Stochastic Variance-Reduced Policy Gradient

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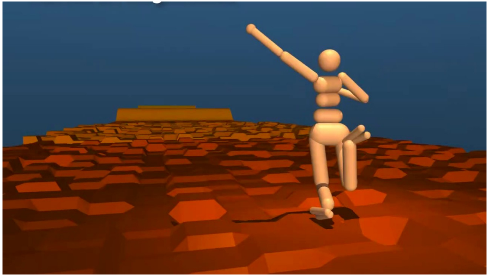
Policy Gradient

An effective **Reinforcement Learning (RL)** solution to **continuous** control problems:



Robotics (Heess et al., 2017)

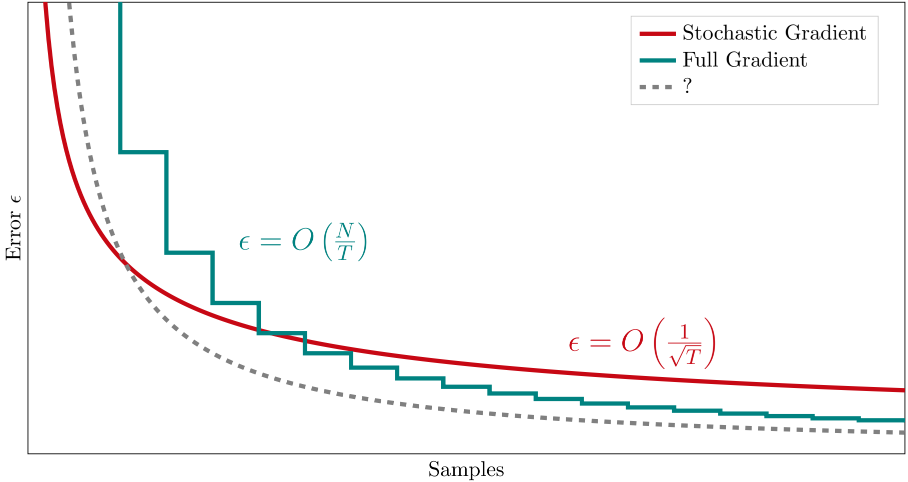
Video games (OpenAI, 2018)

Mostly based on **Stochastic Gradient Ascent** (Robbins and Monro, 1951)

maximize $J(\boldsymbol{\theta})$ by iterating $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \widehat{\nabla} J(\boldsymbol{\theta})$

Full vs Stochastic Gradient

Non-convex optimization



Can we do something better?

Visualization idea from Bach (2016)

SVRG (Johnson and Zhang, 2013)

Stochastic Variance-Reduced Gradient

A solution from **finite-sum optimization**:

$$\max_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \sum_{i=1}^{N} f_i(\boldsymbol{\theta})$$

epoch

iteration

$$\underbrace{\blacktriangledown J(\boldsymbol{\theta})}_{\text{SVRG estimator}} = \underbrace{\nabla J(\widetilde{\boldsymbol{\theta}})}_{\text{FG (\textit{snapshot})}} + \underbrace{\nabla f_i(\boldsymbol{\theta})}_{\text{SG in current parameter}} - \underbrace{\nabla f_i(\widetilde{\boldsymbol{\theta}})}_{\text{Correction term}}$$

- Unbiased
- Linear convergence
- More data-efficient than FG
- **Supervised Learning (SL)**

SVRG for Reinforcement Learning

In **Reinforcement Learing (RL)** we maximize *expected return*:

$$\max_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \int p(\tau|\boldsymbol{\theta})R(\tau)d\tau \qquad \text{(Peters and Schaal, 2008)}$$

SVRG for RL so far:

- Du et al. (2017) apply SVRG to **policy evaluation**
- Xu et al. (2017) apply SVRG to **off-line control**

Our work: **on-policy control**

Nontrivial! There are three **challenges**:

1. **Non-concavity** of $J(\boldsymbol{\theta})$ (Allen-Zhu and Hazan, 2016; Reddi et al., 2016)
2. **Infinite dataset**: we would need *infinite samples* to compute FG (Harikandeh et al., 2015; Bietti and Mairal, 2017)
3. **Non-stationarity**: $\tau \sim p_{\boldsymbol{\theta}}$ (new!)

epoch

iteration

$$\underbrace{\blacktriangledown J(\boldsymbol{\theta})}_{\text{SVRPG estimator}} = \underbrace{\widehat{\nabla}_N J(\widetilde{\boldsymbol{\theta}})}_{\substack{\text{Large N} \\ \text{to approximate FG}}} + \underbrace{\widehat{\nabla}_B J(\boldsymbol{\theta})}_{B \ll N} - \underbrace{\omega(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}}) \widehat{\nabla}_B J(\widetilde{\boldsymbol{\theta}})}_{\substack{\text{Importance weighting} \\ \text{for non-stationarity}}}$$

- Unbiased
- More data-efficient than FG
- **On-policy**: only the correction term is weighted

Convergence Properties

Convergence to **local** optimum:

$$\mathbb{E}\left[\|\nabla J(\boldsymbol{\theta})\|^2\right] \leq \frac{J(\boldsymbol{\theta}^*) - J(\boldsymbol{\theta}_0)}{\psi T} + \underbrace{\frac{\zeta}{\mathbf{N}}}_{\text{Infinite dataset}} + \underbrace{\frac{\xi}{\mathbf{B}}}_{\text{Nonstationarity}}$$

- Linear convergence + **error** (similar to Harikandeh et al., 2015)
- ψ, ζ, ξ depend only on **step size** and **epoch size**

Heuristics

Meta-parameter selection

- **Adaptive step size**: two ADAM (Kingma and Ba, 2014) annealing schedules

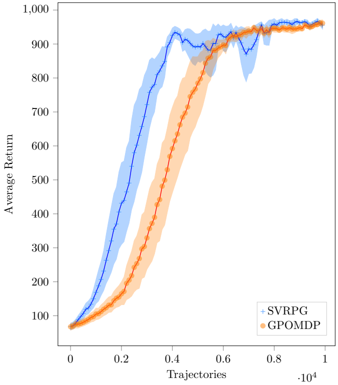
$$\underbrace{\alpha_{FG}}_{\text{used at the snapshot}} \qquad \underbrace{\alpha_{SG}}_{\text{used inside epoch}}$$

- **Adaptive epoch size**: new snapshot when effective step size becomes too small

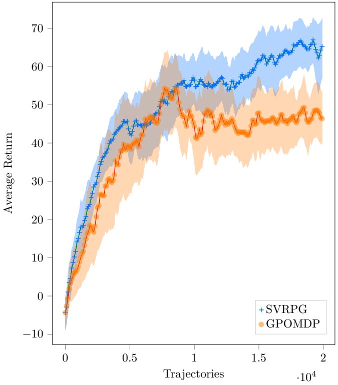
$$\frac{\alpha_{SG}}{B} < \frac{\alpha_{FG}}{N} \implies \textbf{snapshot}$$

Results

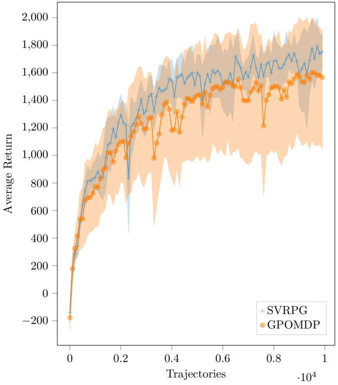
Cart-Pole



Swimmer



Half-Cheetah



SVRPG: $N = 100, B = 10$, ADAM
GPOMDP: $N = 10$, ADAM
Tasks from *rllab* (Duan et al., 2016)

Conclusions

- Efficient policy optimization is challenging
- **SVRPG**: on-policy control based on SVRG
- Meta-parameters still crucial to tame different sources of variance
- Future work: adaptive batch size, natural gradient, actor-critic

Thank you for your attention

- Poster: today 06:15 – 09:00 PM @ **Hall B #65**
- Contact: `matteo.papini@polimi.it`
- Online resources: `t3p.github.io`

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For $s = 1, \dots$

$$
\left.
\begin{array}{l}
\text{Sample } N \text{ trajectories using } \widetilde{\theta} \\
\text{Compute FG} = \widehat{\nabla}_N J(\widetilde{\theta}) \\
\textbf{For } t = 1, \dots, m \\
\quad \left.
\begin{array}{l}
\text{Sample } B \text{ trajectories using } \theta \\
\text{Compute SG} = \widehat{\nabla}_B J(\theta) \\
\text{Compute correction} = \omega(\theta, \widetilde{\theta}) \widehat{\nabla}_B J(\widetilde{\theta}) \\
\text{Update } \theta \leftarrow \theta + \alpha \blacktriangledown J(\theta)
\end{array}
\right\} \text{iteration} \\
\text{Update } \widetilde{\theta} \leftarrow \theta
\end{array}
\right\} \text{epoch}
$$

Adaptive Step Size

ADAM (Kingma and Ba, 2014):

- adapts to gradient variance
- can manage different batch sizes
- **has memory of past gradients (momentum)**

Problem: FG and SG have very different variance magnitudes

$\implies$ spurious momentum

We use two *separate* annealing schedules:

$$\widetilde{\boldsymbol{\theta}} \leftarrow \widetilde{\boldsymbol{\theta}} + \alpha_{FG} \widehat{\nabla}_N J(\widetilde{\boldsymbol{\theta}}) \quad \text{at the snapshot}$$

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha_{SG} \blacktriangledown J(\boldsymbol{\theta}) \quad \text{otherwise}$$

Note that $\widehat{\nabla}_N J(\widetilde{\boldsymbol{\theta}}) \equiv \blacktriangledown J(\boldsymbol{\theta})$ at the snapshot

Adaptive Epoch Size

Epoch size (m) trade-off:

- Large $m \implies$ large importance-weighting variance $\implies$ unstable
- Small $m \implies$ frequent snapshots $\implies$ data-inefficient

Idea: ADAM already relates gradient variance and efficiency

Our stopping criterion:

$$\frac{\alpha_{SG}}{B} < \frac{\alpha_{FG}}{N} \implies \textbf{snapshot}$$

When going on is not *convenient*, take new snapshot

Normalized Importance Weighting

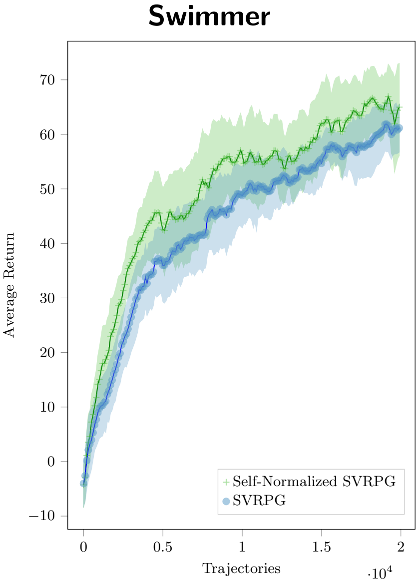
Regular importance weighting (unbiased):

$$\omega(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})\widehat{\nabla}_B J(\widetilde{\boldsymbol{\theta}}) = \frac{1}{B}\sum_{i=1}^{B} \frac{p(\tau_i|\widetilde{\boldsymbol{\theta}})}{p(\tau_i|\boldsymbol{\theta})} \nabla \log p(\tau_i|\widetilde{\boldsymbol{\theta}}) R(\tau_i)$$

Normalized importance weighting:

$$\omega(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})\widehat{\nabla}_B J(\widetilde{\boldsymbol{\theta}}) = \frac{\sum_{i=1}^{B} \frac{p(\tau_i|\widetilde{\boldsymbol{\theta}})}{p(\tau_i|\boldsymbol{\theta})} \nabla \log p(\tau_i|\widetilde{\boldsymbol{\theta}}) R(\tau_i)}{\sum_{i=1}^{B} \frac{p(\tau_i|\widetilde{\boldsymbol{\theta}})}{p(\tau_i|\boldsymbol{\theta})}}$$

- Less variance at the price of small bias
- Only affects the correction term
- Benefits are task-dependent



Critic (or *baseline*): an orthogonal variance-reduction technique

Gradient sample: $$\sum_{t=1}^{H}\left(\sum_{k=1}^{t}\nabla\log\pi_{\boldsymbol{\theta}}(a_t|s_t)\right)(\gamma^t r_t - \underbrace{\mathbf{b}}_{\text{baseline}})$$ (Peters and Schaal, 2008)

Not trivial to combine SVRG with critic: variance reduction is not additive

We combine SVRG with a simple critic from Duan et al. (2016)

Future work: ad hoc critic

The Full Story

- For **Swimmer**, we employ normalized weights in our final result
- For **Half-Cheetah**, we employ normaized weights *and* critic in our final result
- We compare **SVRPG** with GPOMDP (Baxter and Bartlett, 2001) with batch size $B = 10$
- This shows the *advantage* of correcting SG with more data
- However, GPOMDP with batch size $N = 100$ is even worse

Half-Cheetah