Modeling Complex Systems (CS/CSYS 6020), Fall 2023

Assignment #1 of 3
Done in teams of 2 or 3. Due on Blackboard by midnight on Friday, October 6th. Your write-up should contain your answers to the questions, as well as any supporting figures. A brief justification should accompany each answer. Codes should be submitted separately as readable files (e.g. .py, .cpp, .m, .jl, .R) or in notebooks (e.g. .ipynb).

1. In class videos, we derived three cases for the equilibrium state of the Lotka-Volterra competition model with two species. Using a phase space diagram, draw the fourth and final possible outcome for this model. Your diagram should include labeled axes, the isoclines with their intercepts, as well as arrows demonstrating the directions of the populations and any equilibrium points circled. Does this case have a stable equilibrium point? An unstable equilibrium point?

2. You are on an environmental preservation team tasked with ensuring diversity in a environment containing two organisms in the same niche. In similar sites, it has been observed that organism 2 quickly dominates and reduces organism 1 to local extinction. One of your colleagues has proposed that intermittently reducing the number of organism 2 individuals, N_2, could prevent N_1, the number of organism 1 individuals, from crashing. However, your means of reducing N_2 is not very precise, and the disturbance will cause both N_2 and N_1 to decrease by a constant factor $\rho \in (0, 1)$ each time the reduction is applied. That is, at discrete intervals, both N_1 and N_2 will suddenly be reduced to values ρN_1 and ρN_2.

   Do you think your colleague's suggestion could help preserve healthy levels of both N_1 and N_2? Support your argument with a sketch using the phase space diagram from question 1. Is there a range of values which ρ must take on for the intervention to be effective? Suppose you can apply the reduction whenever you like (it doesn't necessarily have to be a fixed schedule).

3. Write your own implementations of Euler's method and Heun's method (aka 2nd order Runge-Kutta, aka improved Euler's method) in the language of your choice. Each function should have a tunable parameter h which represents the step size. What is the relationship between Euler's method and a discrete time model? Chapter 6.4 in the textbook contains a helpful discussion.

4. Numerically solve the continuous SIS model from class using both of your integrators and parameters $N = 100$, $\gamma = 0.25$, and each $\beta \in \{0.03, 0.06, 0.1\}$. Create 9 different plots showing time series from both your Euler's and Heun's method with each of these parameter combinations, as well as using step sizes $h \in \{0.01, 0.5, 2.0\}$. Include legends to distinguish the pairs of β and step sizes, and make sure it is clear which line

corresponds to which method. Run the model for 50 steps each time, and use initial values of $(S, I) = (90, 10)$.

Is there a noticeable difference between the two methods? What might be going on?

5. Show mathematically that the global precision of Heun's method is in h^2 (where h is the step size). HINT: use $x(t + h) = x(t) + hf(x(t)) + \frac{h^2}{2}f'(x(t)) + \dots$, where $f(x(t)) := \frac{\mathrm{d}x}{\mathrm{d}t}$, $f'(x(t)) := \frac{\mathrm{d}f}{\mathrm{d}t}$, etc.

6. The 2-species Lotka-Volterra predator-prey model is very similar to the SIS epidemic model studied in class. Why did we see cycles in the predator-prey model and not the SIS one? What about SIR or SIRS systems? Should we expect chaos in classic epidemic models?

7. Using any system of your choice, come up with your own ODEs to model some phenomena in the system (for example, growth of a population, or spread of a rumor or contagion). The phenomena could be hypothetical, or based on real-world observations that you have studied or read about. Write down what the state variables of your model are, the parameters, and some interesting assumptions you made to simplify the model compared to the real world. Provide several figures of time-series plots with various combinations of parameters and initial conditions. Do you think your model can have cycles and/or stable/unstable equilibria? What do you think your model can tell you about its real-world counterpart?