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SAFELY EXPLORING POLICY GRADIENT

MATTEO PAPINI, ANDREA BATTISTELLO AND MARCELLO RESTELLI

{matteo.papini, andrea.battistello, marcello.restelli}@polimi.it



PROBLEM

- **Reinforcement Learning** for **continuous** control [Deisenroth et al., 2013]
- **Policy Gradient (PG)**: iteratively update **parametric** policy π_θ via **gradient ascent** on performance $J(\theta)$ (expected cumulative reward):

$$\theta^{t+1} \leftarrow \theta^t + \alpha \nabla J(\theta^t)$$

- Convergence to local optimum guaranteed
- **Intermediate policies may be arbitrarily bad!**
- **Safe Exploration** [Amodei et al., 2016]: limit risks/costs of novel behavior

MOTIVATION

- A working controller θ^0 is provided
- **Fine tuning**: improve it online via policy gradient
- Intermediate policies should never be worse than the initial θ^0 (baseline)
- *Economic safety*: losses and gains cancel out

STATE OF THE ART

Existing **safe PG** approaches [Pirrotta et al., 2013, Papini et al., 2017]:

- Apply **only** to **Gaussian** policies with **fixed variance**:

$$\pi_\theta(a|s) \sim \mathcal{N}(\mu_\theta(s), \sigma^2)$$

⚠ The variance parameter regulates exploration and has a big impact on convergence speed

- Focus on **monotonic improvement** guarantees:

$$J(\theta^{t+1}) - J(\theta^t) \geq 0$$

⚠ Too strict for most practical scenarios

CONTRIBUTIONS

- We adopt a more general definition of safety
- We extend the existing guarantees for Gaussian policies to the **adaptive-variance** case
- We introduce a **surrogate objective** for variance updates that explicitly encourages **exploration**
- We provide an algorithm (**SEPG**) for the **fine-tuning** scenario

SETTING

- *Shallow* Gaussian policy parametrization:

$$\pi_\theta(a|s) = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp \left\{ -\frac{1}{2} \left(\frac{a - \mu_\theta(s)}{\sigma_\theta} \right)^2 \right\} \quad \begin{matrix} \mu_\theta(s) = \mathbf{v}^T \phi(s) \\ \sigma_\theta = e^w \end{matrix} \quad \theta = \begin{cases} \mathbf{v} \text{ mean parameter} \\ w \text{ variance parameter} \end{cases}$$

- **Safety requirement** (similar to Thomas et al. [2015]):

$$J(\theta^{t+1}) - J(\theta^t) \geq C^t \quad \text{with probability at least } 1 - \delta$$

$$C^t \geq 0 : \quad \text{required improvement}$$

$$C^t < 0 : \quad \text{bounded worsening}$$

- Base algorithm: REINFORCE with separate mean and variance updates

$$\begin{cases} \mathbf{v}^{t+1} \leftarrow \mathbf{v}^t + \alpha \nabla_{\mathbf{v}} J(\mathbf{v}^t, w^t) \\ w^{t+1} \leftarrow w^t + \eta \nabla_w J(\mathbf{v}^{t+1}, w^t) \end{cases} \quad \text{naive update: } \text{too greedy!}$$

- **Adaptive PG**: we look for the *largest* step sizes guaranteeing safety at each iteration

SAFE-EXPLORATORY UPDATES

We introduce a surrogate **exploration objective** that accounts for long-term advantages of high policy variance:

$$\mathcal{L}(\mathbf{v}, w) = \frac{\|\nabla_{\mathbf{v}} J(\mathbf{v}, w)\|_2^2}{4mc_w} \quad c_w = \frac{\overbrace{\max \text{ reward}}^R \overbrace{\max \text{ feature}}^{M^2}}{(1-\gamma)^2 e^{2w}} \left(\frac{\overbrace{\text{volume of action space}}^{|\mathcal{A}|}}{\sqrt{2\pi} e^w} + \frac{\overbrace{\text{discount factor}}^{\gamma}}{2(1-\gamma)} \right)$$

Largest safe performance improvement obtainable by updating the mean parameter \mathbf{v} when $\sigma = e^w$

We provide a **safe** way to update the variance parameter according to this objective:

$$\begin{cases} \mathbf{v}^{t+1} \leftarrow \mathbf{v}^t + \bar{\alpha} \nabla_{\mathbf{v}} J(\mathbf{v}^t, w^t) \\ w^{t+1} \leftarrow w^t + \bar{\eta} \nabla_w \mathcal{L}(\mathbf{v}^{t+1}, w^t) \end{cases}$$

Largest safe step size $\bar{\alpha}$ from Papini et al. [2017]:

$$\bar{\alpha} = \frac{1}{2c_w} \left(1 + \sqrt{1 - \frac{4c_w C^t}{\|\nabla_{\mathbf{v}} J(\mathbf{v}^t, w^t)\|_\infty^2}} \right)$$

Largest safe-exploratory step size $\bar{\eta}$

$$\eta^* = \frac{\nabla_w J(\mathbf{v}^{t+1}, w^t)}{2d \nabla_w \mathcal{L}(\mathbf{v}^{t+1}, w^t)} \quad (\text{corresponds to naive update})$$

$$\bar{\eta} = \eta^* + |\eta^*| \sqrt{1 - \frac{4dC^t}{\|\nabla_w J(\mathbf{v}^{t+1}, w^t)\|_\infty^2}}$$

$$d = \frac{R}{(1-\gamma)^2} \left(\frac{2(\sqrt{7}-2)e^{\sqrt{7}/2-2}|\mathcal{A}|}{\sqrt{2\pi}e^w} + \frac{\gamma}{1-\gamma} \right)$$

- Policy gradient $\nabla_w J$ (greedy) and surrogate $\nabla_w \mathcal{L}$ (explorative) typically *disagree* $\Rightarrow \eta^*$ typically negative

- Largest safe step size $\bar{\eta}$ can be positive (*exploration is allowed*) or negative (*greediness is required*) according to **safety constraint C^t**

APPROXIMATE FRAMEWORK

- In practice, gradients $\nabla_{\mathbf{v}} J, \nabla_w J, \nabla_w \mathcal{L}$ must be estimated from batches of N trajectories
- We characterize estimation error $\epsilon = \left| \widehat{\nabla}_N J - \nabla J \right|$ using known statistical inequalities
- We obtain corrected step sizes with **high-probability** safety guarantees

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FINE TUNING

- We define the **exploration budget**

$$B^t := J(\theta^t) - J(\theta^0)$$

- "Never do worse than the initial policy" is equivalent to

$$J(\theta^{t+1}) - J(\theta) \geq -B^t$$

- **SEPG** algorithm

1. Keep track of the budget:

$$B^0 \leftarrow 0$$

$$B^{t+1} \leftarrow B^t + J(\theta^{t+1}) - J(\theta^t)$$

2. Update \mathbf{v} and w alternately

3. Select $\bar{\alpha}$ and $\bar{\eta}$ according to $C^t = -B^t$

- **SSEPG** algorithm (*heuristic* variant):

+ Provide initial budget $B^0 > 0$ to encourage initial exploration

+ Test the corresponding **deterministic** policy ($\sigma = 0$) at each iteration to capture long-term advantages

EXPERIMENTS

