The Finite Element Method for Problems in Physics

Coding Assignment 3

Consider the 3D elastostatics problem. Find u such that

PDE	$\sigma_{ij,j} + f_i = 0$ in Ω
Constitutive relation	$\sigma_{ij} = \mathbb{C}_{ijkl}\epsilon_{kl}$
Kinematic relation	$\epsilon_{kl} = \frac{1}{2}\left(\dfrac{\partial u_k}{\partial x_l} + \dfrac{\partial u_l}{\partial x_k}\right)$
Neumann b.c.	$\sigma_{ij}n_j = h_i$ on $\partial\Omega_{h_i}$
Dirichlet b.c.	$u_i = u_i^g$ on $\partial\Omega_{u_i}$

Consider a three-dimensional domain defined by $x_1 = [0, 1]$ m; $x_2 = [0, 1]$ m; $x_3 = [0, 1]$ m (i.e. the unit cube). Use $E = 2.0e11$ Pa and $\nu = 0.3$. Assume traction $h_i = 0$ N.m^{-2} on all surfaces where no other conditions are specified. Use linear basis functions and a 10 x 10 x 10 element mesh for submission.

Apply the following boundary conditions:

$h_1 = h_2 = 0, h_3 = 1.0e9 * x_1$ Pa on the face $x_3 = 1$ m and $u_1 = u_2 = u_3 = 0$ m on the face $x_3 = 0$ m.

Coding Instructions: You will receive the following files:

- main3.cc (the source file)
- FEM3.h (the template header file)
- writeSolutions.h
- CMakeLists.txt

You will also receive the .vtk solution file for a 5 x 5 x 5 mesh. As before, most of your coding will be done in FEM3.h. Do not modify any function names or the names of any class data structure. The only parts of main3.cc that should be modified are the function inputs that define the number of elements in the mesh. Your FEM3.h file must run with the given main3.cc file. Nothing should be changed in writeSolutions.h or CMakeLists.txt.

Submission Instructions: You should submit (through the Coursera website) a .zip file (name it CA3.zip) containing your FEM3.h file and the .h5 solution file. You will have one .h5 solution file. The .h5 file will contain the solution vector. Do not submit any files other than those listed here.