DOCUMENTATION OF TURBINE
DARRIEUS TUBO DI FLUSSO SINGOLO
FUNCTION

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Theory

This function implements the single-streamtubes theory. The principal features of the vertical-axis turbine will be elicited by considering a two-dimensional rotor.

The determination of the induction factor *a* is connected with the assumed functional dependence of *a* with the azimuth angle ϕ. In this approach, *a* is assumed to be independent of ϕ.

From the momentum equation, the force in wind direction D can be derived as:

$$D = 2R\rho 2a(1-a)V^2$$

From the forces on the blade elements of a B-bladed rotor, averaged during one revolution, it follows that:

$$D = \frac{B}{2\pi}\int_0^{2\pi} \frac{1}{2}\rho {V_\infty}^2 c C_l \cos(\phi + \alpha) d\phi$$

For a given geometry of the turbine and introducing the variable *y* with a given value, *a* can be calculated from the following equation:

$$\frac{1}{1-a} = 1 + \frac{Bc}{8R2\pi}\int_0^{2\pi} \{(y+\sin\phi)^2 + cos^2\phi\}\ C_l\ cos\left\{\phi + arc\ tg\ \left[\frac{\cos\phi}{y+\sin\phi}\right]\right\} d\phi$$

With C_l is equal to:

$$C_l = C_{l\alpha} arctan \frac{cos\phi}{y + sin\phi}$$

And the corresponding tip speed ratio can be calculated:

$$\lambda = y(1-a)$$

The generated power is calculated from the tangential force, in which the profile drag is included:

$$P = \Omega R \frac{N}{2\pi}\int_0^{2\pi} \frac{1}{2}\rho V^2 (C_l\ sin\alpha - C_d cos\alpha)\ d\phi$$

The corresponding power coefficient:

$$C_p = \frac{Nc}{4\pi R}\lambda \int_0^{2\pi} \left(\frac{V}{V_\infty}\right)^2 C_l\ sin\alpha(1 - \frac{C_d}{C_l} cot\alpha)\ d\phi$$

Input and Output

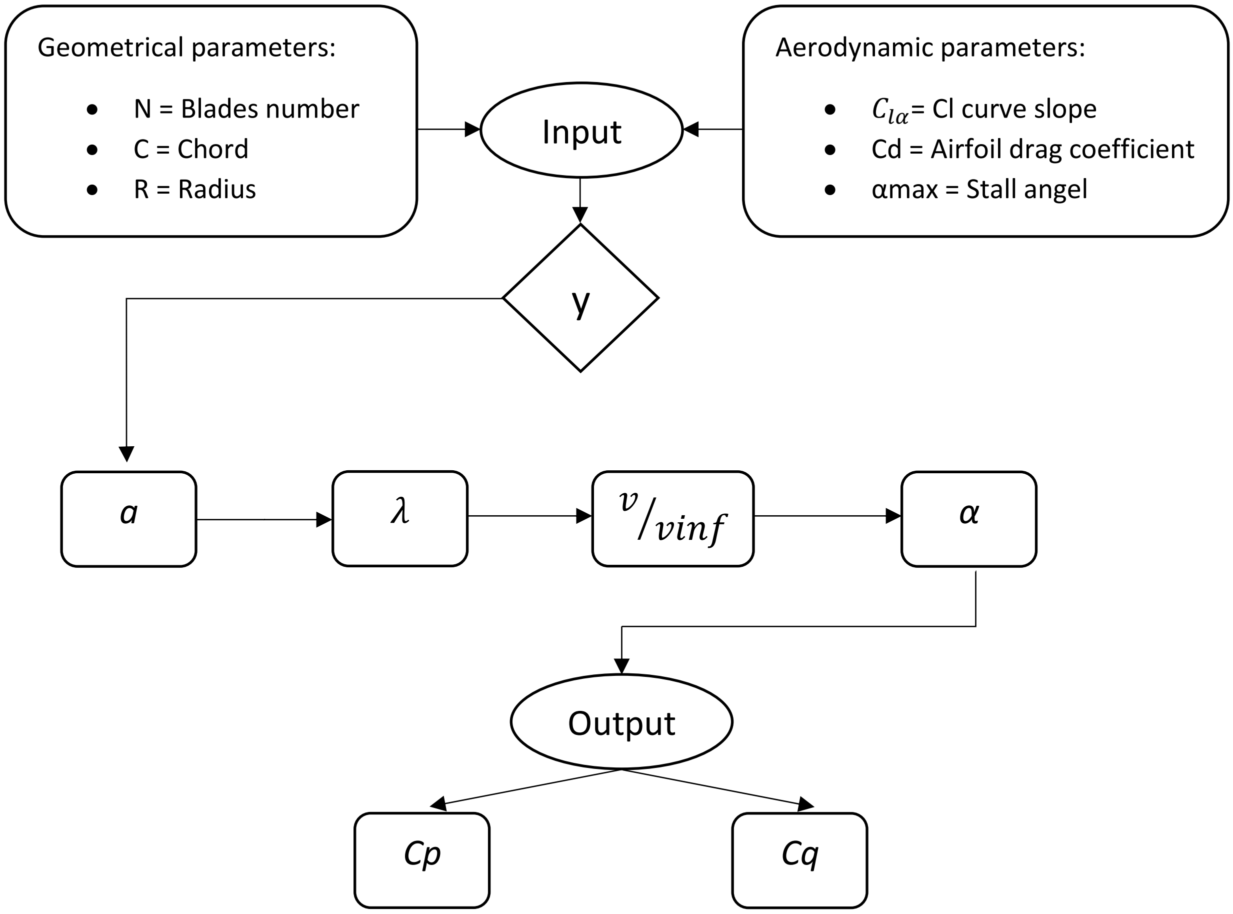

Figure 1. Flow chart of the function code.

Algorithm description

The aim of the following paragraph is to describe the code in every details. First of all, the function receives in input the following parameters:

- $Cl\alpha$, that is the slope of the Cl-α curve of the profile.
- c, that is the chord of the profile.
- R, that is the radius of the rotor.
- N, that is number of blades.
- C_d, that is the drag coefficient.
- α_{max}, that is the angle of stall of the profile.

At the beginning of the function an azimuth angle domain and variable *y* domain are created. The *y* variable is used as proxy variable to evaluate the induction *a*.

```
function [cp2, lambda2] = Turbine_single_tube(Cla, c, R, N, cd,alphamax)

%Domain vector, initial values.
y1=linspace(0.0,20,100);        %Introduction of the variable y
phiv = linspace(0,360,100);     %Phi domain (deg)
phiv = deg2rad(phiv);           %Phi domain (rad)
alphamax = deg2rad(alphamax);
```

Inside of the *for* cycle the variables *v/vinf*, α, *Cp* and *Cq*, considering that they change when ϕ changes, are evaluated starting from the dependence of the induction compared with the tip speed.

```
for i = 1:length(y1)            % a, lambda, cp, cq computing

    y=y1(i);

%   Computing of the induction factor a
   Y= ((y-sin(phiv)).^2+cos(phiv).^2).*Cla.*atan(cos(phiv)./(y+sin(phiv))).*...
        cos(phiv+atan(cos(phiv)./(y+sin(phiv))));
    A =   trapz(phiv,Y);
    x = 1 + (N.*c./(8.*R.*2.*pi)).*A;
    a= 1-1/x;

%   Determination of tip speed ratio
    lambda = y.*(1-a);

%   Determination of velocity ratio
      v_vinf = sqrt((lambda + (1-a).*sin(phiv)).^2 +...
          (1-a).^2 .*cos(phiv).^2);
if a < 0.5

%   Determination of angle of attack
       alpha = atan2(((1-a).*cos(phiv)),(lambda + (1-a).*sin(phiv)));

if alpha < alphamax

       Lambda(i,1)=lambda;

%   Computing Cp - numerical integration
       cost_p = (N*c*lambda)./(4*pi*R);             % Costant part
       cpint =trapz(phiv,(v_vinf.^2.*Cla.*alpha.*sin(alpha).*...
           (1-(cd./(Cla.*alpha)).*cot(alpha))));    % Integral
       cp=cost_p.*cpint;                            % Cp value
       cq = cp/lambda;                              % Cq value

       %Cq and Cp vectors
       Cp(i,1)=cp;
       Cq(i,1)=cq;
end
end
end
```

The trapezoidal numerical integration is used to resolve the integrals. There is a control on the minimum tip speed because the alpha of the blade elements has always to be smaller than alpha max, otherwise the wing will not work properly and will not generate any power.

Test Case

In order to validate the function, some test cases have been run. The obtained results have been compared with the values of De Vries. The table 1 shows the whole parameters taken into account to carry out validation procedure.

Test	σ	α_{max}	c	R	$Cl\alpha$	N	C_d
1	0.1	14	1	30	6.28	3	0.01
2	0.1	14	1	30	6.28	3	0
3	0.2	14	1	15	6.28	3	0.01
4	0.2	14	1	15	6.28	3	0

Table 1. Test case values

Test 1

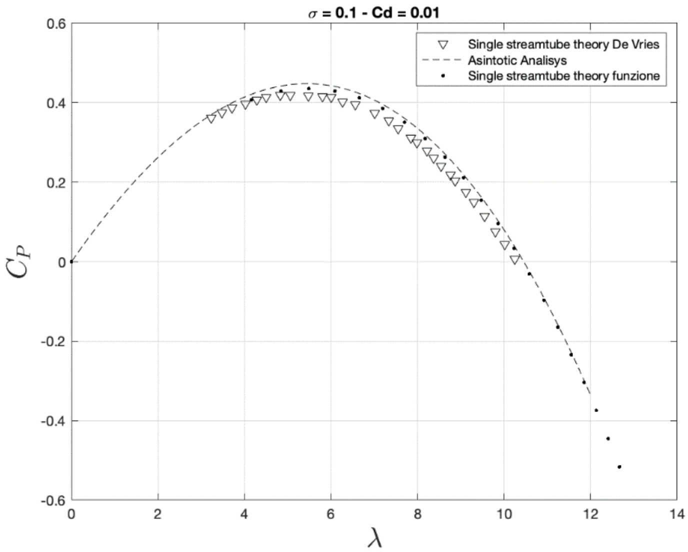

Figure 2. Values compared with single streamtube theory from De Vries and with Asintotic Analisys

Test 2

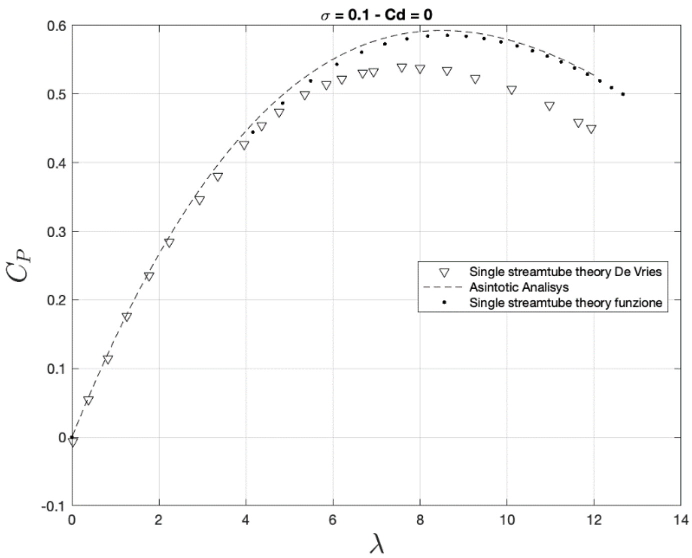

Figure 3. Values compared with single streamtube theory from De Vries and with Asintotic Analisys

Test 3

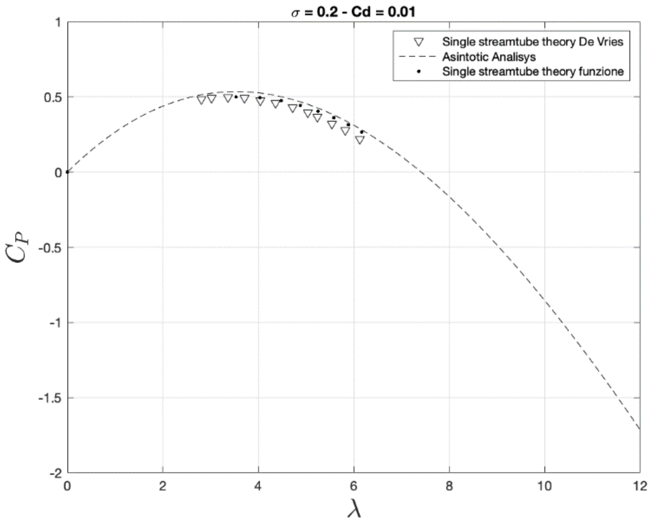

Figure 4. Values compared with single streamtube theory from De Vries and with Asintotic Analisys

Test 4

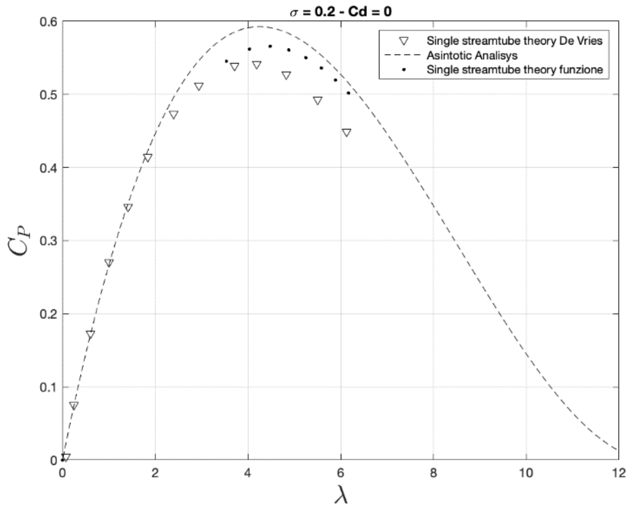

Figure 5. Values compared with single streamtube theory from De Vries and with Asintotic Analisys

Bibliography

[1] Tognaccini R., (2019), “Lezioni di Aerodinamica dell’ala rotante”.

[2]. De Vrues O., (1979), “Fluid Dynamic Aspects of Wing Energy Conversion”,