Mathematical programming with quantors

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1. For all: prime testing

$$\text{isprime}(n) \equiv \forall_{2\leq f<n}\colon \neg\,\text{divides}(f,n)$$

$$\text{isprime}(n) \equiv \neg\,\text{divides}(2,n) \cap \ldots \cap \neg\,\text{divides}(n-1,n)$$

2. in code

```
for (int f=2; f<n; f++)
  isprime = isprime && not divides(f,p)
```

Initialization: for-all over empty set is true:

```
bool isprime{true};
for (int f=2; f<n; f++)
  isprime = isprime && not divides(f,p)
```

3. Prime testing again

$$\mathrm{isprime}(n) \equiv \neg \exists_{2 \leq f < n} \colon \; \mathrm{divides}(f, n)$$

To get a pure quantor, and not a negated one, we write:

$$\mathrm{isnotprime}(n) \equiv \exists_{2 \leq f < n} \colon \; \mathrm{divides}(f, n)$$

Spelling out the exists-quantor as

$$\mathrm{isnotprime}(n) \equiv \mathrm{divides}(2, n) \cup \ldots \cup \mathrm{divides}(n - 1, n)$$

4. In code

applying $\exists_{s\in S}P(s)$ over an empty set S is `false`:

```
bool isnotprime{false};
for (int f=2; f<n; f++)
  isnotprime = isnotprime or divides(f,p)
bool isprime = not isnotprime;
```