

## Measles Outbreak Simulator — Model Details

**Model Structure** — The stochastic compartmental SEIR model tracks the changing numbers of students who are susceptible (S), exposed (E), infectious (I), and recovered (R) over the course of a measles outbreak. N denotes the total number of students in the school (N = S + E + I + R). The model is given by the equations to the right, where  $\beta$ ,  $\sigma$ , and  $\gamma$ , represents the transmission rate, the transition rate from exposed to infectious (reciprocal of the latent period), and the transition rate from infectious to recovered (reciprocal of the infectious period), respectively.

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$

$$\frac{dE}{dt} = -\beta S \frac{I}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

Stochastic transitions between compartments are modeled using the  $\tau$ -leap method with 10 time steps per day <sup>1,2</sup>. Events at each

time-step are assumed to be independent of each other and to not impact the underlying transition rates. The numbers of each type of transition follow Poisson distributions with means equal to the corresponding rate parameters. The model is implemented in Python 3.12.2.

**Initial Conditions** — Let  $\psi$  denote the number of students infected at the start of the outbreak and  $\upsilon$  denote the fraction of students who are vaccinated for measles. Each simulation is initialized with  $\psi$  students in the infectious compartment,  $\upsilon N$  students in the recovered compartment (assuming vaccination provides full immunity), and the remaining N - ( $\psi$  +  $\upsilon N$ ) students are in the susceptible compartment.

**Parameter Settings** — The user can choose values for the model parameters,  $\sigma$  and  $\gamma$ , the initial conditions,  $\psi$  and  $\nu$ , and the basic reproduction number,  $R_0$ . The transmission rate  $\beta$  is then determined by the equation  $\beta = R_0 \cdot \gamma$ . The default parameter values are  $R_0 = 15$ ,  $\sigma = 1/(10.5 \text{ days})$ , and  $\gamma = 1/(8 \text{ days})^{3.4}$ .

## References

- 1. Keeling, M. J. & Rohani, P. *Modeling Infectious Diseases in Humans and Animals*. (Princeton University Press, Princeton, NJ, 2007).
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