

Collective operations

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Overview

In this section you will learn ‘collective’ operations, that combine information from all processes.

Commands learned:

- *MPI_Bcast, MPI_Reduce, MPI_Gather, MPI_Scatter*
- *MPI_All... variants, MPI_....v variants*
- *MPI_Barrier, MPI_Alltoall, MPI_Scan*



Technically

Routines can be ‘collective on a communicator’:

- They involve a communicator;
- if one process calls that routine, every process in that communicator needs to call it
- Mostly about combining data, but also opening shared files, declaring ‘windows’ for one-sided communication.

Collectives

Gathering and spreading information:

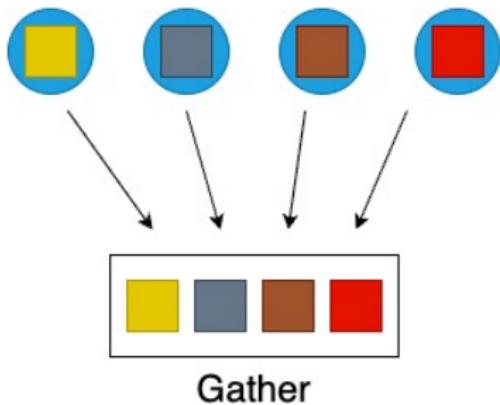
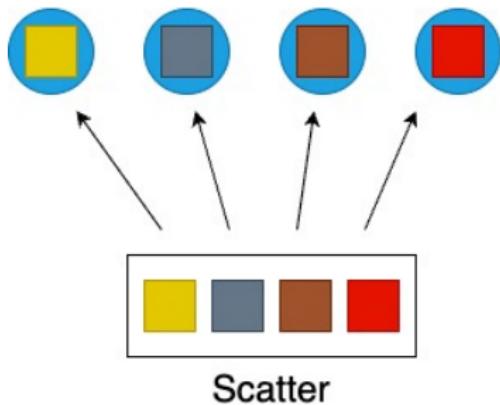
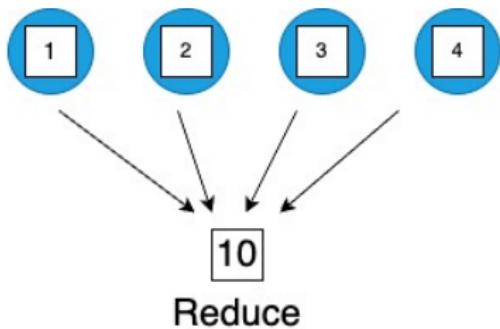
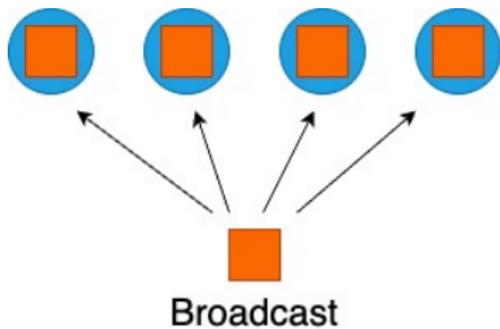
- Every process has data, you want to bring it together;
- One process has data, you want to spread it around.

Root process: the one doing the collecting or disseminating.

Basic cases:

- Collect data: gather.
- Collect data and compute some overall value (sum, max): reduction.
- Send the same data to everyone: broadcast.
- Send individual data to each process: scatter.





Exercise 1

How would you realize the following scenarios with MPI collectives?

1. Let each process compute a random number. You want to print the maximum of these numbers to your screen.
2. Each process computes a random number again. Now you want to scale these numbers by their maximum.
3. Let each process compute a random number. You want to print on what processor the maximum value is computed.

Think about time and space complexity of your suggestions.



Allreduce: reduce-to-all

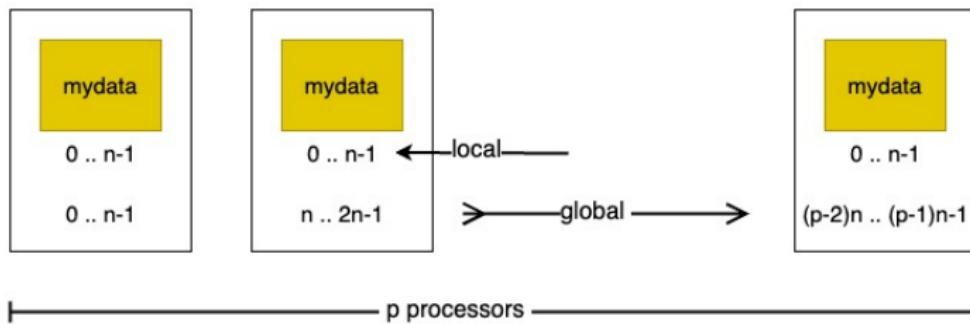
Case 2 in the exercise above contains a common case:
do a reduction, but everyone needs the result.

- `MPI_Allreduce` does the same as:
`MPI_Reduce` (reduction) followed by `MPI_Bcast` (broadcast)
- Same running time as either, half of reduce-followed-by-broadcast
(no proof given here)
- Common use case, symmetrical expression.

Motivation for allreduce

Example: normalizing a vector

$$y \leftarrow x / \|x\|$$



Structure of allreduce

- Vectors x, y are distributed: every process has certain elements
- The norm calculation is an all-reduce: every process gets same value
- Every process scales its part of the vector.
- Question: what kind of reduction do you use for an inf-norm?
One-norm? Two-norm?

Exercise 2

How many objections can you come up to this strategy:

1. Gather vector x on some root process;
2. Compute the reduction on that root;
3. Construct the scaled vector on the root;
4. Scatter the scaled vector.

Another Allreduce

Standard deviation:

$$\sigma = \sqrt{\frac{1}{N} \sum_i^N (x_i - \mu)} \quad \text{where} \quad \mu = \frac{\sum_i^N x_i}{N}$$

and assume that every process stores just one x_i value.

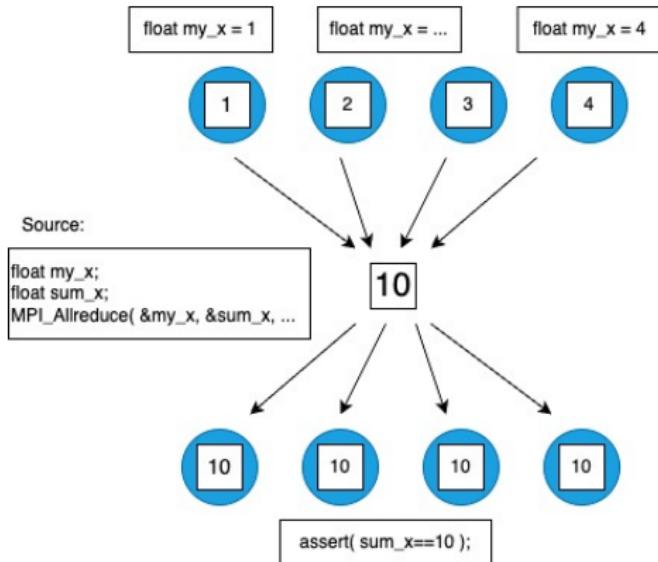
How do we compute this?

1. The calculation of the average μ is a reduction.
2. Every process needs to compute $x_i - \mu$ for its value x_i , so use allreduce operation, which does the reduction and leaves the result on all processes.
3. $\sum_i (x_i - \mu)$ is another sum of distributed data, so we need another reduction operation. Might as well use allreduce.



Conceptual picture

Recall SPMD: every process has the input and output variable



(What actually happens is a different story!)

Allreduce syntax

```
1 int MPI_Allreduce(  
2     const void* sendbuf,  
3     void* recvbuf, int count, MPI_Datatype datatype,  
4     MPI_Op op, MPI_Comm comm)
```

- All processes have send and recv buffer
- (No root argument)
- *count* is number of items in the buffer: 1 for scalar.
 > 1: pointwise application of the reduction operator
- *MPI_Datatype* is *MPI_INT*, *MPI_FLOAT*, *MPI_REAL8* et cetera.
- *MPI_Op* is *MPI_SUM*, *MPI_MAX* et cetera.



MPI_Allreduce

Name	Param name	Explanation	C type	F type
<code>MPI_Allreduce (</code>				
<code> MPI_Allreduce_c (</code>				
	<code> sendbuf</code>	starting address of send buffer	<code>const void*</code>	<code>TYPE(*),</code> <code> DIMENSION(..)</code>
	<code> recvbuf</code>	starting address of receive buffer	<code>void*</code>	<code>TYPE(*),</code> <code> DIMENSION(..)</code>
	<code> count</code>	number of elements in send buffer	<code>[</code> <code> int</code> <code> MPI_Count</code>	<code>INTEGER</code>
	<code> datatype</code>	datatype of elements of send buffer	<code>MPI_Datatype</code>	<code>TYPE(MPI_Datatype)</code>
	<code> op</code>	operation	<code>MPI_Op</code>	<code>TYPE(MPI_Op)</code>
	<code> comm</code>	communicator	<code>MPI_Comm</code>	<code>TYPE(MPI_Comm)</code>
	<code>)</code>			

MPL: MPI_Allreduce

```
1 template<typename T , typename F >
2 void mpl::communicator::allreduce
3     ( F, const T &, T & ) const;
4     ( F, const T *, T *,
5         const contiguous_layout< T > & ) const;
6     ( F, T & ) const;
7     ( F, T *, const contiguous_layout< T > & ) const;
8 F : reduction function
9 T : type
```



Exercise 3 randommax

Let each process compute a random number, and compute the sum of these numbers using the `MPI_Allreduce` routine.

$$\xi = \sum_i x_i$$

Each process then scales its value by this sum.

$$x'_i \leftarrow x_i / \xi$$

Compute the sum of the scaled numbers

$$\xi' = \sum_i x'_i$$

and check that it is 1.



Buffers

Buffers in C

General principle: buffer argument is address in memory of the data.

- Buffer is void pointer:
- write `&x` or `(void*)&x` for scalar
- write `x` or `(void*)x` for array

```
1 double x;
2 MPI_Bcast( &x, .... );
3 double x[5];
4 MPI_Bcast( x, .... );
```



Buffers in C++

- Scalars same as in C.
- Use of `std::vector` or `std::array`:

```
1 vector<float> xx(25);
2 MPI_Send( xx.data(),25,MPI_FLOAT, .... );
3 MPI_Send( &xx[0],25,MPI_FLOAT, .... );
4 MPI_Send( &xx.front(),25,MPI_FLOAT, .... );
```

- Can not send from iterator / let recv determine size/capacity.

Buffers in MPL

Two mechanisms:

1. Scalars; type derived through overloading
2. Automatic (static) arrays; type derived through overloading.
3. Layouts: contiguous or otherwise; see later.

MPL buffers through layout

You can pass a C-style array as buffer, requiring a layout:

```
1 // vector of 50 floats
2 vector<float> ar(50);
3 auto root = 0;
4 auto data = ar.data(); // or &(ar[0]) or &ar.front()
5 auto layout = mpl::contiguous_layout<float>(50)
6 comm_world::bcast( root,data,layout );
```



Large buffers

As of MPI-4 a buffer can be longer than 2^{31} elements.

- Use `MPI_Count` for count
- In C: use `MPI_Reduce_c`
- in Fortran: polymorphism means no change to the call.
- MPL: `long int` and `size_t` supported for layouts.

```
1 MPI_Count buffersize = 1000;  
2 double *indata,*outdata;  
3 indata = (double*) malloc( buffersize*sizeof(double) );  
4 outdata = (double*) malloc( buffersize*sizeof(double) );  
5 MPI_Allreduce_c(indata,outdata,buffersize,  
6 MPI_DOUBLE,MPI_SUM,MPI_COMM_WORLD);
```



Large buffers in MPL

Large buffer communication is supported:

```
1 // bigint.cxx
2 size_t s;
3   vector<char> buffer(s);
4   mpl::contiguous_layout<char> buffersize(s);
5   comm_world.send( buffer.data(),buffersize,processB );
```



Exercise 4

Extend exercise 3 to letting each process have an array.

Collective basics

Elementary datatypes

C	Fortran	Python	meaning
<i>MPI_CHAR</i>	<i>MPI_CHARACTER</i>		only for text
<i>MPI_SHORT</i>	<i>MPI_BYTE</i>		8 bits
<i>MPI_INT</i>	<i>MPI_INTEGER</i>		like the C/F types
<i>MPI_FLOAT</i>	<i>MPI_REAL</i>		
<i>MPI_DOUBLE</i>	<i>MPI_DOUBLE_PRECISION</i>	<i>MPI.DOUBLE</i>	
	<i>MPI_COMPLEX</i>		
	<i>MPI_LOGICAL</i>		
<i>unsigned</i>	extensions		
			<i>MPI_Aint</i>
			<i>MPI_Offset</i>

A bunch more.



MPL datatypes

Elementary types derived through overloading / templating.

Reduction operators

MPI type	meaning	applies to
MPI.Op		
<i>MPI_MAX</i>	<i>MPI.MAX</i>	maximum
<i>MPI_MIN</i>	<i>MPI.MIN</i>	minimum
<i>MPI_SUM</i>	<i>MPI.SUM</i>	sum multilanguage types
<i>MPI_PROD</i>	<i>MPI.PROC</i>	product
<i>MPI_REPLACE</i>	<i>MPI.REPLACE</i>	overwrite
<i>MPI_NO_OP</i>	<i>MPI.OP_NULL</i>	no change
<i>MPI_LAND</i>	<i>MPI.LAND</i>	logical and
<i>MPI_LOR</i>	<i>MPI.LOR</i>	logical or
<i>MPI_LXOR</i>	<i>MPI.LXOR</i>	logical xor
<i>MPI_BAND</i>	<i>MPI.BAND</i>	bitwise and
<i>MPI_BOR</i>	<i>MPI.BOR</i>	bitwise or
<i>MPI_BXOR</i>	<i>MPI.BXOR</i>	bitwise xor
<i>MPI_MAXLOC</i>	<i>MPI.MAXLOC</i>	max value and location
<i>MPI_MINLOC</i>	<i>MPI.MINLOC</i>	min value and location <i>MPI_DOUBLE_INT</i> and such



MPL operators

Operators need to have type:

$T(T\&)$

Elementary operators:

```
comm_world.allreduce(mpl::plus<float>(), rank2p2p1, p2layout);
```

User-defined operator:

```
comm_world.reduce(lcm<int>(), 0, v, result);
```



MPL operators

Available: `max`, `min`, `plus`, `multiplies`, `logical_and`, `logical_or`, `logical_xor`, `bit_and`, `bit_or`, `bit_xor`.

```
1 // separate recv buffer
2 comm_world.allreduce(mpl::plus<float>(), proc_data, reduce_data);
3 // in place
4 comm_world.allreduce(mpl::plus<float>(), proc_data);
```

Reduction to single process

Reduce with a single root process: great for printing out summary information at the end of your job.

Can you think of a case where a rooted reduce is appropriate or unavoidable? (Hint: tree)

Reduction to root

```
1 int MPI_Reduce
2   (void *sendbuf, void *recvbuf,
3    int count, MPI_Datatype datatype,
4    MPI_Op op, int root, MPI_Comm comm)
```

- Buffers: *sendbuf*, *recvbuf* are ordinary variables/arrays.
- Every process has data in its *sendbuf*,
Root combines it in *recvbuf* (ignored on non-root processes).
- *count* is number of items in the buffer: 1 for scalar.
- *MPI_Op* is *MPI_SUM*, *MPI_MAX* et cetera.



In-place operations

```
1 // allreduceinplace.c
2 for (int irand=0; irand<nrandoms; irand++)
3     myrandoms[irand] = (float) rand()/(float)RAND_MAX;
4 // add all the random variables together
5 MPI_Allreduce(MPI_IN_PLACE,myrandoms,
6                 nrandoms,MPI_FLOAT,MPI_SUM,comm);
```



More in-place operations

```
1 if (procno==root)
2   MPI_Reduce(MPI_IN_PLACE,myrandoms,
3             nrandoms,MPI_FLOAT,MPI_SUM,root,comm);
4 else
5   MPI_Reduce(myrandoms,MPI_IN_PLACE,
6             nrandoms,MPI_FLOAT,MPI_SUM,root,comm);
```

or

```
1 float *sendbuf,*recvbuf;
2 if (procno==root) {
3   sendbuf = MPI_IN_PLACE; recvbuf = myrandoms;
4 } else {
5   sendbuf = myrandoms; recvbuf = MPI_IN_PLACE;
6 }
7 MPI_Reduce(sendbuf,recvbuf,
8            nrandoms,MPI_FLOAT,MPI_SUM,root,comm);
```



In-place (MPL)

Scalar:

```
1 // separate recv buffer
2 comm_world.allreduce(mpl::plus<float>(), proc_data, reduce_data);
3 // in place
4 comm_world.allreduce(mpl::plus<float>(), proc_data);
```

Buffer:

```
1 // collectbuffer.cxx
2 vector<float> rank2p2p1{ 2*xrank, 2*xrank+1 }, reduce2p2p1{0,0};
3 mpl::contiguous_layout<float> two_floats(rank2p2p1.size());
4 comm_world.allreduce
5   (mpl::plus<float>(), rank2p2p1.data(), reduce2p2p1.data(), two_floats);
6 if ( iprint )
7   cout << "Got: " << reduce2p2p1.at(0) << ","
8     << reduce2p2p1.at(1) << endl;
```



Broadcast

```
1 int MPI_Bcast(  
2     void *buffer, int count, MPI_Datatype datatype,  
3     int root, MPI_Comm comm )
```

- All processes call with the same argument list
- *root* is the rank of the process doing the broadcast
- Each process allocates buffer space;
root explicitly fills in values,
all others receive values through broadcast call.
- Datatype is *MPI_FLOAT*, *MPI_INT* et cetera, different between C/Fortran.
- *comm* is usually *MPI_COMM_WORLD*

Gauss-Jordan elimination

<https://youtu.be/aQYuwatlWME>

MPI_Bcast

Name	Param name	Explanation	C type	F type
<code>MPI_Bcast (</code>				
<code> MPI_Bcast_c (</code>				
<code>buffer</code>		starting address of buffer	<code>void*</code>	<code>TYPE(*),</code> <code> DIMENSION(..)</code>
<code>count</code>		number of entries in buffer	<code>[</code> <code> int</code> <code> MPI_Count</code>	<code>INTEGER</code>
<code>datatype</code>		datatype of buffer	<code>MPI_Datatype</code>	<code>TYPE(MPI_Datatype)</code>
<code>root</code>		rank of broadcast root	<code>int</code>	<code>INTEGER</code>
<code>comm</code>		communicator	<code>MPI_Comm</code>	<code>TYPE(MPI_Comm)</code>
<code>)</code>				

MPL: MPI_Bcast

```
1 template<typename T >
2 void mpl::communicator::bcast
3   ( int root, T & data ) const
4   ( int root, T * data, const layout< T > & l ) const
```



Exercise 5 jordan

The *Gauss-Jordan algorithm* for solving a linear system with a matrix A (or computing its inverse) runs as follows:

```
for pivot  $k = 1, \dots, n$ 
    let the vector of scalings  $\ell_i^{(k)} = A_{ik}/A_{kk}$ 
    for row  $r \neq k$ 
        for column  $c = 1, \dots, n$ 
             $A_{rc} \leftarrow A_{rc} - \ell_r^{(k)} A_{kc}$ 
```

where we ignore the update of the righthand side, or the formation of the inverse.

Let a matrix be distributed with each process storing one column. Implement the Gauss-Jordan algorithm as a series of broadcasts: in iteration k process k computes and broadcasts the scaling vector $\{\ell_i^{(k)}\}_i$. Replicate the right-hand side on all processors.



Exercise (optional) 6

Bonus exercise: can you extend your program to have multiple columns per process?

Scan



Scan

Scan or ‘parallel prefix’: reduction with partial results

- Useful for indexing operations:
- Each process has an array of n_p elements;
- My first element has global number $\sum_{q < p} n_q$.
- Two variants: MPI_Scan inclusive, and MPI_Exscan exclusive.

In vs Exclusive

process : 0 1 2 \cdots $p - 1$

data : x_0 x_1 x_2 \cdots x_{p-1}

inclusive : x_0 $x_0 \oplus x_1$ $x_0 \oplus x_1 \oplus x_2$ \cdots $\oplus_{i=0}^{p-1} x_i$

exclusive : unchanged x_0 $x_0 \oplus x_1$ \cdots $\oplus_{i=0}^{p-2} x_i$

MPI_Scan

Name	Param name	Explanation	C type	F type
MPI_Scan (
MPI_Scan_c (
sendbuf		starting address of send buffer	const void*	TYPE(*), DIMENSION(..)
recvbuf		starting address of receive buffer	void*	TYPE(*), DIMENSION(..)
count		number of elements in input buffer	[int MPI_Count	INTEGER
datatype		datatype of elements of input buffer	MPI_Datatype	TYPE(MPI_Datatype)
op		operation	MPI_Op	TYPE(MPI_Op)
comm		communicator	MPI_Comm	TYPE(MPI_Comm)
)				

MPL: MPI_Scan

```
1 template<typename T , typename F >
2 void mpl::communicator::scan
3   ( F, const T &, T & ) const;
4   ( F, const T *, T *,
5    const contiguous_layout< T > & ) const;
6   ( F, T & ) const;
7   ( F, T *, const contiguous_layout< T > & ) const;
8 F : reduction function
9 T : type
```

MPI_Exscan

Name	Param name	Explanation	C type	F type
<code>MPI_Exscan (</code>				
<code> MPI_Exscan_c (</code>				
	<code> sendbuf</code>	starting address of send buffer	<code>const void*</code>	<code>TYPE(*),</code> <code>DIMENSION(..)</code>
	<code> recvbuf</code>	starting address of receive buffer	<code>void*</code>	<code>TYPE(*),</code> <code>DIMENSION(..)</code>
	<code> count</code>	number of elements in input buffer	<code>[int MPI_Count</code>	<code>INTEGER</code>
	<code> datatype</code>	datatype of elements of input buffer	<code>MPI_Datatype</code>	<code>TYPE(MPI_Datatype)</code>
	<code> op</code>	operation	<code>MPI_Op</code>	<code>TYPE(MPI_Op)</code>
	<code> comm</code>	intra-communicator	<code>MPI_Comm</code>	<code>TYPE(MPI_Comm)</code>
	<code>)</code>			



MPL: MPI_Exscan

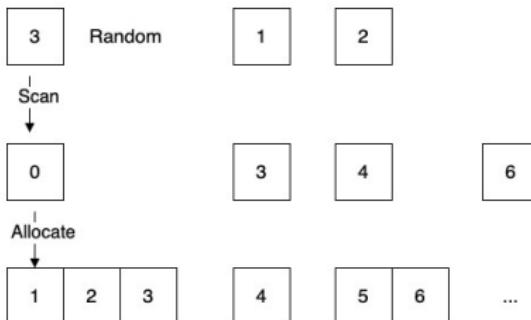
```
1 template<typename T , typename F >
2 void mpl::communicator::exscan
3   ( F, const T &, T & ) const;
4   ( F, const T *, T *,
5    const contiguous_layout< T > & ) const;
6   ( F, T & ) const;
7   ( F, T *, const contiguous_layout< T > & ) const;
8 F : reduction function
9 T : type
```

Exercise 7 scangather

- Let each process compute a random value n_{local} , and allocate an array of that length. Define

$$N = \sum n_{\text{local}}$$

- Fill the array with consecutive integers, so that all local arrays, laid end-to-end, contain the numbers $0 \cdots N - 1$. (See figure 7.)



Gather/Scatter, Barrier, and others

MPI_Gather

Name	Param name	Explanation	C type	F type
MPI_Gather (
MPI_Gather_c (
sendbuf		starting address of send buffer	const void*	TYPE(*), DIMENSION(..)
sendcount		number of elements in send buffer	[int MPI_Count	INTEGER
sendtype		datatype of send buffer elements	MPI_Datatype	TYPE(MPI_Datatype)
recvbuf		address of receive buffer	void*	TYPE(*), DIMENSION(..)
recvcount		number of elements for any single receive	[int MPI_Count	INTEGER
recvtype		datatype of recv buffer elements	MPI_Datatype	TYPE(MPI_Datatype)
root		rank of receiving process	int	INTEGER
comm		communicator	MPI_Comm	TYPE(MPI_Comm)
)				

MPL: MPI_Gather

```
1 void mpl::communicator::gather
2     ( int root_rank, const T * senddata, const layout< T > & sendl ) const
3     ( int root_rank, const T * senddata, const layout< T > & sendl,
4             T * recvdata, const layout< T > & recvl ) const
5 // non-root versions:
6     ( int root_rank, const T & senddata ) const
7     ( int root_rank, const T & senddata, T * recvdata ) const
```



MPI_Scatter

Name	Param name	Explanation	C type	F type
MPI_Scatter (
MPI_Scatter_c (
sendbuf		address of send buffer	const void*	TYPE(*), DIMENSION(..)
sendcount		number of elements sent to each process	[int MPI_Count	INTEGER
sendtype		datatype of send buffer elements	MPI_Datatype	TYPE(MPI_Datatype)
recvbuf		address of receive buffer	void*	TYPE(*), DIMENSION(..)
recvcount		number of elements in receive buffer	[int MPI_Count	INTEGER
recvtype		datatype of receive buffer elements	MPI_Datatype	TYPE(MPI_Datatype)
root		rank of sending process	int	INTEGER
comm		communicator	MPI_Comm	TYPE(MPI_Comm)
)				

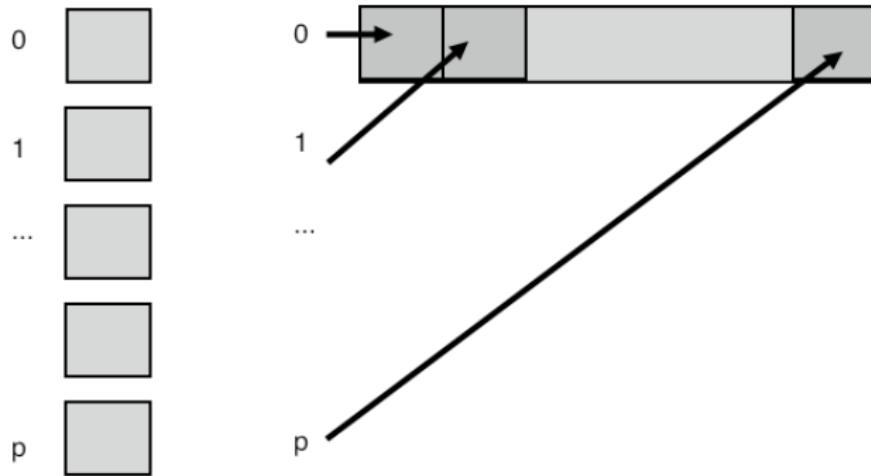
MPL: MPI_Scatter

```
1 void mpl::communicator::scatter
2     ( int root_rank, const T * send_data, const layout< T > & sendl,
3       T * recv_data, const layout< T > & recvl ) const
4     ( int root_rank, const T * send_data,
5       T & recv_data ) const
6     // non-root versions:
7     ( int root_rank, T & recv_data ) const
8     ( int root_rank, T * recv_data, const layout< T > & recvl ) const
```

Gather/Scatter

- Compare buffers to reduce
- Scatter: the sendcount / Gather: the recvcount:
this is not, as you might expect, the total length of the buffer;
instead, it is the amount of data to/from each process.

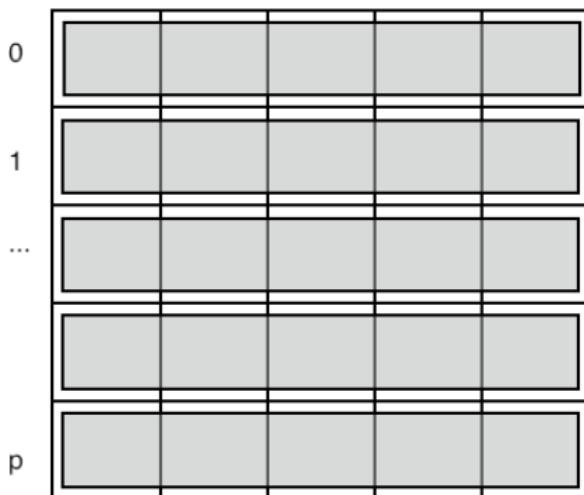
Gather pictured



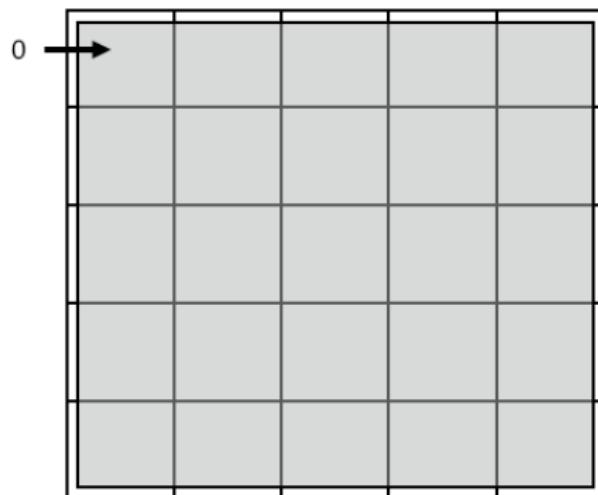
Popular application of gather

Matrix is constructed distributed, but needs to be brought to one process:

distributed matrix



gathered matrix



This is not efficient in time or space. Do this only when strictly necessary.
Remember SPMD: try to keep everything symmetrically parallel.

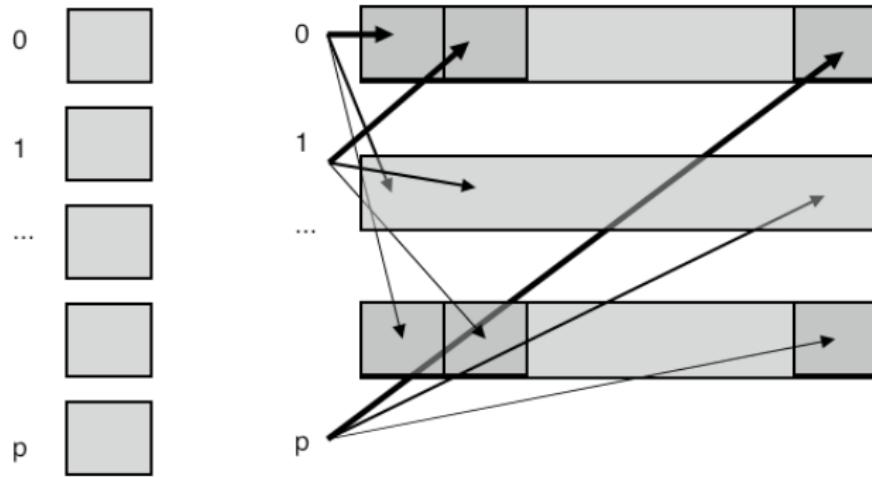
MPI_Allgather

Name	Param name	Explanation	C type	F type
MPI_Allgather (
MPI_Allgather_c (
sendbuf		starting address of send buffer	const void*	TYPE(*), DIMENSION(..)
sendcount		number of elements in send buffer	[int MPI_Count	INTEGER
sendtype		datatype of send buffer elements	MPI_Datatype	TYPE(MPI_Datatype)
recvbuf		address of receive buffer	void*	TYPE(*), DIMENSION(..)
recvcount		number of elements received from any process	[int MPI_Count	INTEGER
recvtype		datatype of receive buffer elements	MPI_Datatype	TYPE(MPI_Datatype)
comm		communicator	MPI_Comm	TYPE(MPI_Comm)
)				

MPL: MPI_Allgather

```
1 void allgather
2     ( const T & send_data, T * recv_data ) const
3     ( const T * send_data, const layout< T > & sendl,
4         T * recv_data, const layout< T > & recvl ) const
```

Allgather pictured



V-type collectives

- Gather/scatter but with individual sizes
- Requires displacement in the gather/scatter buffer

MPI_Gatherv

Name	Param name	Explanation	C type	F type
<code>MPI_Gatherv (</code>				
<code> MPI_Gatherv_c (</code>				
<code>sendbuf</code>		starting address of send buffer	<code>const void*</code>	<code>TYPE(*), DIMENSION(..)</code>
<code>sendcount</code>		number of elements in send buffer	<code>[int MPI_Count]</code>	<code>INTEGER</code>
<code>sendtype</code>		datatype of send buffer elements	<code>MPI_Datatype</code>	<code>TYPE(MPI_Datatype)</code>
<code>recvbuf</code>		address of receive buffer	<code>void*</code>	<code>TYPE(*), DIMENSION(..)</code>
<code>recvcounts</code>		non-negative integer array (of length group size) containing the number of elements that are received from each process	<code>[const int[] MPI_Count[]]</code>	<code>INTEGER(*)</code>
<code>displs</code>		integer array (of length group size). Entry i specifies the displacement relative to recvbuf at which to place the incoming data from process i	<code>[const int[] MPI_Aint[]]</code>	<code>INTEGER(*)</code>
<code>recvtype</code>		datatype of recv buffer elements	<code>MPI_Datatype</code>	<code>TYPE(MPI_Datatype)</code>
<code>root</code>		rank of receiving process	<code>int</code>	<code>INTEGER</code>
<code>comm</code>		communicator	<code>MPI_Comm</code>	<code>TYPE(MPI_Comm)</code>
<code>)</code>				



MPL: MPI_Gatherv

```
1 template<typename T>
2 void gatherv
3     (int root_rank, const T *senddata, const layout<T> &sendl,
4      T *recvdata, const layouts<T> &recvls, const displacements &recvdispls) const
5     (int root_rank, const T *senddata, const layout<T> &sendl,
6      T *recvdata, const layouts<T> &recvls) const
7     (int root_rank, const T *senddata, const layout<T> &sendl ) const
```

Exercise 8 scangather

Take the code from exercise 7 and extend it to gather all local buffers onto rank zero. Since the local arrays are of differing lengths, this requires `MPI_Gatherv`.

How do you construct the lengths and displacements arrays?

Review 1

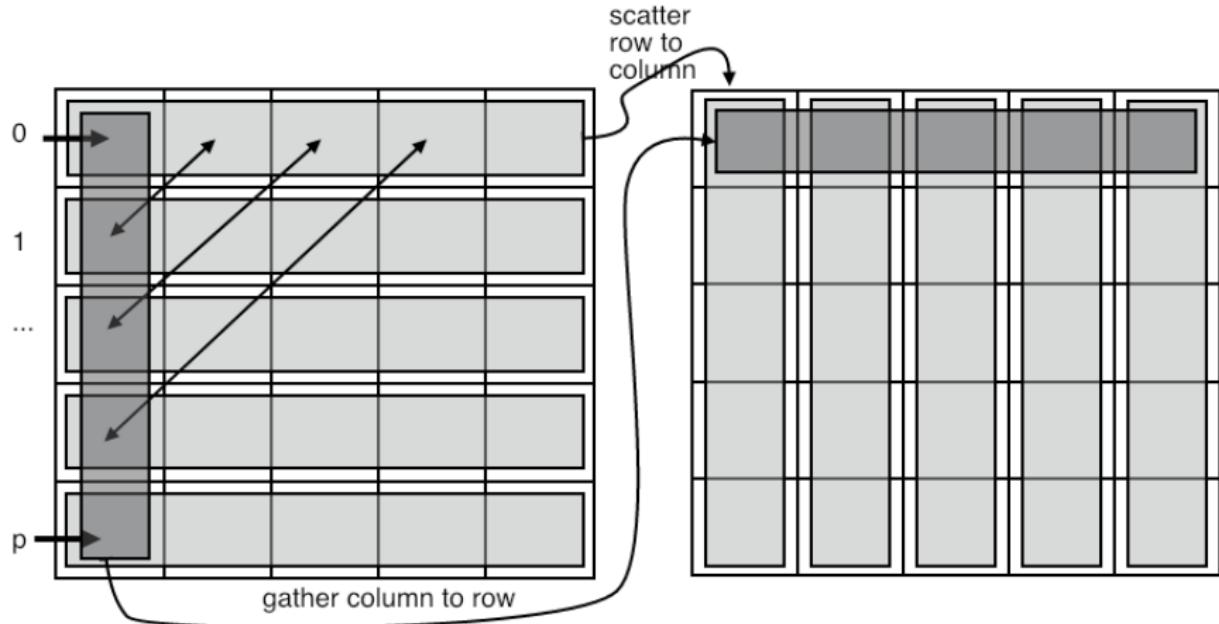
An *MPI_Scatter* call puts the same data on each process

```
/poll "A scatter call puts the same data on each process" "T" "F"
```

All-to-all

- Every process does a scatter;
- (equivalently: every process gather)
- each individual data, but amounts are identical
- Example: data transposition in FFT

Data transposition



Example: each process knows who to send to,
all-to-all gives information who to receive from

All-to-allv

- Every process does a scatter or gather;
- each individual data and individual amounts.
- Example: radix sort by least-significant digit.

Radix sort

Sort 4 numbers on two processes:

array binary	proc0		proc1	
	2	5	7	1
010	101	111	001	
stage 1				
last digit	0	1	1	1
(this serves as bin number)				
sorted	010		101	111 001
stage 2				
next digit	1		0 1	0
(this serves as bin number)				
sorted	101	001	010 111	
stage 3				
next digit	1	0	0 1	
(this serves as bin number)				
sorted	001	010	101 111	
decimal	1	2	5	7

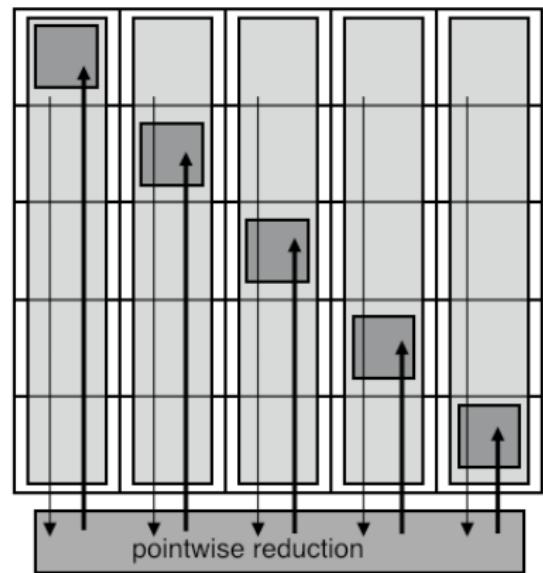
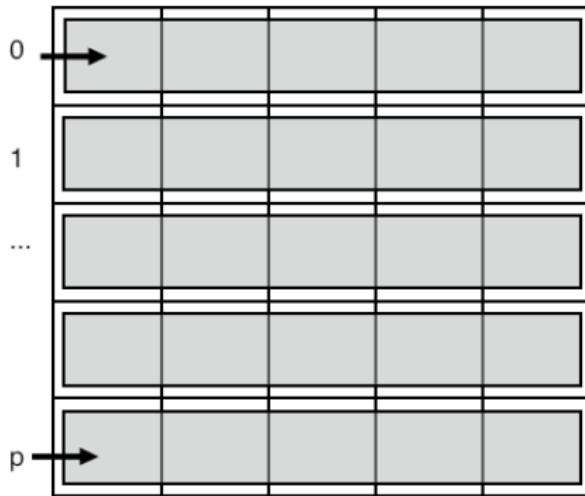


Reduce-scatter

- Pointwise reduction (one element per process) followed by scatter
- Somewhat related to all-to-all: data transpose but reduced information, rather than gathered.
- Applications in both sparse and dense matrix-vector product.

Example: sparse matrix setup

Example: each process knows who to send to,
all-to-all gives information how many messages to expect
reduce-scatter leaves only relevant information



Barrier

```
1 int MPI_BARRIER( MPI_Comm comm )
```

- Synchronize processes:
- each process waits at the barrier until all processes have reached the barrier
- **This routine is almost never needed:**
collectives are already a barrier of sorts, two-sided communication is a local synchronization
- One conceivable use: timing



User-defined operators

MPI Operators

Define your own reduction operator

- Define operator between partial result and new operand

```
1 typedef void MPI_User_function
2   (void *invec, void *inoutvec, int *len,
3    MPI_Datatype *datatype);
```

- Don't forget to free:

```
1 int MPI_Op_free(MPI_Op *op)
```

- Make your own reduction scheme *MPI_Reduce_local*

MPI_Op_create

Name	Param name	Explanation	C type	F type
MPI_Op_create (
MPI_Op_create_c (
user_fn		user defined function	[MPI_User_function* MPI_User_function_c*	PROCEDURE (MPI_User_function)
commute		true if commutative; false otherwise.	int	LOGICAL
op		operation	MPI_Op*	TYPE(MPI_Op)
)				

MPL: MPI_Op_create

Missing MPL proto: mpi::op::create

Example

Smallest nonzero:

```
1 *(int*)inout = m;  
2 }
```



Review 2

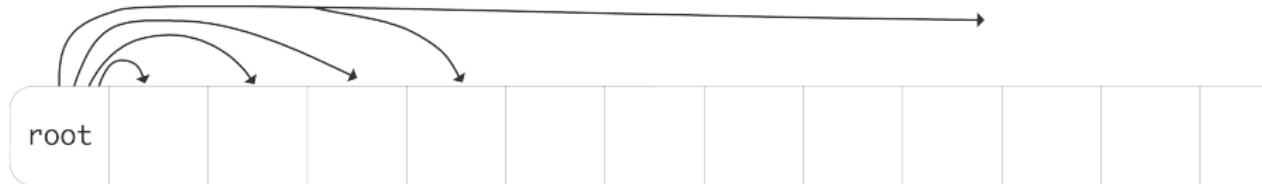
The $\|\cdot\|_2$ norm (sum of squares) needs a custom operator.

```
/poll "The sum of squares norm needs a custom operators" "T" "F"
```

Performance of collectives

Naive realization of collectives

Broadcast:



Single message:

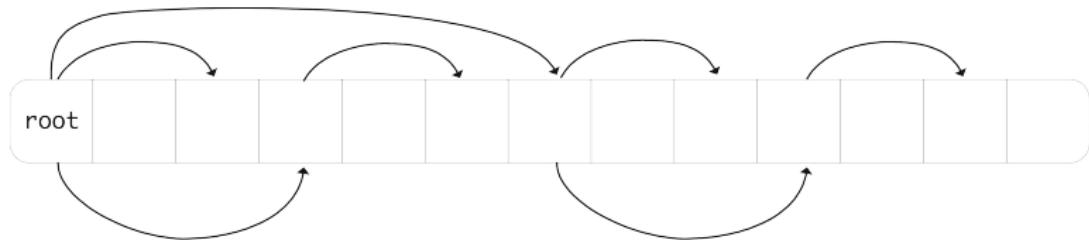
$$\alpha = \text{message startup} \approx 10^{-6} s, \quad \beta = \text{time per word} \approx 10^{-9} s$$

- Time for message of n words:

$$\alpha + \beta n$$

- Time for collective? Can you improve on that?

Better implementation of collective



- What is the running time now?
- Can you come up with lower bounds on the α, β terms? Are these achieved here?
- How about the case of really long buffers?

Implementation of Reduce

	$t = 1$	$t = 2$	$t = 3$
p_0	$x_0^{(0)}, x_1^{(0)}, x_2^{(0)}, x_3^{(0)}$	$x_0^{(0:1)}, x_1^{(0:1)}, x_2^{(0:1)}, x_3^{(0:1)}$	$x_0^{(0:3)}, x_1^{(0:3)}, x_2^{(0:3)}, x_3^{(0:3)}$
p_1	$x_0^{(1)} \uparrow, x_1^{(1)} \uparrow, x_2^{(1)} \uparrow, x_3^{(1)} \uparrow$		
p_2	$x_0^{(2)}, x_1^{(2)}, x_2^{(2)}, x_3^{(2)}$	$x_0^{(2:3)} \uparrow, x_1^{(2:3)} \uparrow, x_2^{(2:3)} \uparrow, x_3^{(2:3)} \uparrow$	
p_3	$x_0^{(3)} \uparrow, x_1^{(3)} \uparrow, x_2^{(3)} \uparrow, x_3^{(3)} \uparrow$		

Implementation of Allreduce

	$t = 1$	$t = 2$	$t = 3$
p_0	$x_0^{(0)} \downarrow, x_1^{(0)} \downarrow, x_2^{(0)} \downarrow, x_3^{(0)} \downarrow$	$x_0^{(0:1)} \downarrow\downarrow, x_1^{(0:1)} \downarrow\downarrow, x_2^{(0:1)} \downarrow\downarrow, x_3^{(0:1)} \downarrow\downarrow$	$x_0^{(0:3)}, x_1^{(0:3)}, x_2^{(0:3)}, x_3^{(0:3)}$
p_1	$x_0^{(1)} \uparrow, x_1^{(1)} \uparrow, x_2^{(1)} \uparrow, x_3^{(1)} \uparrow$	$x_0^{(0:1)} \downarrow\downarrow, x_1^{(0:1)} \downarrow\downarrow, x_2^{(0:1)} \downarrow\downarrow, x_3^{(0:1)} \downarrow\downarrow$	$x_0^{(0:3)}, x_1^{(0:3)}, x_2^{(0:3)}, x_3^{(0:3)}$
p_2	$x_0^{(2)} \downarrow, x_1^{(2)} \downarrow, x_2^{(2)} \downarrow, x_3^{(2)} \downarrow$	$x_0^{(2:3)} \uparrow\uparrow, x_1^{(2:3)} \uparrow\uparrow, x_2^{(2:3)} \uparrow\uparrow, x_3^{(2:3)} \uparrow\uparrow$	$x_0^{(0:3)}, x_1^{(0:3)}, x_2^{(0:3)}, x_3^{(0:3)}$
p_3	$x_0^{(3)} \uparrow, x_1^{(3)} \uparrow, x_2^{(3)} \uparrow, x_3^{(3)} \uparrow$	$x_0^{(2:3)} \uparrow\uparrow, x_1^{(2:3)} \uparrow\uparrow, x_2^{(2:3)} \uparrow\uparrow, x_3^{(2:3)} \uparrow\uparrow$	$x_0^{(0:3)}, x_1^{(0:3)}, x_2^{(0:3)}, x_3^{(0:3)}$

Review 3

True or false: there are collectives that do not communicate data

```
/poll "there are collectives that do not communicate data" "T" "F"
```



Reduction operators

User-defined operators

Given a reduction function:

```
1 typedef void user_function  
2     (void *invec, void *inoutvec, int *len,  
3      MPI_Datatype *datatype);
```

create a new operator:

```
1 MPI_Op rwz;  
2 MPI_Op_create(user_function,1,&rwz);  
3 MPI_Allreduce(data+procno,&positive_minimum,1,MPI_INT,rwz,comm);
```

Exercise 9 onenorm

Write the reduction function to implement the *one-norm* of a vector:

$$\|x\|_1 \equiv \sum_i |x_i|.$$