Documentation of RVortexInt

Massimiliano Olino, Giuseppe Marino

Contents

1 Introduction

The function RVortexInt evaluates the dimensionless induced velocity of a ring vortex by integration of the Biot-Savart law:

$$V(P) = -\frac{\Gamma}{4\pi}\int_0^{2\pi} \frac{\underline{R} \otimes d\underline{l}}{R^3} \tag{1.1}$$

Where:

$$\underline{R} = (x - \bar{x})\underline{i} + (y - \bar{y})\underline{j} + z\underline{k} \tag{1.2}$$

Since the ring vortex equations are:

$$\bar{x} = \bar{r}\cos(\vartheta), \quad \bar{y} = \bar{r}\sin(\vartheta), \quad \bar{z} = 0 \tag{1.3}$$

The vector product is developed in this way:

$$\underline{dl} = \frac{d\underline{s}}{d\vartheta}d\vartheta = (-\bar{r}\sin\vartheta\underline{i} + \bar{r}\cos\vartheta\underline{j})d\vartheta \tag{1.4}$$

$$\underline{R} \otimes d\underline{l} = -z\bar{r}\cos\vartheta\underline{i} + z\bar{r}\sin\vartheta + [(x - \bar{x})\bar{r}\cos\vartheta) + (y - \bar{y})\bar{r}\sin\vartheta)]\underline{k} \tag{1.5}$$

And:

$$R^3 = [(x - \bar{x})^2 + (y - \bar{y})^2 + z^2]^{\frac{3}{2}} \tag{1.6}$$

The three induced velocities are:

$$V_x(x, y, z) = \frac{\Gamma}{4\pi}\bar{r}z\int_0^{2\pi} \frac{\cos\vartheta d\vartheta}{R^3} \tag{1.7}$$

$$V_y(x, y, z) = -\frac{\Gamma}{4\pi}\bar{r}z\int_0^{2\pi} \frac{\sin\vartheta d\vartheta}{R^3} \tag{1.8}$$

$$V_z(x, y, z) = -\frac{\Gamma}{4\pi}\bar{r}\int_0^{2\pi} \frac{[(x - \bar{x})\bar{r}\cos\vartheta) + (y - \bar{y})\bar{r}\sin\vartheta)]d\vartheta}{R^3} \tag{1.9}$$

If the origin of the coordinate system is placed on the ring vortex, The distance between the ring vortex and the velocity control point is $z = \frac{3}{4}$ of the length shroud less $\frac{1}{4}$ of the same. The velocity component of interests is V_x calculated imposing $y = 0$ and $x = D_{\frac{1}{4}}/2$ because it provides exactly the radial component.

$$V_x(x, 0, z) = \frac{\Gamma}{4\pi}\bar{r}z\int_0^{2\pi} \frac{\cos\vartheta d\vartheta}{[(x - \bar{r}\cos\vartheta)^2 + (\bar{r}\sin\vartheta)^2 + z^2]^{\frac{3}{2}}} \tag{1.10}$$

This model is based on the McCornick's Theory: The point at 3/4 of chord of the shroud is classified as a control point and of course it is the point where the velocity of interests is evaluated. Instead the Ring Vortex is placed at 1/4 of the shroud.

$$f\left(\frac{c}{D_{\frac{1}{4}}}, \frac{D_{\frac{3}{4}}}{D_{\frac{1}{4}}}\right) = \frac{V_x \pi D_{\frac{1}{4}}}{\Gamma} = \frac{1}{4} \bar{r} z D_{\frac{1}{4}} \int_0^{2\pi} \frac{\cos\vartheta d\vartheta}{[(x - \bar{r}\cos\vartheta)^2 + (\bar{r}\sin\vartheta)^2 + z^2]^{\frac{3}{2}}} \tag{1.11}$$

The results that are expected are represented on the following picture for different ratio of $\frac{D_{\frac{3}{4}}}{D_{\frac{1}{4}}}$ and $\frac{C}{D_{\frac{1}{4}}}$. [1],[2]

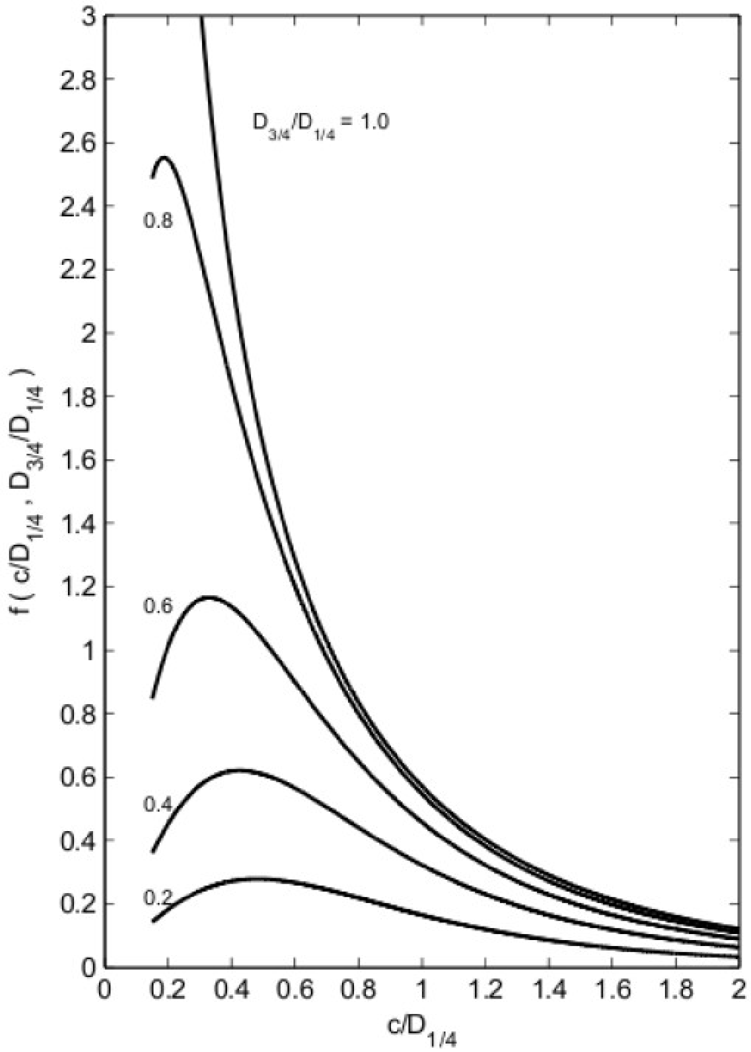

Figure 1.1: *Velocity induced by vortex ring*

2 Algorithm description

The user has to give as inputs three different values: the shroud length and the diameter at the $\frac{1}{4}$ and $\frac{3}{4}$ of the shroud too.
In this way, it could correctly place the ring vortex at $\frac{1}{4}$ of the shroud and the velocity control point at $\frac{3}{4}$ of the shroud length. The ring vortex radius is equaled to the radius at $1/4$ of the shroud.
The code performs the Biot-Savart's integrals on the three directions using anonymous functions based on the value of ϑ. The MatLab function *Integral* is used for the calculations.

```
% Anonymous Function
R1=@(t) cos(t)./(((x-r.*cos(t)).^2 +(y-r.*sin(t)).^2+z(i).^2)).^(3/2);
R2=@(t) sin(t)./(((x-r.*cos(t)).^2 +(y-r.*sin(t)).^2+z(i).^2)).^(3/2);
R3=@(t)((x-r.*cos(t)).*cos(t)+(y-r.*sin(t)).*sin(t))./(((x-r.*cos(t)).^2+(y-r.*sin(t)).^2+z(i).^2)).^(3/2);
    % Integrals
Ix(i)=integral(R1,0,2*pi);
Iy(i)=integral(R2,0,2*pi);
Iz(i)=integral(R3,0,2*pi);
```

The code evaluates the dimensionless components of the velocity induced by the ring vortex based on the Biot-Savart's Law at $3/4$ of the shroud.

```
%Dimensionless;
fx(i)=(r*z(i)*D14/4)*Ix(i);
fy(i)=-(r*z(i)*D14/4)*Iy(i);
fz(i)=-(r*D14/4)*Iz(i);
```

The code's output is the value of f_x. It has been developed another version of the code that gives as output the curves for different Ratio of $D_{\frac{3}{4}}/D_{\frac{1}{4}}$ and $C/D_{\frac{1}{4}}$.

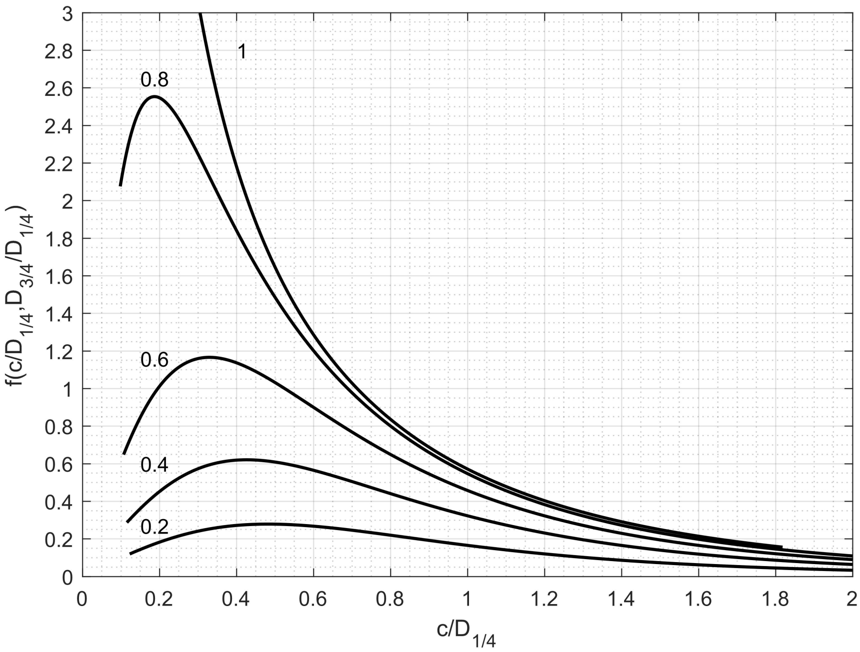

Figure 2.1: *Induced speed for different ratios- MatLab*

3 Warnings indicators

There is a warning when the user runs the code. According to the input data there are three different warnings pop-ups that advice the user of which shroud is being analyzed.

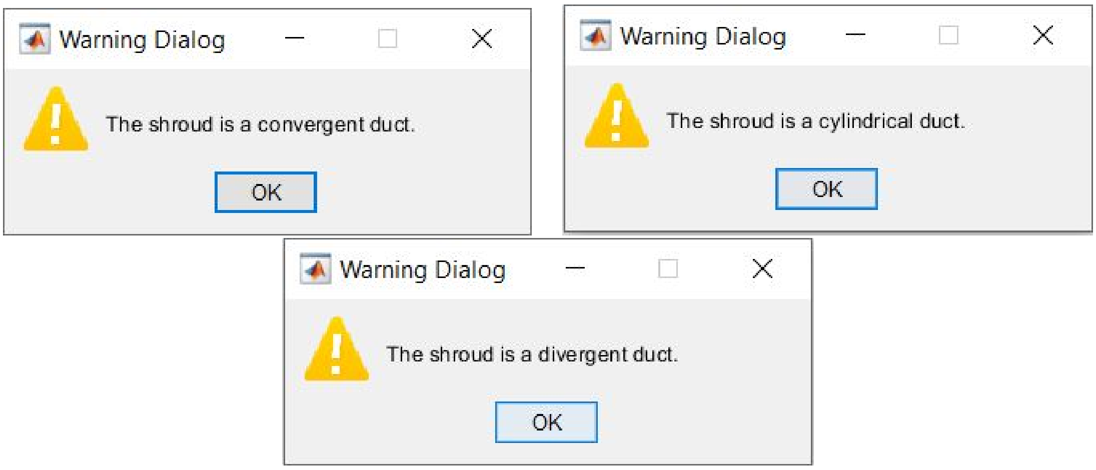

Figure 3.1: *Code's warnings for shroud geometry.*

4 Test Case

In order to validate the function it is done a comparison between the results obtained with the integration to the ones of reference on the [2].

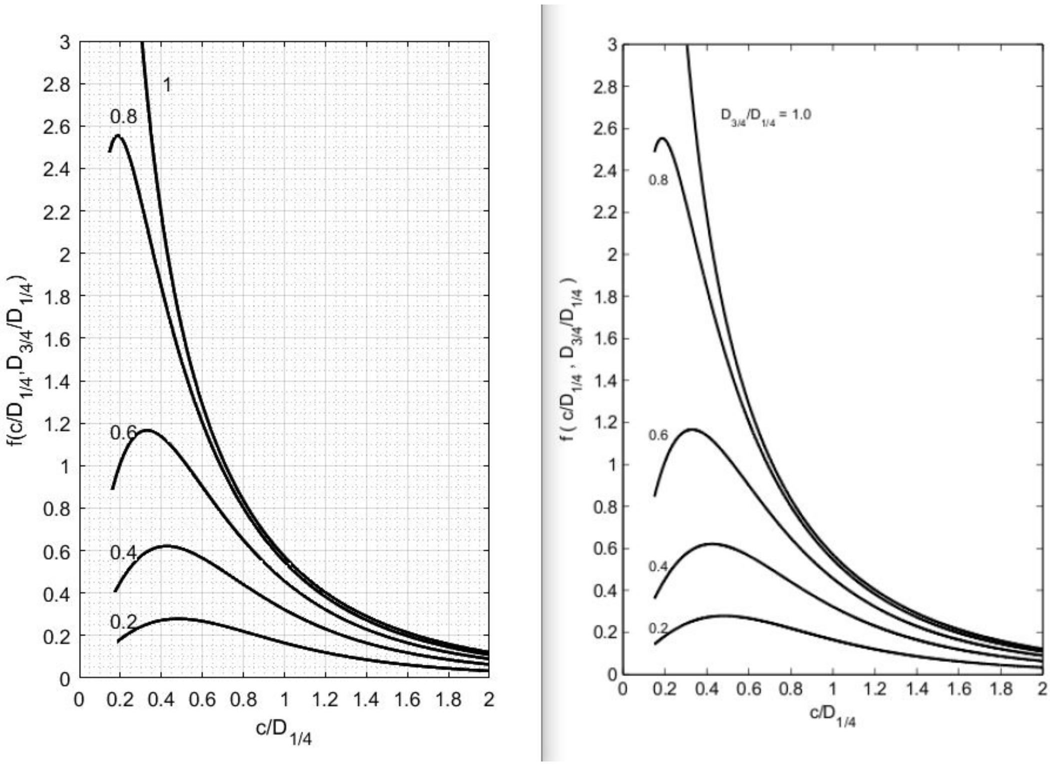

Figure 4.1: *Test cases: Left: MatLab code output. Right: Reference data*

There is a good overlap between the two graphics.

5 Appendix: Code

```
% % \RVortexInt.m
%  \The function evaluates the dimensionless radial velocity component
%   of the ring vortex by means of integration of Biot-Savart law
%  \Olino Massimiliano, Marino Giuseppe
%  \1.0
%
% Eli-TAARG is free software; you can redistribute it and/or
% modify it under the terms of the GNU General Public
% License as published by the Free Software Foundation; either
% version 3 of the License, or (at your option) any later version.
%
% Eli-TAARG is developed by the TAARG Educational organization for
% educational purposes only.
% Theoretical and Applied Aerodynamic Research Group - University of Naples Federico II.
%
% Eli-TAARG GitHub link: <https://github.com/TAARG-Education/Eli-TAARG>
%
% Eli-TAARG is distributed in the hope that it will be useful,
% but WITHOUT ANY WARRANTY; without even the implied warranty of
% MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU
% General Public License for more details.
% <http://www.gnu.org/licenses/>.
%
% ==============================================================================================
% Name        : RVortexInt.m
% Author      : Olino Massimiliano, Marino Giuseppe
%               University of Naples Federico II.
% Version     : 1.0
% Date        : 01/12/2020
% Modified    : 22/01/2021
% Description : The function evaluates the dimensionless radial velocity component
%               of the ring vortex by means of integration of Biot-Savart law
% Reference   : McCormick, B.W.,(1967), Aerodynamics of V/STOL Flight, Academic Press.
%               Tognaccini, R., (2020), Lezioni di AERODINAMICA DELL'ALA ROTANTE.
% Input       : ch (chord) D14, D34 (The diameters at 1/4 and 3/4 of the shroud)
% Output      : fx (the dimensionless radial velocity component of the ring vortex )
% Note        :
% ==============================================================================================

function [fx] = RVortexInt(ch,D14,D34)
c_D14=ch/D14;                 % Ratio between chord and diameter at 1/4
RatioD=D34/D14;               % Ratio between diameter at 3/4 and 1/4

r=(D14/2);                    % flow radius
x=D34/2;                      % x coordinate of the Speed control point
z=ch*(3/4)-ch*(1/4);          % distance along the z-axis between the vortex and the control point
y=0;                          % y coordinate of the Speed control point

% Anonymous Function
R1=@(t) cos(t)./(((x-r.*cos(t)).^2 +((y-r.*sin(t)).^2)+z^2)).^(3/2);
R2=@(t) sin(t)./(((x-r.*cos(t)).^2 +(y-r.*sin(t)).^2+z^2)).^(3/2);
R3=@(t) ((x-r.*cos(t)).*cos(t)+(y-r.*sin(t)).*sin(t))./(((x-r.*cos(t)).^2 +(y-r.*sin(t)).^2+z^2)).^(3/2);

% Integrals
Ix=integral(R1,0,2*pi);
Iy=integral(R2,0,2*pi);
Iz=integral(R3,0,2*pi);

% Adimesionalisation of the velocity components

fx=(r*z*D14/4)*Ix;            % Component of interests
fy=-(r*z*D14/4)*Iy;
fz=-(r*D14/4)*Iz;

% Vector sum
```

```
f=sqrt(fx^2+fy^2+fz^2);

% Results
figure(1);
plot(f,f)
axis ([0 1 0 1]);
text(0.25,1,'Velocity induced by vortex ring:');
text(0.25,0.90,['fx= ',num2str(fx)]);
axis off;

% Warnings
    if D14>D34
        warndlg('The shroud is a convergent duct.');
    elseif D14<D34
     warndlg('The shroud is a divergent duct.');
    else D14=D34;
     warndlg('The shroud is a cylindrical duct.');
    end
end
```

Bibliography

[1] McCORMICK, B.W.,(1967), *Aerodynamics of V/STOL Flight*, Academic Press.
[2] TOGNACCINI, R., (2020), *Lezioni di AERODINAMICA DELL'ALA ROTANTE.*