{p_{\boldsymbol{\theta}_A}(\tau)} R(\tau) \right]$$

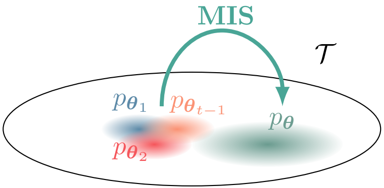
The OPTIMIST index [Papini et al., 2019]

- A **UCB-like** index:

$$\boldsymbol{\theta}_t = \arg\max_{\boldsymbol{\theta}\in\Theta} \underbrace{\check{J}_t(\boldsymbol{\theta})}_{\textbf{ESTIMATE}} + \underbrace{C\sqrt{\frac{d_2(p_{\boldsymbol{\theta}}\|\Phi_t)\log\frac{1}{\delta_t}}{t}}}_{\textbf{EXPLORATION BONUS:}}$$

a **robust multiple** importance sampling estimator

distributional distance from previous solutions



Robust Multiple Importance Sampling Estimator

- Use **Multiple** Importance Sampling (MIS) [Veach and Guibas, 1995] to reuse *all* past experience

- Use **dynamic truncation** to prevent **heavy-tails** [Bubeck et al., 2013, Metelli et al., 2018]

$$\widehat{J}_t(\boldsymbol{\theta}) = \frac{1}{t}\sum_{k=0}^{t-1} \underbrace{\frac{p_{\boldsymbol{\theta}}(\tau_k)}{\Phi_t(\tau_k)}}_{\textbf{MIS weight}} R(\tau_k), \qquad \underbrace{\Phi_t(\tau) = \frac{1}{t}\sum_{k=0}^{t-1} p_{\boldsymbol{\theta}_k}(\tau)}_{\textbf{mixture}}$$

Robust Multiple Importance Sampling Estimator

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$$\check{J}_t(\boldsymbol{\theta}) = \frac{1}{t} \sum_{k=0}^{t-1} \min \left\{ M_t, \frac{p_{\boldsymbol{\theta}}(\tau_k)}{\Phi_t(\tau_k)} \right\} R(\tau_k), \qquad \underbrace{M_t = \sqrt{\frac{t d_2(p_{\boldsymbol{\theta}} \| \Phi_t)}{\log(1/\delta_t)}}}_{\textbf{threshold}}$$

Exploration Bonus

- Measure novelty with the *exponentiated* **Rényi divergence** [Cortes et al., 2010, Metelli et al., 2018]

$$d_2(p_{\boldsymbol{\theta}} \| \Phi_t) = \int \left(\frac{\mathrm{d}p_{\boldsymbol{\theta}}}{\mathrm{d}\Phi_t} \right)^2 \mathrm{d}\Phi_t$$

- Used to **upper bound** the true value (OFU):

$$J(\boldsymbol{\theta}) \quad \leqslant \quad \check{J}_t(\boldsymbol{\theta}) \quad + \quad C\sqrt{\frac{d_2(p_{\boldsymbol{\theta}} \| \Phi_t) \log \frac{1}{\delta_t}}{t}}$$ with high probability

Sublinear Regret

- $Regret(T) = \sum_{t=0}^{T} J(\boldsymbol{\theta}^*) - J(\boldsymbol{\theta}_t)$
- **Compact**, d-dimensional parameter space Θ
- Under **mild assumptions** on the policy class, with high probability:

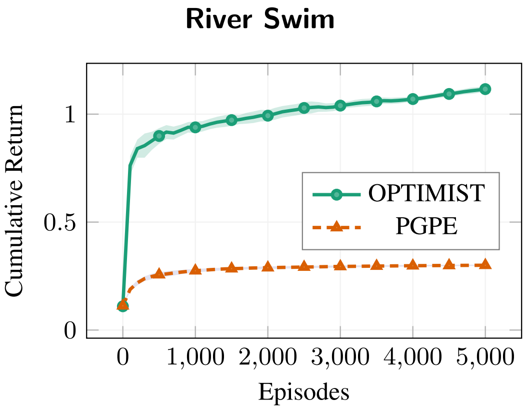
$$Regret(T) = \tilde{\mathcal{O}}\left(\sqrt{dT}\right)$$

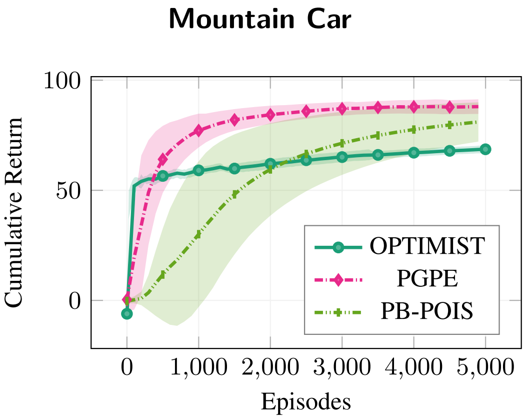
OPTIMIST in Practice

- Easy implementation only for *parameter-based exploration* Sehnke et al. [2008]
- Difficult index optimization $\implies$ **discretization**
- Computational time can be traded-off with regret

$$\widetilde{\mathcal{O}}\left(dT^{(1-\epsilon/d)}\right) \text{ regret } \implies \mathcal{O}\left(t^{(1+\epsilon)}\right) \text{ time}$$

Empirical Results



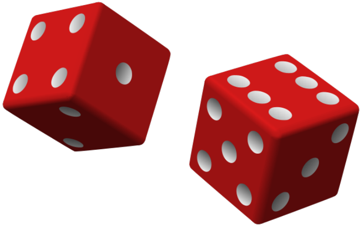


Future Work

- Extend to action-based exploration
- Improve index optimization
- Posterior sampling [Thompson, 1933]

The Abstract Problem [Chu et al., 2019]

- Outcome space $\mathcal{Z}$
- Decision set $\mathcal{P} \subseteq \Delta(\mathcal{Z})$
- Payoff $f : \mathcal{Z} \rightarrow \mathbb{R}$
- $\max_{p \in \mathcal{P}} \mathbb{E}_{z \sim p} \left[f(z) \right]$



Thank you for your attention!

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Web page: `t3p.github.io/icml19`

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