C++ course

Victor Eijkhout, Susan Lindsey

Fall 2022

last formatted: August 28, 2022



1: Basics



Basics



1. Two kinds of files

In programming you have two kinds of files:

- source files, which are understandable to you, and which you create with an editor such as vi or emacs; and
- binary files, which are understandable to the computer, and unreadable to you.

Your source files are translated to binary by a compiler, which 'compiles' your source file.



Exercise 1

Make a file zero.cc with the following lines:

```
#include <iostream>
using std::cout;
int main() {
 return 0;
and compile it. Intel compiler:
icpc -o zeroprogram zero.cc
Run this program (it gives no output):
./zeroprogram
```



2. Anatomy of the compile line

- icpc : compiler. Alternative: use g++ or clang++
- -o zeroprogram : output into a binary name of your choosing
- zero.cc : your source file.



Exercise 2

Add this line:

```
cout << "Hello world!" << "\n";</pre>
```

(copying from the pdf file is dangerous! please type it yourself)

Compile and run again.

(Did you indent the 'hello world' line? Did your editor help you with the indentation?)



3. C++ versions

- The compiler by default uses C++98.
- This course explains C++17. You need tell your compiler about this.
- On isp.tacc.utexas.edu 'icpc' uses this by default.
- On your own machine you need to do

```
g++ -std=c++17 [other options]
```

or



Review quiz 1

True or false?

- 1. The programmer only writes source files, no binaries. /poll "A programmer writes sources, no binaries" "T" "F"
- 2. The computer only executes binary files, no human-readable files.

```
/poll "Computer executes binaries, no readable files" "T" "F"
```



4. Error types

- 1. Syntax or compile-time errors: these arise if what you write is not according to the language specification. The compiler catches these errors, and it refuses to produce a binary file.
- Run-time errors: these arise if your code is syntactically correct, and the compiler has produced an executable, but the program does not behave the way you intended or foresaw. Examples are divide-by-zero or indexing outside the bounds of an array.



5. File names

File names can have extensions: the part after the dot. (The part before the dot is completely up to you.)

- program.cxx or program.cc are typical extensions for C++ sources.
- program.cpp is also used, but there is a possible confusion with 'C PreProcessor'.
- Using program without extension usually indicates an executable. (What happens if you leave the -o myprogram part off the compile line?)



Statements



6. Program statements

- A program contains statements, each terminated by a semicolon.
- 'Curly braces' can enclose multiple statements.
- A statement can correspond to some action when the program is executed.
- Some statements are definitions, of data or of possible actions.
- Comments are 'Note to self', short:



Exercise 3

Take the 'hello world' program you wrote above, and duplicate the hello-line. Compile and run.

Does it make a difference whether you have the two hellos on the same line or on different lines?

Experiment with other changes to the layout of your source. Find at least one change that leads to a compiler error. Can you relate the message to the error?



7. Fixed elements

You see that certain parts of your program are inviolable:

- There are keywords such as return or cout; you can not change their definition.
- Curly braces and parentheses need to be matched.
- There has to be a main keyword.
- The *iostream* and std are usually needed.



Exercise 4

Experiment with the **cout** statement. Replace the string by a number or a mathematical expression. Can you guess how to print more than one thing, for instance:

- the string One third is, and
- the result of the computation 1/3,

with the same cout statement? Do you get anything unexpected?

Review quiz 2

True or false?

- If your program compiles correctly, it is correct.

 /poll "If it compiles it is correct" "T" "F"
- If you run your program and you get the right output, it is correct.

/poll "Program runs, no errors, therefore it is correct" "T" "F"



Variables



8. What's a variable?

Programs usually contain data, which is stored in a variable. A variable has

- a datatype,
- a name, and
- a value.

These are defined in a variable declaration and/or variable assignment.



9. Example variable lifetime

```
int i,j; // declaration
i = 5; // set a value
i = 6; // set a new value
j = i+1; // use the value of i
i = 8; // change the value of i
// but this doesn't affect j:
// it is still 7.
```



10. Variable names

- A variable name has to start with a letter;
- a name can contains letters and digits, but not most special characters, except for the underscore.
- For letters it matters whether you use upper or lowercase: the language is case sensitive.
- Words such as main or return are reserved words.
- Usually i and j are not the best variable names: use row and column, or other meaningful names, instead.
- (Strictly speaking, a variable name can start with an underscore, but it's better not to do that.)



11. Declaration

A variable declaration establishes the name and the type of a variable:

```
int n;
float x;
int n1,n2;
double re_part,im_part;
```



12. Where do declarations go?

Declarations can go pretty much anywhere in your program, but they need to come before the first use of the variable.

Note: it is legal to define a variable before the main program but such global variables are usually not a good idea. Please only declare variables *inside* main (or inside a function et cetera).



Review quiz 3

Which of the following are legal variable names?

```
    mainprogram
/poll "Legal mainprogram?" "T" "F"
    main
/poll "Legal main?" "T" "F"
    Main
/poll "Legal Main?" "T" "F"
    1forall
/poll "Legal iforall?" "T" "F"
    one4all
/poll "Legal one4all?" "T" "F"
    one_for_all
/poll "Legal one for_all?" "T" "F"
```

/poll "Legal onefor{all}?" "T" "F"

7. onefor{all}



13. Datatypes

Variables come in different types;

- We call a variable of type int, float, double a numerical variable.
- Complex numbers will be discussed later.
- For characters: char. Strings are complicated; see later.
- Truth values: bool
- You can make your own types. Later.



Assignments



14. Assignment

Once you have declared a variable, you need to establish a value. This is done in an assignment statement. After the above declarations, the following are legitimate assignments:

```
n = 3;

x = 1.5;

n1 = 7; n2 = n1 * 3;
```

These are not math equations: variable on the lhs gets the value of the rhs.

You see that you can assign both a simple value or an expression.



15. Special forms

Update:

```
x = x+2; y = y/3;
// can be written as
x += 2; y /= 3;
```

Integer add/subtract one:

```
i++; j--; /* same as: */ i=i+1; j=j-1;
```



Review quiz 4

Which of the following are legal? If they are, what is their meaning?

```
1. n = n;
  /poll "Legal 'n = n;'?" "T" "F"
```



16. Initialization syntax

There are (at least) two ways of initializing a variable

```
int i = 5;
int j{6};
```

Note that writing

```
int i;
i = 7;
```

is not an initialization: it's a declaration followed by an assignment.

If you declare a variable but not initialize, you can not count on its value being anything, in particular not zero. Such implicit initialization is often omitted for performance reasons.



Review quiz 5

```
#include <iostream>
using std::cout;
int main() {
   int i;
   int j = i+1;
   cout << j << "\n";
   return 0;
}</pre>
```

What happens?

- 1. Compiler error
- 2. Output: 1
- 3. Output is undefined
- 4. Error message during running the program.

/poll "What happens:" "Compiler error" "Output: 1" "Output undefined" "Runtime error"



17. Floating point constants

• Default: double

• Float: 3.14f or 3.14F

• Long double: 1.221 or 1.22L.



18. Warning: floating point arithmetic

Floating point arithmetic is full of pitfalls.

- Don't count on 3*(1./3) being exactly 1.
- Not even associative.

(See Eijkhout, Introduction to High Performance Computing, chapter 3.)



19. Truth values

So far you have seen integer and real variables. There are also boolean values which represent truth values. There are only two values: true and false.

```
bool found{false};
found = true;
```



Input/Output



20. Terminal output

You have already seen cout:

```
float x = 5;
cout << "Here is the root: " << sqrt(x) << "\n";</pre>
```



21. Terminal input

To make a program run dynamic, you can set starting values from keyboard input. For this, use cin, which takes keyboard input and puts it in a numerical (or string) variable.

```
// add at the top of your program:
using std::cin;

// then in your main:
int i;
cin >> i;
```



22. Quick intro to strings

• Add the following at the top of your file:

```
#include <string>
using std::string;
```

• Declare string variables as

```
string name;
```

And you can now cin and cout them.



(Just what is a console?)





Exercise 5

Write a program that asks for the user's first name, uses cin to read that, and prints something like Hello, Susan! in response.

What happens if you enter first and last name?



Expressions



23. Arithmetic expressions

Expression looks pretty much like in math.
 With integers: 2+3
 with reals: 3.2/7

• Use parentheses to group 25.1*(37+42/3.)

Careful with types.

• There is no 'power' operator: library functions.

Modulus: %



24. Math library calls

Math function in cmath:

```
#include <cmath>
.....
x = pow(3,.5);
```

For squaring, usually better to write x*x than pow(x,2).



Boolean expressions

We'll do that in the lecture on conditionals.



Exercise 6

Write a program that :

- displays the message Type a number,
- accepts an integer number from you (use cin),
- makes another variable that is three times that integer plus one,
- and then prints out the second variable.



Turn it in!

- If you have compiled your program, do: coe_3np1 yourprogram.cc
 where 'yourprogram.cc' stands for the name of your source file.
- Is it reporting that your program is correct? If so, do: coe_3np1 -s yourprogram.cc
 where the -s flag stands for 'submit'.

Note: this will send your file to the instructors with a **time stamp**. If you submit again after the deadline, you will be recorded as a late submission.



25. Conversion and casting

Real to integer: round down:

```
double x, y; x = ....; y = ....;
int i; i = x+y:
```

Dangerous:

```
int i, j; i = ...; j = ...; double x; x = 1+i/j;
```

The fraction is executed as integer division. For floating point result do:

```
(1.*i)/j
```



Optional exercise 7

Write a program that asks for two integer numbers n1,n2.

- Assign the integer ratio n_1/n_2 to an integer variable.
- Can you use this variable to compute the modulus

 $n_1 \mod n_2$

(without using the % modulus operator!) Print out the value you get.

- Also print out the result from using the modulus operator:%.
- Investigate the behaviour of your program for negative inputs.
 Do you get what you were expecting?



Optional exercise 8

Write two programs, one that reads a temperature in Centigrade and converts to Fahrenheit, and one that does the opposite conversion.

$$C = (F - 32) \cdot 5/9, \qquad F = 9/5 C + 32$$

Check your program for the freezing and boiling point of water. (Do you know the temperature where Celsius and Fahrenheit are the same?)

Can you use Unix pipes to make one accept the output of the other?



Review quiz 6

True of false?

- 1. Within a certain range, all integers are available as values of an integer variable.
- 2. Within a certain range, all real numbers are available as values of a float variable.
- 3. 5(7+2) is equivalent to 45.
- 4. 1--1 is equivalent to zero.
- 5. int i = 5/3; The variable i is 2.
- 6. float x = 2/3; The variable x is approximately 0.6667.



2: Control structures



Conditionals



26. If-then-else

A conditional is a test: 'if something is true, then do this, otherwise maybe do something else'. The C++ syntax is

```
if ( something ) {
   // do something;
} else {
   // do otherwise;
}
```

- The 'else' part is optional
- You can leave out braces in case of single statement.



27. Complicated conditionals

Chain:

```
if ( /* some test */ ) {
} else if ( /* other test */ ) {
Nest:
if ( /* some test */ ) {
  if ( /* other test */ ) {
 } else {
    . . .
```



28. What are logical expressions?

```
logical_expression ::
   comparison_expression
   | NOT comparison_expression
   | logical_expression CONJUNCTION comparison_expression
comparison_expression ::
   numerical_expression COMPARE numerical_expression
numerical_expression ::
   quantity
   | numerical_expression OPERATOR quantity
quantity :: number | variable
```



29. Comparison and logical operators

Here are the most common logic operators and comparison operators:

Operator	meaning	example
==	equals	x==y-1
! =	not equals	x*x!=5
>	greater	y>x-1
>=	greater or equal	sqrt(y)>=7
<,<=	less, less equal	
&&,	and, or	x<1 && x>0
and,or	and, or	x<1 and $x>0$
!	not	!(x>1 && x<2)
not		not ($x>1$ and $x<2$)

Precedence rules of operators are common sense. When in doubt, use parentheses.



Exercise 9

The following code claims to detect if an integer has more than 2 digits.

```
Output
[basic] if:
... with 50 as input
....
... with 150 as
input ....
That number 150 had
more than 2
digits
```

Fix the small error in this code. Also add an 'else' part that prints if a number is negative.

You can base this off the file if.cxx in the repository



Review quiz 7

True or false?

- The tests if (i>0) and if (0<i) are equivalent.

 /poll "Same tests: 'i>0' and '0<i' ?" "T" "F"
- The test

The test

```
if (0<i<1)
  cout << "foo"</pre>
```

prints foo if *i* is between zero and one.

/poll "'if (0<i<1)' true if i between 0 and 1" "T" "F"



Review quiz 8

Any comments on the following?

```
bool x;
// ... code with x ...
if ( x == true )
   // do something
```



Exercise 10

Read in an integer. If it is even, print 'even', otherwise print 'odd':

```
if ( /* your test here */ )
  cout << "even" << endl;
else
  cout << "odd" << endl;</pre>
```

Then, rewrite your test so that the true branch corresponds to the odd case.



Exercise 11

Read in a positive integer. If it's a multiple of three print 'Fizz!'; if it's a multiple of five print 'Buzz!'. It it is a multiple of both three and five print 'Fizzbuzz!'. Otherwise print nothing.

Note:

- Capitalization.
- Exclamation mark.
- Your program should display at most one line of output.



Turn it in!

- If you have compiled your program, do: coe_fizzbuzz yourprogram.cc
 where 'yourprogram.cc' stands for the name of your source file.
- Is it reporting that your program is correct? If so, do: coe_fizzbuzz -s yourprogram.cc where the -s flag stands for 'submit'.

Note: this will send your file to the instructors with a **time stamp**. If you submit again after the deadline, you will be recorded as a late submission.



Project Exercise 12

Read two numbers and print a message stating whether the second is as divisor of the first:

```
Code:
  int number, divisor;
  bool is_a_divisor;
 /* ... */
  if (
  /* ... */
    cout << "Indeed, " << divisor
         << " is a divisor of "
         << number << "\n":
 } else {
    cout << "No. " << divisor
         << " is not a divisor of "
         << number << "\n";
```

```
Output
[primes] division:
( echo 6 ; echo 2 )
    | 1divisiontest
Enter a number:
Enter a trial
    divisor:
Indeed, 2 is a
    divisor of 6
( echo 9 ; echo 2 )
    | 1divisiontest
Enter a number:
Enter a trial
    divisor:
No, 2 is not a
```

divisor of 9



30. Switch statement example

Cases are executed consecutively until you 'break' out of the switch statement:

```
Code:
switch (n) {
case 1:
case 2:
  cout << "very small" << "\n";</pre>
  break:
case 3:
 cout << "trinity" << "\n";</pre>
  break:
default :
  cout << "large" << "\n";</pre>
```

```
Output
[basic] switch:
for v in 1 2 3 4 5;
    do \
          echo $v |
    ./switch ; \
        done
very small
very small
trinity
large
large
```

31. Local variables in conditionals

The curly brackets in a conditional allow you to define local variables:

```
if ( something ) {
   int i;
   .... do something with i
}
// the variable 'i' has gone away.
```

Good practice: only define variable where needed.

Braces induce a scope.



For loops



32. 'For' statement

Sometimes you need to repeat a statement a number of times. That's where the loop comes in. A loop has a counter, called a loop variable, which (usually) ranges from a lower bound to an upper bound.

Here is the syntax in the simplest case:



33. Loop syntax: variable

The loop variable is usually an integer:

```
for ( int index=0; index<max_index; index=index+1) {
   ...
}</pre>
```

But other types are allowed too:

```
for ( float x=0.0; x<10.0; x+=delta ) {
   ...
}</pre>
```

Beware the stopping test for non-integral variables!



34. Loop syntax: test

- If this boolean expression is true, do the next iteration.
- Done before the first iteration too!
- Test can be empty. This means no test is applied.

```
for ( int i=0; i<N; i++) {...}
for ( int i=0; ; i++ ) {...}
```



35. Loop syntax: increment

Increment performed after each iteration. Most common:

- i++ for a loop that counts forward;
- i-- for a loop that counts backward;

Others:

- i+=2 to cover only odd or even numbers, depending on where you started;
- i*=10 to cover powers of ten.

Even optional:

```
for (int i=0; i<N; ) {
    // stuff
    if ( something ) i+=1; else i+=2;
}</pre>
```



Review quiz 9

For each of the following loop headers, how many times is the body executed? (You can assume that the body does not change the loop variable.)

```
for (int i=0; i<7; i++)

/poll "for (int i=0; i<7; i++)" "6 iterations" "7" "8"

for (int i=0; i<=7; i++)

/poll "for (int i=0; i<=7; i++)" "6 iterations" "7" "8"

for (int i=0; i<0; i++)

/poll "for (int i=0: i<0; i++)" "0 iterations" "1" "inf"
```



Review quiz 10

What is the last iteration executed?

```
for (int i=1: i<=2: i=i+2)
/poll "for (int i=1: i<=2: i=i+2) last iteration" "i=1" "i=2" "i=3" "i=4"
     for (int i=1: i<=5: i*=2)
/poll "for (int i=1; i<=5; i*=2) last iteration" "4" "5" "8"
     for (int i=0: i<0: i--)
/poll "for (int i=0; i<0; i--) last iteration" "none" "0" "-1" "-inf"
     for (int i=5; i>=0; i--)
/poll "for (int i=5; i>=0; i--) last iteration" "0" "1" "-1" "4"
     for (int i=5: i>0: i--)
```

Exercise 13

Take this code:

```
int sum_of_squares{0};
for (int var=low; var<upper; var++) {
    sum_of_squares += var*var;
}
cout << "The sum of squares from "
    << low << " to " << upper
    << " is " << sum_of_squares << "\n";</pre>
```

and modify it to sum only the squares of every other number, starting at *low*.

Can you find a way to sum the squares of the even numbers $\geq 10W$?



Project Exercise 14

Read an integer and set a boolean variable to determine whether it is prime by testing for the smaller numbers if they divide that number.

Print a final message

Your number is prime

or

Your number is not prime: it is divisible by

where you report just one found factor.



36. Nested loops

```
Traversing a matrix
(we will discuss actual matrix data structures later):

for (int row=0; row<m; row++)
  for (int col=0; col<n; col++)
   ...
```

This is called 'loop nest', with

row: outer loop col: inner loop.



37. Indefinite looping

Sometimes you want to iterate some statements not a predetermined number of times, but until a certain condition is met. There are two ways to do this.

First of all, you can use a 'for' loop and leave the upperbound unspecified:

```
for (int var=low; ; var=var+1) { ... }
```



38. Break out of a loop

This loop would run forever, so you need a different way to end it. For this, use the break statement:

```
for (int var=low; ; var=var+1) {
  statement;
  if (some_test) break;
  statement;
}
```



Exercise 15

The integer sequence

$$u_{n+1} = \begin{cases} u_n/2 & \text{if } u_n \text{ is even} \\ 3u_n + 1 & \text{if } u_n \text{ is odd} \end{cases}$$

leads to the Collatz conjecture: no matter the starting guess u_1 , the sequence $n \mapsto u_n$ will always terminate at 1.

$$5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \cdots$$

(What happens if you keep iterating after reaching 1?)

Try all starting values $u_1 = 1, ..., 1000$ to find the values that lead to the longest sequence: every time you find a sequence that is longer than the previous maximum, print out the starting number.



39. Where did the break happen?

Suppose you want to know what the loop variable was when the break happened. You need the loop variable to be global:

```
int var;
... code that sets var ...
for ( ; var<upper; var++) {
    ... statements ...
    if (some condition) break
     ... more statements ...
}
... code that uses the breaking value of var ...</pre>
```

In other cases: define the loop variable in the header!



40. Test in the loop header

If the test comes at the start or end of an iteration, you can move it to the loop header:

```
bool need_to_stop{false};
for (int var=low; !need_to_stop ; var++) {
    ... some code ...
    if ( some condition )
        need_to_stop = true;
}
```



Exercise 16

Write an i, j loop nest that prints out all pairs with

$$1 \le i, j \le 10, \quad j \le i.$$

Output one line for each i value.

Now write an i, j loop that prints all pairs with

$$1 \le i, j \le 10, \quad |i - j| < 2,$$

again printing one line per *i* value. Food for thought: this exercise is definitely easiest with a conditional in the inner loop, but can you do it without?



Optional exercise 17

Find all triples of integers u, v, w under 100 such that $u^2 + v^2 = w^2$. Make sure you omit duplicates of solutions you have already found.



Exercise 18

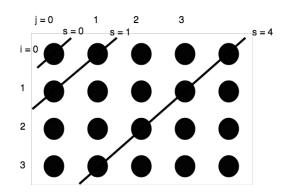
Write a double loop over $0 \le i, j < 10$ that prints the first pair where the product of indices satisfies $i \cdot j > N$, where N is a number your read in. A good test case is N = 40.

Secondly, find a pair with $i \cdot j > N$, but with the smallest value for i+j. (If there is more than one pair, report the one with lower i value.) Can you traverse the i,j indices such that they first enumerate all pairs i+j=1, then i+j=2, then i+j=3 et cetera? Hint: write a loop over the sum value $1,2,3,\ldots$, then find i,j.

You program should print out both pairs, each on a separate line, with the numbers separated with a comma, for instance 8,5.

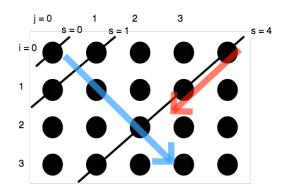


Suggestive picture 1





Suggestive picture 2





Turn it in!

- If you have compiled your program, do: coe_ij yourprogram.cc
 where 'yourprogram.cc' stands for the name of your source file.
- Is it reporting that your program is correct? If so, do: coe_ij -s yourprogram.cc where the -s flag stands for 'submit'.



41. Skip iteration

```
for (int var=low; var<N; var++) {</pre>
  statement;
  if (some_test) {
    statement:
    statement:
Alternative:
for (int var=low; var<N; var++) {</pre>
  statement:
  if (!some_test) continue;
  statement:
  statement;
```

The only difference is in layout.



While loops



42. While loop

```
Syntax:
while ( condition ) {
   statements;
}

or
do {
   statements;
} while ( condition );
```



43. Pre-test while loop

```
float money = inheritance();
while ( money < 1.e+6 )
  money += on_year_savings();</pre>
```



44. While syntax 1

```
Code:
cout << "Enter a positive number: " :</pre>
cin >> invar; cout << "\n";</pre>
cout << "You said: " << invar << "\n";</pre>
while (invar<=0) {
  cout << "Enter a positive number: "</pre>
  cin >> invar; cout << "\n";</pre>
  cout << "You said: " << invar <<
     "\n":
cout << "Your positive number was "</pre>
     << invar << "\n":
```

```
Output
[basic] whiledo:
Enter a positive
    number:
You said: -3
Enter a positive
    number:
You said: 0
Enter a positive
    number:
You said: 2
Your positive number
    was 2
```

Problem: code duplication.



45. While syntax 2

```
code:
int invar;
do {
   cout << "Enter a positive number: "
   ;
   cin >> invar; cout << "\n";
   cout << "You said: " << invar <<
        "\n";
} while (invar<=0);
cout << "Your positive number was: "
   << invar << "\n";</pre>
```

```
Output
[basic] dowhile:
Enter a positive
    number:
You said: -3
Enter a positive
    number:
You said: 0
Enter a positive
    number:
You said: 2
Your positive number
    was: 2
```

The post-test syntax leads to more elegant code.



Optional exercise 19

A horse is tied to a post with a 1 meter elastic band. A spider that was sitting on the post starts walking to the horse over the band, at 1 cm/sec. This startles the horse, which runs away at 1 m/sec. Assuming that the elastic band is infinitely stretchable, will the spider ever reach the horse?



3: Functions



Function basics



46. Why functions?

Functions are an abstraction mechanism.

- Code fragment with clear function:
- Turn into *subprogram*: function *definition*.
- Use by single line: function call.
- Abstraction: you have introduced a name for a section of code.



47. Introducing a function $p_{program}$

```
float foo_compute(float x) {
int main() {
                                                                // foo computation
   x = ...
                                                                 xtmp = ... x ...
                                                                 ytmp = ... x ... xtmp ....
   // foo computation
                                                                 return .... xtmp .... ytmp ..
   xtmp = ... x ...
   ytmp = ... x ... xtmp ....
   y = \dots xtmp .... ytmp ....
                                                               int main() {
                                                                  x = ...
   .... y ....
                                                                  y = foo_compute(x);
```



48. Program without functions

Example: zero-finding through bisection.

?:
$$f(x) = 0$$
, $f(x) = x^3 - x^2 - 1$

(where the question mark quantor stands for 'for which x').

First attempt at coding this: everything in the main program.

```
Code:
float left{0.},right{2.},
    mid;
while (right-left>.1) {
    mid = (left+right)/2.;
    float fmid =
        mid*mid*mid - mid*mid-1;
    if (fmid<0)
        left = mid;
    else
        right = mid;
}
cout << "Zero happens at: " << mid <<</pre>
```

```
Output
[func] bisect1:

Zero happens at:
1.4375
```



49. Introducing functions, step 1

```
Introduce a function for the
expression m*m*m - m*m-1:
float f(float x) {
  return x*x*x - x*x-1;
};
```

Used in main:

```
while (right-left>.1) {
  mid = (left+right)/2.;
  float fmid = f(mid);
  if (fmid<0)
    left = mid;
  else
    right = mid;
}</pre>
```

50. Introducing functions, step 2

Function:

```
float f(float x) {
  return x*x*x - x*x-1;
};
float find_zero_between
    (float 1,float r) {
  float mid;
  while (r-1>.1) {
    mid = (1+r)/2.;
    float fmid = f(mid);
    if (fmid<0)
      1 = mid:
    else
      r = mid:
  return mid;
};
```

New main:



Exercise 20

Make the bisection algorithm more elegant by introducing functions new_1, new_r used as:

```
1 = new_1(1,mid,fmid);
r = new_r(r,mid,fmid);
```

You can base this off the file bisect.cxx in the repository

Question: you could leave out *fmid* from the functions. Write this variant. Why is this not a good idea?



51. Why functions?

- Easier to read: use application terminology
- Shorter code: reuse
- Cleaner code: local variables are no longer in the main program.
- Maintainance and debugging



52. Code reuse

Suppose you do the same computation twice:

```
double x,y, v,w;
y = ..... computation from x .....
w = ..... same computation, but from v .....
```

With a function this can be replaced by:

```
double computation(double in) {
  return .... computation from 'in' ....
}

y = computation(x);
w = computation(y);
```



53. Code reuse

Example: multiple norm calculations:

Repeated code:

```
float s = 0;
for (int i=0; i<x.size(); i++)
    s += abs(x[i]);
cout << "One norm x: " << s <<
        endl;
s = 0;
for (int i=0; i<y.size(); i++)
    s += abs(y[i]);
cout << "One norm y: " << s <<
        endl;</pre>
```

becomes:

```
float OneNorm( vector<float> a
  ) {
 float sum = 0;
 for (int i=0; i<a.size();</pre>
    i++)
    sum += abs(a[i]);
 return sum;
int main() {
  ... // stuff
  cout << "One norm x: "
       << OneNorm(x) << endl;
  cout << "One norm y: "</pre>
       << OneNorm(y) << endl;
```

Review quiz 11

True or false?

- The purpose of functions is to make your code shorter.

 /poll "Functions are to make your code shorter" "T" "F"
- Using functions makes your code easier to read and understand.

/poll "Functions make your code easier to understand" "T" "F"

- Functions have to be defined before you can use them.
 /poll "Functions have to be defined before use" "T" "F"
- Function definitions can go inside or outside the main program.

/poll "Function defitions can go in or out main" "T" "F"



54. Declaration first, definition last

```
int my_computation(int);
int main() {
   int result;
   result = my_computation(5);
   return 0;
};
int my_computation(int i) {
   return i+3;
}
```



55. Anatomy of a function definition

```
void write_to_file(int i,double x) { /* ... */ }
float euler_phi(int i,bool tf) { /* ... */ return x; }
```

- Result type: what's computed.
 void if no result
- Name: make it descriptive.
- Parameters: zero or more.
 int i,double x,double y

 These act like variable declarations.
- Body: any length. This is a scope.
- Return statement: usually at the end, but can be anywhere; the computed result. Not necessary for a void function.



56. Function call

The function call

- 1. copies the value of the function argument to the function parameter;
- 2. causes the function body to be executed, and
- 3. the function call is replaced by whatever you return.
- 4. (If the function does not return anything, for instance because it only prints output, you declare the return type to be void.)



Review quiz 12

True or false?

- A function can have only one input /poll "Function can have only one input" "T" "F"
- A function can have only one return result /poll "Function can have only one return result" "T" "F"
- A void function can not have a return statement.
 /poll "Void function can not have 'return'" "T" "F"



Write a function with (float or double) inputs x, y that returns the distance of point (x, y) to the origin.

Test the following pairs: 1,0; 0,1; 1,1; 3,4.



Project Exercise 22

Write a function test_if_prime that has an integer parameter, and returns a boolean corresponding to whether the parameter was prime.

```
int main() {
  bool isprime;
  isprime = test_if_prime(13);
```

Read the number in, and print the value of the boolean.

Does your function have one or two return statements? Can you imagine what the other possibility looks like? Do you have an argument for or against it?



Project Exercise 23

Take your prime number testing function test_if_prime, and use it to write a program that prints multiple primes:

- Read an integer how_many from the input, indicating how many (successive) prime numbers should be printed.
- Print that many successive primes, each on a separate line.
- (Hint: keep a variable number_of_primes_found that is increased whenever a new prime is found.)



Turn it in!

- If you have compiled your program, do: coe_primes yourprogram.cc
 where 'yourprogram.cc' stands for the name of your source file.
- Is it reporting that your program is correct? If so, do: coe_primes -s yourprogram.cc where the -s flag stands for 'submit'.
- If you don't manage to get your code working correctly, you can submit as incomplete with coe_primes -i yourprogram.cc
- (Like all good unix programs, the tester also accepts a -h flag for 'help'.)



57. Background: square roots by Newton's

Early computers had no hardware for computing a square root. Instead, they used Newton's method. Suppose you have a value y and you want want to compute $x = \sqrt{y}$. This is equivalent to finding the zero of

$$f(x) = x^2 - y$$

where y is fixed. To indicate this dependence on y, we will write $f_v(x)$. Newton's method then finds the zero by evaluating

$$x_{\text{next}} = x - f_y(x)/f_y'(x)$$

until the guess is accurate enough, that is, until $f_v(x) \approx 0$.



Optional exercise 24

- Write functions f(x,y) and deriv(x,y), that compute $f_y(x)$ and $f_y'(x)$ for the definition of f_y above.
- Read a value y and iterate until $|f(x,y)| < 10^{-5}$. Print x.
- Second part: write a function newton_root that computes \(\sqrt{y} \).



Parameter passing



58. Mathematical type function

Pretty good design:

- pass data into a function,
- return result through return statement.
- Parameters are copied into the function. (Cost of copying?)
- pass by value
- 'functional programming'



59. Pass by value example

Note that the function alters its parameter x:

```
Code:
double squared( double x ) {
  double v = x * x;
  return y;
  /* ... */
  number = 5.1:
  cout << "Input starts as: "</pre>
       << number << "\n":
  other = squared(number);
  cout << "Output var is: "</pre>
       << other << "\n";
  cout << "Input var is now: "</pre>
       << number << "\n";
```

```
Output
[func] passvalue:
Input starts as: 5.1
Output var is: 26.01
Input var is now: 5.1
```

but the argument in the main program is not affected.



60. Reference

A reference is indicated with an ampersand in its definition, and it acts as an alias of the thing it references.

```
Code:
int i;
int &ri = i;
i = 5;
cout << i << "," << ri << "\n";
i *= 2;
cout << i << "," << ri << "\n";
ri -= 3;
cout << i << "," << ri << "\n";</pre>
```

```
Output [basic] ref: 5,5 10,10 7,7
```

(You will not use references often this way.)



61. Parameter passing by reference

The function parameter n becomes a reference to the variable i in the main program:

```
void f(int &n) {
  n = /* some expression */;
};
int main() {
  int i;
  f(i);
  // i now has the value that was set in the function
}
```



62. Results other than through return

Also good design:

- Return no function result,
- or return return status (0 is success, nonzero various informative statuses), and
- return other information by changing the parameters.
- pass by reference
- Parameters are sometimes classified 'input', 'output', 'throughput'.



63. Pass by reference example 1

```
Code:
void f( int &i ) {
   i = 5;
}
int main() {
   int var = 0;
   f(var);
   cout << var << "\n";</pre>
```

```
Output
[basic] setbyref:
5
```

Compare the difference with leaving out the reference.



64. Pass by reference example 2

```
bool can read value( int &value ) {
  // this uses functions defined elsewhere
  int file_status = try_open_file();
  if (file status==0)
    value = read_value_from_file();
  return file status==0:
int main() {
  int n;
  if (!can read value(n)) {
    // if you can't read the value, set a default
    n = 10;
  ..... do something with 'n' ....
```



Write a void function swap of two parameters that exchanges the input values:

```
Code:
cout << i << "," << j << "\n";
swap(i,j);
cout << i << "," << j << "\n";</pre>
```

```
Output
[func] swap:
1,2
2,1
```



Write a divisibility function that takes a number and a divisor, and gives:

- a bool return result indicating that the number is divisible, and
- a remainder as output parameter.

```
Output
[func] divisible:

8 has remainder 2
from 3

8 is divisible by 4
```



Write a function with inputs x, y, θ that alters x and y corresponding to rotating the point (x, y) over an angle θ .

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Your code should behave like:

```
Output
[geom] rotate:

Rotated halfway:
    (0.707107,0.707107)

Rotated to the
    y-axis: (0,1)
```



Recursion



65. Recursion

A function is allowed to call itself, making it a recursive function. For example, factorial:

$$5! = 5 \cdot 4 \cdot \cdots \cdot 1 = 5 \times 4!$$

You can define factorial as

$$F(n) = n \times F(n-1)$$
 if $n > 1$, otherwise 1

```
int factorial( int n ) {
  if (n==1)
    return 1;
  else
    return n*factorial(n-1);
}
```



The sum of squares:

$$S_n = \sum_{n=1}^N n^2$$

can be defined recursively as

$$S_1 = 1,$$
 $S_n = n^2 + S_{n-1}.$

Write a recursive function that implements this second definition. Test it on numbers that are input interactively.

Then write a program that prints the first 100 sums of squares.

How many squares do you need to sum before you get overflow? Can you estimate this number without running your program?



It is possible to define multiplication as repeated addition:

```
code:
int times( int number,int mult ) {
  cout << "(" << mult << ")";
  if (mult==1)
    return number;
  else
    return number +
       times(number,mult-1);
}</pre>
```

```
Output
[func] mult:

Enter number and
    multiplier
recursive
    multiplication
of 7 and 5:
    (5)(4)(3)(2)(1)35
```

Extend this idea to define powers as repeated multplication.

You can base this off the file mult.cxx in the repository



The Egyptian multiplication algorithm is almost 4000 years old. The result of multiplying $x \times n$ is:

```
if n is even:
twice the multiplication x \times (n/2);
otherwise:
x plus the multiplication x \times (n-1)
```

Extend the code of exercise 29 to implement this.

Food for thought: discuss the computational aspects of this algorithm to the traditional one of repeated addition.



Write a recursive function for computing Fibonacci numbers:

$$F_0 = 1,$$
 $F_1 = 1,$ $F_n = F_{n-1} + F_{n-2}$

First write a program that computes F_n for a value n that is input interactively.

Then write a program that prints out a sequence of Fibonacci numbers; set interactively how many.



More about functions



66. Default arguments

Functions can have default argument(s):

```
double distance( double x, double y=0. ) {
  return sqrt( (x-y)*(x-y) );
}
...
  d = distance(x); // distance to origin
  d = distance(x,y); // distance between two points
```

Any default argument(s) should come last in the parameter list.



67. Polymorphic functions

You can have multiple functions with the same name:

```
double average(double a,double b) {
  return (a+b)/2; }
double average(double a,double b,double c) {
  return (a+b+c)/3; }
```

Distinguished by type or number of input arguments: can not differ only in return type.

```
int f(int x);
string f(int x); // DOES NOT WORK
```



68. Useful idiom

Don't trace a function unless I say so:

```
void dosomething(double x,bool trace=false) {
  if (trace) // report on stuff
};
int main() {
  dosomething(1); // this one I trust
  dosomething(2); // this one I trust
  dosomething(3,true); // this one I want to trace!
  dosomething(4); // this one I trust
  dosomething(5); // this one I trust
```



Scope



69. Lexical scope

Visibility of variables

```
int main() {
  int i;
  if ( something ) {
    int j;
    // code with i and j
  }
  int k;
  // code with i and k
}
```



70. Shadowing

```
int main() {
  int i = 3;
  if ( something ) {
    int i = 5;
  }
  cout << i << endl; // gives 3
  if ( something ) {
    float i = 1.2;
  }
  cout << i << endl; // again 3
}</pre>
```

Variable *i* is shadowed: invisible for a while.

After the lifetime of the shadowing variable, its value is unchanged from before.



What is the output of this code?

```
bool something{false};
int i = 3;
if ( something ) {
  int i = 5;
  cout << "Local: " << i << "\n";</pre>
cout << "Global: " << i << "\n";</pre>
if ( something ) {
  float i = 1.2;
  cout << i << "\n";
  cout << "Local again: " << i << "\n";</pre>
cout << "Global again: " << i << "\n";</pre>
```



71. Life time vs reachability

Even without shadowing, a variable can exist but be unreachable.

```
void f() {
    ...
}
int main() {
    int i;
    f();
    cout << i;
}</pre>
```



4: Arrays



Vectors



72. What are vectors?

- Contiguous, indexed, storage of items: (often called 'array' but that has other meanings)
 - items of any type (but the same for all elements of one vector)
 - potentially very many items
- Indexed set of items
- ... but if you don't need the index: collection of items



73. Short vectors

Short vectors can be created by enumerating their elements:

```
#include <vector>
using std::vector;

int main() {
   vector<int> evens{0,2,4,6,8};
   vector<float> halves = {0.5, 1.5, 2.5};
   auto halfloats = {0.5f, 1.5f, 2.5f};
   cout << evens.at(0) << "\n";
   return 0;
}</pre>
```



Exercise 33

- 1. Take the above snippet, supply the missing header lines, compile, run.
- 2. Add a statement that alters the value of a vector element. Check that it does what you think it does.
- 3. Add a vector of the same length, containing odd numbers, which are the even values plus 1?

You can base this off the file shortvector.cxx in the repository



74. Range over elements

You can write a range-based for loop, which considers the elements as a collection.

```
for ( float e : my_data )
  // statement about element e
for ( auto e : my_data )
  // same, with type deduced by compiler
```

```
Code:
vector<int> numbers = {1,4,2,6,5};
int tmp_max = -2000000000;
for (auto v : numbers)
   if (v>tmp_max)
       tmp_max = v;
cout << "Max: " << tmp_max
       << " (should be 6)" << "\n";</pre>
```

```
Output
[array] dynamicmax:

Max: 6 (should be 6)
```



Exercise 34

Find the element with maximum absolute value in a vector. Use:

```
Hint:
#include <cmath>
...
absx = abs(x);
```

vector<int> numbers = {1,-4,2,-6,5};

75. Range over vector denotation

```
Code:
for ( auto i : {2,3,5,7,9} )
   cout << i << ",";
cout << "\n";</pre>
```

```
Output
[array] rangedenote:
2,3,5,7,9,
```



76. Vector definition

Definition and/or initialization:

```
#include <vector>
using std::vector;

vector<type> name;
vector<type> name(size);
vector<type> name(size,init_value);
```

where

- vector is a keyword,
- type (in angle brackets) is any elementary type or class name,
- name of the vector is up to you, and
- size is the (initial size of the vector). This is an integer, or more precisely, a size_t parameter.
- Initialize all elements to init_value.
- If no default given, zero is used for numeric types.



77. Accessing vector elements

Square bracket notation (zero-based):

```
Code:
vector<int> numbers = {1,4};
numbers[0] += 3;
numbers[1] = 8;
cout << numbers[0] << ","</pre>
     << numbers[1] << "\n";
```

```
Output
[array] assignbracket:
4,8
```

With bound checking:

```
Code:
vector<int> numbers = {1,4};
numbers.at(0) += 3;
numbers.at(1) = 8;
cout << numbers.at(0) << ","</pre>
     << numbers.at(1) << "\n";
```

```
Output
[array] assignatfun:
4.8
```



78. Vector elements out of bounds

Square bracket notation:

```
Output
[array]
assignoutofboundbracket:
1,4
```

With bound checking:

```
Output
[array]
assignoutofboundatfun:
libc++abi:
terminating with
uncaught
exception of
type
```



79. Range over elements by reference

Range-based loop indexing makes a copy of the vector element. If you want to alter the vector, use a reference:

```
for ( auto &e : my_vector)
  e = ....
```

```
Code:
    vector<float> myvector
    = {1.1, 2.2, 3.3};
    for ( auto &e : myvector )
        e *= 2;
    cout << myvector.at(2) << "\n";</pre>
```

```
Output [array] vectorrangeref: 6.6
```

(Can also use const auto& e to prevent copying, but also prevent altering data.)



80. Indexing the elements

You can write an indexed for loop, which uses an index variable that ranges from the first to the last element.

```
for (int i= /* from first to last index */ )
  // statement about index i
```

Example: find the maximum element in the vector, and where it occurs.

```
Output
[array] vecidxmax:

Max: 6.6 at index: 3
```



81. A philosophical point

Conceptually, a vector can correspond to a set of things, and the fact that they are indexed is purely incidental, or it can correspond to an ordered set, and the index is essential. If your algorithm requires you to access all elements, it is important to think about which of these cases apply, since there are two different mechanism.



Exercise 35

Find the location of the first negative element in a vector.

Which mechanism do you use?



Exercise 36

Create a vector x of float elements, and set them to random values. (Use the C random number generator for now.)

Now normalize the vector in L_2 norm and check the correctness of your calculation, that is,

1. Compute the L_2 norm of the vector:

$$||v|| \equiv \sqrt{\sum_i v_i^2}$$

- 2. Divide each element by that norm;
- 3. The norm of the scaled vector should now by 1. Check this.
- 4. Bonus: your program may be printing 1, but is it actually 1? Investigate.

What type of loop are you using?



82. Indexing with pre/post increment

Indexing in while loop and such:

```
x = a.at(i++); /* is */ x = a.at(i); i++;

y = b.at(++i); /* is */ i++; y = b.at(i);
```



83. Vector copy

Vectors can be copied just like other datatypes:

```
Output
[array] vectorcopy:
3.5,7
```



84. Vector methods

A vector is an object, with methods.

Given vector<sometype> x:

- Get elements, including bound checking, with ar.at(3). Note: (zero-based indexing).
- (also get elements with ar[3]: see later discussion.)
- Size: ar.size().
- Other functions: front, back, empty.
- With iterators (see later): insert, erase



85. Your first encounter with templates

vector is a 'templated class': vector<X> is a vector-of-X.

Code behaves as if there is a class definition for each type:

```
class vector<int> {
    class vector<float> {
    public:
        size(); at(); // stuff
    }
    size(); at(); // stuff
}
```

Actual mechanism uses templating: the type is a parameter to the class definition. More later.



Dynamic behaviour



86. Dynamic vector extension

Extend a vector's size with push_back:

Similar functions: pop_back, insert, erase. Flexibility comes with a price.



87. When to push back and when not

```
Known vector size:
```

```
int n = get_inputsize();
vector<float> data(n);
for ( int i=0; i<n; i++ ) {
  auto x = get_item(i);
  data.at(i) = x;
}</pre>
```

Unknown vector size:

```
vector<float> data;
float x;
while ( next_item(x) ) {
  data.push_back(x);
}
```

If you have a guess as to size: data.reserve(n).



88. Filling in vector elements

You can push elements into a vector:

```
vector<int> flex;
/* ... */
for (int i=0; i<LENGTH; i++)
  flex.push_back(i);</pre>
```

If you allocate the vector statically, you can assign with at:

```
vector<int> stat(LENGTH);
/* ... */
for (int i=0; i<LENGTH; i++)
    stat.at(i) = i;</pre>
```



89. Filling in vector elements

With subscript:

```
vector<int> stat(LENGTH);
/* ... */
for (int i=0; i<LENGTH; i++)
    stat[i] = i;

You can also use new to allocate*:
  int *stat = new int[LENGTH];
/* ... */
for (int i=0; i<LENGTH; i++)
  stat[i] = i;</pre>
```

*Considered bad practice. Do not use.



90. Timing the ways of filling a vector

```
Flexible time: 2.445
Static at time: 1.177
```

Static assign time: 0.334

Static assign time to new: 0.467



Vectors and functions



91. Vector as function argument

You can pass a vector to a function:

```
double slope( vector<double> v ) {
  return v.at(1)/v.at(0);
};
```

Vectors, like any argument, are passed by value, so the vector is actually copied into the function.



92. Vector pass by value example

```
Code:
    void set0
        ( vector<float> v,float x )
{
        v.at(0) = x;
}
    /* ... */
    vector<float> v(1);
    v.at(0) = 3.5;
    set0(v,4.6);
    cout << v.at(0) << "\n";</pre>
```

```
Output
[array] vectorpassnot:
3.5
```

- Vector is copied
- 'Original' in the calling environment not affected
- Cost of copying?



93. Vector pass by reference

If you want to alter the vector, you have to pass by reference:

```
Code:
void set0
  ( vector<float> &v,float x )
{
   v.at(0) = x;
}

/* ... */
vector<float> v(1);
v.at(0) = 3.5;
set0(v,4.6);
cout << v.at(0) << "\n";</pre>
```

```
Output
[array] vectorpassref:
4.6
```

- Parameter vector becomes alias to vector in calling environment
 - \Rightarrow argument *can* be affected.
- No copying cost
- What if you want to avoid copying cost, but need not alter



94. Vector pass by const reference

Passing a vector that does not need to be altered:

```
int f( const vector<int> &ivec ) { ... }
```

- Zero copying cost
- Not alterable, so: safe!
- (No need for pointers!)



Exercise 37

Revisit exercise 36 and introduce a function for computing the L_2 norm.



95. Vector as function return

You can have a vector as return type of a function.

Example: this function creates a vector, with the first element set to the size:

```
Code:
vector<int> make vector(int n) {
  vector<int> x(n);
  x.at(0) = n;
 return x;
 /* ... */
  vector<int> x1 = make_vector(10):
 // "auto" also possible!
  cout << "x1 size: " << x1.size() <<</pre>
    "\n":
  cout << "zero element check: " <<
    x1.at(0) << "\n";
```

```
Output
[array] vectorreturn:

x1 size: 10
zero element check:
10
```



Exercise 38

Write code to take a vector of integers, and construct two vectors, one containing all the odd inputs, and one containing all the even inputs. So:

```
input:
    5,6,2,4,5
output:
    5,5
    6,2,4
```

Can you write a function that accepts a vector and produces two vectors as described?



(hints for the next exercise)

```
// high up in your code:
#include <random>
using std::rand;

// in your main or function:
float r = 1.*rand()/RAND_MAX;
// gives random between 0 and 1
```



Exercise 39

Write functions random_vector and sort to make the following main program work:

```
int length = 10;
vector<float> values = random_vector(length);
vector<float> sorted = sort(values);
```

This creates a vector of random values of a specified length, and then makes a sorted copy of it.

Instead of making a sorted copy, sort in-place (overwrite original data with sorted data):

```
int length = 10;
vector<float> values = random_vector(length);
sort(values); // the vector is now sorted
```

Find arguments for/against that approach.

(Note: C++ has sorting functions built in.)



Vectors in classes



96. Can you make a class around a vector?

You may want a class of objects that contain a vector. For instance, you may want to name your vectors.

```
class named_field {
private:
   vector<double> values;
   string name;
```

The problem here is when and how that vector is going to be created



97. Create the contained vector

Use initializers for creating the contained vector:

```
class named field {
private:
  string name;
  vector<double> values;
public:
  named_field( string name,int nelements )
    : name(name),
      values(vector<double>(n)) {
 };
};
Less desirable method is creating in the constructor:
  named_field( string uname, int nelements ) {
    name = uname;
    values = vector<double>(n);
  };
```



Multi-dimensional arrays



98. Multi-dimensional vectors

Multi-dimensional is harder with vectors:

```
vector<float> row(20);
vector<vector<float>> rows(10,row);
```

Create a row vector, then store 10 copies of that: vector of vectors.



99. Matrix class

```
class matrix {
private:
  vector<vector<double>> elements;
public:
 matrix(int m,int n) {
    elements =
      vector<vector<double>>(m, vector<double>(n));
  void set(int i,int j,double v) {
    elements.at(i).at(j) = v;
  };
  double get(int i,int j) {
    return elements.at(i).at(j);
 };
```



Exercise 40

Write rows() and cols() methods for this class that return the number of rows and columns respectively.



Exercise 41

Write a method void set(double) that sets all matrix elements to the same value.

Write a method double totalsum() that returns the sum of all elements.

You can base this off the file matrix.cxx in the repository



100. Matrix class; better design

Better idea:

```
class Matrix {
private:
  int rows, cols;
  vector<double> elements;
private:
  Matrix( int m, int n )
  : rows(m), cols(n),
    elements(vector < double > (rows * cols))
    {};
  double get(int i,int j) {
    return elements.at(i*cols+j);
```

(Old-style solution: use cpp macro)



Exercise 42

In the matrix class of the previous slide, why are m,n stored explicitly, and not in the previous case?



Exercise 43

Add methods such as transpose, scale to your matrix class. Implement matrix-matrix multiplication.



101. Pascal's triangle

Pascal's triangle contains binomial coefficients:

```
R.ow
      1:
R.ow
      2:
Row
      3:
Row
      4:
      5:
R.ow
      5: 1 4 0 4 1
6: 1 5 10 10 5 1
7: 1 6 15 20 15 6 1
Row
R.ow
      8: 1 7 21 35 35 21 7 1
R.ow
Row
     9: 1 8 28 56 70 56 28 8 1
R.ow
     10:
           1 9 36 84 126 126 84 36 9 1
```

where

$$p_{rc} = \binom{r}{c} = \frac{r!}{c!(r-c)!}.$$

The coefficients can be computed from the recurrence

$$p_{rc} = \begin{cases} 1 & c \equiv 1 \lor c \equiv r \\ p_{r-1,c-1} + p_{r-1,c} \end{cases}$$

(There are other formulas. Why are they less preferable?)



Exercise 44

- Write a class pascal so that pascal(n) is the object containing
 n rows of the above coefficients. Write a method get(i,j) that
 returns the (i,j) coefficient.
- Write a method print that prints the above display.
- First print out the whole pascal triangle; then:
- Write a method print(int m) that prints a star if the coefficient modulo m is nonzero, and a space otherwise.



Optional exercise 45

Extend the Pascal exercise:

Optimize your code to use precisely enough space for the coefficients.



Turn it in!

- Write a program that accepts:
 - one integer: the height of the triangle. You should use this to construct a PascalTriangle object that contains the binomial coefficients. Then:
 - 2. a number of modulos with which to print the triangle. A value of zero indicates that your program should stop.

The tester will search for stars in your output and test that you have the right number in each line.

- If you have compiled your program, do a test run: coe_pascal yourprogram.cc
- Is it Submit if it is correct: coe_pascal -s yourprogram.cc
- If you don't manage to get your code working correctly, you can submit as incomplete with coe_pascal -i yourprogram.cc



Static arrays



Array creation

```
C-style arrays still exist,
{
   int numbers[] = {5,4,3,2,1};
   cout << numbers[3] << "\n";
}
{
   int numbers[5]{5,4,3,2,1};
   numbers[3] = 21;
   cout << numbers[3] << "\n";
}</pre>
```

but you shouldn't use them. Prefer to use array class (not in this course) or span (C++20; very advanced)



Ranging

You can range over static arrays same as for vector



Advanced material



Iterators



102. Beyond begin/end

- An iterator is a little like a pointer (into anything iteratable)
- beginend
- pointer-arithmetic and 'dereferencing':

```
auto element_ptr = my_vector.begin();
element_ptr++;
cout << *element_ptr;</pre>
```

• allows operations (erase, insert) on containers



103. Erase at/between iterators

Erase from start to before-end:

```
Code:
vector<int> counts{1,2,3,4,5,6};
vector<int>::iterator second =
    counts.begin()+1;
auto fourth = second+2; // easier
    than 'iterator'
counts.erase(second,fourth);
cout << counts[0] << "," << counts[1]
    << "\n";</pre>
```

```
Output
[iter] erase2:
1,4
```

(Also single element without end iterator.)



104. Insert at iterator

Insert at iterator: value, single iterator, or range:

```
Code:
                                               Output
                                               [iter] insert2:
vector<int>
     counts{1,2,3,4,5,6},zeros{0,0};
                                               0,2,3,0
auto after_one = zeros.begin()+1;
zeros.insert(
     after_one, counts.begin()+1, counts.begin()+3
    ):
//vector<int>::insert(
     after_one,counts.begin()+1,counts.begin()+3
    ):
cout << zeros[0] << "," << zeros[1]</pre>
     << ","
     << zeros[2] << "," << zeros[3]
     << "\n":
```



5: I/O



Formatted output



105. Formatted output

- cout uses default formatting
- Possible: pad a number, use limited precision, format as hex, etc
- Many of these output modifiers need

#include <iomanip>



106. Default unformatted output

```
Code:
for (int i=1; i<200000000; i*=10)
   cout << "Number: " << i << "\n";
cout << "\n";</pre>
```

```
Output
[io] cunformat:

Number: 1
Number: 10
Number: 100
Number: 1000
Number: 10000
Number: 100000
Number: 1000000
Number: 10000000
Number: 100000000
Number: 100000000
```



107. Reserve space

You can specify the number of positions, and the output is right aligned in that space by default:

```
Code:
#include <iomanip>
using std::setw;
 /* ... */
  cout << "Width is 6:" << "\n":</pre>
 for (int i=1; i<200000000; i*=10)
    cout << "Number: "
         << setw(6) << i << "\n":
  cout << "\n":
  // 'setw' applies only once:
  cout << "Width is 6:" << "\n":
  cout << ">"
       << setw(6) << 1 << 2 << 3 <<
    "\n":
  cout << "\n";
```

```
Output
[io] width:
Width is 6:
Number: 1
Number: 10
Number: 100
Number: 1000
Number: 10000
Number: 100000
Number: 1000000
Number: 10000000
Number: 100000000
Width is 6:
     123
```

108. Padding character

Normally, padding is done with spaces, but you can specify other characters:

```
Output
[io] formatpad:

Number: ....1

Number: ...100

Number: ..1000

Number: .10000

Number: 100000

Number: 1000000

Number: 10000000

Number: 10000000

Number: 100000000

Number: 100000000
```

Note: single quotes denote characters, double quotes denote strings.



109. Left alignment

Instead of right alignment you can do left:

```
Output
[io] formatleft:

Number: 1....

Number: 10...

Number: 100...

Number: 1000..

Number: 10000.

Number: 100000

Number: 1000000

Number: 10000000

Number: 10000000
```



110. Number base

Finally, you can print in different number bases than 10:

```
Code:
#include <iomanip>
using std::setbase;
using std::setfill;
 /* ... */
 cout << setbase(16)
      << setfill(' ');
 for (int i=0; i<16; i++) {
   for (int j=0; j<16; j++)
      cout << i*16+; << " ";
   cout << "\n":
```

```
Output
[io] format16:
0 1 2 3 4 5 6 7 8 9 a b c d e f
10 11 12 13 14 15 16 17 18 19 1a
    1b 1c 1d 1e 1f
20 21 22 23 24 25 26 27 28 29 2a
    2b 2c 2d 2e 2f
30 31 32 33 34 35 36 37 38 39 3a
    3b 3c 3d 3e 3f
40 41 42 43 44 45 46 47 48 49 4a
    4b 4c 4d 4e 4f
50 51 52 53 54 55 56 57 58 59 5a
    5b 5c 5d 5e 5f
60 61 62 63 64 65 66 67 68 69 6a
    6b 6c 6d 6e 6f
70 71 72 73 74 75 76 77 78 79 7a
    7b 7c 7d 7e 7f
80 81 82 83 84 85 86 87 88 89 8a
```



COE 322 - 2022 - - 208

Exercise 46

Make the first line in the above output align better with the other lines:

```
00 01 02 03 04 05 06 07 08 09 0a 0b 0c 0d 0e 0f 10 11 12 13 14 15 16 17 18 19 1a 1b 1c 1d 1e 1f 20 21 22 23 24 25 26 27 28 29 2a 2b 2c 2d 2e 2f etc
```



Exercise 47

Use integer output to print real numbers aligned on the decimal:

Use four spaces for both the integer and fractional part; test only with numbers that fit this format.



111. Hexadecimal

Hex output is useful for addresses (chapter ??):

```
Output
[pointer] coutpoint:

address of i,

decimal:

140732703427524

address of i, hex
: 0x7ffee2cbcbc4
```

Back to decimal:

```
cout << hex << i << dec << j;</pre>
```



Floating point formatting



112. Floating point precision

Use setprecision to set the number of digits before and after decimal point:

```
Code:
#include <iomanip>
using std::left;
using std::setfill;
using std::setw;
using std::setprecision;
 /* ... */
  x = 1.234567;
 for (int i=0; i<10; i++) {
    cout << setprecision(4) << x <<</pre>
     "\n":
    x *= 10:
```

```
Output
[io] formatfloat:
1.235
12.35
123.5
1235
1.235e+04
1.235e+05
1.235e+06
1.235e+07
1.235e+08
1.235e+09
```

This mode is a mix of fixed and floating point. See the scientific option below for consistent use of floating point format.



113. Fixed point precision

Fixed precision applies to fractional part:

```
Code:
x = 1.234567;
cout << fixed;
for (int i=0; i<10; i++) {
   cout << setprecision(4) << x <<
        "\n";
   x *= 10;
}</pre>
```

```
Output
[io] fix:
1.2346
12.3457
123.4567
1234,5670
12345.6700
123456.7000
1234567.0000
12345670.0000
123456700.0000
1234567000.0000
```

(Notice the rounding)



114. Aligned fixed point output

Combine width and precision:

```
Output
[io] align:
    1.2346
   12.3457
  123 4567
 1234.5670
12345.6700
123456.7000
1234567.0000
12345670.0000
123456700.0000
1234567000.0000
```



115. Scientific notation

Combining width and precision:

```
Output
[io] iofsci:
1.2346e+00
1.2346e+01
1.2346e+02
1.2346e+03
1.2346e+04
1.2346e+05
1.2346e+06
1.2346e+07
1.2346e+08
1.2346e+09
```



File output



116. Text output to file

The *iostream* is just one example of a stream: general mechanism for converting entities to exportable form. In particular: file output works the same as screen output.

Use:

```
Code:
#include <fstream>
using std::ofstream;
   /* ... */
   ofstream file_out;
   file_out.open
      ("fio_example.out");
   /* ... */
   file_out << number << "\n";
   file_out.close();</pre>
```

```
Output
[io] fio:

echo 24 | ./fio ; \
cat
fio_example.out
A number please:
Written.
24
```

Compare: cout is a stream that has already been opened to your terminal 'file'.



117. Binary output

Binary output: write your data byte-by-byte from memory to file. (Why is that better than a printable representation?)

```
Code:
  ofstream file_out;
  file_out.open
    ("fio_binary.out",ios::binary);
/* ... */
  file_out.write( (char*)(&number),4);
```

```
Output
[io] fiobin:
echo 25 | ./fiobin;
          od
    fio_binary.out
A number please:
    Written.
0000000
           000031
    000000
0000004
```



Cout on classes (for future reference)



118. Redefine less-less

If you want to output a class that you wrote yourself, you have to define how the << operator deals with your class.

```
class container {
  /* ... */
  int value() const {
  /* ... */
  /* ... */
ostream & operator << (ostream & os, const container & i) {
  os << "Container: " << i.value():
  return os;
}:
  /* ... */
  container eye(5);
  cout << eye << "\n";</pre>
```



6: Strings



Characters



119. Characters and ints

- Type char;
- represents '7-bit ASCII': printable and (some) unprintable characters.
- Single quotes: char c = 'a'



120. Char / int equivalence

Equivalent to (short) integer:

```
Output
[string] intchar:

x is at position 120
one further lies y
```

Also: 'x'-'a' is distance a--x



Exercise 48

Write a program that accepts an integer $1 \cdots 26$ and prints the so-manieth letter of the alphabet.

Extend your program so that if the input is negative, it prints the minus-so-manieth uppercase letter of the alphabet.



Strings



121. String declaration

```
#include <string>
using std::string;
// .. and now you can use 'string'
(Do not use the C legacy mechanisms.)
```



122. String creation

A string variable contains a string of characters.

```
string txt;
```

You can initialize the string variable or assign it dynamically:

```
string txt{"this is text"};
string moretxt("this is also text");
txt = "and now it is another text";
```



123. Quotes in strings

You can escape a quote, or indicate that the whole string is to be taken literally:

```
Code:
string
  one("a b c"),
  two("a \"b\" c"),
  three( R"("a ""b """c)" );
cout << one << "\n";
cout << two << "\n";
cout << the control of the cont
```

```
Output
[string] quote:
a b c
a "b" c
"a ""b """c
```



124. Concatenation

Strings can be concatenated:

```
Output [string] stringadd:

foo bar: 7
```



125. String indexing

You can query the *size*:

```
Code:
string five_text{"fiver"};
cout << five_text.size() << "\n";</pre>
```

```
Output
[string] stringsize:
5
```

or use subscripts:

```
Output
[string] stringsub:

char three: 2

char four: 3
```



126. Ranging over a string

Same as ranging over vectors.

Range-based for:

```
Code:
cout << "By character: ";
for ( char c : abc )
   cout << c << " ";
cout << "\n";</pre>
```

```
Output
[string] stringrange:

By character: a b c
```

Ranging by index:

```
Code:
string abc = "abc";
cout << "By character: ";
for (int ic=0; ic<abc.size(); ic++)
    cout << abc[ic] << " ";
cout << "\n";</pre>
```

```
Output
[string] stringindex:

By character: a b c
```



127. Range with reference

Range-based for makes a copy of the element You can also get a reference:

```
Code:
for ( char &c : abc )
   c += 1;
cout << "Shifted: " << abc << "\n";</pre>
```

```
Output
[string] stringrangeset:
Shifted: bcd
```



Review quiz 13

True or false?

- "O" is a valid value for a char variable /poll "double-quote 0 is a valid char" "T" "F"
- 3. "O" is a valid value for a string variable $_{\mbox{\scriptsize /poll "double-quote 0 is a valid string" "T" "F"}}$
- 4. 'a'+'b' is a valid value for a char variable $_{/poll\ "adding\ single-quote\ chars\ is\ a\ valid\ char''\ "T"\ "F"}$



Exercise 49

The oldest method of writing secret messages is the Caesar cypher. You would take an integer s and rotate every character of the text over that many positions:

$$s \equiv 3$$
: "acdz" \Rightarrow "dfgc".

Write a program that accepts an integer and a string, and display the original string rotated over that many positions.



128. More vector methods

Other methods for the vector class apply: insert, empty, erase, push_back, et cetera.

```
Code:
string five_chars;
cout << five_chars.size() << "\n";
for (int i=0; i<5; i++)
  five_chars.push_back(' ');
cout << five_chars.size() << "\n";</pre>
Output
[string]
O
5
```

```
Output
[string] stringpush:

0
5
```

Methods only for string: find and such.

http://en.cppreference.com/w/cpp/string/basic_string



Exercise 50

Write a function to print out the digits of a number: 156 should print one five six. You need to convert a digit to a string first; can you think of more than one way to do that?

Start by writing a program that reads a single digit and prints its name.

For the full program it is easiest to generate the digits last-to-first. Then figure out how to print them reversed.



Optional exercise 51

Write a function to convert an integer to a string: the input 215 should give two hundred fifteen, et cetera.



129. String stream

Like cout (including conversion from quantity to string), but to object, not to screen.

- Use the << operator to build it up; then
- use the str method to extract the string.

```
#include <sstream>
stringstream s;
s << "text" << 1.5;
cout << s.str() << endl;</pre>
```



Exercise 52

Use integer output to print real numbers aligned on the decimal:

```
Output
[io] quasifix:

1.5

12.32

123.456

1234.5678
```

Use four spaces for both the integer and fractional part; test only with numbers that fit this format.



7: Objects



Classes



130. Definition of object/class

An object is an entity that you can request to do certain things. These actions are the *methods*, and to make these possible the object probably stores data, the *members*.

When designing a class, first ask yourself: 'what functionality should the objects support'.

A class is a user-defined type; an object is an instance of that type.



131. Running example

We are going to build classes for points/lines/shapes in the plane.

```
class Point {
    /* stuff */
};
int main () {
    Point p; /* stuff */
}
```



Exercise 53

Thought exercise: what are some of the actions that a point object should be capable of?



132. Object functionality

Small illustration: point objects.

```
Output
[object] functionality:

distance to origin
    2.23607

distance to origin
    4.47214

and angle 1.10715
```

Note the 'dot' notation.



Exercise 54

Thought exercise: What data does the object need to store to be able to calculate angle and distance to the origin? Is there more than one possibility?



133. The object workflow

• First define the class, with data and function members:

```
class MyObject {
   // define class members
   // define class methods
};
(details later) typically before the main.
```

• You create specific objects with a declaration

```
MyObject
object1( /* .. */ ),
object2( /* .. */ );
```

You let the objects do things:

```
object1.do_this();
x = object2.do_that( /* ... */ );
```



134. Constructor and data initialization

Use the constructor to create an object of a class: function with same name as the class. (but no return type!)

Constructors are typically used to initialize data members.



135. Private and public

Best practice we will use:

```
class MyClass {
private:
   // data members
public:
   // methods
}
```

- Data is private: not visible outside of the objects.
- Methods are public: can be used in the code that uses objects.



Methods



136. Class methods

Let's define method distance.

Definition in the class:

```
Use in the program:
```

```
class Point {
   /* stuff */
   double distance_to_origin() {
     return sqrt(x*x + y*y); };
}
Point pt(5,12);
double
s = pt.distance_to_origin();
```

- Methods look like ordinary functions,
- except that they can use the data members of the class, for instance x, y;
- Methods can only be used on an object with the 'dot' notation. They are not independently defined.



Exercise 55

Add a method *angle* to the *Point* class. How many parameters does it need?



Hint: use the function atan or atan2.

You can base this off the file pointclass.cxx in the repository



Exercise 56

Make a class <code>GridPoint</code> which can have only integer coordinates. Implement a function <code>manhattan_distance</code> which gives the distance to the origin counting how many steps horizontal plus vertical it takes to reach that point.



137. Member default values

Class members can have default values, just like ordinary variables:

```
class Point {
private:
   float x=3., y=.14;
public:
   // et cetera
}
```

Each object will have its members initialized to these values.



138. Food for thought: constructor vs data

The arguments of the constructor imply nothing about what data members are stored!

Example: create a vector from x,y cartesian coordinates, but store r, theta polar coordinates:

```
#include <cmath>
class Point {
private: // members
  double r,theta;
public: // methods
  Point( double x,double y ) {
    r = sqrt(x*x+y*y);
    theta = atan2(y/x);
}
```

Note: no change to outward API.



Exercise 57

Discuss the pros and cons of this design:

```
class Point {
private:
    double x,y,alpha;
public:
    Point(double x,double y)
    : x(x),y(y) {
        alpha = // something trig
    };
    double angle() { return alpha; };
};
```



139. Data access in methods

You can access data members of other objects of the same type:

```
class Point {
private:
    double x,y;
public:
    void flip() {
        Point flipped;
        flipped.x = y; flipped.y = x;
        // more
    };
};
```

(Normally, data members should not be accessed directly from outside an object)



Exercise 58

Extend the Point class of the previous exercise with a method: distance that computes the distance between this point and another: if p,q are Point objects,

p.distance(q)

computes the distance between them.



Review quiz 14

T/F?

- A class is primarily determined by the data it stores.
 /poll "Class determined by its data" "T" "F"
- A class is primarily determing by its methods.

 /poll "Class determined by its methods" "T" "F"
- If you change the design of the class data, you need to change the constructor call.

```
/poll "Change data, change constructor proto too" "T" "F"
```



140. Methods that alter the object

For instance, you may want to scale a vector by some amount:

```
Code:
class Point {
 /* ... */
 void scaleby( double a ) {
   x *= a; y *= a; };
 /* ... */
};
 /* ... */
 Point p1(1.,2.);
  cout << "p1 to origin "</pre>
       << p1.length() << "\n";
  p1.scaleby(2.);
  cout << "p1 to origin "
       << p1.length() << "\n";
```

```
Output
[geom] pointscaleby:

p1 to origin 2.23607

p1 to origin 4.47214
```



Interaction between objects



141. Methods that create a new object

```
Code:
class Point {
 /* ... */
 Point scale(double a) {
    auto scaledpoint =
        Point( x*a, y*a );
    return scaledpoint;
  }:
 /* ... */
  cout << "p1 to origin "
       << p1.dist_to_origin() << "\n";</pre>
  Point p2 = p1.scale(2.);
  cout << "p2 to origin "
       << p2.dist_to_origin() << "\n";</pre>
```

```
Output
[geom] pointscale:

p1 to origin 2.23607
p2 to origin 4.47214
```



142. Anonymous objects

Create a point by scaling another point:

```
new_point = old_point.scale(2.81);
```

Two ways of handling the return statement:

Naive:

```
Point Point::scale( double a )
  Point scaledpoint =
    Point( x*a, y*a );
 return scaledpoint;
```

Creates point, copies it to new_point

'move semantics'

Better:

```
Point Point::scale( double a )
 return Point( x*a, y*a );
};
```

Creates point, moves it directly to new_point



};

Optional exercise 59

Write a method halfway that, given two Point objects p,q, construct the Point halfway, that is, (p+q)/2:

```
Point p(1,2.2), q(3.4,5.6);
Point h = p.halfway(q);
```

You can write this function directly, or you could write functions Add and Scale and combine these.

(Later you will learn about operator overloading.)

How would you print out a *Point* to make sure you compute the halfway point correctly?



143. Default constructor

Point p1(1.5, 2.3);

Point p2;

Consider this code that looks like variable declaration, but for objects:



144. Default constructor

The problem is with p2:

```
Point p1(1.5, 2.3);
Point p2;
```

- p1 is created with the constructor;
- p2 uses the default constructor:

```
Point() {};
```

- as soon as you define a constructor, the default constructor goes away;
- you need to redefine the default constructor:

```
Point() {};
Point( double x,double y )
    : x(x),y(y) {};
```

(but only if you really need it.)



145. Public versus private

- Interface: public functions that determine the functionality of the object; effect on data members is secondary.
- Implementation: data members, keep private: they only support the functionality.

This separation is a Good Thing:

- Protect yourself against inadvertant changes of object data.
- Possible to change implementation without rewriting calling code.



Exercise 60

```
Make a class LinearFunction with a constructor:
LinearFunction( Point input_p1,Point input_p2 );
and a member function
float evaluate_at( float x );
which you can use as:
LinearFunction line(p1,p2);
cout << "Value at 4.0: " << line.evaluate_at(4.0) << endl;</pre>
```



146. Classes for abstract objects

Objects can model fairly abstract things:

```
Code:
class Stream {
private:
  int last_result{0};
public:
  int next() {
   return last_result++; };
};
int main() {
  Stream ints:
  cout << "Next: "
       << ints.next() << "\n":
  cout << "Next: "
       << ints.next() << "\n";
  cout << "Next: "
       << ints.next() << "\n";
```

```
Output
[object] stream:

Next: 0
Next: 1
Next: 2
```



Project Exercise 61

Write a class primegenerator that contains:

- Methods how_many_primes_found and nextprime;
- Also write a function isprime that does not need to be in the class.

Your main program should look as follows:

```
cin >> nprimes;
primegenerator sequence;
while (sequence.number_of_primes_found()<nprimes) {
  int number = sequence.nextprime();
  cout << "Number " << number << " is prime" << "\n";
}</pre>
```



Turn it in!

- If you have compiled your program, do: coe_primes yourprogram.cc
 where 'yourprogram.cc' stands for the name of your source file.
- Is it reporting that your program is correct? If so, do: coe_primes -s yourprogram.cc where the -s flag stands for 'submit'.
- If you don't manage to get your code working correctly, you can submit as incomplete with coe_primes -i yourprogram.cc
- If you don't understand what the script is telling you, try the debug flag:
 - coe_primes -d yourprogram.cc



Project Exercise 62

The Goldbach conjecture says that every even number, from 4 on, is the sum of two primes p+q. Write a program to test this for the even numbers up to a bound that you read in. Use the primegenerator class you developed in exercise 272.

This is a great exercise for a top-down approach!

- 1. Make an outer loop over the even numbers e.
- 2. For each e, generate all primes p.
- 3. From p + q = e, it follows that q = e p is prime: test if that q is prime.

For each even number e then print e,p,q, for instance:

The number 10 is 3+7

If multiple possibilities exist, only print the first one you find.



Turn it in!

- If you have compiled your program, do: coe_goldbach yourprogram.cc
 where 'yourprogram.cc' stands for the name of your source file.
- Is it reporting that your program is correct? If so, do: coe_goldbach -s yourprogram.cc where the -s flag stands for 'submit'.
- If you don't manage to get your code working correctly, you can submit as incomplete with
 coe_goldbach -i yourprogram.cc



Advanced stuff about constructors



147. Member initializer lists

```
Other syntax for initialization:
    use initializer list.

class Point {
    private:
        double x,y;
    public:
        Point( double userx,double usery )
            : x(userx),y(usery) {
```



148. Advantage of initializer list

Allows for reuse of names:

```
Code:
class Point {
private:
  double x, y;
public:
  Point( double x,double y )
      : x(x), y(y) \{
  /* ... */
  Point p1(1.,2.);
  cout << "p1 = "
       << p1.getx() << "," <<
    p1.gety()
       << "\n";
```

```
Output
[geom] pointinitxy:
p1 = 1,2
```



149. Constructors and contained classes

Finally, if a class contains objects of another class,

```
class Inner {
public:
    Inner(int i) { /* ... */ }
};
class Outer {
private:
    Inner contained;
public:
};
```



150. Intent checking

Compiler checks your intent against your implementation. This code is not legal:

```
subroutine ArgIn(x)
  implicit none
  real,intent(in) :: x
  x = 5 ! compiler complains
end subroutine ArgIn
```



151. When are contained objects created?

```
Outer( int n ) {
  contained = Inner(n);
};
```

- 1. This first calls the default constructor
- then calls the Inner(n) constructor,
- 3. then copies the result over the *contained* member.

```
Outer( int n )
  : contained(Inner(n)) {
    /* ... */
};
```

- This creates the Inner(n) object,
- 2. placed it in the *contained* member,
- 3. does the rest of the constructor, if any.



152. Copy constructor

 Default defined copy and 'copy assignment' constructors:

```
some_object x(data);
some_object y = x;
some_object z(x);
```

- They copy an object:
 - simple data, including pointers
 - included objects recursively.
- You can redefine them as needed.

```
class has int {
private:
  int mine{1};
public:
  has_int(int v) {
    cout << "set: " << v <<
    "\n":
    mine = v; };
  has int( has int &h ) {
    auto v = h.mine;
    cout << "copy: " << v <<
    "\n":
    mine = v; };
  void printme() {
    cout << "I have: " << mine
         << "\n"; };
};
```

153. Copy constructor in action

```
Code:
has_int an_int(5);
has_int other_int(an_int);
an_int.printme();
other_int.printme();
```

```
Output
[object] copyscalar:
set: 5
copy: 5
I have: 5
I have: 5
```



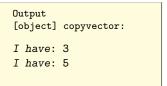
154. Copying is recursive

Class with a vector:

Copying is recursive, so the copy has its own vector:

```
Code:

has_vector a_vector(5);
has_vector other_vector(a_vector);
a_vector.set(3);
a_vector.printme();
other_vector.printme();
```





155. Destructor

- Every class myclass has a destructor ~myclass defined by default.
- The default destructor does nothing:

```
~myclass() {};
```

A destructor is called when the object goes out of scope.
 Great way to prevent memory leaks: dynamic data can be released in the destructor. Also: closing files.



156. Destructor example

Just for tracing, constructor and destructor do cout:



157. Destructor example

Destructor called implicitly:

```
Output
[object] destructor:
Before the nested
    scope
calling the
    constructor
Inside the nested
    scope
calling the
    destructor
After the nested
    scope
```



Other object stuff



158. String an object

- 1. Define a function that yields a string representing the object, and
- 2. redefine the less-less operator to use this.

```
#include <sstream> #include <string>
using std::stringstream; using std::string;
```



159. Class declarations

Header file:

```
class something {
private:
   int i;
public:
   double dosomething( int i, char c );
};

Implementation file:
double something::dosomething( int i, char c ) {
   // do something with i, c
};
```



160. Hierarchical object relations

 Hierarchical relations between classes: each object in class A is also in class B.



161. Example of class hierarchy

• Class Employee:

```
class Employee {
private:
   int number,salary;
/* ... */
};
```

- class Manager is subclass of Employee
 (every manager is an employee, with number and salary)
- Manager has extra field n_minions

How do we implement this?



162. Another example: multiple subclasses

- Example: both triangle and square are polygons.
- You can implement a method draw for both triangle/square
- ... or write it once for polygon, and then use that.



163. Terminology

- Polygon / Employee is the base class.
- Triangle / Manager is a derived class.
- Derived classes *inherit* data and methods from the base class: they are accessible in objects of the derived class.



164. Examples for base and derived cases

- Base case: employee. Has: salary, employee number.
 Special case: manager. Has in addition: underlings.
- Base case: shape in drawing program. Has: extent, area, drawing routine.

Special case: square et cetera; has specific drawing routine.



165. General case, special case

You can have classes where an object of one class is a special case of the other class. You declare that as

```
class General {
protected: // note!
  int g;
public:
  void general_method() {};
};

class Special : public General {
public:
  void special_method() { g = ... };
};
```



166. Inheritance: derived classes

Derived class Special inherits methods and data from base class General:

```
int main() {
   Special special_object;
   special_object.general_method();
   special_object.special_method();
}
```

Members of the base class need to be protected, not private, to be inheritable.



167. Constructors

When you run the special case constructor, usually the general constructor needs to run too. By default the 'default constructor', but usually explicitly invoked:

```
class General {
public:
    General( double x,double y ) {};
};
class Special : public General {
public:
    Special( double x ) : General(x,x+1) {};
};
```



168. Access levels

Methods and data can be

- private, because they are only used internally;
- public, because they should be usable from outside a class object, for instance in the main program;
- protected, because they should be usable in derived classes.



Exercise 63

Take your code where a Rectangle was defined from one point, width, and height.

Make a class Square that inherits from Rectangle. It should have the function area defined, inherited from Rectangle.

First ask yourself: what should the constructor of a Square look like?



Exercise 64

Revisit the LinearFunction class. Add methods slope and intercept.

Now generalize LinearFunction to StraightLine class. These two are almost the same except for vertical lines. The slope and intercept do not apply to vertical lines, so design StraightLine so that it stores the defining points internally. Let LinearFunction inherit.



169. Overriding methods

- A derived class can inherit a method from the base class.
- A derived class can define a method that the base class does not have.
- A derived class can *override* a base class method:

```
class Base {
public:
    virtual f() { ... };
};
class Deriv : public Base {
public:
    virtual f() override { ... };
};
```



170. More

- Multiple inheritance: an X is-a A, but also is-a B.
 This mechanism is somewhat dangerous.
- Virtual base class: you don't actually define a function in the base class, you only say 'any derived class has to define this function'.



171. Has-a relationship

A class usually contains data members. These can be simple types or other classes. This allows you to make structured code.

```
class Person {
    string name;
    ....
};
class Course {
private:
    Person the_instructor;
    int year;
};'
```

This is called the has-a relation:

Course has-a Person



172. Literal and figurative has-a

A line segment has a starting point and an end point.

```
or store one and derive the other:
A Segment class can store those
points:
                                   class Segment {
class Segment {
                                   private:
private:
                                      Point starting_point;
  Point.
                                      float length, angle;
    starting_point, ending_point;
                                   public:
public:
                                      Point get_the_end_point() {
  Point get_the_end_point() {
                                        /* some computation
    return ending_point; };
                                           from the
                                           starting point */ };
int main() {
  Segment somesegment;
  Point somepoint =
    somesegment.get_the_end_point();
```

Implementation vs API: implementation can be very different from user



173. Constructors in has-a case

Class for a person:

```
class Person {
private:
    string name;
public:
    Person( string name ) {
        /* ... */
    };
};
```

Class for a course, which contains a person:

```
class Course {
private:
    Person instructor;
    int enrollment;
public:
    Course( string instr,int n )
      {
        /* ???? */
    };
};
```

You want to use this as Course("Eijkhout",65);



174. Constructors in the has-a case

Possible constructor:

```
Course( string teachname,int nstudents ) {
  instructor = Person(teachname);
  enrollment = nstudents;
};

Preferred:

Course( string teachname,int nstudents )
  : instructor(Person(teachname)),
    enrollment(nstudents) {
};
```



Exercise 65

1. Make a class Rectangle (sides parallel to axes) with a constructor:

```
Rectangle(Point botleft,float width,float height);
```

The logical implementation is to store these quantities. Implement methods:

```
float area(); float rightedge_x(); float topedge_y();
and write a main program to test these.
```

2. Add a second constructor

```
Rectangle(Point botleft, Point topright);
```

Can you figure out how to use member initializer lists for the constructors?



Optional exercise 66

Make a copy of your solution of the previous exercise, and redesign your class so that it stores two Point objects. Your main program should not change.



8: Pointers



175. Recursive data structures

Naive code:

```
class Node {
private:
  int value;
  Node tail;
  /* ... */
};
```

This does not work: would take infinite memory.

Indirect inclusion: only 'point' to the tail:

```
class Node {
private:
  int value;
  PointToNode tail;
  /* ... */
};
```



Pointer types

- Smart pointers. You will see 'shared pointers'.
- There are 'unique pointers'. Those are tricky.
- Please don't use old-style C pointers.
- Unless you become very advanced.



176. Simple example

Simple class that stores one number:

```
class HasX {
private:
   double x;
public:
   HasX( double x) : x(x) {};
   auto get() { return x; };
   void set(double xx) { x = xx; };
};
```



177. Creating a pointer

Allocation and pointer in one:

```
shared_ptr<Obj> X =
    make_shared<Obj>( /* constructor args */ );
    // or simpler:
auto X = make_shared<Obj>( /* args */ );
```



178. Headers for smart pointers

Using smart pointers requires at the top of your file:

```
#include <memory>
using std::shared_ptr;
using std::make_shared;
// not in this lecture:
using std::unique_ptr;
using std::make_unique;
```



179. What's the point of pointers?

Pointers make it possible for two variables to own the same object.

```
Code:
auto xptr = make_shared<HasX>(5);
auto yptr = xptr;
cout << xptr->get() << "\n";
yptr->set(6);
cout << xptr->get() << "\n";</pre>
```

```
Output
[pointer] twopoint:
5
```



Automatic memory management



Memory leaks

C has a 'memory leak' problem

```
// the variable 'array' doesn't exist
{
    // attach memory to 'array':
    double *array = new double[N];
    // do something with array
}
// the variable 'array' does not exist anymore
// but the memory is still reserved.
```

The application 'is leaking memory'.

(even worse if you do this in a loop!)

Java/Python have 'garbage collection': runtime impact

C++ has the best solution: smart pointers with reference counting.



180. Illustration

We need a class with constructor and destructor tracing:

```
class thing {
public:
   thing() { cout << ".. calling constructor\n"; };
   ~thing() { cout << ".. calling destructor\n"; };
};</pre>
```



181. Illustration 1: pointer overwrite

Let's create a pointer and overwrite it:

```
Output
[pointer] ptr1:
set pointer1
.. calling
    constructor
overwrite pointer
.. calling destructor
```



182. Illustration 2: pointer copy

```
Code:
cout << "set pointer2" << "\n";</pre>
auto thing_ptr2 =
  make shared<thing>():
cout << "set pointer3 by copy"</pre>
     << "\n":
auto thing_ptr3 = thing_ptr2;
cout << "overwrite pointer2"</pre>
     << "\n":
thing_ptr2 = nullptr;
cout << "overwrite pointer3"</pre>
     << "\n":
thing_ptr3 = nullptr;
```

```
Output
[pointer] ptr2:
set pointer2
.. calling
    constructor
set pointer3 by copy
overwrite pointer2
overwrite pointer3
.. calling destructor
```

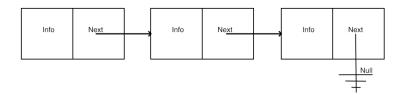
- The object counts how many pointers there are:
- 'reference counting'
- A pointed-to object is deallocated if no one points to it.



Example: linked lists



Linked list





183. Linked lists

The prototypical example use of pointers is in linked lists. Consider a class Node with

- a data value to store, and
- a pointer to another Node, or nullptr if none.

Constructor sets the data value: Set next / test if there is a next:

```
class Node {
    private:
        int datavalue{0};
    shared_ptr<Node>
        tail_ptr{nullptr};
public:
    Node() {}
    Node(int value)
        i datavalue(value) {};
    int value() { return
        datavalue; };
}
```



184. List usage

Example use:

```
Output
[tree] simple:

List <<23,45>> has
length 2
```



185. Linked lists and recursion

Many operations on linked lists can be done recursively:

```
int Node::list_length() {
  if (!has_next()) return 1;
  else return 1+tail_ptr->list_length();
};
```



Exercise 67

Write a method set_tail that sets the tail of a node.

```
Node one;
one.set_tail( two ); // what is the type of 'two'?
cout << one.list_length() << endl; // prints 2</pre>
```



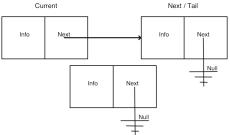
Exercise 68

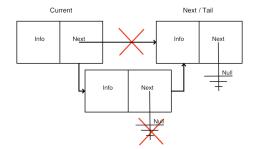
Write a recursive append method that appends a node to the end of a list:

```
Code:
auto
   first = make_shared<Node>(23),
   second = make_shared<Node>(45),
   third = make_shared<Node>(32);
first->append(second);
first->append(third);
first->print();
```



Insertion







Exercise 69

Write a recursive *insert* method that inserts a node in a list, such that the list stays sorted:

```
code:
auto
   first = make_shared<Node>(23),
   second = make_shared<Node>(45),
   third = make_shared<Node>(32);
first->insert(second);
first->insert(third);
first->print();
```

```
Output
[tree] insert:

Insert 45 on 23
gives <<23,45>>

Insert 32 gives
<<23,32,45>>
```

Assume that the new node always comes somewhere after the head node.



Pointers and addresses



C and F pointers

```
C++ and Fortran have a clean reference/pointer concept: a reference or pointer is an 'alias' of the original object
```

```
C/C++ also has a very basic pointer concept: a pointer is the address of some object (including pointers)
```

```
If you're writing C++ you should not use it. (until you get pretty advanced) if you write C, you'd better understand it.
```



186. Memory addresses

If you have an int i, then &i is the address of i.

An address is a (long) integer, denoting a memory address. Usually it is rendered in hexadecimal notation.

```
Output
[pointer] coutpoint:

address of i,
    decimal:
    140732703427524

address of i, hex
    : 0x7ffee2cbcbc4
```



187. Same in C

Using purely C:

```
Output
[pointer] printfpoint:

address of i:
    140732690693076

same in hex:
    7ffee2097bd4
```



188. Address types

```
The type of '&i' is int*, pronounced 'int-star', or more formally: 'pointer-to-int'.
```

You can create variables of this type:

```
int i;
int* addr = &i;
// exactly the same:
int *addr = &i;
```

Now addr contains the memory address of i.



189. Dereferencing

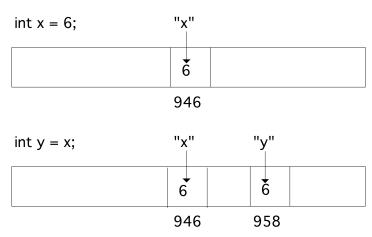
Using *addr 'dereferences' the pointer: gives the thing it points to; the value of what is in the memory location.

```
Code:
int i;
int* addr = &i;
i = 5;
cout << *addr << "\n";
i = 6;
cout << *addr << "\n";</pre>
```

```
Output
[pointer] cintpointer:
5
6
```

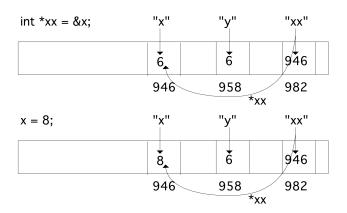


190. illustration





191. illustration





192. Star stuff

Equivalent:

- int* addr: addr is an int-star, or
- int *addr: *addr is an int.



Addresses and parameter passing



193. C++ pass by reference

C++ style functions that alter their arguments:

```
void inc(int &i) {
    i += 1;
}
int main() {
    int i=1;
    inc(i);
    cout << i << endl;
    return 0;
}</pre>
```



194. C-style pass by reference

In C you can not pass-by-reference like this. Instead, you pass the address of the variable i by value:

```
void inc(int *i) {
   *i += 1;
}
int main() {
   int i=1;
   inc(&i);
   cout << i << endl;
   return 0;
}</pre>
```

Now the function gets an argument that is a memory address: i is an int-star. It then increases *i, which is an int variable, by one.



Exercise 70

Write another version of the swap function:

```
void swap( /* something with i and j */ {
    /* your code */
}
int main() {
    int i=1, j=2;
    swap( /* something with i and j */ );
    cout << "check that i is 2: " << i << endl;
    cout << "check that j is 1: " << i << endl;
    return 0;
}</pre>
```

Hint: write C++ code, then insert stars where needed.



Arrays and pointers



195. Array and pointer equivalence

Array and memory locations are largely the same:

```
Code:
double array[5] = {11,22,33,44,55};
double *addr_of_second = &(array[1]);
cout << *addr_of_second << "\n";
array[1] = 7.77;
cout << *addr_of_second << "\n";</pre>
```

```
Output
[pointer] arrayaddr:
22
7.77
```



196. Array passing to function

When an array is passed to a function, it behaves as an address:

```
Output
[pointer] arraypass:
1.41....1.41
```

Note that these arrays don't know their size, so you need to pass it.



Multi-dimensional arrays



197. Multi-dimensional arrays

After

```
double x[10][20];
a row x[3] is a double*, so is x a double**?
Was it created as:
double **x = new double*[10];
for (int i=0; i<10; i++)
   x[i] = new double[20];</pre>
```

No: multi-d arrays are contiguous.



Dynamic allocation



198. Problem with static arrays

Create an array with size depending on something:

```
if ( something ) {
  double ar[25];
} else {
  double ar[26];
}
ar[0] = // there is no array!
```

This Does Not Work



199. Declaration and allocation

Now dynamic allocation:
double *array;
if (something) {
 array = new double[25];
} else {
 array = new double[26];
}
Don't forget:
delete array;



200. Allocation, C vs C++

```
C allocates in bytes:
double *array;
array = (double*) malloc( 25*sizeof(double) );
C++ allocates an array:
double *array;
array = new double[25];
Don't forget:
free(array); // C
delete array; // C++
```



201. De-allocation

Memory allocated with malloc / new does not disappear when you leave a scope. Therefore you have to delete the memory explicitly:

```
free(array);
delete(array);
```

The C++ *vector* does not have this problem, because it obeys scope rules.



202. Memory leak1

```
void func() {
  double *array = new double[large_number];
  // code that uses array
}
int main() {
  func();
};
```

- The function allocates memory
- After the function ends, there is no way to get at that memory
- \Rightarrow memory leak.



203. Memory leaks

```
for (int i=0; i<large_num; i++) {
  double *array = new double[1000];
  // code that uses array
}</pre>
```

Every iteration reserves memory, which is never released: another memory leak.

Your code will run out of memory!

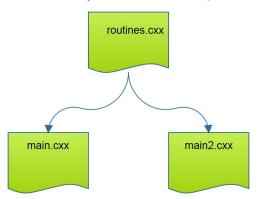


9: Prototypes, separate compilation



204. Include files

Reuse code by include it in multiple mains.



We will develop a better scenario.



205. Forward declarations, 1

A first use of declarations is forward declarations.

Some people like defining functions after the main:



206. Forward declarations, 2

You also need forward declaration for mutually recursive functions:

```
int f(int);
int g(int i) { return f(i); }
int f(int i) { return g(i); }
```



207. Declarations for separate compilation

```
// file: def.cxx
int tester(float x) {
   .....
}
```

```
// file : main.cxx
int tester(float);

int main() {
  int t = tester(...);
  return 0;
}
```



208. Compiling and linking

Your regular compile line

```
icpc -o yourprogram yourfile.cc
```

actually does two things: compilation, and linking. You can do those separately:

- First you compile
 icpc -c yourfile.cc
 which gives you a file yourfile.o, a so-called object file; and
- 2. Then you use the compiler as linker to give you the executable file:
 - icpc -o yourprogram yourfile.o



209. Dealing with multiple files

Compile each file separately, then link:

```
icpc -c mainfile.cc
icpc -c functionfile.cc
icpc -o yourprogram mainfile.o functionfile.o
```



210. Declarations and header files

```
Header file contains only
declaration: // file: def.h
int tester(float);
```

The header file gets included both in the definitions file and the main program:

```
// file: def.cxx
#include "def.h"
int tester(float x) {
    ....
    int main() {
        int t = tester(...);
        return 0;
    }
```

What happens if you leave out the **#include** "def.h" in both cases?



211. Class declarations

Header file:

```
class something {
private:
   int i;
public:
   double dosomething( int i, char c );
};

Implementation file:
double something::dosomething( int i, char c ) {
   // do something with i, c
};
```



Review quiz 15

For each of the following answer: is this a valid function definition or function declaration.

```
• int foo();
• int foo() {};
• int foo(int) {};
• int foo(int bar) {};
• int foo(int) { return 0; };
• int foo(int bar) { return 0; };
```



212. Make

Good idea to learn the Make utility for project management.

(Also Cmake.)



10: Advanced topics



213. A simple example

You can define a function and apply it:

```
double sum(float x,float y) { return x+y; }
cout << sum( 1.2, 3.4 );</pre>
```

or you can apply the function recipe directly:

```
Code:
[] (float x,float y) -> float {
  return x+y; } ( 1.5, 2.3 )
```

```
Output
[func] lambdadirect:
3.8
```



214. Lambda syntax

```
[capture] ( inputs ) -> outtype { definition };
[capture] ( inputs ) { definition };
```

- The square brackets are how you recognize a lambda; we will get to the 'capture' later.
- Inputs: like function parameters
- Result type specification -> outtype: can be omitted if compiler can deduce it;
- Definition: function body.



215. Assign lambda to variable

```
Code:
    auto summing =
      [] (float x,float y) -> float {
      return x+y; };
    cout << summing ( 1.5, 2.3 ) << "\n";</pre>
```

```
Output
[func] lambdavar:
3.8
```

- This is a variable declaration.
- Uses auto for technical reasons; see later.

Return type could have been ommitted:

```
auto summing =
[] (float x,float y) { return x+y; };
```



Exercise 71

The Newton method (see HPC book) for finding the zero of a function f, that is, finding the x for which f(x) = 0, can be programmed by supplying the function and its derivative:

```
double f(double x) { return x*x-2; };
double fprime(double x) { return 2*x; };

and the algorithm:
double x{1.};
while ( true ) {
  auto fx = f(x);
  cout << "f( " << x << " ) = " << fx << "\n";
  if (std::abs(fx)<1.e-10 ) break;
  x = x - fx/fprime(x);
}</pre>
```

Rewrite this code to use lambda functions for f and g.

You can base this off the file newton.cxx in the repository



216. Lambdas as parameter: the problem

Lambdas are in a class that is dynamically generated, so you can not write a function that takes a lambda as argument, because you don't have the class name.

```
void do_something( /* what? */ f ) {
    f(5);
}
int main() {
    do_something
    ( [] (double x) { cout << x; } );
}</pre>
```

(Do not use C-style function pointer syntax.)



217. Lambdas as parameter: the solution

```
#include <functional>
using std::function;
```

With this, you can declare parameters by their signature:

```
double find_zero
  ( function< double(double) > f,
    function< double(double) > fprime ) {
```

This states that f is in the class of double(double) functions.



Exercise 72

Rewrite the Newton exercise above to use a function with prototype

```
double root = find_zero( f,g );
```

Call the function

- 1. first with the lambda variables you already created;
- 2. but in a better variant, directly with the lambda expressions as arguments, that is, without assigning them to variables.



218. Capture parameter

Capture value and reduce number of arguments:

```
int exponent=5;
auto powerfive =
  [exponent] (float x) -> float {
    return pow(x, exponent); };
```

Now powerfive is a function of one argument, which computes that argument to a fixed power.

```
Output
[func] lambdait:

To the power 5
1:1
2:32
3:243
4:1024
5:3125
```



Exercise 73

Extend the newton exercise to compute roots in a loop:

Without lambdas, you would define a function

```
double squared_minus_n( double x,int n ) {
  return x*x-n; }
```

However, the $find_zero$ function takes a function of only a real argument. Use a capture to make f dependent on the integer parameter.



Exercise 74

You don't need the gradient as an explicit function: you can approximate it as

$$f'(x) = (f(x+h) - f(x))/h$$

for some value of h.

Write a version of the root finding function

double find_zero(function< double(double)> f)

that uses this. You can use a fixed value h=1e-6. Do not reimplement the whole newton method: instead create a lambda for the gradient and pass it to the function find_zero you coded earlier.



219. Turn it in!

Write a program that

- 1. reads an integer from the commandline
- 2. prints a line:

The root of this number is 1.4142 which contains the word root and the value of the square root of the input in default output format.

Your program should

- have a subroutine newton_root as described above.
- (8/10 credit): call it with two lambda expressions: one for the function and one for the derivative, *or*
- (10/10 credit) call it with a single lambda expression for the function and approximate the derivative as described above.

The tester is coe_newton, options as usual.



220. Capture by value

Normal capture is by value:

```
Code:
int one=1;
auto one_more =
   [one] ( int input ) -> void {
       cout << input+one << "\n";
};
one_more (5);
one_more (6);
one_more (7);</pre>
```

```
Output
[func] lambdavalue:
6
7
```



221. Capture by value/reference

Capture by reference:

```
Code:
int stride = 1;
auto more and more =
  [&stride] ( int input ) -> void {
    cout << input << "=>" <<
    input+stride << "\n";</pre>
    stride++;
};
more and more(5):
more_and_more(6);
more and more(7):
more and more(8):
more_and_more(9);
cout << "stride is now: " << stride
    << "\n":
```

```
Output
[func] lambdareference:

5=>6
6=>8
7=>10
8=>12
9=>14
stride is now: 6
```



222. Capture a reduction variable

This mechanism is useful

```
int count=0;
auto count_if_f = [&count] (int i) {
    if (f(i)) count++; }
for ( int i : int_data )
    count_if_f(i);
cout << "We counted: " << count;</pre>
```



223. Lambdas vs function pointers

```
Code:
int cfun_add1( int i ) { return i+1;
    }:
int apply_to_5( int(*f)(int) ) {
 return f(5);
}:
//codesnippet end
 /* ... */
  auto lambda_add1 = [] (int i) {
    return i+1; };
  cout << "C ptr:</pre>
       << apply_to_5(&cfun_add1) <<
     "\n":
  cout << "Lambda: "
       << apply_to_5(lambda_add1) <<</pre>
     "\n":
```

```
Output
[func] lambdacptr:

C ptr: 6
Lambda: 6
```



Lambda in algorithms



224. For each, very simple example

Apply something to each array element:

```
Output
[iter] each:
13
14
15
```



225. For any

See if any element satisfies a boolean test:

```
Code:
    vector<int>
        ints{2,3,4,5,7,8,13,14,15};
    bool there_was_an_8 =
        any_of( ints.begin(),ints.end(),
            [] ( int i ) -> bool {
            return i==8;
            }
            );
    cout << "There was an 8: " <<
        boolalpha << there_was_an_8 <<
        "\n";</pre>
```

```
Output
[iter] each:
13
14
15
```



226. Capture by reference

Capture variables are normally by value, use ampersand for reference. This is often used in algorithm header.

```
Output
[stl] counteach:
number of even: 3
```



227. For each, with capture

Capture by reference, to update with the array elements.

```
Output
[iter] each:
13
14
15
```



228. Sorting

```
lterator syntax:
(see later for ranges)
sort( myvec.begin(),myvec.end() );
```

The comparison used by default is ascending. You can specify other compare functions:

```
sort( myvec.begin(),myvec.end(),
      [] (int i,int j) { return i>j; }
);
```



Namespaces



229. You have already seen namespaces

Safest:

```
#include <vector>
int main() {
   std::vector<stuff> foo;
}
```

Drastic:

```
#include <vector>
using namespace std;
int main() {
   vector<stuff> foo;
}
```

Prudent:

```
#include <vector>
using std::vector;
int main() {
   vector<stuff> foo;
}
```



230. Why not 'using namespace std'?

This compiles, but should not:

```
#include <iostream>
using namespace std;

def swop(int i,int j) {};

int main() {
   int i=1,j=2;
   swap(i,j);
   cout << i << "\n";
   return 0;
}</pre>
```

This gives an error:

```
#include <iostream>
using std::cout;

def swop(int i,int j) {};

int main() {
   int i=1,j=2;
   swap(i,j);
   cout << i << "\n";
   return 0;</pre>
```



231. Defining a namespace

You can make your own namespace by writing

```
namespace a_namespace {
   // definitions
   class an_object {
   };
}
```



232. Namespace usage

```
Qualify type with namespace:
a_namespace::an_object myobject();
or
using namespace a_namespace;
an_object myobject();
or
using a_namespace::an_object;
an_object myobject();
or
using namespace abc = space_a::space_b::space_c;
abc::func(x)
```



Templates



233. Templated type name

If you have multiple routines that do 'the same' for multiple types, you want the type name to be a variable. Syntax:

```
template <typename yourtypevariable>
// ... stuff with yourtypevariable ...
```



234. Example: function

Definition:

```
template<typename T>
void function(T var) { cout << var << end; }

Usage:
int i; function(i);
double x; function(x);</pre>
```

and the code will behave as if you had defined function twice, once for int and once for double.



Exercise 75

Machine precision, or 'machine epsilon', is sometimes defined as the smallest number ϵ so that $1+\epsilon>1$ in computer arithmetic.

Write a templated function epsilon so that the following code prints out the values of the machine precision for the float and double type respectively:

```
Output
[template] eps:

Epsilon float:
    1.0000e-07

Epsilon double:
    1.0000e-15
```



235. Templated vector

The Standard Template Library (STL) contains in effect

```
template<typename T>
class vector {
private:
    T *vectordata; // internal data
public:
    T at(int i) { return vectordata[i] };
    int size() { /* return size of data */ };
    // much more
}
```



Exceptions



236. Exception throwing

Throwing an exception is one way of signalling an error or unexpected behaviour:

```
void do_something() {
  if ( oops )
    throw(5);
}
```



237. Catching an exception

It now becomes possible to detect this unexpected behaviour by *catching* the exception:

```
try {
   do_something();
} catch (int i) {
   cout << "doing something failed: error=" << i << endl;
}</pre>
```



238. Exception classes

```
class MyError {
public :
  int error_no; string error_msg;
  MyError( int i,string msg )
  : error_no(i),error_msg(msg) {};
throw( MyError(27, "oops");
try {
  // something
} catch ( MyError &m ) {
  cout << "My error with code=" << m.error_no</pre>
    << " msg=" << m.error_msg << endl;
```

You can use exception inheritance!



239. Multiple catches

You can multiple catch statements to catch different types of errors:

```
try {
   // something
} catch ( int i ) {
   // handle int exception
} catch ( std::string c ) {
   // handle string exception
}
```



240. Catch any exception

Catch exceptions without specifying the type:

```
try {
   // something
} catch ( ... ) { // literally: three dots
   cout << "Something went wrong!" << endl;
}</pre>
```



241. More about exceptions

• Functions can define what exceptions they throw:

```
void func() throw( MyError, std::string );
void funk() throw();
```

- Predefined exceptions: bad_alloc, bad_exception, etc.
- An exception handler can throw an exception; to rethrow the same exception use 'throw;' without arguments.
- Exceptions delete all stack data, but not new data. Also, destructors are called: section ??.
- There is an implicit try/except block around your main.
 You can replace the handler for that. See the exception header file.
- Keyword noexcept:

```
void f() noexcept { ... };
```

• There is no exception thrown when dereferencing a nullptr.



242. Destructors and exceptions

The destructor is called when you throw an exception:

```
Code:
class SomeObject {
public:
  SomeObject() {
    cout << "calling the constructor"</pre>
          << "\n"; };
  ~SomeObject() {
    cout << "calling the destructor"</pre>
         << "\n": }:
};
 /* ... */
  trv {
    SomeObject obj;
    cout << "Inside the nested scope"</pre>
     << "\n":
    throw(1);
 } catch (...) {
    cout << "Exception caught" <<</pre>
     "\n";
```

```
Output
[object] exceptdestruct:

calling the
    constructor

Inside the nested
    scope

calling the
    destructor

Exception caught
```

Auto



243. Type deduction



244. Type deduction in functions

Return type can be deduced in C++17:

```
auto equal(int i,int j) {
  return i==j;
};
```



245. Auto and references, 1

auto discards references and such:

```
Code:
A my_a(5.7);
auto get_data = my_a.access();
get_data += 1;
my_a.print();
```

```
Output
[auto] plainget:
data: 5.7
```



246. Auto and references, 2

Combine auto and references:

```
Code:
A my_a(5.7);
auto &get_data = my_a.access();
get_data += 1;
my_a.print();
```

```
Output
[auto] refget:
data: 6.7
```



247. Auto and references, 3

For good measure:

```
A my_a(5.7);
const auto &get_data = my_a.access();
get_data += 1; // WRONG does not compile
my_a.print();
```



248. Auto iterators

```
is actually short for:
vector<int> myvector(20);
for ( auto copy_of_int :
    myvector )
                                   for ( std::vector<int>
  s += copy_of_int;
                                        iterator
for ( auto &ref_to_int :
                                       it=myvector.begin();
    myvector )
                                        it!=myvector.end() ; ++it
  ref_to_int = s;
for ( const auto&
                                      s += *it ; // note the deref
    copy_of_thing : myvector )
  s += copy_of_thing.f();
```

Range iterators can be used with anything that is iteratable (vector, map, your own classes!)



Random



249. Random floats

```
// seed the generator
std::random_device r;
// std::seed_seq ssq{r()};
// and then passing it to the engine does the same

// set the default random number generator
std::default_random_engine generator{r()};

// distribution: real between 0 and 1
std::uniform_real_distribution<float> distribution(0.,1.);

cout << "first rand: " << distribution(generator) << "\n";</pre>
```



250. Dice throw

```
// set the default generator
std::default_random_engine generator;

// distribution: ints 1..6
std::uniform_int_distribution<int> distribution(1,6);

// apply distribution to generator:
int dice_roll = distribution(generator);
   // generates number in the range 1..6
```



251. Poisson distribution

Another distribution is the Poisson distribution:

```
std::default_random_engine generator;
float mean = 3.5;
std::poisson_distribution(int> distribution(mean);
int number = distribution(generator);
```



252. Global engine

Wrong approach:

```
code:
int nonrandom_int(int max) {
   std::default_random_engine engine;
   std::uniform_int_distribution<>
      ints(1,max);
   return ints(engine);
};
```

```
Output [rand] nonrandom:

Three ints: 15, 15, 15.
```

Good approach:

```
code:
int realrandom_int(int max) {
   static   std::default_random_engine
       static_engine;
   std::uniform_int_distribution<>
       ints(1,max);
   return ints(static_engine);
```

```
Output [rand] truerandom:

Three ints: 15, 98, 70.
```

11: Libraries



Commandline arguments



253. Traditional commandline parsing

```
Output
[args] argcv:
./argcv 5 12
Program name: ./argcv
arg 1: 5 => 5
arg 2: 12 => 12
./argcv abc 3.14 foo
Program name: ./argcv
arg 1: abc => 0
arg 2: 3.14 => 3
arg 3: foo => 0
```

Can you imagine coding

```
myprog --debug -f "input.txt" -n 10,20,40
```

7



254. Example: cxxopts

https://github.com/jarro2783/cxxopts

Find the 2.2.1 release.

Use wget or curl to download straight to the class machine.

Unpack it.



255. Cmake based installation

The cxxopts-2.2.1 directory has a file CMakeLists.txt

```
mkdir build
cd build
cmake -D CMAKE_INSTALL_PREFIX:PATH=${HOME}/mylibs \
    ..
make
make install
```

(This is an 'in-source' build. I don't like it: prefer to have the build directory elsewhere to keep the source untouched.)



256. Let's use this library

```
#include "cxxopts.hpp"
int main() {
    cxxopts::Options
    options("programname", "Program description");
}

compile

icpc -o program source.cpp \
    -I/path/to//cxxopts/installdir/include
```

Can you compile and run this?



257. Commandline options

Write code to test this.

Can you add more types of options?



12: Software development



258. Programming and correctness

Find your favorite example of costly programming mistakes . . .

What to do about it?

- Never make mistakes.
- Prove that your program is correct.
- Test your program before deploying it.
- Handle errors as they occur.



Error handling



259. Assertions to catch logic errors

Sanity check on things 'that you just know are true':

```
#include <cassert>
...
assert( bool expression )

Example:
x = sin(2.81);
y = x*x;
z = y * (1-y);
assert( z>=0. and z<=1. );</pre>
```



260. Using assertions

Check on valid input parameters:

```
#include <cassert>
// this function requires x<y
// it computes something positive
float f(x,y) {
  assert( x<y );</pre>
  return /* some result */;
Check on valid results:
float positive_outcome = f(x,y);
assert( positive_outcome>0 );
```



261. Example

```
int collatz_next( int current ) {
   assert( current>0 );
   int next{-1};
   if (current%2==0) {
      next = current/2;
      assert(next<current);
   } else {
      next = 3*current+1;
      assert(next>current);
   }
   return next;
}
```



262. Use assertions during development

Assertions are disabled by

#define NDEBUG

before the include.

You can pass this as compiler flag: icpc -DNDEBUG yourprog.cxx



263. Exceptions

Not every error is fatal:

Exception
$$\equiv$$

$$\begin{cases} \text{'this should not happen'} \\ \text{but we can handle it} \end{cases}$$

- 1. recover from the problem
- 2. graceful exit



264. Exceptions

Have you seen the following?

```
Code:
vector<float> x(5);
x.at(5) = 3.14;
```

```
Output
[except] boundthrow:

libc++abi.dylib: terminating with
   uncaught exception of type
   std::out_of_range: vector
```

The STL can generate many exceptions.

- You can let your program crash, and start debugging
- You can try to catch and handle them yourself.



265. Exception structure

Code with problem:

```
if ( /* some problem */ )
  throw(5);
  /* or: throw("error"); */
```

```
try {
  /* code that can go wrong */
} catch (...) { // literally
    three dots!
  /* code to deal with the
    problem */
}
```



266. Exceptions

Assume a routine only works for certain values, and you want to generate an error if called with an inappropriate value.

```
double compute_root(double x) {
   if (x<0) throw(1);
   return sqrt(x);
}
int main() {
   try {
      y = compute_root(x);
   } catch (...) {
      /* handle error */
      cout << "Root failed, using default\n";
      y = 0;
   }</pre>
```

See book for more details.



Unit testing and test-driven development (TDD)



267. Dijkstra quote

Today a usual technique is to make a program and then to test it. But: program testing can be a very effective way to show the presence of bugs, but is hopelessly inadequate for showing their absence. (cue laughter)

Still ...



268. Types of testing

- Unit tests that test a small part of a program by itself;
- System tests test the correct behavior of the whole software system; and
- Regression tests establish that the behavior of a program has not changed by adding or changing aspects of it.



269. Unit testing

- Every part of a program should be testable
- ullet \Rightarrow good idea to have a function for each bit of functionality
- Positive tests: show that code works when it should
- Negative tests: show that the code fails when it should



270. Unit testing

- Every part of a program should be testable
- Do not write the tests after the program: write tests while you develop the program.
- Test-driven development:
 - 1. design functionality
 - 2. write test
 - 3. write code that makes the test work



271. Principles of TDD

Develop code and tests hand-in-hand:

- Both the whole code and its parts should always be testable.
- When extending the code, make only the smallest change that allows for testing.
- With every change, test before and after.
- Assure correctness before adding new features.



272. Unit testing frameworks

Testing is important, so there is much software to assist you.

Popular choice with C++ programmers: Catch2

https://github.com/catchorg



273. Toy example

Function and tester:

```
double f(int n) { return n*n+1; }

#define CATCH_CONFIG_MAIN
#include "catch2/catch_all.hpp"

TEST_CASE( "test that f always returns positive" ) {
  for (int n=0; n<1000; n++)
     REQUIRE( f(n)>0 );
}
```

(accept the define and include as magic)



274. Compiling toy example

```
icpc -o tdd tdd.cxx \
   -I${TACC_CATCH2_INC} -L${TACC_CATCH2_LIB} \
   -1Catch2Main -1Catch2
```

Files:

```
icpc -o tdd tdd.cxx
```

• Path to include and library files:

```
-I${TACC_CATCH2_INC} -L${TACC_CATCH2_LIB}
```

• Libraries:

-1Catch2Main -1Catch2



Exercise 76: Extend the toy example

1. Write a function

```
double f(int n) \{ /* .... */ \} with values in the range (0,1).
```

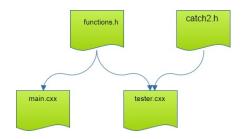
2. Write a unit test for this.

You can base this off the file tdd.cxx in the repository



275. Realistic setup

- All program functionality in a 'library' file
- Main program really short
- Tester file with only tests.
- (Tester also needs the catch2 stuff included)





276. Slightly realistic example

Example: we use a function that

- only works for positive inputs;
- returns input +1.

Program that uses this:

Note the include file!



277. Function to be developed

We know the structure:

```
int increment_positive_only( int i ) {
   // this function returns one more than the input
   // input has to be positive, error otherwise
   /* ... */
}
```

function body to be developed.



278. Functionality testing

File tester.cxx:

Same include file for the functionality; the testing framework creates its own main.

```
#include "functions.h"

#define CATCH_CONFIG_MAIN
#include "catch2/catch_all.hpp"

TEST_CASE( "test the increment function" ) {
   /* ... */
}
```



279. Compiling the tester at TACC

One-line solution:

```
icpc -o tester test_main.cxx \
   -I${TACC_CATCH2_INC} -L${TACC_CATCH2_LIB} \
   -1Catch2Main -1Catch2
```



Exercise 77: File structure

Make three files:

- 1. Include file with the functions.
- 2. Main program that uses the functions.
- 3. Tester main file, contents to be determined.



280. Correctness through 'require' clause

```
Tests go in tester.cxx:
TEST_CASE( "test that f always returns positive" ) {
  for (int n=0; n<1000; n++)
    REQUIRE( f(n)>0 );
}
```

- TEST_CASE acts like independent program.
- REQUIRE is like assert but more sophisticated
- Can contain (multiple) tests for correctness.



281. **Tests**

Boolean:

```
REQUIRE( some_test(some_input) );
REQUIRE( not some_test(other_input) );
Integer:
REQUIRE( integer_function(1)==3 );
REQUIRE( integer_function(1)!=0 );
Beware floating point:
REQUIRE( real_function(1.5)==Catch::Approx(3.0) );
REQUIRE( real_function(1)!=Catch::Approx(1.0) );
```

In general exact tests don't work.



282. Output for failing tests

Run the tester:

```
test the increment function
test.cxx:25
test.cxx:29: FATLED:
  REQUIRE( increment_positive_only(i)==i+1 )
with expansion:
  1 == 2
test cases: 1 | 1 failed
assertions: 1 | 1 failed
```



283. Diagnostic information for failing tests

```
INFO: print out information at a failing test
```

```
TEST_CASE( "test that f always returns positive" ) {
  for (int n=0; n<1000; n++)
    INFO( "function fails for " << n );
    REQUIRE( f(n)>0 );
}
```



Exercise 78: Positive tests

Continue with the example of slide 448: add a positive TEST_CASE

```
for (int i=1; i<10; i++)
   REQUIRE( increment_positive_only(i)==i+1 );</pre>
```

Make the function satisfy this test.



284. Test for exceptions

Suppose function g(n)

```
 succeeds for input n > 0
```

 fails for input n ≤ 0: throws exception

```
TEST_CASE( "test that g only works for positive" ) {
  for (int n=-100; n<+100; n++)
    if (n<=0)
        REQUIRE_THROWS( g(n) );
    else
        REQUIRE_NOTHROW( g(n) );
}
```



Exercise 79: Negative tests

Make sure your function throws an exception at illegal inputs:

```
for (int i=0; i>-10; i--)
   REQUIRE_THROWS( increment_positive_only(i) );
```



285. Tests with code in common

Use SECTION if tests have intro/outtro in common:

```
TEST_CASE( "commonalities" ) {
 // common setup:
 double x, y, z;
 REQUIRE_NOTHROW(y = f(x));
 // two independent tests:
 SECTION( "g function" ) {
   REQUIRE_NOTHROW( z = g(y) );
 SECTION( "h function" ) {
    REQUIRE_NOTHROW(z = h(y));
 // common followup
 REQUIRE( z>x );
```

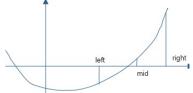
(sometimes called setup/teardown)



TDD example: Bisection



286. Root finding by bisection



• Start with bounds where the function has opposite signs.

$$x_{-} < x_{+}, \qquad f(x_{-}) \cdot f(x_{+}) < 0,$$

- Find the mid point;
- Adjust either left or right bound.



287. Coefficient handling

$$f(x) = c_d x^d + \dots + c_1 x^1 + c_0$$

We implement this by storing the coefficients in a *vector*<double>. Proper:

```
TEST_CASE( "coefficients are polynomial","[1]" ) {
  auto coefficients = set_coefficients();
  REQUIRE( coefficients.size()>0 );
  REQUIRE( coefficients.front()!=0. );
}
```



Exercise 80: Proper polynomials

Write a routine set_coefficients that constructs a vector of coefficients:

```
vector<double> coefficients = set_coefficients();
```

and make it satisfy the above conditions.

At first write a hard-coded set of coefficients, then try reading them from the command line.



Exercise 81: One test for properness

Write a function *proper_polynomial* as described, and write unit tests for it, both passing and failing.



288. Test on polynomials evaluation

```
// correct interpretation: 2x^2 + 1
vector<double> second{2,0,1};
REQUIRE( proper_polynomial(second) );
REQUIRE( evaluate_at(second,2) == Catch::Approx(9) );
// wrong interpretation: 1x^2 + 2
REQUIRE( evaluate_at(second,2) != Catch::Approx(6) );
```



Exercise 82: Implementation

Write a function evaluate_at which computes

$$y \leftarrow f(x)$$
.

and confirm that it passes the above tests.

For bonus points, look up Horner's rule and implement it.



Exercise 83: Odd degree polynomials only

With odd degree you can always find bounds x_-, x_+ . Reject even degree polynomials:

```
if ( not is_odd(coefficients) ) {
   cout << "This program only works for odd-degree polynomials\n";
   exit(1);
}</pre>
```

Gain confidence by unit testing:

```
vector<double> second{2,0,1}; // 2x^2 + 1
REQUIRE( not is_odd(second) );
vector<double> third{3,2,0,1}; // 3x^3 + 2x^2 + 1
REQUIRE( is_odd(third) );
```



Exercise 84: Find bounds

Write a function find_outer which computes x_-, x_+ such that

$$f(x_{-}) < 0 < f(x_{+})$$
 or $f(x_{+}) < 0 < f(x_{-})$

(can you write that more compactly?)
Unit test:

```
right = left+1;
vector<double> second{2,0,1}; // 2x^2 + 1
REQUIRE_THROWS( find_initial_bounds(second,left,right) );
vector<double> third{3,2,0,1}; // 3x^3 + 2x^2 + 1
REQUIRE_NOTHROW( find_initial_bounds(third,left,right) );
REQUIRE( left<right );</pre>
```

How would you test the function values?



Exercise 85: Put it all together

Make this call work:

Add an optional precision argument to the root finding function.

Design unit tests, including on the precision attained, and make sure your code passes them.



Turn it in!

 If you think your functions pass all tests, subject them to the tester:

```
coe_bisection yourprogram.cc
where 'yourprogram.cc' stands for the name of your source
file.
```

- Is it reporting that your program is correct? If so, do: coe_bisection -s yourprogram.cc where the -s flag stands for 'submit'.
- If you don't manage to get your code working correctly, you can submit as incomplete with coe_bisection -i yourprogram.cc
- If you want feedback on what the tester thinks about your code do
 coe_bisection -d yourprogram.cc
 with the -d flag for 'debug.

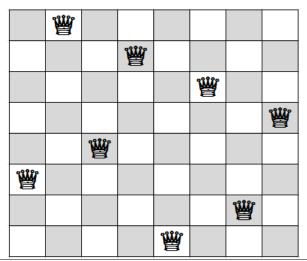


Eight queens problem



289. Problem statement

Can you place eight queens on a chess board so that no pair threatens each other?





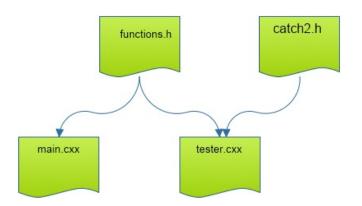
290. Sort of test-driven development

You will solve the 'eight queens' problem by

- designing tests for the functionality
- then implementing it



291. File structure





292. Basic object design

Object constructor of an empty board:

```
ChessBoard(int n);
Test how far we are:
int next row to be filled()
First test:
TEST_CASE( "empty board","[1]" ) {
  constexpr int n=10;
  ChessBoard empty(n);
  REQUIRE( empty.next_row_to_be_filled()==0 );
```



Exercise 86: Board object

Start writing the board class, and make it pass the above test.



Exercise 87: Board method

```
Write a method for placing a queen on the next row,
void place_next_queen_at_column(int i);
and make it pass this test (put this in a TEST_CASE):
REQUIRE_THROWS( empty.place_next_queen_at_column(-1) );
REQUIRE_THROWS( empty.place_next_queen_at_column(n) );
REQUIRE_NOTHROW( empty.place_next_queen_at_column(0) );
REQUIRE( empty.next_row_to_be_filled()==1 );
```



Exercise 88: Test for collisions

Write a method that tests if a board is collision-free:

```
bool feasible()
```

This test has to work for simple cases to begin with. You can add these lines to the above tests:

```
ChessBoard empty(n);
REQUIRE( empty.feasible() );
ChessBoard one = empty;
one.place_next_queen_at_column(0);
REQUIRE( one.next_row_to_be_filled()==1 );
REQUIRE( one.feasible() );
ChessBoard collide = one;
// place a queen in a 'colliding' location
collide.place_next_queen_at_column(0);
// and test that this is not feasible
REQUIRE( not collide.feasible() );
```



Exercise 89: Test full solutions

Make a second constructor to 'create' solutions:

```
ChessBoard( int n,vector<int> cols );
ChessBoard( vector<int> cols );
```

Now we test small solutions:

```
ChessBoard five( {0,3,1,4,2} );
REQUIRE( five.feasible() );
```



Exercise 90: No more delay: the hard stuff!

Write a function that takes a partial board, and places the next queen:

```
optional<ChessBoard> place_queens()

Test that the last step works:
ChessBoard almost( 4, {1,3,0} );
auto solution = almost.place_queens();
REQUIRE( solution.has_value() );
REQUIRE( solution->filled() );

Alternative to using optional:
bool place_queen( const board& current, board &next );
```

// true if possible, false is not



Exercise 91: Test that you can find solutions

Test that there are no 3×3 solutions: TEST_CASE("no 3x3 solutions", "[9]") { ChessBoard three(3): auto solution = three.place_queens(); REQUIRE(not solution.has_value()); but 4×4 solutions do exist: TEST CASE("there are 4x4 solutions"."[10]") { ChessBoard four(4): auto solution = four.place_queens(); REQUIRE(solution.has_value());



Dijkstra quote, part 1

Today a usual technique is to make a program and then to test it. But: program testing can be a very effective way to show the presence of bugs, but is hopelessly inadequate for showing their absence. (cue laughter)



Dijkstra quote, part 2

The only effective way to raise the confidence level of a program significantly is to give a convincing proof of its correctness. But one should not first make the program and then prove its correctness, because then the requirement of providing the proof would only increase the poor programmer's burden. On the contrary: the programmer should let correctness proof and program grow hand in hand.



Matrix-vector product

$$y = Ax$$

Partitioned:

$$\begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} A_T \\ A_B \end{pmatrix} (x)$$

Two equations:

$$\begin{cases} y_T = A_T x \\ y_B = A_B x \end{cases}$$



Inductive construction

$$\begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} A_T \\ A_B \end{pmatrix} (x)$$

Assume only equation

$$y_T = A_T x$$

is satisfied, and grow the T block by one row.

Algorithm outline

$$\begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} A_T \\ A_B \end{pmatrix} (x)$$

While *T* is not the whole system

Predicate: $y_T = A_T x$ true

Update: grow T block by one

Predicate: $y_T = A_T x$ true for new/bigger T block

Note initial and final condition.



Inductive step

Here is the big trick Before

$$\begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} A_T \\ A_B \end{pmatrix} (x)$$

split:

$$\begin{pmatrix} Y_1 \\ \cdots \\ y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} A_1 \\ \cdots \\ a_2 \\ A_3 \end{pmatrix} (x)$$

Then the update step, and After

$$\begin{pmatrix} Y_1 \\ y_2 \\ \dots \\ Y_3 \end{pmatrix} = \begin{pmatrix} A_1 \\ a_2 \\ \dots \\ A_3 \end{pmatrix} (x)$$

and unsplit

$$\begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} A_T \\ A_B \end{pmatrix} (x)$$



Before the update:

$$\begin{pmatrix} Y_1 \\ \dots \\ y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} A_1 \\ \dots \\ a_2 \\ A_3 \end{pmatrix} (x)$$

SO

$$Y_1 = A_1 x$$

is true

Then the update step, and

After

$$\begin{pmatrix} Y_1 \\ y_2 \\ \dots \\ Y_3 \end{pmatrix} = \begin{pmatrix} A_1 \\ a_2 \\ \dots \\ A_3 \end{pmatrix} (x)$$

so

$$\begin{cases} Y_1 = A_1 x & \text{we had this} \\ y_2 = a_2 x & \text{we need this} \end{cases}$$



Resulting algorithm

While *T* is not the whole system

Predicate: $y_T = A_T x$ true

Update: $y_2 = a_2 x$

Predicate: $y_T = A_T x$ true for new/bigger T block

In traditional notation:

Do
$$n = 0..N - 1$$

Predicate: $y_{0:n} = A_{0:n,*}x$ true

Compute: $y_n = a_{n,*}x$

Predicate: $y_{0:n+1} = A_{0:n+1,*}x$ true



So what?

We already know this stuff, right?

Well, we made an assumption on how to partition the matrix.

What if we partition it differently?



Matrix-vector product, the other way around

$$y = Ax$$

Partitioned:

$$(y) = \begin{pmatrix} A_L & A_R \end{pmatrix} \begin{pmatrix} x_T \\ x_B \end{pmatrix}$$

Equation:

$$\Big\{y = A_L x_T + A_R x_B$$



Inductive construction

$$(y) = (A_L \quad A_R) \begin{pmatrix} x_T \\ x_B \end{pmatrix}$$

Assume

$$y = A_L x_T$$

is constructed, and grow the T block.

Inductive step

Before

$$(y) = \begin{pmatrix} A_L & A_R \end{pmatrix} \begin{pmatrix} x_T \\ x_B \end{pmatrix}$$

split:

$$(y) = \begin{pmatrix} A_1 & \vdots & A_2 & A_3 \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_2 \\ x_3 \end{pmatrix}$$

Then the update step, and

After

$$(y) = \begin{pmatrix} A_1 & A_2 & \vdots & A_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_3 \end{pmatrix}$$

and unsplit

$$(y) = \begin{pmatrix} A_L & A_R \end{pmatrix} \begin{pmatrix} x_T \\ x_B \end{pmatrix}$$



Derivation of the update

Before the update:

$$(y) = \begin{pmatrix} A_1 & \vdots & A_2 & A_3 \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_2 \\ x_3 \end{pmatrix}$$

so

$$y = A_1 x_1$$

is true

Then the update step, and

After

$$(y) = \begin{pmatrix} A_1 & A_2 & \vdots & A_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_3 \end{pmatrix}$$

so

$$y = A_1 x_1 + A_2 x_2$$

in other words, we need





Resulting algorithm

While T is not the whole system

Predicate: $y = A_L x_T$ true

Update: $y \leftarrow y + A_2 x_2$

Predicate: $y = A_L x_T$ true for new/bigger T block



Two algorithms

for
$$r = 1, m$$

 $y_r = A_{r,*} x_*$

$$y \leftarrow 0$$
for $c = 1, n$

$$y \leftarrow y + A_{*,c}x_c$$



Take it from here

This gets more interesting if you take other algorithms:

- Matrix-matrix multiplication
- Matrix inversion
- Sylvester equation
- Iterative linear system solving

https://shpc.oden.utexas.edu/

