สูตรสำหรับสอบกลางภาค วิชาสนามแม่เหล็กไฟฟ้า 1
ปีการศึกษา 2568

ขนาดของเวกเตอร์

$$\mathbf{B} = B_x\hat{\mathbf{a}}_x + B_y\hat{\mathbf{a}}_y + B_z\hat{\mathbf{a}}_z,\ |\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2},\ \hat{\mathbf{a}}_B = \frac{\mathbf{B}}{|\mathbf{B}|}$$

ผลคูณจุด (dot product)

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB}$$

$$\mathbf{A} = A_x\hat{\mathbf{a}}_x + A_y\hat{\mathbf{a}}_y + A_z\hat{\mathbf{a}}_z,\ \mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z,\ \mathbf{A}\cdot\mathbf{A} = |\mathbf{A}|^2$$

ผลคูณไขว้ (cross product)

$$\mathbf{A}\times\mathbf{B} = \hat{\mathbf{a}}_N|\mathbf{A}||\mathbf{B}|\sin\theta_{AB}$$

$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\hat{\mathbf{a}}_x + (A_zB_x - A_xB_z)\hat{\mathbf{a}}_y + (A_xB_y - A_yB_x)\hat{\mathbf{a}}_z$$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{a}}_x & \hat{\mathbf{a}}_y & \hat{\mathbf{a}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

ระบบพิกัดทรงกระบอก

$$x = \rho\cos\phi,\ y = \rho\sin\phi,\ z = z,\ \rho = \sqrt{x^2 + y^2},\ \phi = \tan^{-1}\frac{y}{x}$$

$$\begin{bmatrix} \hat{\mathbf{a}}_\rho \\ \hat{\mathbf{a}}_\phi \\ \hat{\mathbf{a}}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}}_x \\ \hat{\mathbf{a}}_y \\ \hat{\mathbf{a}}_z \end{bmatrix},\ \begin{bmatrix} \hat{\mathbf{a}}_\rho \\ \hat{\mathbf{a}}_\phi \\ \hat{\mathbf{a}}_z \end{bmatrix} \equiv [A]_{rc} \begin{bmatrix} \hat{\mathbf{a}}_x \\ \hat{\mathbf{a}}_y \\ \hat{\mathbf{a}}_z \end{bmatrix}$$

$$\begin{bmatrix} \hat{\mathbf{a}}_x \\ \hat{\mathbf{a}}_y \\ \hat{\mathbf{a}}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}}_\rho \\ \hat{\mathbf{a}}_\phi \\ \hat{\mathbf{a}}_z \end{bmatrix},\ \begin{bmatrix} \hat{\mathbf{a}}_x \\ \hat{\mathbf{a}}_y \\ \hat{\mathbf{a}}_z \end{bmatrix} \equiv [A]_{cr} \begin{bmatrix} \hat{\mathbf{a}}_\rho \\ \hat{\mathbf{a}}_\phi \\ \hat{\mathbf{a}}_z \end{bmatrix}$$

$$[A]_{rc}^{-1} = [A]_{rc}^T,\ [A]_{cr} = [A]_{rc}^T$$

$$\mathbf{A} = A_x\hat{\mathbf{a}}_x + A_y\hat{\mathbf{a}}_y + A_z\hat{\mathbf{a}}_z,\ \mathbf{A} = A_\rho\hat{\mathbf{a}}_\rho + A_\phi\hat{\mathbf{a}}_\phi + A_z\hat{\mathbf{a}}_z$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix},\ \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \equiv [A]_{rc} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}, \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \equiv [A]_{cr} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

ระบบพิกัดทรงกลม

$$x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2},\ \theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}},\ \phi = \tan^{-1}\frac{y}{x}$$

$$\begin{bmatrix} \hat{\mathbf{a}}_r \\ \hat{\mathbf{a}}_\theta \\ \hat{\mathbf{a}}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}}_x \\ \hat{\mathbf{a}}_y \\ \hat{\mathbf{a}}_z \end{bmatrix}, \begin{bmatrix} \hat{\mathbf{a}}_r \\ \hat{\mathbf{a}}_\theta \\ \hat{\mathbf{a}}_\phi \end{bmatrix} \equiv [A]_{rs} \begin{bmatrix} \hat{\mathbf{a}}_x \\ \hat{\mathbf{a}}_y \\ \hat{\mathbf{a}}_z \end{bmatrix}$$

$$\begin{bmatrix} \hat{\mathbf{a}}_x \\ \hat{\mathbf{a}}_y \\ \hat{\mathbf{a}}_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}}_r \\ \hat{\mathbf{a}}_\theta \\ \hat{\mathbf{a}}_\phi \end{bmatrix}, \begin{bmatrix} \hat{\mathbf{a}}_x \\ \hat{\mathbf{a}}_y \\ \hat{\mathbf{a}}_z \end{bmatrix} \equiv [A]_{sr} \begin{bmatrix} \hat{\mathbf{a}}_r \\ \hat{\mathbf{a}}_\theta \\ \hat{\mathbf{a}}_\phi \end{bmatrix}$$

$$[A]_{rs}^{-1} = [A]_{rs}^T,\ [A]_{sr} = [A]_{rs}^T$$

$$\mathbf{A} = A_x\hat{\mathbf{a}}_x + A_y\hat{\mathbf{a}}_y + A_z\hat{\mathbf{a}}_z,\ \mathbf{A} = A_r\hat{\mathbf{a}}_r + A_\theta\hat{\mathbf{a}}_\theta + A_\phi\hat{\mathbf{a}}_\phi$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}, \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} \equiv [A]_{rs} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}, \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \equiv [A]_{sr} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

กฎของคูลอมบ์ (Coulomb's law)

$$\mathbf{F}_2 = k\frac{Q_1Q_2}{|\mathbf{R}_{12}|^2}\hat{\mathbf{a}}_{12},\ \mathbf{R}_{12} = -\mathbf{r}_1 + \mathbf{r}_2 = \mathbf{r}_2 - \mathbf{r}_1,\ \hat{\mathbf{a}}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|},\ \mathbf{F}_1 = k\frac{Q_2Q_1}{|\mathbf{R}_{21}|^2}\hat{\mathbf{a}}_{21}$$

$$k = \frac{1}{4\pi\varepsilon_0},\ \varepsilon_0 = 8.854\times10^{-12}\ \text{F/m}$$

สนามไฟฟ้าเนื่องจากประจุแบบจุด

$$\mathbf{E} = k\frac{Q}{|\mathbf{R}|^2}\hat{\mathbf{a}}_R,\ \mathbf{R} = \mathbf{r} - \mathbf{r}',\ \hat{\mathbf{a}}_R = \frac{\mathbf{R}}{|\mathbf{R}|}$$

สนามไฟฟ้าเนื่องจากประจุแบบจุดหลายตัว

$$\mathbf{E} = \sum_{i=1}^{N} k \frac{Q_i}{|\mathbf{R}_i|^2} \hat{\mathbf{a}}_{R,i}$$

การกระจายตัวของประจุ และความหนาแน่นประจุ

$$\rho_l = \lim_{\Delta l \to 0} \frac{\Delta Q}{\Delta l},\ \rho_s = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s},\ \rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

สนามไฟฟ้าเนื่องจากการกระจายตัวของประจุ

$$d\mathbf{E} = \hat{\mathbf{a}}_R k \frac{dQ}{|\mathbf{R}|^2},\ dQ = \rho_l dl',\ \mathbf{R} = -\mathbf{r}' + \mathbf{r} = \mathbf{r} - \mathbf{r}',\ \hat{\mathbf{a}}_R = \frac{\mathbf{R}}{|\mathbf{R}|}$$

$$\int \cos\psi d\psi = \sin\psi,\ \frac{d\tan\psi}{d\psi} = \sec^2\psi$$

$$\mathbf{E} = \int_S \hat{\mathbf{a}}_R k \frac{\rho_s}{|\mathbf{R}|^2} ds',\ \mathbf{E} = \int_V \hat{\mathbf{a}}_R k \frac{\rho_v}{|\mathbf{R}|^2} dv'$$

ส่วนย่อยปริมาณน้อย ๆ ของพื้นที่ และปริมาตร

$dv = dxdydz,\ ds = dxdy,\ ds = dydz,\ ds = dxdz$ ระบบพิกัดฉาก

$dv = \rho d\rho d\phi dz,\ ds = \rho d\rho d\phi,\ ds = d\rho dz,\ ds = \rho d\phi dz$ ระบบพิกัดทรงกระบอก

$dv = r^2 \sin\theta dr d\theta d\phi,\ ds = r\sin\theta dr d\phi,\ ds = r dr d\theta,\ ds = r^2 \sin\theta d\theta d\phi$

ระบบพิกัดทรงกลม

ฟลักซ์ไฟฟ้า และความหนาแน่นฟลักซ์

$$\Psi = Q,\ \mathbf{D} = \frac{\Psi}{4\pi r^2}\hat{\mathbf{a}}_r = \frac{Q}{4\pi r^2}\hat{\mathbf{a}}_r$$

กฎของเกาส์ (Gauss's law)

$$\Psi = Q,\ \Psi = \oint_S \mathbf{D}_s \cdot d\mathbf{s},\ \Psi = \oint_S \mathbf{D}_s \cdot d\mathbf{s} = Q$$

การประยุกต์กฎของเกาส์และไดเวอร์เจนซ์

$$\mathbf{D}_0 = D_{x0}\hat{\mathbf{a}}_x + D_{y0}\hat{\mathbf{a}}_y + D_{z0}\hat{\mathbf{a}}_z$$

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q,\ \oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{\text{front}} + \int_{\text{back}} + \int_{\text{left}} + \int_{\text{right}} + \int_{\text{top}} + \int_{\text{bottom}}$$

$$\int_{\text{front}} = \mathbf{D}_{\text{front}} \cdot \Delta\mathbf{s}_{\text{front}}$$

$$\mathbf{D}_{\text{front}} = D_{x,\text{front}}\hat{\mathbf{a}}_x + D_{y,\text{front}}\hat{\mathbf{a}}_y + D_{z,\text{front}}\hat{\mathbf{a}}_z,\ \Delta\mathbf{s}_{\text{front}} = \Delta y \Delta z \hat{\mathbf{a}}_x$$

$$\int_{\text{front}} + \int_{\text{back}} = \frac{\partial D_x}{\partial x}\Delta x \Delta y \Delta z$$

$$\nabla \cdot \mathbf{D} = \rho_v$$

ไดเวอร์เจนซ์ (divergence)

$$\nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$ ระบบพิกัดฉาก

$$\nabla \cdot \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$ พิกัดทรงกระบอก

$$\nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$$

ระบบพิกัดทรงกลม

พลังงานที่ใช้ในการเคลื่อนประจุแบบจุดท่ามกลางสนามไฟฟ้า

$$\mathbf{F}_E = \mathbf{E}Q,\ \mathbf{F}_{\text{appl}} = -\mathbf{F}_E = -\mathbf{E}Q,\ dW = \mathbf{F}_{\text{appl}} \cdot d\mathbf{l} = -\mathbf{E}Q \cdot d\mathbf{l},\ W = -Q\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{l}$$

$$\frac{W}{Q} = -\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{l},\ \text{potential difference} = -\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{l}$$

$$V_{AB} = -\int_B^A \mathbf{E} \cdot d\mathbf{l}$$

$$d\mathbf{l} = dx\hat{\mathbf{a}}_x + dy\hat{\mathbf{a}}_y + dz\hat{\mathbf{a}}_z$$ ระบบพิกัดฉาก

$$d\mathbf{l} = d\rho\hat{\mathbf{a}}_\rho + \rho d\phi\hat{\mathbf{a}}_\phi + dz\hat{\mathbf{a}}_z$$ ระบบพิกัดทรงกระบอก

$$d\mathbf{l} = dr\hat{\mathbf{a}}_r + rd\theta\hat{\mathbf{a}}_\theta + r\sin\theta d\phi\hat{\mathbf{a}}_\phi$$ ระบบพิกัดทรงกลม

ศักย์ไฟฟ้า (potential)

$$V_{AB} = V_A - V_B,\ V \equiv V_A = V_{AB}$$

การหาสนามไฟฟ้าในจากเกรเดียนต์ (gradient)

$$\mathbf{E} = -\nabla V$$

$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{a}}_x + \frac{\partial V}{\partial y}\hat{\mathbf{a}}_y + \frac{\partial V}{\partial z}\hat{\mathbf{a}}_z$$ ระบบพิกัดฉาก

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{\mathbf{a}}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{\mathbf{a}}_\phi + \frac{\partial V}{\partial z}\hat{\mathbf{a}}_z$$ ระบบพิกัดทรงกระบอก

$$\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{a}}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\mathbf{a}}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\mathbf{a}}_\phi$$ ระบบพิกัดทรงกลม