Measles Outbreak Simulator — Model Details

Model Structure — The stochastic compartmental model tracks the number of students in various epidemiological states over the course of a measles outbreak. Unvaccinated students are categorized as susceptible (S), exposed (E), infectious (I), and recovered (R); fully vaccinated students are similarly categorized as susceptible (S_V), exposed (E_V), infectious (I_V), and recovered (R_V). Partially vaccinated students are not explicitly modeled. The total student population is given by $N = S + E + I + R + S_V + E_V + I_V + R_V$. The model is governed by the equations below, where β, σ, and γ represent the transmission rate, the rate of progression from exposed to infectious (i.e., the reciprocal of the average latent period), and the recovery rate (i.e., the reciprocal of the average infectious period), respectively. The parameters ϵ_s and ϵ_i capture the impact of vaccination on susceptibility and infectiousness, respectively, with lower values indicating greater protection. For example, a vaccine efficacy of 99% corresponds to $\epsilon = 0.01$.

Stochastic transitions between compartments are modeled using the τ-leap method with 10 time steps per day [1,2]. Events at each time-step are assumed to be independent of each other and to not impact the underlying transition rates. The numbers of each type of transition follow Poisson distributions with means equal to the corresponding rate parameters. The model is implemented in Python 3.12.2.

$$\frac{dS}{dt} = -\beta S \cdot \left(\frac{I + \epsilon_i I_v}{N}\right) \qquad \frac{dS_V}{dt} = -\beta \epsilon_s S_V \cdot \left(\frac{I + \epsilon_i I_v}{N}\right)$$

$$\frac{dE}{dt} = \beta S \cdot \left(\frac{I + \epsilon_i I_v}{N}\right) - \sigma E \qquad \frac{dE_V}{dt} = \beta \epsilon_s S_V \cdot \left(\frac{I + \epsilon_i I_v}{N}\right) - \sigma E_V$$

$$\frac{dI}{dt} = \sigma E - \gamma I \qquad \frac{dI_V}{dt} = \sigma E_V - \gamma I_V$$

$$\frac{dR}{dt} = \gamma I \qquad \frac{dR_V}{dt} = \gamma I_V$$

Initial Conditions — Let I_0 denote the number of students infected at the start of the outbreak and let υ denote the fraction of students who are vaccinated against measles. Each simulation is initialized with I_0 students in the infectious compartment (I), υN students in the vaccinated susceptible compartment (S_V), and the remaining N - (I_0 - νN) students in the susceptible unvaccinated compartment (S).

Parameter Settings — The user can choose values for the model parameters, σ, γ, ϵ_s and ϵ_i, the initial conditions, I_0 and ν, and the basic reproduction number, R_0. The transmission rate β is then determined by the equation $\beta = R_0 \cdot \gamma$. The default parameter values are $R_0 = 15$, $\sigma = 1/(10.5 \text{ days})$, $\gamma = 1/(8 \text{ days})$, $\epsilon_s = 1 - 0.997 = 0.003$ and $\epsilon_i = 1 - 0.95 = 0.05$ [3–6].

References

1. Keeling MJ, Rohani P. Modeling infectious diseases in humans and animals. Princeton, NJ: Princeton University Press; 2007.
2. Gillespie DT. Approximate accelerated stochastic simulation of chemically reacting systems. J Chem Phys. 2001;115: 1716–1733. doi:10.1063/1.1378322
3. CDC. Measles Clinical Diagnosis Fact Sheet. In: Measles (Rubeola) [Internet]. 5 Aug 2024 [cited 25 Feb 2025].
4. Factsheet about measles. In: European Centre for Disease Prevention and Control [Internet]. 24 Feb 2017 [cited 25 Feb 2025].
5. van Boven M, Kretzschmar M, Wallinga J, O'Neill PD, Wichmann O, Hahné S. Estimation of measles vaccine efficacy and critical vaccination coverage in a highly vaccinated population. J R Soc Interface. 2010;7: 1537–1544. doi:10.1098/rsif.2010.0086
6. Tranter I, Smoll N, Lau CL, Williams D-L, Neucom D, Barnekow D, et al. Onward virus transmission after measles secondary vaccination failure. Emerg Infect Dis. 2024;30: 1747–1754. doi:10.3201/eid3009.240150