Documentation of Axial_Descent_Ascent_Operating_Curves_Rotor Function

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Chapter 1

Documentation

1.1 Algorithm Introduction

The function plots $\mathrm{w}(\mathrm{V}_\infty)$ and $\mathrm{P}(\mathrm{V}_\infty)$ curves according to rotor's simply impulsive theory.
To achieve this objective in the first step we defined the following vectors:

- ALTITUDE
- ASCENT VELOCITY
- DESCENT VELOCITY

Density is then calculated from the altitude vector with the MATLAB function *atmosisa*.
After calculated the density, the axial hovering induction is obtained from the following relation derived by rotor's simply impulsive theory:

$$\mathrm{w_h} = \sqrt{\frac{\mathrm{Mg}}{2\rho(\mathrm{h})\pi \mathrm{r}^2}} \tag{1.1}$$

From this value, we calculated the non-dimensional variables:

- $\widetilde{\mathrm{V}} = \frac{\mathrm{V}_\infty}{\mathrm{w_h}}$
- $\widetilde{\mathrm{w}}_{\mathrm{ascent}} = -\frac{\widetilde{\mathrm{V}}}{2} + \sqrt{\frac{\widetilde{\mathrm{V}}}{4}^2 + 1}$
- $\widetilde{\mathrm{w}}_{\mathrm{descent}} = -\frac{\widetilde{\mathrm{V}}}{2} + \sqrt{\frac{\widetilde{\mathrm{V}}}{4}^2 - 1}$
- $\widetilde{\mathrm{P}}_{\mathrm{ascent}} = \widetilde{\mathrm{V}} + \widetilde{\mathrm{w}}_{\mathrm{ascent}}$
- $\widetilde{\mathrm{P}}_{\mathrm{descent}} = \widetilde{\mathrm{V}} + \widetilde{\mathrm{w}}_{\mathrm{descent}}$

Then, we used some cycles to obtain the matrices including the values of induction and power that are two variables functions.
Subsequently these functions are plotted. In the code the user has also the possibility to insert the interest altitude.

1.2 Algorithm Description

The code begins at line 40 with the function call, where are defined the inputs as well as described in section 1.3.
Dimensional variables, as the altitude and the velocity, are defined in the lines [41-43].
The density as a function of altitude is calculated in line 45 through the MATLAB function *atmosisa*. In line 46 this variable is interpolated in a function handle to obtain a numerical law to use whit any altitude that the user choice in lines 149 and 214.
From the line 50 to line 54 the code calculates the non-dimensional variables as mentioned in section 1.1.
At line 56 if the user has insert only 2 variables the code calculated the power and the induction distributions. Indeed At lines [62-63] the matrices which will contain the numerical value of induction as a function of velocity and altitude are initialized . Subsequently (lines [67-82]) those matrices are filled with *for loops* as show below, where you can see that is respected the limit of simply impulsive rotor theory:

```
for i = 1 : length(hh)
    for j = 1 : length(VVs)
        WTS(i,j) = w_tilde_salita(VVs(j),hh(i));
    end
end

for i = 1 : length(hh)
    for j = 1 : length(VVd)
        if V_tilde(VVd(j),hh(i)) < -2
        WTD(i,j) = w_tilde_discesa(VVd(j),hh(i));
        aa(1,i) = j;
        else
            WTD(i,j) = 0;
        end
    end
end
```

Subsequently (lines [85-105]) the same thing is done for the non-dimensional power
The first type of outputs are the 3D plots of the induction and power as a function of altitude and velocity, which are in line [108-141]. After that, in lines [148-152] the user has the possibility to choose the altitude of interest to obtain the 2d plots of induction and power as function of only velocity at the entered altitude. In the following lines [154-185] the matrices containg the numerical value of induction and power are initialized and till as function of velocity at the entered altitude as show below:

```
prompt = {'Insert interest altitude in metres [min=0,Max=6000]: '};
dlgtitle = 'Altitude';
dims = [1 35];
answer = inputdlg(prompt,dlgtitle,dims);
hnew = str2double(answer{1});

WTSnew = zeros(1,length(VVs));
WTDnew = zeros(1,length(VVd));

% Matrices fill
for j = 1 : length(VVs)
    WTSnew(1,j) = w_tilde_salita(VVs(j),hnew);
end

for j = 1 : length(VVd)
    if V_tilde(VVd(j),hh(i)) < -2
        WTDnew(1,j) = w_tilde_discesa(VVd(j),hnew);
        aanew = j;
    else
        break
    end
end

PTSnew = zeros(1,length(VVs));
PTDnew = zeros(1,length(VVd));


for j = 1 : length(VVs)
    PTSnew(1,j) =  P_tilde_salita(VVs(j),hnew);
end

for j = 1 : length(VVd)
    if V_tilde(VVd(j),hh(i)) < -2
        PTDnew(1,j) = P_tilde_discesa(VVd(j),hnew);
        bbnew = j;
    else
        break
    end
end
```

The values of induction and power as function of velocity at the entered altitude are plotted through the command in line[188-207].
While, in lines [208-234], if the user insert three inputs (rotorcraft's mass, rotor's radius and ascent or descent velocity) the code calculated and shows only the power and the induction at the interest altitude at the velocity inserted as third input.

1.3 Input and Output

The function takes in input:

- Rotorcraft's mass
- Rotor's radius
- Ascent or Descent Velocity

If the inputs values are two (Rotorcraft's mass and Rotor's radius) the outputs are the plots of induction and power and these are reported in section *Test Case*. If the inputs values are all three the outputs are only the value of power and induction at the interest altitude.

1.4 Error Indicators

The plots in axial discent operating condition have been obtained within the validity limits of simple impulsive rotor theory, that's until when $V_\infty < -2w$. Indeed for these values in inputs:

- $V_\infty = -10[m/s]$
- $h = 1000[m]$

the code shows this error message:

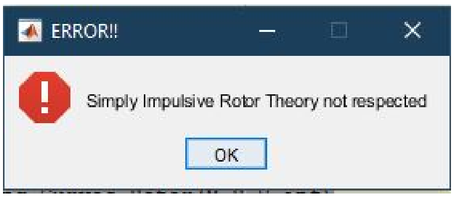


Figure 1.1: *Error Message*

1.5 Test Case

The numerical values in input for those plots are:

- Rotorcraft's mass = 5000kg
- Rotor's radius = 7m

The outputs of the test case are the following plots:

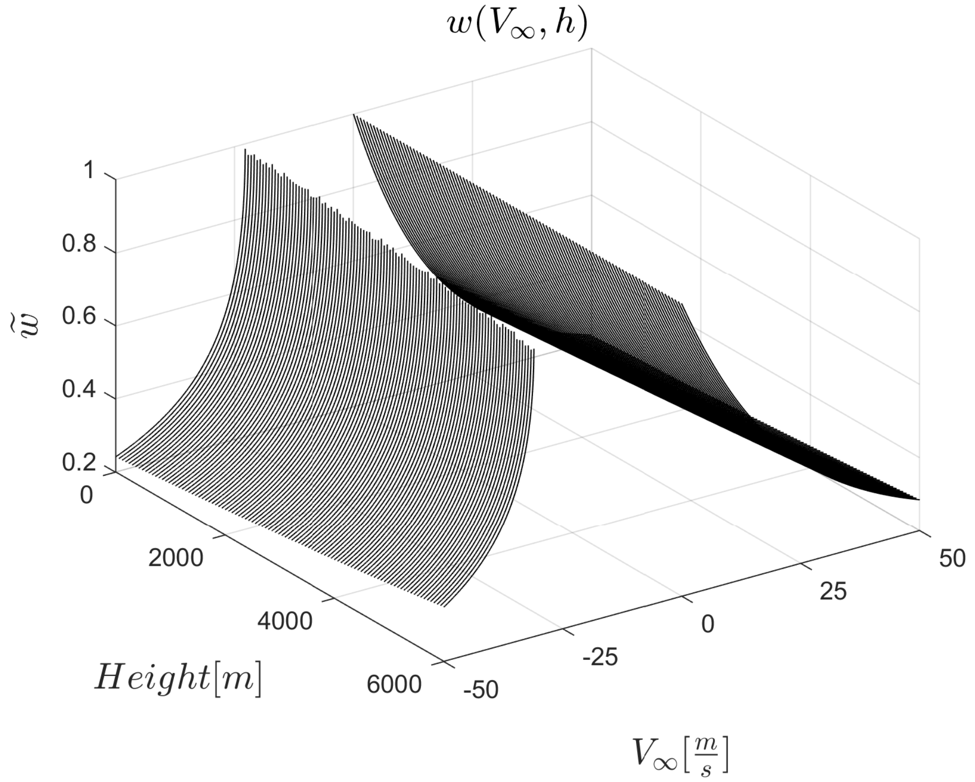


Figure 1.2: *Induction*

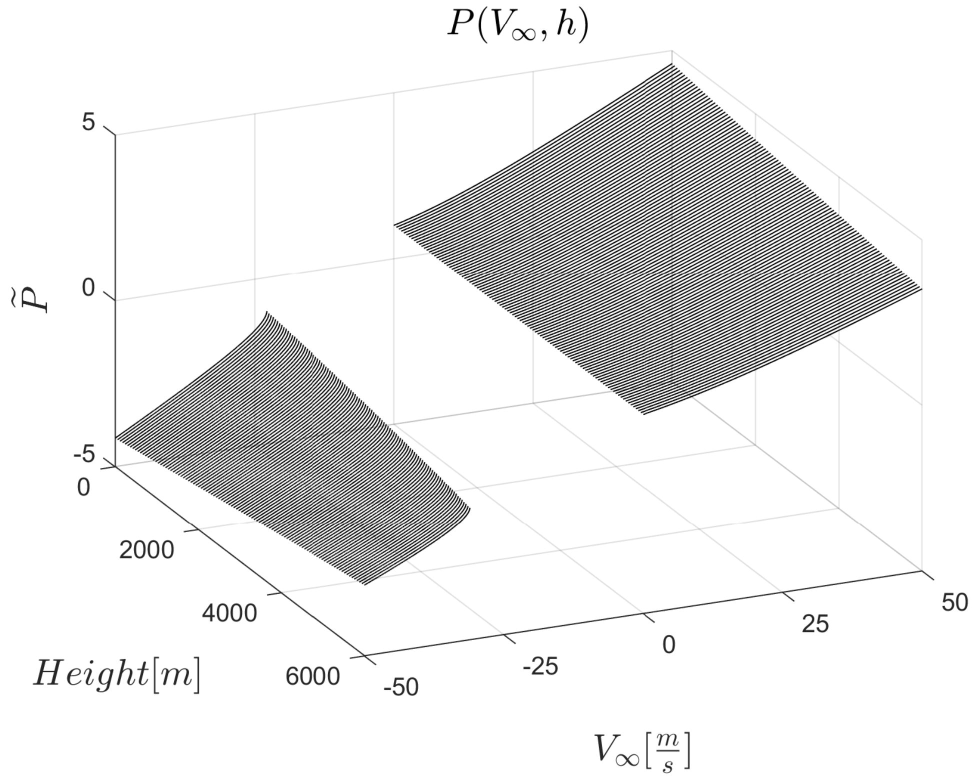


Figure 1.3: *Power*

In the code the user has also the possibility to insert the interest altitude: for example, at sea level we are obtained the following curves. In order to prove the validity of the code the curves were compared with some examples reported in [1] as follow.

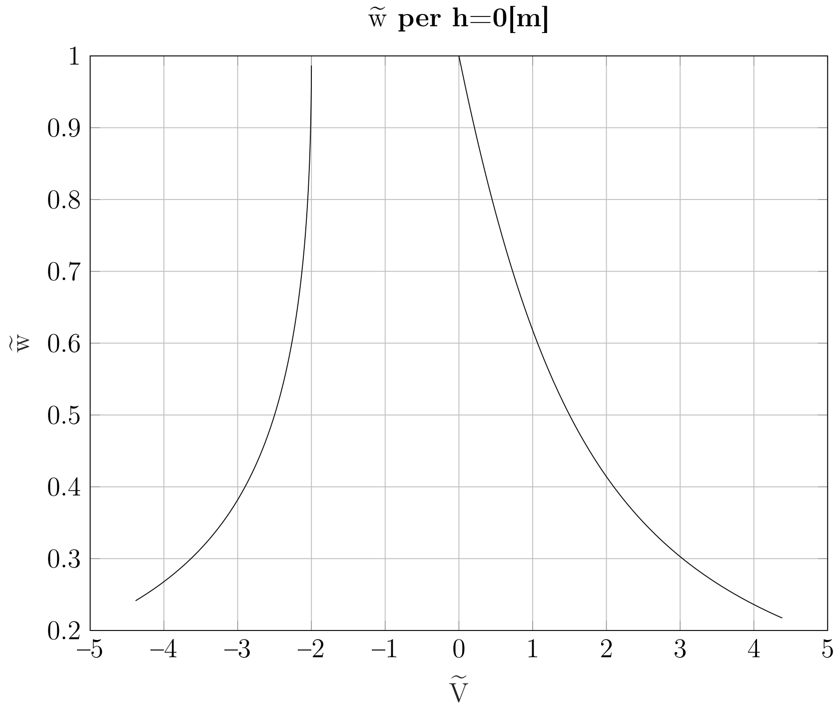
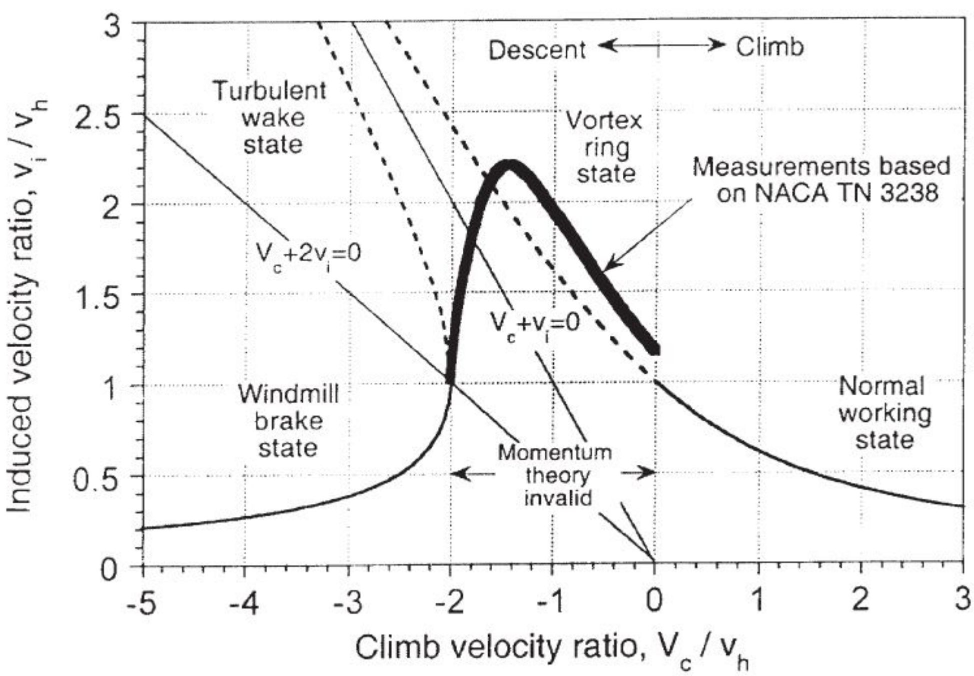


Figure 1.4: *Induction*



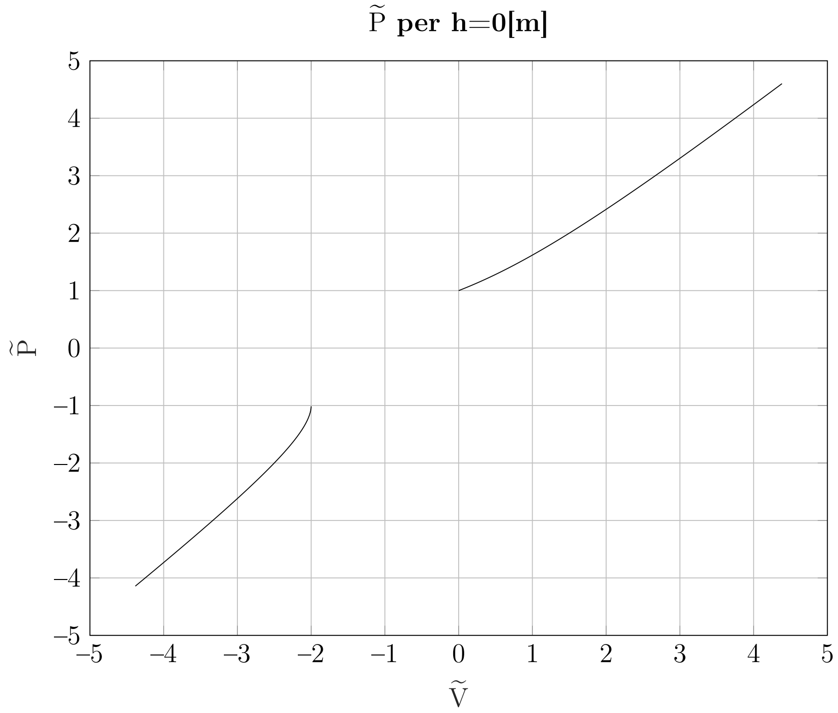
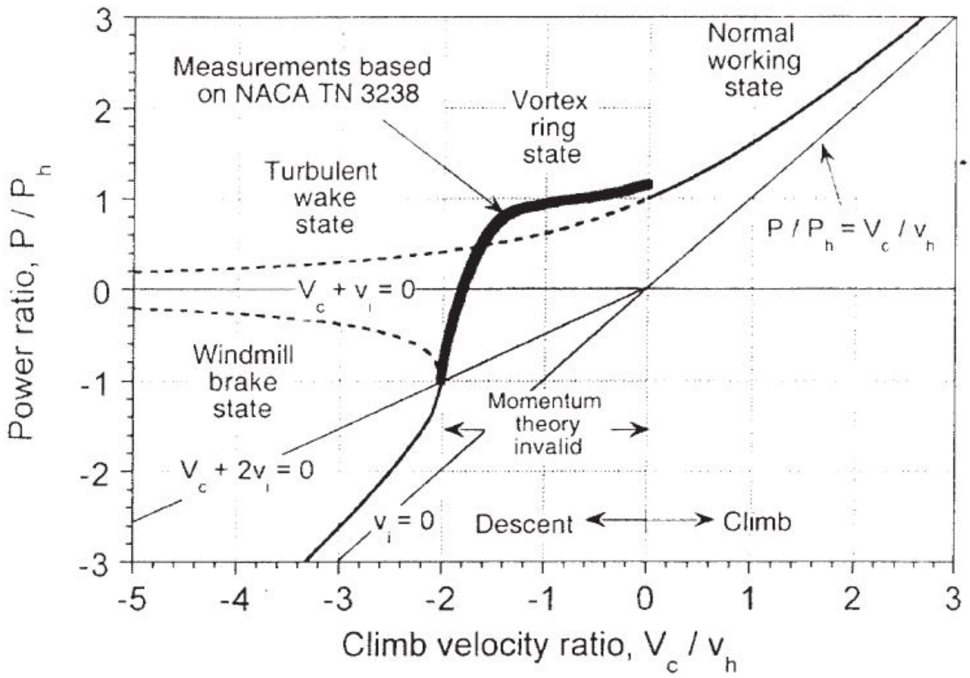


Figure 1.5: *Power*



Appendices

Appendix A

Function's code

```
%% \Axial_Descent_Ascent_Operating_Curves_Rotor.m
%  \brief: the function plots w(V_infty) and P(V_infty) curves according to
%  Impulsive theory.
%  aerodynamic model
%  \author: ColledÃ  Moreno, Veneruso Salvatore
%  \version: 1.00
%
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%
--------------------------------------------------------------------------------
% Name        : Axial_Descent_Ascent_Operating_Curves_Rotor.m
% Author      : ColledÃ  Moreno, Veneruso Salvatore
%               University of Naples Federico II.
% Version     : 1.00
% Date        : 21/12/2020
% Modified    : 21/12/2020
% Description : the function plots w(V_infty) and P(V_infty) curves according to
%                Rotor Simply Impulsive theory.
% Reference   : Renato Tognaccini. Appunti Aerodinamica dell'ala rotante.
%                UniversitÃ  degli studi di Napoli Federico II. a.a.2020/2021
% Input       : * the inputs must be Mass of rotorcraft and radius of rotor
% Output      : w(V_infty) and P(V_infty) plots
% Note        :
--------------------------------------------------------------------------------
function [Power,Induction] =Axial_Descent_Ascent_Operating_Curves_Rotor(M,R,V_inf)
g = 9.81;
%Altitude Range
hh    = linspace(0,6000,100);
%Axial Induction
[~, ~, ~, rho1] = atmosisa(hh);
rho = @(h) interp1(hh,rho1,h, 'pchip');
wh  = @(h) sqrt((M*g)/(2*rho(h)*pi*R^2));

% Non-Dimensional Variables Definition
V_tilde = @(V,h) V/wh(h);
w_tilde_salita  = @(V,h) (-V_tilde(V,h)/2) + sqrt((V_tilde(V,h)/2)^2+1);
w_tilde_discesa = @(V,h) (-V_tilde(V,h)/2) - sqrt((V_tilde(V,h)/2)^2-1);
P_tilde_salita   = @(V,h) V_tilde(V,h) + w_tilde_salita(V,h);
P_tilde_discesa  = @(V,h) V_tilde(V,h) + w_tilde_discesa(V,h);

if nargin==2
    VVs    = linspace(0, 50, 100);
    VVd    = linspace(-50, 0, 7000);

    %Non Dimensional Induction

    WTS = zeros(length(hh),length(VVs)); %Ascent Induction
    WTD = zeros(length(hh),length(VVd)); %Descent Induction
    aa  = zeros(1,length(hh));           %Control Parameter

    % Matrices fill
    for i = 1 : length(hh)
        for j = 1 : length(VVs)
            WTS(i,j) = w_tilde_salita(VVs(j),hh(i));
        end
    end

    for i = 1 : length(hh)
        for j = 1 : length(VVd)
```

```
        if V_tilde(VVd(j),hh(i)) <= -2            %Validity limit of simply impulsive theory;
            WTD(i,j) = w_tilde_discesa(VVd(j),hh(i));
            aa(1,i) = j;
        else
            WTD(i,j) = 0;
        end
    end
end

%Non-Dimensional Power
PTS = zeros(length(hh),length(VVs)); %Ascent Power
PTD = zeros(length(hh),length(VVd)); %Descent Power
bb  = zeros(1,length(hh));           %Control Paramenter

% Matrices fill
for i = 1 : length(hh)
    for j = 1 : length(VVs)
        PTS(i,j) = P_tilde_salita(VVs(j),hh(i));
    end
end

for i = 1 : length(hh)
    for j = 1 : length(VVd)
        if V_tilde(VVd(j),hh(i)) <= -2            %Validity limit of simply impulsive theory;
            PTD(i,j) = P_tilde_discesa(VVd(j),hh(i));
            bb(1,i) = j;
        else
            break
        end
    end
end

%% 3D Plots of Induction [output]
figure(1)
plot3(hh(1)*ones(1,length(WTS(1,:))),VVs,WTS(1,:),'k')
hold on
for i = 2 : length(hh)
    plot3(hh(i)*ones(1,length(WTS(i,:))),VVs,WTS(i,:),'k')
end
for i = 1 : length(hh)
    plot3(hh(i)*ones(1,length(WTD(i,1:aa(i)))),VVd(1,1:aa(i)),WTD(i,1:aa(i)),'k')
end
yticks([-50 -25 0 25 50])
xlabel('$Height[m]$','Interpreter','latex', 'FontSize',15)
ylabel('$V_{\infty}[\frac{m}{s}]$','Interpreter','latex', 'FontSize',15)
zlabel('$\widetilde{w}$','Interpreter','latex', 'FontSize',15)
grid on
title('$w (V_{\infty}, h)$','Interpreter','latex', 'FontSize', 15)
view(71,32)

% 3D Plots of Power [output]
figure(2)
plot3(hh(1)*ones(1,length(PTS(1,:))),VVs,PTS(1,:),'k')
hold on
for i = 2 : length(hh)
    plot3(hh(i)*ones(1,length(PTS(i,:))),VVs,PTS(i,:),'k')
end
for i = 1 : length(hh)
    plot3(hh(i)*ones(1,length(PTD(i,1:bb(i)))),VVd(1,1:bb(i)),PTD(i,1:bb(i)),'k')
end
yticks([-50 -25 0 25 50])
xlabel('$Height[m]$','Interpreter','latex', 'FontSize',15)
ylabel('$V_{\infty}[\frac{m}{s}]$','Interpreter','latex', 'FontSize',15)
zlabel('$\widetilde{P}$','Interpreter','latex', 'FontSize',15)
grid on
title('$P (V_{\infty}, h)$','Interpreter','latex', 'FontSize', 15)
view(71,32)

%% Insertion of Interes Altitude;

prompt = {'Insert interest altitude in metres [min=0,Max=6000]: '};
dlgtitle = 'Altitude';
dims = [1 35];
answer = inputdlg(prompt,dlgtitle,dims);
hnew = str2double(answer{1});

WTSnew = zeros(1,length(VVs));  %Vector Inizialization of axial ascent induction related to velocity
WTDnew = zeros(1,length(VVd));  %Vector Inizialization of axial descent induction related to velocity

% Matrices fill
for j = 1 : length(VVs)
    WTSnew(1,j) = w_tilde_salita(VVs(j),hnew);
end

for j = 1 : length(VVd)
    if V_tilde(VVd(j),hnew) <= -2                 %Validity limit of simply impulsive theory;
        WTDnew(1,j) = w_tilde_discesa(VVd(j),hnew);
        aanew = j;
    else
        break
    end
end

PTSnew = zeros(1,length(VVs));
PTDnew = zeros(1,length(VVd));


for j = 1 : length(VVs)
    PTSnew(1,j) =  P_tilde_salita(VVs(j),hnew);
end

for j = 1 : length(VVd)
    if V_tilde(VVd(j),hnew) <= -2                 %Validity limit of simply impulsive theory;
```

```
            PTDnew(1,j) = P_tilde_discesa(VVd(j),hnew);
            bbnew = j;
        else
            break
        end
    end

    %% 2D Plots of induction [Output]
    figure(3)
    plot(V_tilde(VVs,hnew), WTSnew, '-k')
    hold on
    plot(V_tilde(VVd(1:aanew),hnew), WTDnew(1:aanew), '-k')
    grid on
    xlabel('$\widetilde{V}$','Interpreter','latex','FontSize',15)
    ylabel('$\widetilde{w}$','Interpreter','latex','FontSize',15)
    title(['$\widetilde{w}$ per h=' num2str(hnew) '[m]'],'Interpreter','latex')

    % 2D Plots of power [Output]
    figure(4)
    plot(V_tilde(VVs,hnew), PTSnew, '-k')
    hold on
    plot(V_tilde(VVd(1:aanew),hnew), PTDnew(1:bbnew), '-k')
    grid on
    xlabel('$\widetilde{V}$','Interpreter','latex','FontSize',15)
    ylabel('$\widetilde{P}$','Interpreter','latex','FontSize',15)
    title(['$\widetilde{P}$ per h=' num2str(hnew) '[m]'],'Interpreter','latex')
    %% Numerical Output of Power and Induction for the interest altitude;
    Power = [PTDnew(1:bbnew), PTSnew];
    Induction = [WTDnew(1:aanew), WTSnew];

elseif nargin==3 % In Output only value of Power and Induction
                 % at altitude and velocity of interest;
    prompt = {'Insert interest altitude in metres [min=0,Max=6000]: '};
    dlgtitle = 'Altitude';
    dims = [1 35];
    answer = inputdlg(prompt,dlgtitle,dims);
    hnew = str2double(answer{1});

    if V_tilde(V_inf,hnew) >= 0
        w = w_tilde_salita(V_inf,hnew)*wh(hnew);
        P   = (M*g)*(V_inf + w);
    elseif V_tilde(V_inf,hnew)<= -2
        w= w_tilde_discesa(V_inf,hnew)*wh(hnew);
        P  = (M*g)*(V_inf + w);
    else
        errordlg('Simply Impulsive Rotor Theory not respected','ERROR!!');
        Power = [];
        Induction = [];
        return
    end
    ff = msgbox(sprintf('Power= %d [kW], Induction= %d [m/s]', P/1000, w),...
        'Power and Induction at the altitude and velocity of interest');
    set(ff, 'position', [500 250 400 50]);

    Power = P;
    Induction = w;
end

end
--------------------------------------------------------------
```

Bibliography

[1] Renato Tognaccini, *Appunti Aerodinamica dell'Ala Rotante*. Univeristà degli studi di Napoli Federico II, a.a. 2020/2021