

Arrays and Vectors

Victor Eijkhout, Susan Lindsey

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Vectors

What are vectors?

- Linear storage of items:
(often called 'array' but that has other meanings)
 - items of any type (but the same for all elements of one vector)
 - potentially very many items
- Indexed set of items
- ... but if you don't need the index: collection of items

Short vectors

Short vectors can be created by enumerating their elements:

```
#include <vector>
using std::vector

vector<int> evens{0,2,4,6,8};
vector<float> halves = {0.5, 1.5, 2.5};
vector<Point> diagonal =
    { {0.,0.}, {1.,1.}, {2.,2.}, {3.,3.} };
```

Range over elements

You can write a range-based for loop, which considers the elements as a collection.

```
for ( float e : array )  
    // statement about element e  
for ( auto e : array )  
    // same, with type deduced by compiler
```

Code:

```
vector<int> numbers = {1,4,2,6,5};  
int tmp_max = -2000000000;  
for (auto v : numbers)  
    if (v>tmp_max)  
        tmp_max = v;  
cout << "Max: " << tmp_max  
    << " (should be 6)" << endl;
```

Output

[array] dynamicmax:

Max: 6 (should be 6)

Exercise 1

Find the element with maximum absolute value in a vector. Use:

```
vector<int> numbers = {1,-4,2,-6,5};
```

Hint:

```
#include <cmath>
..
absx = abs(x);
```

Range over vector denotation

Code:

```
for ( auto i : {2,3,5,7,9} )  
    cout << i << ", ";  
cout << endl;
```

Output

[array] rangedenote:

2,3,5,7,9,

Vector definition

Definition and/or initialization:

```
#include <vector>
using std::vector;

vector<type> name;
vector<type> name(size);
vector<type> name(size, init_value);
```

where

- *vector* is a keyword,
- type (in angle brackets) is any elementary type or class name,
- name of the vector is up to you, and
- size is the (initial size of the vector). This is an integer, or more precisely, a `size_t` parameter.
- Initialize all elements to `init_value`.

Accessing vector elements

Square bracket notation (zero-based):

Code:

```
vector<int> numbers = {1,4};  
numbers[0] += 3;  
numbers[1] = 8;  
cout << numbers[0] << ", "  
      << numbers[1] << endl;
```

Output

[array] assignbracket:

4,8

With bound checking:

Code:

```
vector<int> numbers = {1,4};  
numbers.at(0) += 3;  
numbers.at(1) = 8;  
cout << numbers.at(0) << ", "  
      << numbers.at(1) << endl;
```

Output

[array] assignatfun:

4,8

Safer, slower.

(Remember Knuth about optimization.)

Vector elements out of bounds

Square bracket notation:

Code:

```
vector<int> numbers = {1,4};  
numbers[-1] += 3;  
numbers[2] = 8;  
cout << numbers[0] << ", "  
      << numbers[1] << endl;
```

Output

[array] assignoutof-
boundbracket:

1,4

With bound checking:

Code:

```
vector<int> numbers = {1,4};  
numbers.at(-1) += 3;  
numbers.at(2) = 8;  
cout << numbers.at(0) << ", "  
      << numbers.at(1) << endl;
```

Output

[array] assignoutof-
boundatfun:

libc++abi.dylib: terminating wi

Safer, slower.

(Remember Knuth about optimization.)

Range over elements by reference

Range-based loop indexing makes a copy of the vector element. If you want to alter the vector, use a reference:

```
for ( auto &e : my_vector)
    e = ....
```

Code:

```
vector<float> myvector
    = {1.1, 2.2, 3.3};
for ( auto &e : myvector )
    e *= 2;
cout << myvector.at(2) << endl;
```

Output

[array] vectorrangeref:

6.6

(Can also use `const auto& e` to prevent copying, but also prevent altering data.)

Indexing the elements

You can write an indexed for loop, which uses an index variable that ranges from the first to the last element.

```
for (int i= /* from first to last index */ )  
    // statement about index i
```

Example: find the maximum element in the array, and where it occurs.

Code:

```
int tmp_idx = 0;  
int tmp_max = numbers.at(tmp_idx);  
for (int i=0; i<numbers.size(); i++) {  
    int v = numbers.at(i);  
    if (v>tmp_max) {  
        tmp_max = v; tmp_idx = i;  
    }  
}  
  
cout << "Max: " << tmp_max  
      << " at index: " << tmp_idx <<
```

Output

[array] vecidxmax:

Max: 6.6 at index: 3

A philosophical point

Conceptually, a *vector* can correspond to a set of things, and the fact that they are indexed is purely incidental, or it can correspond to an ordered set, and the index is essential. If your algorithm requires you to access all elements, it is important to think about which of these cases apply, since there are two different mechanism.

Exercise 2

Find the location of the first negative element in a vector.

Which mechanism do you use?

Exercise 3

Create a vector x of `float` elements, and set them to random values.

Now normalize the vector in L_2 norm and check the correctness of your calculation, that is,

1. Compute the L_2 norm of the vector:

$$\|v\| \equiv \sqrt{\sum_i v_i^2}$$

2. Divide each element by that norm;
3. The norm of the scaled vector should now be 1. Check this.
4. Bonus: your program may be printing 1, but is it actually 1? Investigate.

What type of loop are you using?

Indexing with pre/post increment

Indexing in `while` loop and such:

```
x = a.at(i++); /* is */ x = a.at(i); i++;  
y = b.at(++i); /* is */ i++; y = b.at(i);
```


Vector copy

Vectors can be copied just like other datatypes:

Code:

```
vector<float> v(5,0), vcopy;  
v.at(2) = 3.5;  
vcopy = v;  
vcopy.at(2) *= 2;  
cout << v.at(2) << ", "  
      << vcopy.at(2) << endl;
```

Output

[array] vectorcopy:

3.5,7

Vector methods

A vector is an object, with methods.

Given `vector<sometype> x`:

- Get elements, including bound checking, with `ar.at(3)`. Note: (zero-based indexing).
- (also get elements with `ar[3]`: see later discussion.)
- Size: `ar.size()`.
- Other functions: `front`, `back`, `empty`.

Your first encounter with templates

`vector` is a 'templated class': `vector<X>` is a vector-of-X.

Code behaves as if there is a class definition for each type:

```
class vector<int> {  
public:  
    size(); at(); // stuff  
}
```

```
class vector<float> {  
public:  
    size(); at(); // stuff  
}
```

Actual mechanism uses templating: the type is a parameter to the class definition. More later.

Dynamic behaviour

Dynamic vector extension

Extend a vector's size with `push_back`:

Code:

```
vector<int> mydata(5,2);  
mydata.push_back(35);  
cout << mydata.size() << endl;  
cout << mydata[mydata.size()-1] << endl  
;
```

Output

[array] vectorend:

6
35

Similar functions: *pop_back*, *insert*, *erase*.

Flexibility comes with a price.

When to push back and when not

Known vector size:

```
int n = get_inputsize();
vector<float> data(n);
for ( int i=0; i<n; i++ ) {
    auto x = get_item(i);
    data.at(i) = x;
}
```

Unknown vector size:

```
vector<float> data;
float x;
while ( next_item(x) ) {
    data.push_back(x);
}
```

If you have a guess as to size: `data.reserve(n)`.

Filling in vector elements

You can push elements into a vector:

```
vector<int> flex;  
/* ... */  
for (int i=0; i<LENGTH; i++)  
    flex.push_back(i);
```

If you allocate the vector statically, you can assign with at:

```
vector<int> stat(LENGTH);  
/* ... */  
for (int i=0; i<LENGTH; i++)  
    stat.at(i) = i;
```

Filling in vector elements

With subscript:

```
vector<int> stat(LENGTH);  
/* ... */  
for (int i=0; i<LENGTH; i++)  
    stat[i] = i;
```

You can also use new to allocate*:

```
int *stat = new int[LENGTH];  
/* ... */  
for (int i=0; i<LENGTH; i++)  
    stat[i] = i;
```

*Considered back practice. Do not use.

Timing the ways of filling a vector

Flexible time: 2.445

Static at time: 1.177

Static assign time: 0.334

Static assign time to new: 0.467

Vectors and functions

Vector as function argument

You can pass a vector to a function:

```
double slope( vector<double> v ) {  
    return v.at(1)/v.at(0);  
};
```

Vectors, like any argument, are passed by value, so the vector is actually copied into the function.

Vector pass by value example

Code:

```
void set0  
    ( vector<float> v,float x )  
{  
    v.at(0) = x;  
}  
/* ... */  
vector<float> v(1);  
v.at(0) = 3.5;  
set0(v,4.6);  
cout << v.at(0) << endl;
```

Output

[array] vectorpassnot:

3.5

Vector pass by reference

If you want to alter the vector, you have to pass by reference:

Code:

```
void set0  
    ( vector<float> &v, float x )  
{  
    v.at(0) = x;  
}  
/* ... */  
vector<float> v(1);  
v.at(0) = 3.5;  
set0(v,4.6);  
cout << v.at(0) << endl;
```

Output

[array] vectorpassref:

4.6

Exercise 4

Revisit exercise 3 and introduce a function for computing the L_2 norm.

Vector as function return

You can have a vector as return type of a function.

Example: this function creates a vector, with the first element set to the size:

Code:

```
vector<int> make_vector(int n) {  
    vector<int> x(n);  
    x.at(0) = n;  
    return x;  
}  
  
/* ... */  
vector<int> x1 = make_vector(10);  
// "auto" also possible!  
cout << "x1 size: " << x1.size() <<  
    endl;  
cout << "zero element check: " << x1.  
    at(0) << endl;
```

Output

[array] vectorreturn:

x1 size: 10
zero element check: 10

Exercise 5

Write code to take a vector of integers, and construct two vectors, one containing all the odd inputs, and one containing all the even inputs. So:

input:

5,6,2,4,5

output:

5,5

6,2,4

Can you write a function that accepts a vector and produces two vectors as described?

(hints for the next exercise)

```
// high up in your code:  
#include <random>  
using std::rand;  
  
// in your main or function:  
float r = 1.*rand()/RAND_MAX;  
// gives random between 0 and 1
```

Exercise 6

Write functions `random_vector` and `sort` to make the following main program work:

```
int length = 10;
vector<float> values = random_vector(length);
vector<float> sorted = sort(values);
```

This creates a vector of random values of a specified length, and then makes a sorted copy of it.

Instead of making a sorted copy, sort in-place (overwrite original data with sorted data):

```
int length = 10;
vector<float> values = random_vector(length);
sort(values); // the vector is now sorted
```

Find arguments for/against that approach.

(Note: C++ has sorting functions built in.)

Vectors in classes

Can you make a class around a vector?

You may want a class of objects that contain a vector. For instance, you may want to name your vectors.

```
class named_field {  
private:  
    vector<double> values;  
    string name;
```

The problem here is when and how that vector is going to be created.

Create the contained vector

Use initializers for creating the contained vector:

```
class named_field {  
private:  
    vector<int> values;  
public:  
    named_field( int n )  
        : values(vector<int>(n)) {  
    };  
};
```

Less desirable method is creating in the constructor:

```
named_field( int n ) {  
    values = vector<int>(n);  
};
```

Multi-dimensional arrays

Multi-dimensional vectors

Multi-dimensional is harder with vectors:

```
vector<float> row(20);  
vector<vector<float>> rows(10,row);  
// alternative:  
vector<vector<float>> rows(10);  
for ( auto &row : rows )  
    row = vector<float>(20);
```

Create a row vector, then store 10 copies of that:
vector of vectors.

Matrix class

```
class matrix {  
private:  
    vector<vector<double>> elements;  
public:  
    matrix(int m,int n) {  
        elements =  
            vector<vector<double>>(m,vector<double>(n));  
    }  
    void set(int i,int j,double v) {  
        elements.at(i).at(j) = v;  
    };  
    double get(int i,int j) {  
        return elements.at(i).at(j);  
    };  
};
```


Exercise 7

Write `rows()` and `cols()` methods for this class that return the number of rows and columns respectively.

Matrix class; better design

Better idea:

```
class Matrix {  
private:  
    int rows,cols;  
    vector<double> elements;  
private:  
    Matrix( int m,int n )  
        : rows(m),cols(n),  
          elements = vector<double>(rows*cols)  
        {};  
    ...  
    double get(int i,int j) {  
        return elements.at(i*cols+j);  
    }  
};
```

(Old-style solution: use cpp macro)

Exercise 8

In the matrix class of the previous slide, why are m, n stored explicitly, and not in the previous case?

Exercise 9

Add methods such as transpose, scale to your matrix class.
Implement matrix-matrix multiplication.

Pascal's triangle

Pascal's triangle contains binomial coefficients:

Row	1:																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
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where

$$p_{rc} = \binom{r}{c} = \frac{r!}{c!(r-c)!}.$$

The coefficients can easily be computed from the recurrence

$$p_{rc} = \begin{cases} 1 & c \equiv 1 \vee c \equiv r \\ p_{r-1,c-1} + p_{r-1,c} & \end{cases}$$

Exercise 10

- Write a class `pascal` so that `pascal(n)` is the object containing n rows of the above coefficients. Write a method `get(i,j)` that returns the (i,j) coefficient.
- Write a method `print` that prints the above display.
- Write a method `print(int m)` that prints a star if the coefficient modulo m is nonzero, and a space otherwise.

```
      *
     * *
    *  *
   * * * *
  *     *
 * *       * *
*   *   *   *
* * * * * * * *
 *           *
* *           * *
```

- The object needs to have an array internally. The easiest solution is to make an array of size $n \times n$.

Optional exercise 11

Extend the Pascal exercise:

Optimize your code to use precisely enough space for the coefficients.

Turn it in!

- Write a program that accepts:
 1. one integer: the height of the triangle. You should use this to construct a `PascalTriangle` object that contains the binomial coefficients. Then:
 2. a number of modulus with which to print the triangle. A value of zero indicates that your program should stop.

The tester will search for stars in your output and test that you have the right number in each line.

- If you have compiled your program, do a test run:
`coe_pascal yourprogram.cc`
- Is it Submit if it is correct:
`coe_pascal -s yourprogram.cc`
- If you don't manage to get your code working correctly, you can submit as incomplete with
`coe_pascal -i yourprogram.cc`

Static arrays

Array creation

C-style arrays still exist,

```
{  
    int numbers[] = {5,4,3,2,1};  
    cout << numbers[3] << endl;  
}  
  
{  
    int numbers[5]{5,4,3,2,1};  
    numbers[3] = 21;  
    cout << numbers[3] << endl;  
}
```

but you shouldn't use them.

Prefer to use `array` class (not in this course)

or `span` (C++20; very advanced)

Ranging

You can range over static arrays same as for vector