
# Optimistic Policy Optimization via Multiple Importance Sampling

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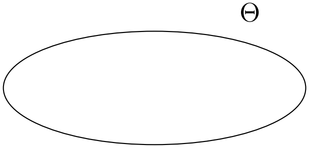
# Policy Optimization

- **Parameter space** $\Theta \subseteq \mathbb{R}^d$




- A parametric **policy** for each $\boldsymbol{\theta} \in \Theta$
- Each inducing a distribution $p_{\boldsymbol{\theta}}$ over **trajectories**
- A **return** $R(\tau)$ for every trajectory $\tau$
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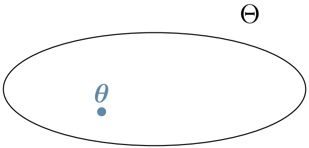

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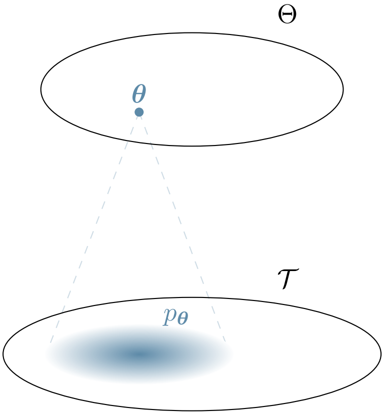

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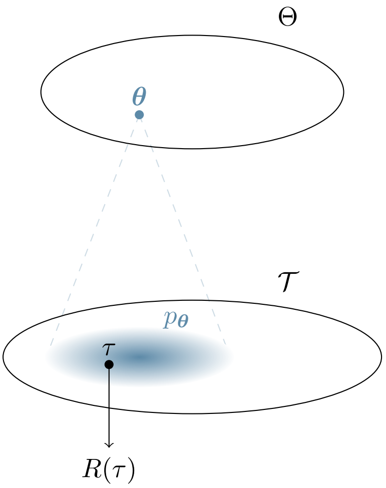

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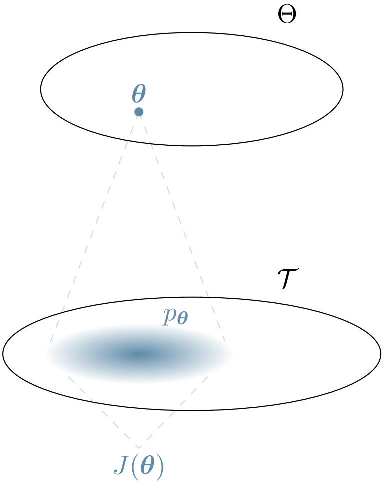

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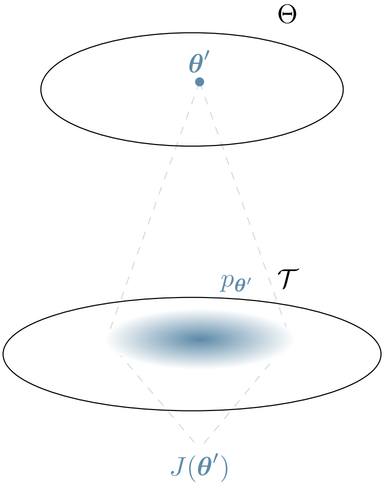

# Exploration in Policy Optimization



- **Continuous** decision process $\implies$ difficult
- Policy gradient methods tend to be **greedy** (e.g., TRPO [6], PGPE [7])
- Mainly **undirected** (e.g., entropy bonus [2])
- **Lack of theoretical guarantees**

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**If only this were a Correlated Multi-Armed Bandit...**

# Policy Optimization as a Correlated MAB

- **Arms:** parameters $\boldsymbol{\theta}$


- **Payoff:** expected return $J(\boldsymbol{\theta})$
- **Continuous MAB** [3]: we *need* structure
- **Arm correlation** [5] through trajectory distributions
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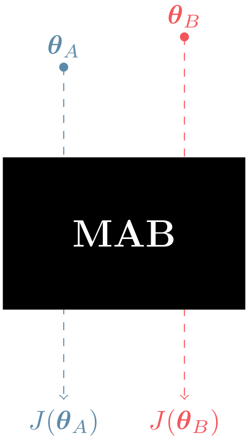

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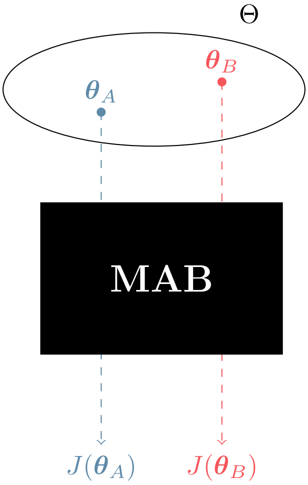

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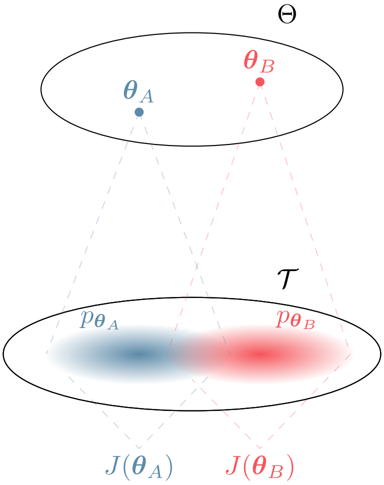

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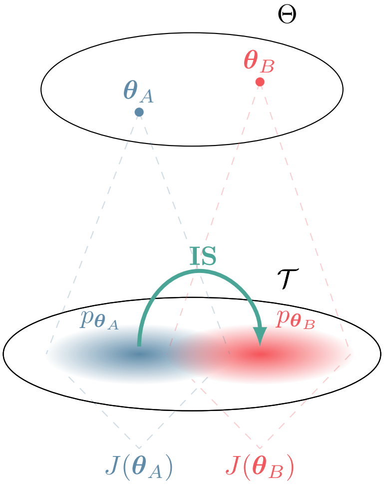
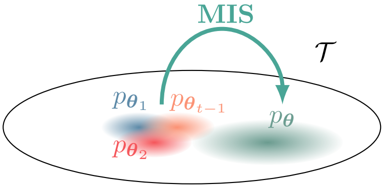


- A **UCB-like** index [4]:

$$B_t(\boldsymbol{\theta}) \quad = \quad \underbrace{\check{J}_t(\boldsymbol{\theta})}_{\textbf{ESTIMATE}}$$

a **truncated** **multiple**
importance sampling estimator [8, 1]

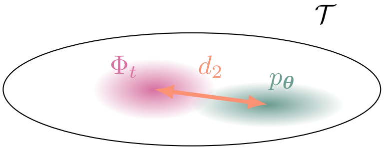


- A **UCB-like** index [4]:

$$B_t(\boldsymbol{\theta}) = \underbrace{\check{J}_t(\boldsymbol{\theta})}_{\textbf{ESTIMATE}} + \underbrace{C\sqrt{\frac{d_2(p_{\boldsymbol{\theta}} \| \Phi_t) \log \frac{1}{\delta_t}}{t}}}_{\textbf{EXPLORATION BONUS:}}$$

a **truncated multiple** importance sampling estimator [8, 1]

**distributional** distance from previous solutions



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- Select $\boldsymbol{\theta}_t = \arg\max_{\boldsymbol{\theta}\in\Theta} B_t(\boldsymbol{\theta})$

# Sublinear Regret

- $Regret(T) = \sum_{t=0}^{T} J(\boldsymbol{\theta}^*) - J(\boldsymbol{\theta}_t)$
- **Compact**, $d$-dimensional parameter space $\Theta$
- Under **mild assumptions** on the policy class, with high probability:

$$Regret(T) = \widetilde{\mathcal{O}}\left(\sqrt{dT}\right)$$

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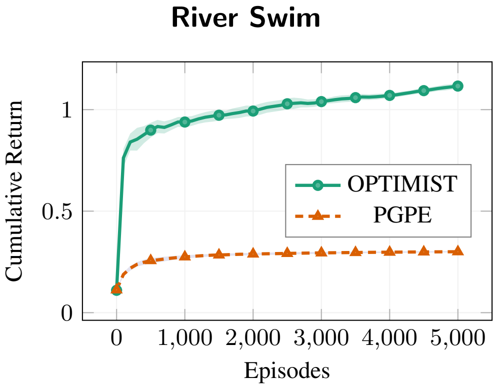
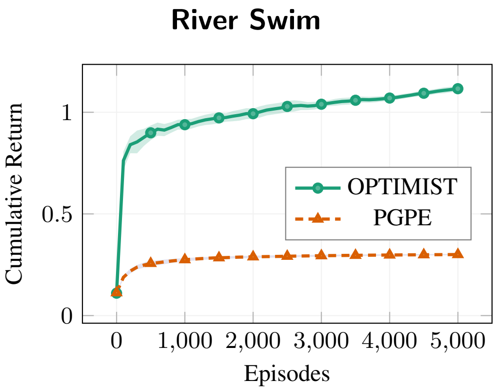
# Empirical Results






### Caveats

- Easy implementation only for parameter-based exploration [7]
- Difficult optimization $\implies$ discretization
- ...

# Thank You for Your Attention!

Poster **#103**

Code: `github.com/WolfLo/optimist`

Contact: matteo.papini@polimi.it

Web page: `t3p.github.io/icml19`

# References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