Documentation of RVortexInt

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Contents

1 Introduction

The function RVortexInt evaluates the dimensionless induced velocity of a ring vortex by integration of the Biot-Savart law:

$$\mathrm{V(P)} = -\frac{\Gamma}{4\pi}\int_0^{2\pi}\frac{\underline{\mathrm{R}}\otimes \mathrm{d}\underline{\mathrm{l}}}{\mathrm{R}^3} \tag{1.1}$$

Where:

$$\underline{\mathrm{R}} = (\mathrm{x}-\bar{\mathrm{x}})\underline{\mathrm{i}} + (\mathrm{y}-\bar{\mathrm{y}})\underline{\mathrm{j}} + \mathrm{z}\underline{\mathrm{k}} \tag{1.2}$$

Since the ring vortex equations are:

$$\bar{\mathrm{x}} = \bar{\mathrm{r}}\cos(\vartheta), \quad \bar{\mathrm{y}} = \bar{\mathrm{r}}\sin(\vartheta), \quad \bar{\mathrm{z}} = 0 \tag{1.3}$$

The vector product is developed in this way:

$$\mathrm{d}\underline{\mathrm{l}} = \frac{\mathrm{d}\underline{\mathrm{s}}}{\mathrm{d}\vartheta}\mathrm{d}\vartheta = (-\bar{\mathrm{r}}\sin\vartheta\underline{\mathrm{i}} + \bar{\mathrm{r}}\cos\vartheta\underline{\mathrm{j}})\mathrm{d}\vartheta \tag{1.4}$$

$$\underline{\mathrm{R}}\otimes \mathrm{d}\underline{\mathrm{l}} = -\mathrm{z}\bar{\mathrm{r}}\cos\vartheta\underline{\mathrm{i}} + \mathrm{z}\bar{\mathrm{r}}\sin\vartheta + [(\mathrm{x}-\bar{\mathrm{x}})\bar{\mathrm{r}}\cos\vartheta) + (\mathrm{y}-\bar{\mathrm{y}})\bar{\mathrm{r}}\sin\vartheta)]\underline{\mathrm{k}} \tag{1.5}$$

And:

$$\mathrm{R}^3 = [(\mathrm{x}-\bar{\mathrm{x}})^2 + (\mathrm{y}-\bar{\mathrm{y}})^2 + \mathrm{z}^2]^{\frac{3}{2}} \tag{1.6}$$

The three induced velocities are:

$$\mathrm{V_x(x,y,z)} = \frac{\Gamma}{4\pi}\bar{\mathrm{r}}\mathrm{z}\int_0^{2\pi}\frac{\cos\vartheta \mathrm{d}\vartheta}{\mathrm{R}^3} \tag{1.7}$$

$$\mathrm{V_y(x,y,z)} = -\frac{\Gamma}{4\pi}\bar{\mathrm{r}}\mathrm{z}\int_0^{2\pi}\frac{\sin\vartheta \mathrm{d}\vartheta}{\mathrm{R}^3} \tag{1.8}$$

$$\mathrm{V_z(x,y,z)} = -\frac{\Gamma}{4\pi}\bar{\mathrm{r}}\int_0^{2\pi}\frac{[(\mathrm{x}-\bar{\mathrm{x}})\bar{\mathrm{r}}\cos\vartheta) + (\mathrm{y}-\bar{\mathrm{y}})\bar{\mathrm{r}}\sin\vartheta)]\mathrm{d}\vartheta}{\mathrm{R}^3} \tag{1.9}$$

The velocity component of interests is $\mathrm{V_x}$ calculated imposing $\mathrm{y} = 0$ because it provide exactly the radial component:

$$\mathrm{V_x(x,0,z)} = \frac{\Gamma}{4\pi}\bar{\mathrm{r}}\mathrm{z}\int_0^{2\pi}\frac{\cos\vartheta \mathrm{d}\vartheta}{[(\mathrm{x}-\bar{\mathrm{r}}\cos\vartheta)^2 + (\bar{\mathrm{r}}\sin\vartheta)^2 + \mathrm{z}^2]^{\frac{3}{2}}} \tag{1.10}$$

This model is based on the McCornick's Theory: The point at 3/4 of chord of the shroud is classified as a control point and of course it is the point where the velocity of interests is evaluated. Instead the Ring Vortex is placed at 1/4 of the shroud.

$$f(\frac{c}{D_{1/4}}, \frac{D_{3/4}}{D_{1/4}}) = \frac{V_x \pi D_{1/4}}{\Gamma} = \frac{1}{4}\bar{r}zD_{1/4}\int_0^{2\pi} \frac{\cos\vartheta d\vartheta}{[(x-\bar{r}\cos\vartheta)^2 + (\bar{r}\sin\vartheta)^2 + z^2]^{\frac{3}{2}}} \tag{1.11}$$

The results that are expected are represented on the following picture for different ratio of $\frac{D_{\frac{3}{4}}}{D_{\frac{1}{4}}}$ and $\frac{C}{D_{\frac{1}{4}}}$. [1],[2]

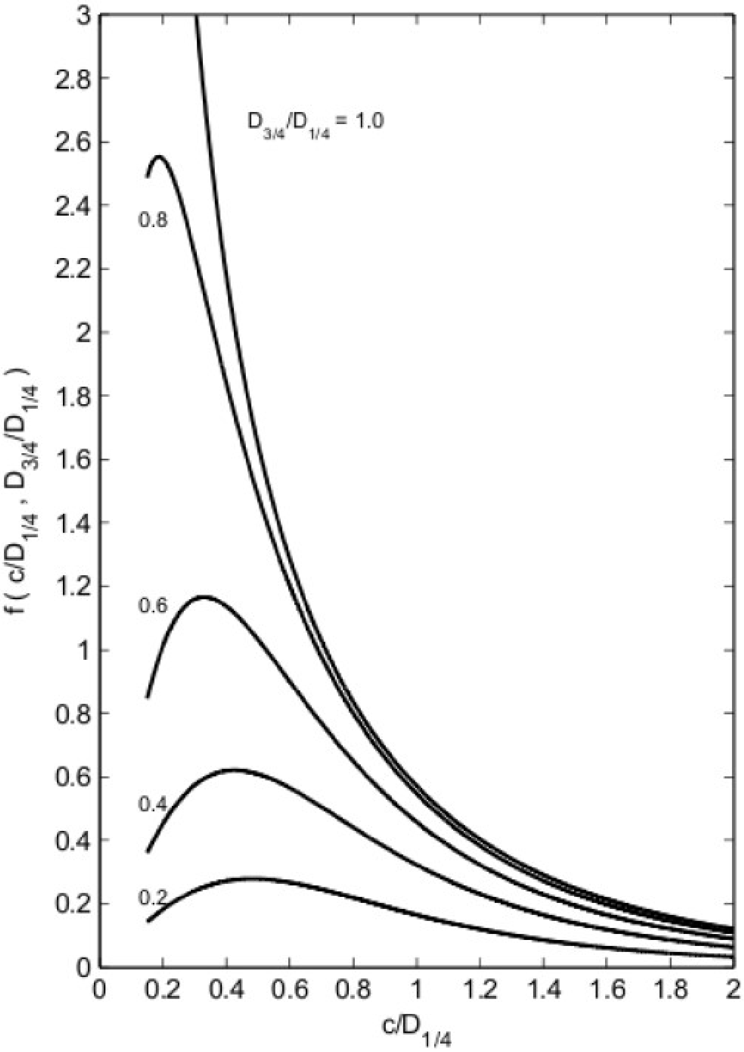

Figure 1.1: *Velocity induced by vortex ring*

2 Algorithm description

The user has to give as inputs in the GUI three different values: the shroud length and the diameter at the beginning and at the end of the shroud too.

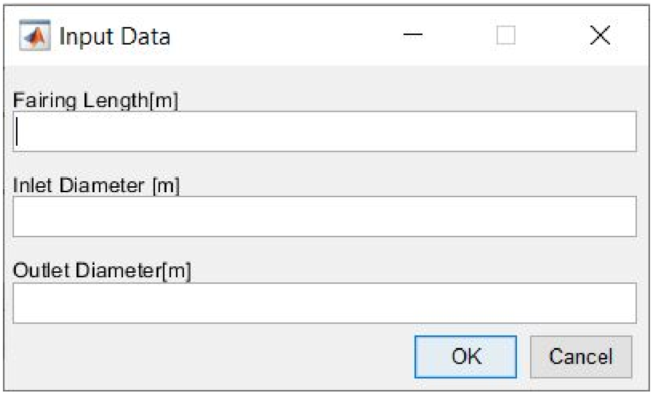

Figure 2.1: *Code's GUI*

The first part of the code rebuilds the shroud and it evaluates the diameters at 1/4 and at 3/4 of the shroud. In this way, it could correctly place the radius of the ring vortex at 1/4 and the velocity control point at 3/4 of the shroud length.

```
t=linspace(0,1,1000); % Recreating the shroud
    for i=1:length(t)
        xs(i)=0+t(i)*(ch);
        ys(i)=Di/2+t(i)*(De/2-Di/2);
    end

D14=2*ys(250); % Diameter at 1/4 of the shroud
D34=2*ys(750); % Diameter at 1/4 of the shroud
r=D14/2; % Ring Vortex radius
x=D34/2; % Speed Control Point
z=ch*(3/4); % Three quarters of the shroud
y=0;
```

The code performs the Biot-Savart's integrals on the three directions using anonymous functions based on the value of ϑ. The MatLab function *Integral* is used for the calculations.

```
% Anonymous Function
R1=@(t) cos(t)./(((x-r.*cos(t)).^2 +(y-r.*sin(t)).^2+z(i).^2)).^(3/2);
R2=@(t) sin(t)./(((x-r.*cos(t)).^2 +(y-r.*sin(t)).^2+z(i).^2)).^(3/2);
R3=@(t)((x-r.*cos(t)).*cos(t)+(y-r.*sin(t)).*sin(t))./(((x-r.*cos(t)).^2+(y-r.*sin(t)).^2+z(i).^2)).^(3/2);
    % Integrals
Ix(i)=integral(R1,0,2*pi);
Iy(i)=integral(R2,0,2*pi);
Iz(i)=integral(R3,0,2*pi);
```

The code evaluates the dimensionless components of the velocity induced by the ring vortex based on the Biot-Savart's Law at 3/4 of the shroud.

```
%Dimensionless;
fx(i)=(r*z(i)*D14/4)*Ix(i);
fy(i)=-(r*z(i)*D14/4)*Iy(i);
fz(i)=-(r*D14/4)*Iz(i);
```

The code's output is the value of $\mathrm{f_x}$. It has been developed another version of the code that gives as output the curves of different Ratio of $\frac{\mathrm{D}_{\frac{3}{4}}}{\mathrm{D}_{\frac{1}{4}}}$ and $\frac{\mathrm{C}}{\mathrm{D}_{\frac{1}{4}}}$.

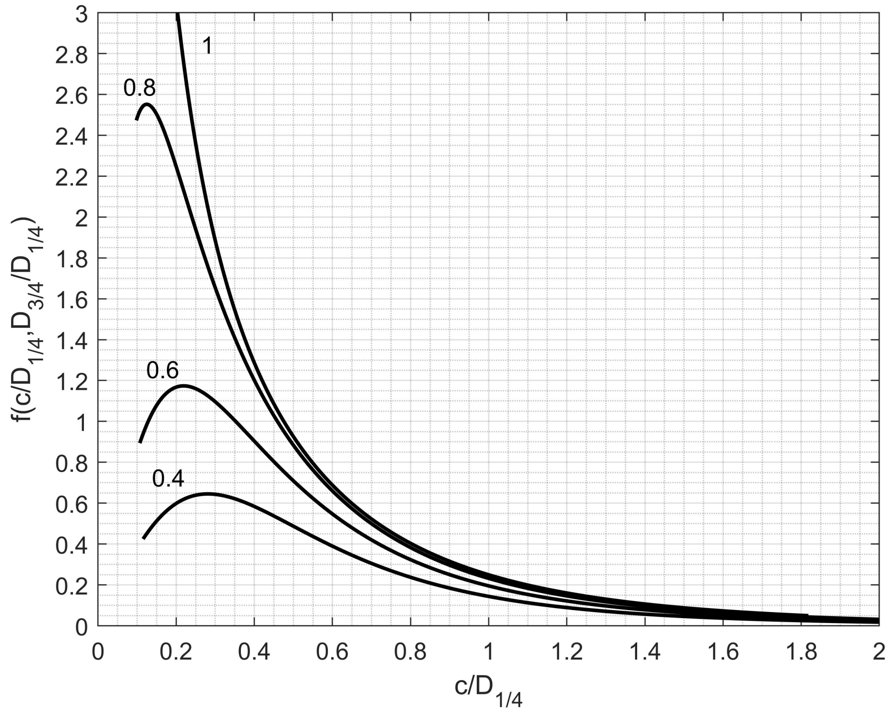

Figure 2.2: *Induced speed for different ratios- MatLab*

3 Warnings indicators

There is a warning when the user runs the code. According to the input data there are three different warnings pop-ups that advice the user of which shroud is being analyzed.

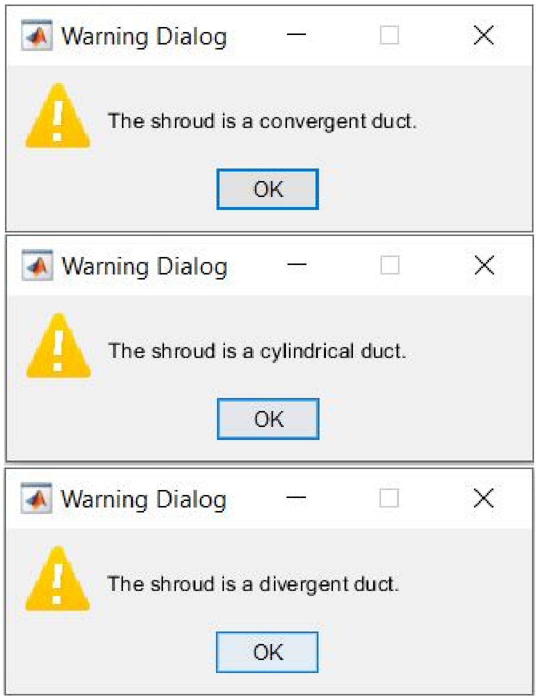

Figure 3.1: *Code's warnings for shroud geometry.*

4 Test Case

In order to validate the function it is done a comparison between the results obtained with the integration to the ones of reference on the [2].

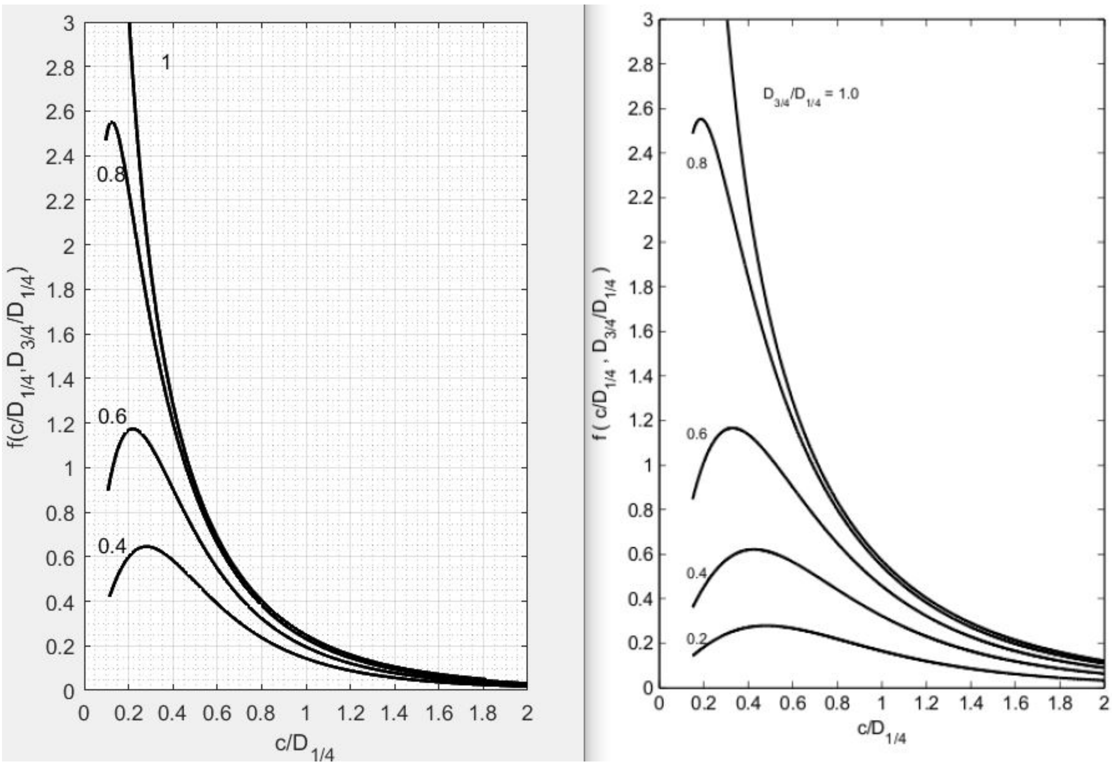

Figure 4.1: *Test cases: Left: MatLab code output. Right: Reference data*

There is a good overlap between the two graphics.

5 Appendix: Code

```
function [fx] = RVortexInt(ch,Di,De)
% % \RVortexInt.m
%  \The function evaluates the dimensionless radial velocity component
%   of the ring vortex by means of integration of Biot-Savart law
%  \Olino Massimiliano, Marino Giuseppe
%  \1.0
%
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%
% ==============================================================================================
% Name        : RVortexInt.m
% Author      : Olino Massimiliano, Marino Giuseppe
%               University of Naples Federico II.
% Version     : 1.0
% Date        : 01/12/2020
% Modified    : 20/01/2021
% Description : The function evaluates the dimensionless radial velocity component
%               of the ring vortex by means of integration of Biot-Savart law
% Reference   : McCormick, B.W.,(1967), Aerodynamics of V/STOL Flight, Academic Press.
%               Tognaccini, R., (2020), Lezioni di AERODINAMICA DELL'ALA ROTANTE.
% Input       : ch (chord), Di (inlet diameter), De (exit diameter) of the shroud
% Output      : fx (the dimensionless radial velocity component of the ring vortex )
% Note        :
% ==============================================================================================
%
%Gui for the input by users.
title='Input Data';
prompt={'Shroud chord [m]','Inlet diameter [m]','Exit diameter [m]'};
answer=inputdlg(prompt,title,[1 55;1 55;1 55]);
ch=str2double(answer{1});
Di=str2double(answer{2});
De=str2double(answer{3});

% Creation of shroud geometry
t=linspace(0,1,1000);
    for i=1:length(t)
        xs(i)=0+t(i)*(ch);
        ys(i)=Di/2+t(i)*(De/2-Di/2);
    end
D14=2*ys(250);               % Diameter at 1/4 of the shroud
D34=2*ys(750);               % Diameter at 3/4 of the shroud
c_D14=ch/D14 ;               % Ratio between chord and diameter at 1/4
RatioD=D34/D14;              % Ratio between diameter at 3/4 and 1/4

r=(D14/2);                   % flow radius
x=D34/2;                     %
z=ch*(3/4);                  %  Speed control point coordinates
y=0;                         %

% Anonymous Function
```

```
R1=@(t) cos(t)./(((x-r.*cos(t)).^2 +((y-r.*sin(t)).^2)+z^2)).^(3/2);
R2=@(t) sin(t)./(((x-r.*cos(t)).^2 +(y-r.*sin(t)).^2+z^2)).^(3/2);
R3=@(t) ((x-r.*cos(t)).*cos(t)+(y-r.*sin(t)).*sin(t))./(((x-r.*cos(t)).^2 +(y-r.*sin(t)).^2+z^2)).^(3/2);

% Integrals
Ix=integral(R1,0,2*pi);
Iy=integral(R2,0,2*pi);
Iz=integral(R3,0,2*pi);

% Adimesionalisation of the velocity components

fx=(r*z*D14/4)*Ix;           % Component of interests
fy=-(r*z*D14/4)*Iy;
fz=-(r*D14/4)*Iz;

% Vector sum
f=sqrt(fx^2+fy^2+fz^2);

% Results
figure(1);
plot(f,f)
axis ([0 1 0 1]);
text(0.25,1,'Velocity induced by vortex ring:');
text(0.25,0.90,['fx= ',num2str(fx)]);
axis off;

% Warnings
    if Di>De
        warndlg('The shroud is a convergent duct.');
    elseif Di<De
     warndlg('The shroud is a divergent duct.');
    else De=Di;
     warndlg('The shroud is a cylindrical duct.');
    end
end
```

Bibliography

[1] McCORMICK, B.W.,(1967), *Aerodynamics of V/STOL Flight*, Academic Press.
[2] TOGNACCINI, R., (2020), *Lezioni di AERODINAMICA DELL'ALA ROTANTE.*