# The Finite Element Method for Problems in Physics

**Coding Assignment 4**

Consider a three-dimensional domain defined by $x_1 = [0, 1]$ m; $x_2 = [0, 1]$ m; $x_3 = [0, 0.1]$ m. Solve the steady state and transient heat conduction problems with the following boundary conditions and initial conditions. Use $\rho = 3.8151$ x $10^6$ N.m$^{-2}$K$^{-1}$ (specific heat per unit volume), $\kappa = 385$ watt.m$^{-1}$K$^{-1}$, where $\kappa_{ij} = \kappa\delta_{ij}$. Assume $j = 0$ watt.m$^{-2}$ on all edges/surfaces where no temperature/flux conditions are specified. Use a mesh of 20 x 20 x 1 elements.

1. (Steady State problem): Boundary conditions $u = 300$ K along $x_1 = 0$ m and $u = 310$ K along $x_1 = 1$m.
2. (Transient problem): Boundary conditions $u = 300$ K along $x_1 = 0$ m, $u = 310$ K along $x_1 = 1$ m. Initial conditions $u = 300$ K for $x_1 < 0.5$ m and $u = 300 + 20 * (x_1 - 0.5)$ K for $x_1 \geq 0.5$ m.

**Coding Instructions:** You will receive the following files:

- main4.cc (the source file)
- FEM4.h (the template header file)
- writeSolutions.h
- CMakeLists.txt

You will also receive the .vtk files for the transient solution at $t = 0$ and $t = 3000s$ for a 10 x 10 x 1 mesh, with $\alpha = 0$, $\frac{1}{2}$, and 1. The corresponding $L^2$ norm at $t = 0$ and $t = 3000s$ is given at the end of the instructions.

Your code should solve both the steady state and transient heat conduction problems using linear basis functions. You should use the $v$-method to solve for the transient solution. Your code should be able to use Backward Euler, Forward Euler, and Mid-Point schemes. Your code should also calculate the $L_2$ norm of $(u_{steadystate} - u_{t_n})$ at a given time step.

As before, most of your coding will be done in FEM4.h. Do not modify any function names or the names of any class data structure. The only parts of main3.cc that should be modified are the function inputs that define the number of elements in the mesh. Your FEM4.h file must run with the given main4.cc file. Nothing should be changed in writeSolutions.h or CMakeLists.txt.

| $\alpha$ | t=0 | t=3000s |
|---|---|---|
| 0 | 0.912871 | 0.0444862 |
| $\frac{1}{2}$ | 0.912871 | 0.0445536 |
| 1 | 0.912871 | 0.044621 |

Table 1: $L^2$ norm of the error for a 10 element mesh.

**Submission Instructions:** Submit results using a mesh of 20 x 20 x 1 elements, with $\alpha = 0$, $\frac{1}{2}$, and 1.You should submit (through the Coursera website) a .zip file (name it CA4.zip) containing your FEM4.h file and the .h5 solution files. You will have three .h5 solution file (one each for $\alpha = 0$, $\frac{1}{2}$, and 1). Each .h5 file will contain the transient solution vector at $t = 3000s$, and the $L_2$ norm of $(u_{steadystate} - u_{t_n})$ at $t = 3000s$. Do not submit any files other than those listed here.