Stochastic Variance-Reduced Policy Gradient

Matteo Papini
Damiano Binaghi Giuseppe Canonaco
Matteo Pirotta Marcello Restelli

35th International Conference on Machine Learning, Stockholm, Sweden

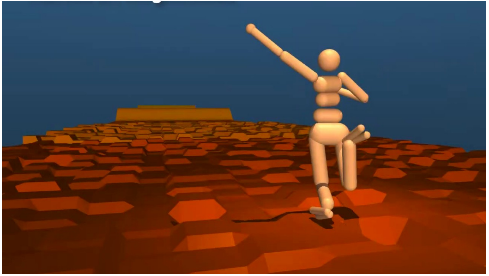
Outline

Stochastic Variance-Reduced (Policy) Gradient

- **SVRG** for Reinforcement Learning
  - Motivation
  - Challenges
- **SVRPG**
  - Convergence Properties
  - Heuristics
  - Experiments

Policy Gradient

An effective **Reinforcement Learning (RL)** solution to **continuous** control problems:



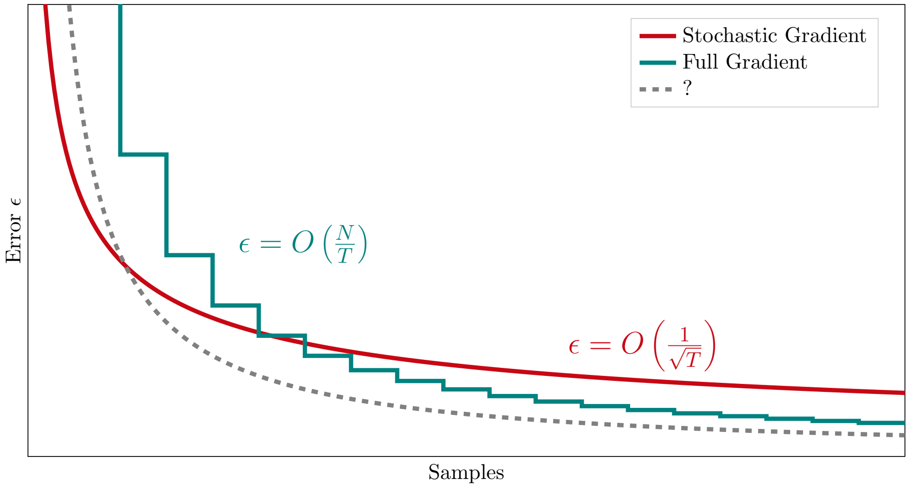
Robotics (Heess et al., 2017)

Video games (OpenAI, 2018)

Mostly based on **Stochastic Gradient Ascent** (Robbins and Monro, 1951)

Full vs Stochastic Gradient

Non-convex optimization



Can we do something better?

A solution from **finite-sum optimization** (Johnson and Zhang, 2013):

$$\underbrace{\blacktriangledown J(\boldsymbol{\theta})}_{\text{SVRG estimator}} = \underbrace{\nabla J(\widetilde{\boldsymbol{\theta}})}_{\text{FG in snapshot parameter}} + \underbrace{\nabla J(\boldsymbol{\theta})\big|_{\tau_i}}_{\text{SG in current parameter}} - \underbrace{\nabla J(\widetilde{\boldsymbol{\theta}})\big|_{\tau_i}}_{\text{Correction term}}$$

- Unbiased
- Linear convergence
- More data-efficient than FG
- Easily applicable to **Supervised Learning (SL)**

Not trivial! There are three **challenges**:

1. **Non-concavity** of $J(\boldsymbol{\theta})$ (Allen-Zhu and Hazan, 2016; Reddi et al., 2016)
2. **Infinite dataset**: we would need *infinite samples* to compute FG (Harikandeh et al., 2015; Bietti and Mairal, 2017)
3. **Non-stationarity**: $\tau \sim p_{\boldsymbol{\theta}}$ (new!)

RL so far: *policy evaluation* (Du et al., 2017) and *off-policy control* (Xu et al., 2017)

Our work: **on-policy control**

$$\underbrace{\blacktriangledown J(\boldsymbol{\theta})}_{\text{SVRPG estimator}} = \underbrace{\widehat{\nabla}_N J(\widetilde{\boldsymbol{\theta}})}_{\substack{\text{Large N} \\ \text{to approximate FG}}} + \underbrace{\widehat{\nabla}_B J(\boldsymbol{\theta})}_{B<<N} - \underbrace{\omega(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}}) \widehat{\nabla}_B J(\widetilde{\boldsymbol{\theta}})}_{\substack{\text{Importance weighting} \\ \text{for non-stationarity}}}$$

- Unbiased
- More data-efficient than FG
- **On-policy**: only the correction term is weighted

Convergence to **local** optimum:

$$\mathbb{E}\left[\|\nabla J(\boldsymbol{\theta})\|^2\right] \leq \frac{J(\boldsymbol{\theta}^*) - J(\boldsymbol{\theta}_0)}{\psi T} + \underbrace{\frac{\zeta}{N}}_{\text{Infinite dataset}} + \underbrace{\frac{\xi}{B}}_{\text{Nonstationarity}}$$

- Linear convergence + error (similar to Harikandeh et al., 2015)

Heuristics

Meta-parameter selection

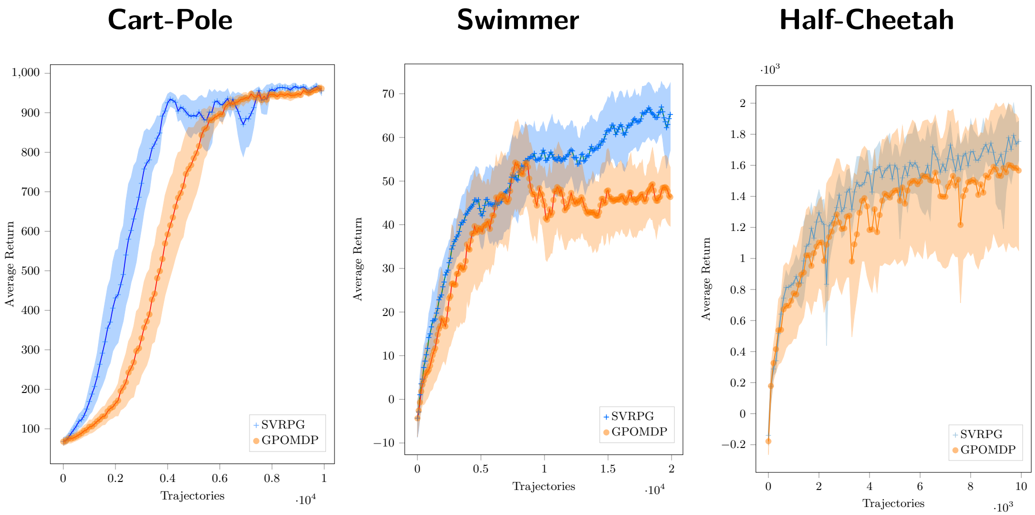
- **Adaptive step size**: two ADAM (Kingma and Ba, 2014) annealing schedules

$$\underbrace{\alpha_{FG}}_{\text{used at the snapshot}} \qquad \underbrace{\alpha_{SG}}_{\text{used inside epoch}}$$

- **Adaptive epoch size**: take new snapshot when the effective step size becomes too small

$$\frac{\alpha_{SG}}{B} < \frac{\alpha_{FG}}{N} \implies \textbf{snapshot}$$

Results


Cart-Pole
Swimmer
Half-Cheetah
Average Return
Trajectories
$\cdot 10^4$
$\cdot 10^3$
SVRPG
GPOMDP

Conclusions

- Efficient policy optimization is challenging
- **SVRPG**: on-policy control based on SVRG
- Meta-parameters still crucial to tame different sources of variance
- Future work: adaptive batch size, natural gradient, actor-critic

Thank You

Thank you for your attention

- Poster: today 06:15 – 09:00 PM @ **Hall B #65**
- Contact: `matteo.papini@polimi.it`
- Online resources: `t3p.github.io`

References

Allen-Zhu, Z. and Hazan, E. (2016). Variance reduction for faster non-convex optimization. In *International Conference on Machine Learning*, pages 699–707.

Baxter, J. and Bartlett, P. L. (2001). Infinite-horizon policy-gradient estimation. *Journal of Artificial Intelligence Research*, 15:319–350.

Bietti, A. and Mairal, J. (2017). Stochastic optimization with variance reduction for infinite datasets with finite sum structure. In *Advances in Neural Information Processing Systems*, pages 1622–1632.

Du, S. S., Chen, J., Li, L., Xiao, L., and Zhou, D. (2017). Stochastic variance reduction methods for policy evaluation. In *ICML*, volume 70 of *Proceedings of Machine Learning Research*, pages 1049–1058. PMLR.

Duan, Y., Chen, X., Houthooft, R., Schulman, J., and Abbeel, P. (2016). Benchmarking deep reinforcement learning for continuous control. In *International Conference on Machine Learning*, pages 1329–1338.

Harikandeh, R., Ahmed, M. O., Virani, A., Schmidt, M., Konečnỳ, J., and Sallinen, S. (2015). Stopwasting my gradients: Practical svrg. In *Advances in Neural Information Processing Systems*, pages 2251–2259.

Heess, N., Sriram, S., Lemmon, J., Merel, J., Wayne, G., Tassa, Y., Erez, T., Wang, Z., Eslami, A., Riedmiller, M., et al. (2017). Emergence of locomotion behaviours in rich environments. *arXiv preprint arXiv:1707.02286*.

Johnson, R. and Zhang, T. (2013). Accelerating stochastic gradient descent using predictive variance reduction. In *Advances in neural information processing systems*, pages 315–323.

Kingma, D. P. and Ba, J. (2014). Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*.

References (cont.)

OpenAI (2018). Openai five.

Peters, J. and Schaal, S. (2008). Reinforcement learning of motor skills with policy gradients. *Neural networks*, 21(4):682–697.

Reddi, S. J., Hefny, A., Sra, S., Poczos, B., and Smola, A. (2016). Stochastic variance reduction for nonconvex optimization. In *International conference on machine learning*, pages 314–323.

Robbins, H. and Monro, S. (1951). A stochastic approximation method. *The annals of mathematical statistics*, pages 400–407.

Xu, T., Liu, Q., and Peng, J. (2017). Stochastic variance reduction for policy gradient estimation. *CoRR*, abs/1710.06034.

SVRPG

Pseudo-code

For $s = 1, \dots$

Sample N trajectories using $\widetilde{\theta}$

Compute FG $= \widehat{\nabla}_N J(\widetilde{\theta})$

For $t = 1, \dots, m$

Sample B trajectories using θ

Compute SG $= \widehat{\nabla}_B J(\theta)$

Compute correction $= \omega(\theta, \widetilde{\theta}) \widehat{\nabla}_B J(\widetilde{\theta})$

Update $\theta \leftarrow \theta + \alpha \blacktriangledown J(\theta)$

iteration

Update $\widetilde{\theta} \leftarrow \theta$

epoch

Adaptive Step Size

ADAM (Kingma and Ba, 2014):

- adapts to gradient variance
- can manage different batch sizes
- **has memory of past gradients (momentum)**

Problem: FG and SG updates have very different variance magnitudes
$\implies$ spurious momentum

We use two *separate* annealing schedules:

$$\widetilde{\boldsymbol{\theta}} \leftarrow \widetilde{\boldsymbol{\theta}} + \alpha_{FG} \widehat{\nabla}_N J(\widetilde{\boldsymbol{\theta}}) \quad \text{at the snapshot}$$
$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha_{SG} \blacktriangledown J(\boldsymbol{\theta}) \quad \text{otherwise}$$

Note that $\widehat{\nabla}_N J(\widetilde{\boldsymbol{\theta}}) \equiv \blacktriangledown J(\boldsymbol{\theta})$ at the snapshot

Adaptive Epoch Size

Epoch size m trade-off:

- Large $m \implies$ policy very far from snapshot $\implies$ importance weighting introduces more variance than it reduces $\implies$ instability
- Small $m \implies$ frequent snapshot $\implies$ data-inefficient

Idea: ADAM already relates gradient variance and efficiency

Our stopping criterion:

$$\frac{\alpha_{SG}}{B} < \frac{\alpha_{FG}}{N} \implies \textbf{snapshot}$$

When going on with the current snapshot is not *convenient*, take a new one

Normalized Importance Weighting

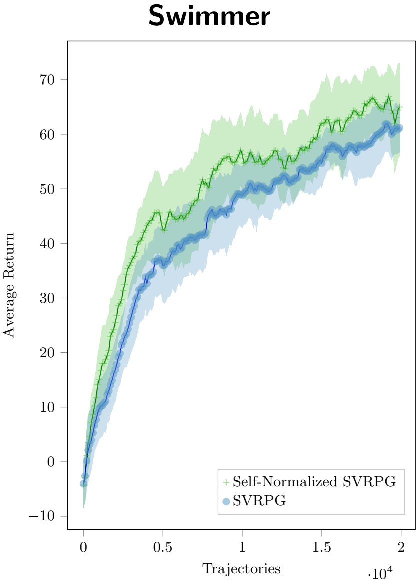
Regular importance weighting (unbiased):

$$\omega(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})\widehat{\nabla}_B J(\widetilde{\boldsymbol{\theta}}) = \frac{1}{B}\sum_{i=1}^{B} \frac{p(\tau_i|\widetilde{\boldsymbol{\theta}})}{p(\tau_i|\boldsymbol{\theta})} g(\tau_i|\widetilde{\boldsymbol{\theta}})$$

Normalized importance weighting:

$$\omega(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})\widehat{\nabla}_B J(\widetilde{\boldsymbol{\theta}}) = \frac{\sum_{i=1}^{B} \frac{p(\tau_i|\widetilde{\boldsymbol{\theta}})}{p(\tau_i|\boldsymbol{\theta})} g(\tau_i|\widetilde{\boldsymbol{\theta}})}{\sum_{i=1}^{B} \frac{p(\tau_i|\widetilde{\boldsymbol{\theta}})}{p(\tau_i|\boldsymbol{\theta})}}$$

- Reduces variance at the price of introducing a small bias
- Only affects the correction term
- Effectiveness is task-dependent



Critic (or *baseline*): an orthogonal variance-reduction technique

$$g(\tau_i|\boldsymbol{\theta}) = \sum_{t=1}^{H} \left(\sum_{k=1}^{t} \nabla \log \pi_{\boldsymbol{\theta}}(a_t|s_t) \right) (\gamma^t r_t - \underbrace{\mathbf{b}}_{\text{baseline}})$$ (Peters and Schaal, 2008)

Not trivial to combine SVRG with critic: variance reductions are not additive

We combine SVRG with a simple critic suggested in Duan et al. (2016) on the **Half-Cheetah** task

Ad-hoc critic left for future work

The Full Story

- For Swimmer, we employ normalized weights in our final result
- For Half-Cheetah, we employ both normaized weights and critic in our final result
- We compare SVRPG with GPOMDP (Baxter and Bartlett, 2001) with batch size $= B$
- This is a natural comparison since we want to evaluate the *convenience* of correcting the SG update
- However, GPOMDP with batch size $= N$ is even worse

Half-Cheetah