Eli - TAARG

- Ducted propeller's function - Total thrust

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Contents

1 Algorithm

In this brief document, we will describe the algorithm of the function `ducted_prop_thrust.m` based on the semi - empirical method proposed by McCormick. First, the function that calculates the universal function f is called.

```
% Use function Feature_RVortexInt to evaluate universal function f
f = Feature_RvortexInt(ch,D14,D34);
% -----------------------------------------------------------------
```

Listing 1: Calling the `Feature_RVortexInt.m` function

Then, the Propeller's induced velocity is calculated.

```
% eq 4.24
w0 = .5*(-Vinf + sqrt(Vinf^2 + 2*TR/(rho*pi*R^2)));
```

Listing 2: Propeller's induced velocity. `ducted_prop_thrust.m`

Is now possible to evaluate the elicoidal vortex induced axial velocity at $(3/4)c$.

```
% eq 3.15
w34 = w0*[1 + (0.75*ch/R)/sqrt(1+(0.75*ch/R)^2)];
```

Listing 3: w at $(3/4)c$. `ducted_prop_thrust.m`

Next, is calculated the rotor's induced radial velocity at $(3/4)c$.

```
% eq 4.23
vi34 = -0.5*(D34/2)*w0*(R^2/((R^2 + (0.25*ch)^2)^1.5));
```

Listing 4: Induced radial velocity v_{iR} at $(3/4)c$. `ducted_prop_thrust.m`

Anular vortex induced velocity is calculated using tangency condition at $(3/4)c$.

```
% eq 4.25
vi = -vi34 - theta*(w34 + Vinf);
```

Listing 5: Anular vortex induced velocity at $(3/4)c$. `ducted_prop_thrust.m`

Next, the circulation around the shroud is calculated.

```
% eq. 4.26
Gamma = (pi*D14/f)*[-vi34 - theta*(Vinf + w34)];
```

Listing 6: Circulation around the shroud Γ. `ducted_prop_thrust.m`

Rotor's induced radial velocity at $c/4$ is calculated.

```
% eq 4.23
vi14 = -0.5*(D14/2)*w0*(R^2/((R^2 + (-0.25*ch)^2)^1.5));
```

Listing 7: Induced radial velocity v_{iR} at $c/4$. `ducted_prop_thrust.m`

At the end of the code, thrust's calculations are performed.

```
% -----------------------------------------------------------------
% Thrust component due to the shroud
TS = -rho*vi14*Gamma*pi*D14;
% -----------------------------------------------------------------
% Total thrust
T = TR + TS;
% -----------------------------------------------------------------
end
```

Listing 8: Thrust's calculations. `ducted_prop_thrust.m`

1.1 Inputs

The function accepts the following inputs:

- flow density ρ;
- shroud chord c;
- stream velocity V_∞;
- quarter diameter D_{14};
- $(3/4)c$ diameter D_{34};
- tangency condition's angle θ
- rotor's radius R;
- free rotor's thrust T_R.

1.2 Outputs

The function generates the following outputs:

- total thrust T_{total};
- shroud thrust T_S.

A test case for the function `ducted_prop_thrust.m` is shown, with relative outputs.

```
% ==========================================
% |Input        :                           |
% |                (rho) = 1.225  [kg/m^3]  |
% |                (ch) = 2.40    [m]       |
% |                (Vinf) = 35    [m/s]     |
% |                (D14) = 1.50   [m]       |
% |                (D34) = 1.50   [m]       |
% |                (theta) = 0    [deg]     |
% |                (R) = 0.75     [m]       |
% |                (TR) = 500     [N]       |
% |Output       : (T) = 566.3129 [N]       |
% |                (TS) = 66.3129 [N]       |
% |Note         :                           |
% ==========================================

rho   = 1.225;
ch    = 2.40;
Vinf  = 35;
D14   = 1.5;
D34   = 1.5;
theta = 0;
R     = 0.75;
TR    = 500;

[T,TS] = ducted_prop_thrust(rho,Gamma,Vinf,Dquarter,R,TR)
```

Listing 9: Test case for the `ducted_prop_thrust.m`

1.3 Use of the function

This function must be used in conjunction with another program that provides ring vortex circulation and, thus, the isolated rotor's thrust; in this case, the function `Feature_RVortexInt.m` has been used. Here is also shown a MATLAB workspace output when the function is launched.

2 Code listing

```
%% \ducted_prop_thrust.m
%  \brief:  A function that calculates total thrust of a ducted propeller.
%   It generates a vector with total thrust and shroud thrust as output.
%  \author: Claudio Mirabella, Christian Salzano
%  \version: 1.04
%
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%
% ======================================================================
% |Name        : ducted_prop_thrust.m                                  |
% |Author      : Claudio Mirabella, Christian Salzano                  |
% |              University of Naples Federico II.                     |
% |Version     : 1.04                                                  |
% |Date        : 25/11/2020                                            |
% |Modified    : 18/02/2021                                            |
% |Description : A function that calculates total thrust of a         |
% |              ducted propeller.                                     |
% |              It generates a vector with total thrust and           |
% |              shroud thrust as output.                              |
% |Reference   : Lezioni di Aerodinamica dell'Ala Rotante              |
% |Input       :                                                       |
% |              (rho) = Density                                       |
% |              (ch) = Chord of the shroud                            |
% |              (Vinf) = Stream velocity                              |
% |              (D14) = Shroud diameter at c/4                        |
% |              (D34) = Shroud diameter at 3c/4                       |
% |              (theta) = Angle of tangent between velocity and mean  |
% |                        camber line at 3c/4                         |
% |              (R) = Rotor radius                                    |
% |              (TR) = Isolated rotor thrust                          |
% |Output      : (T) = Total thrust generated                          |
% |              (TS) = Thrust generated by the shroud                 |
% |Note        :                                                       |
% =======================================================================
```
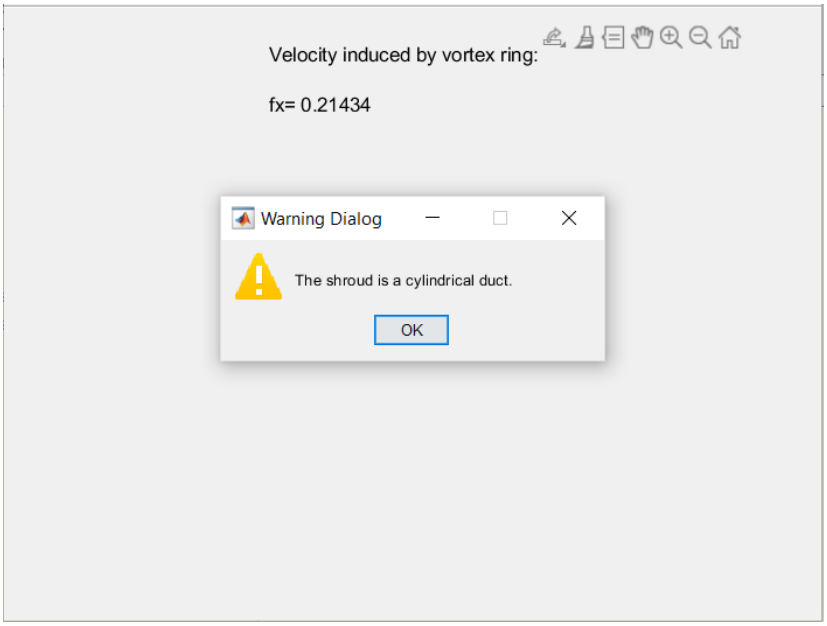
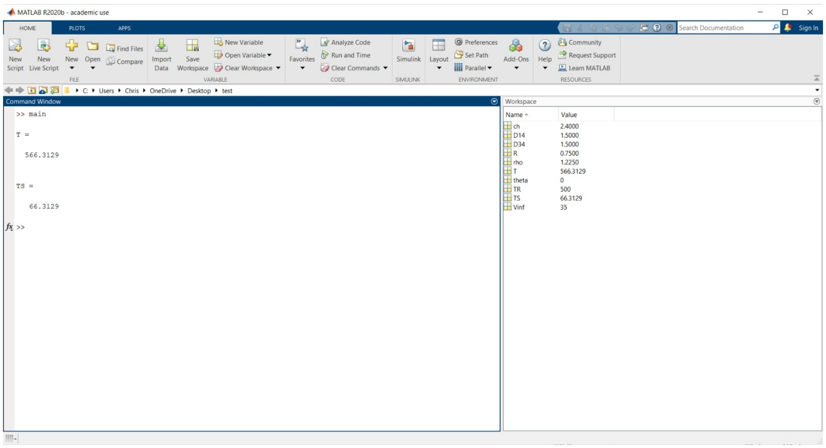

```
function [T, TS] = ducted_prop_thrust(rho, ch, Vinf, D14, D34, theta, R, TR)
% Use function Feature_RVortexInt to evaluate universal function f
f = Feature_RvortexInt(ch,D14,D34);
% -------------------------------------------------------------------
% Propeller's induced velocity calculations
% eq 4.24
w0 = .5*(-Vinf + sqrt(Vinf^2 + 2*TR/(rho*pi*R^2)));
% -------------------------------------------------------------------
% Elicoidal vortex induced axial velocity at 3c/4
% eq 3.15
w34 = w0*[1 + (0.75*ch/R)/sqrt(1+(0.75*ch/R)^2)];
% -------------------------------------------------------------------
% Rotor's induced radial velocity at 3c/4
% eq 4.23
vi34 = -0.5*(D34/2)*w0*(R^2/((R^2 + (0.25*ch)^2)^1.5));
% -------------------------------------------------------------------
% Anular vortex induced velocity calculated using tangency condition at
% 3c/4
% eq 4.25
vi = -vi34 - theta*(w34 + Vinf);
% -------------------------------------------------------------------
% Circulation around the shroud
% eq. 4.26
Gamma = (pi*D14/f)*[-vi34 - theta*(Vinf + w34)];
% -------------------------------------------------------------------
% Rotor's induced radial velocity at c/4
% eq 4.23
vi14 = -0.5*(D14/2)*w0*(R^2/((R^2 + (-0.25*ch)^2)^1.5));
% -------------------------------------------------------------------
% Thrust component due to the shroud
TS = -rho*vi14*Gamma*pi*D14;
% -------------------------------------------------------------------
% Total thrust
T = TR + TS;
% -------------------------------------------------------------------
end
```

Listing 10: Function `ducted_prop_thrust.m`



(a) *Output from the anular's vortex function calculation.*



(b) *Final output.*

Figure 1: Workspace output when the function `ducted_prop_thrust.m` is launched.

Listings

References

[1] Tognaccini Renato. *Lezioni di Aerodinamica dell'ala rotante*. Università degli Studi Ferico II, 2020.