



Scientific and Technical Computing

Hardware and Code Optimization

Lars Koesterke
UT Austin, 9/29/22 & ...

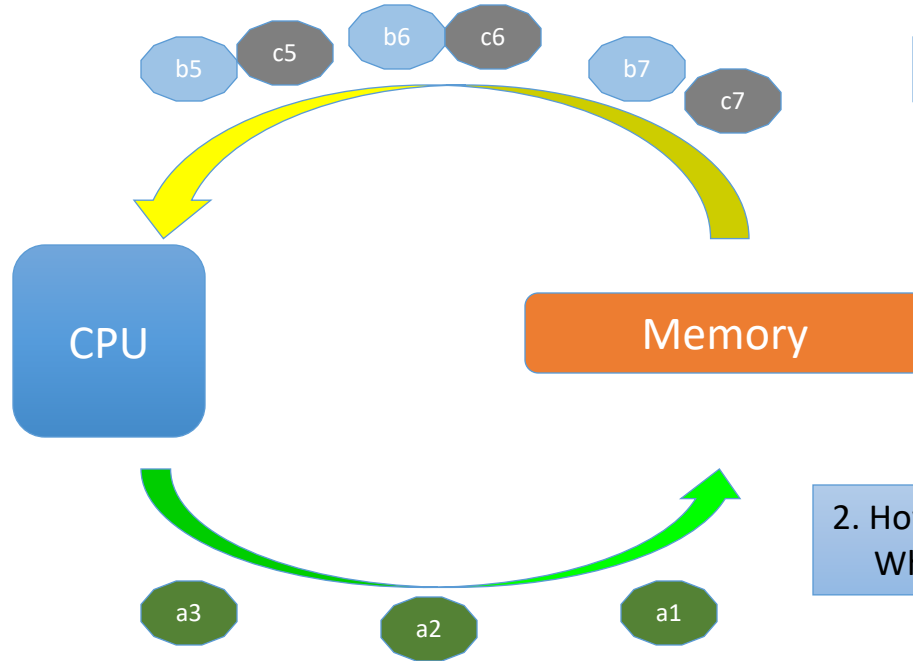
Tentative(!) Schedule

Total of 9 class periods

1. 9/29: Slide deck 3 (1) (Today, 5th lecture of 'Hardware')
 2. 10/6: Slide deck 3 (2)
 3. 10/13: Slide deck 4 (1)
 4. 10/18: Finish lecture and Quiz review. (Tuesday!)
 5. 10/20: Quiz
-
- 10/27: Quiz return (30 minutes)
Important: You must finish the quiz before(!) I return it

Remember this? Data streams

$$a(i) = b(i) + c(i)$$



1. How much data has to be 'en route'?

Operation: 3 cycles
Data transfer: 300 cycles
Ratio: 1/100

100 elements of a, b, and c

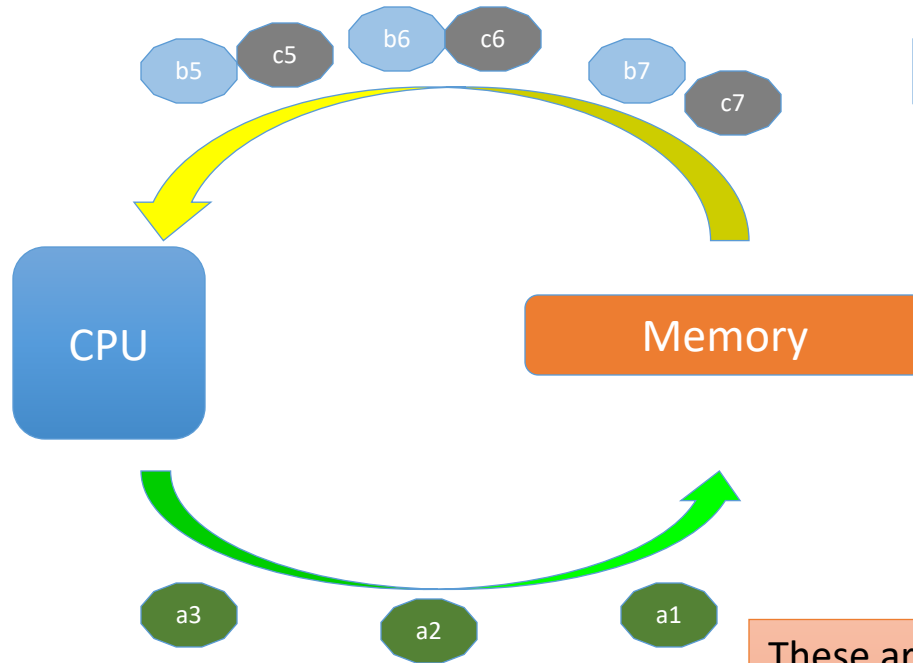
2. How many results (a) do we get per cycle?
What is the speedup?

1 result every 3 cycles
Speedup is 200×

What do the designations 'a3',
'b5', 'c7' now stand for?

Remember this? Data streams

$$a(i) = b(i) + c(i)$$



1. How much data has to be 'en route'?

Operation: 3 cycles
Data transfer: 300 cycles
Ratio: 1/100

100 elements of a, b, and c

2. How many results (a) do we get per cycle?
What is the speedup?

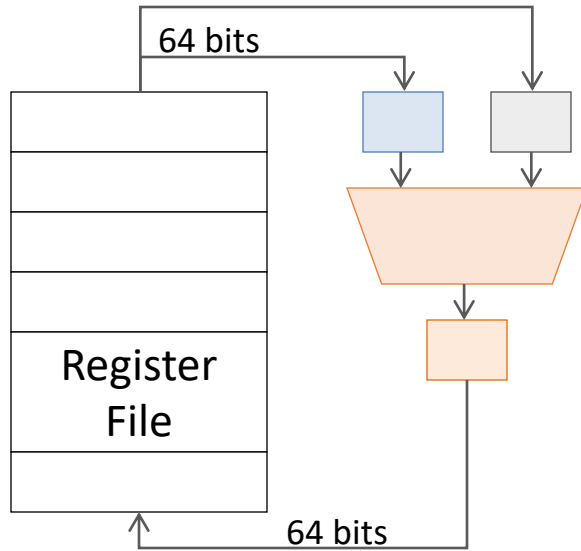
1 result every 3 cycles
Speedup is 200x

What do the designations 'a3', 'b5', 'c7' now stand for?

These are now 'names/designations' of whole cache lines (512 bit)
And we will use this to our advantage to **increase computational speed**

Scalar Hardware

$$a(i) = b(i) + c(i)$$



Scalar Unit

Input: 2 words (single or double precision)

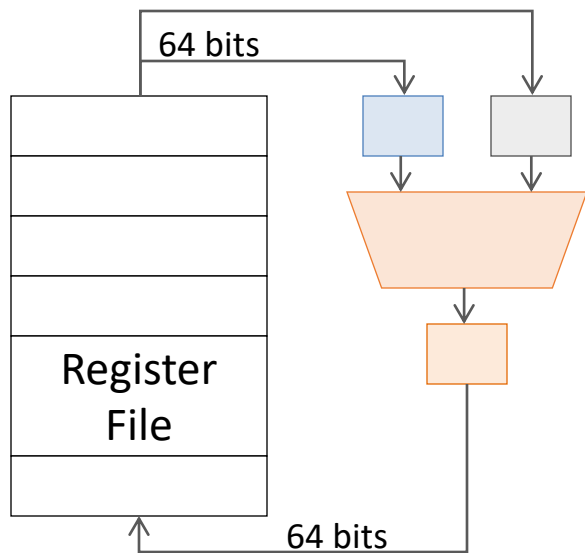
Output: 1 word

Operations: 1 operation → 1 result

Vector Hardware

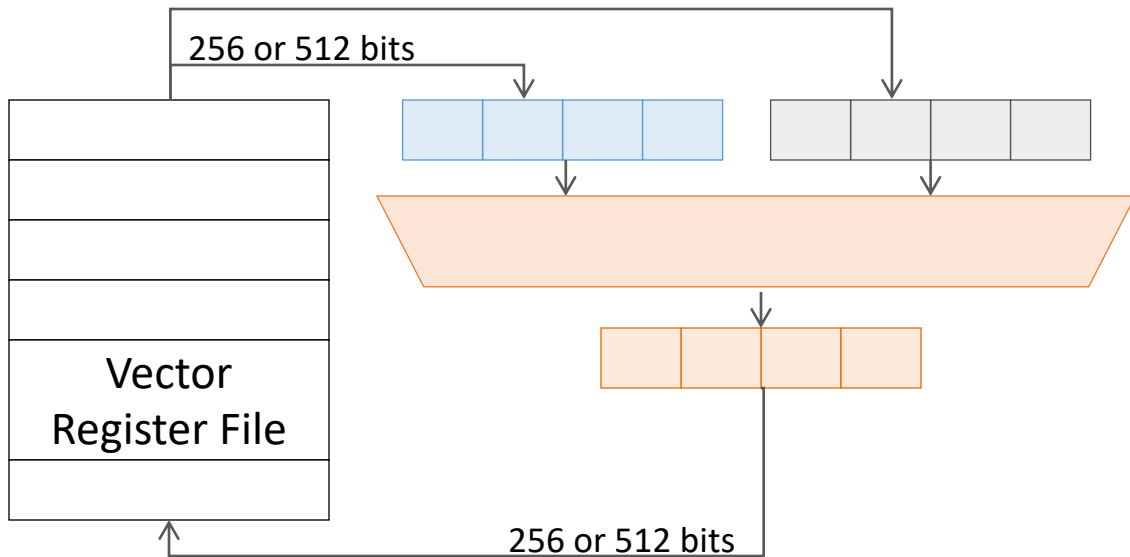
$$a(i) = b(i) + c(i)$$

Example for vector length = 4 words



Scalar Unit

Input: 2 words (single or double precision)
Output: 1 word
Operations: 1 operation → 1 result



Vector Unit

Input: 2 cache lines
Output: 1 cache line
Operations: 1 operation → 8/16 results (dp/sp)

Vectorization

The whole process is called vectorization

- Vector registers
- Vector instructions
- Loading a cache line into vector register with one instruction
- Operating on vector registers with one instruction

Using vector instructions

- Compiler creates vector instructions when possible (and
- Compiler creates scalar instructions when necessary
- Programmers help by
 - Writing vectorizable code
 - Adding hints to the source code (OpenMP)

Status

- Cache line: 512 bits
- Vector width/length: 512 bits (width of the vector register)
- In the past: vector length shorter than cache line

Reminder

All 'concurrency' topics come with these questions:

- What is it?
- Why do we need it?
- How is it implemented?
- Is there a specific trick?
- How does this increase concurrency?

Overarching idea

Single transaction is limited in speed

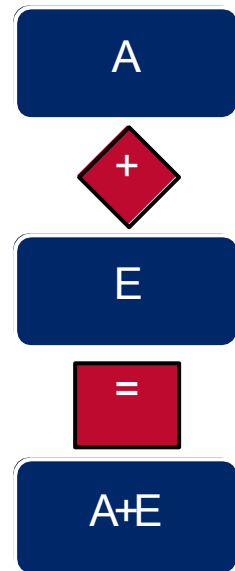
Increasing **concurrency** is our goal

Memory transactions and (floating point) operations

Vectorization

Basics

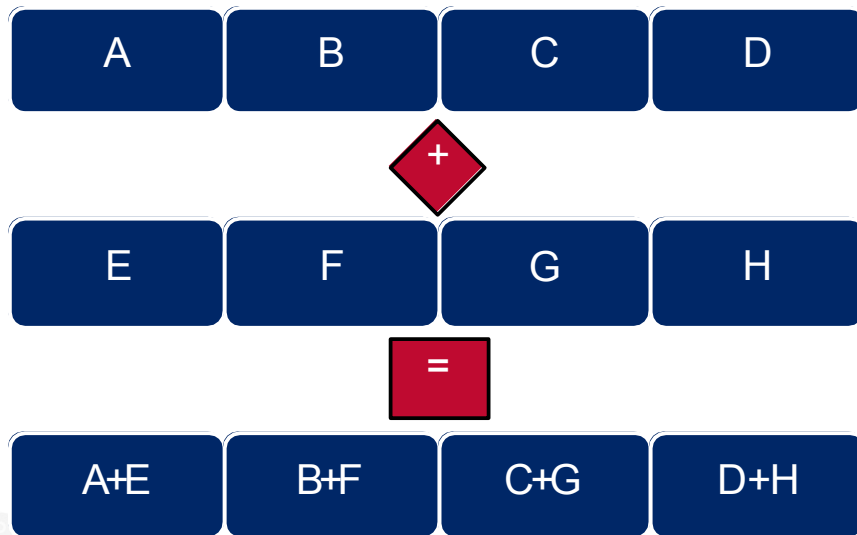
- Cores have registers
- Data must be moved from cache/memory into registers before operating on
- To add 2 floating point (fp) numbers:
 1. Move first fp 'A' from cache to a register
 2. Move second fp 'E' from cache to a different register
 3. Add fp's and place result in yet a different register
 4. Move result to cache/memory
- Frequencies are limited so a single fp operation can only go so fast
- Why not move and add several fp's simultaneously?
 - SIMD: Single Instruction Multiple Data



Vectorization

Make registers wider → vector registers

- Same motivations as multicore: more compute at comparable power
- Increase # computations by increasing number of ops per cycle



Evolution of SIMD Hardware

2001 – 2017: Vector length has increased by a factor of 4

2017: Vector length = length of a cache line

Year	Width (bits)	ISA Name	Data (register names)
1997	80	x87 + MMX	float+Integer
1999	128	SSE1	SP FP SIMD (xMM0-8)
2001	128	SSE2	DP FP SIMD (xMM0-8)
2004	128	SSE3	DP FP SIMD (xMM0-8)
2006	128	SSE4	DP FP SIMD (xMM0-8)
2010	256	AVX	DP FP SIMD (yMM0-16)
2013	256	AVX2	DP FP SIMD (yMM0-16)
2016	512	MIC-AVX512, COMMON-AVX512	DP FP SIMD (zMM0-32)
2017	512	CORE-AVX512	DP FP SIMD (zMM0-32)

Stampede2 and Frontera

Vector Registers

- Different types of processors have different width vector registers
- SP/DP are 32b/64b (4B/8B)
- Vector instructions are required to use vector registers

SSE/SSE2/3/4 2DP

8Bytes 8Bytes

AVX/AVX2 4DP

8Bytes 8Bytes 8Bytes 8Bytes

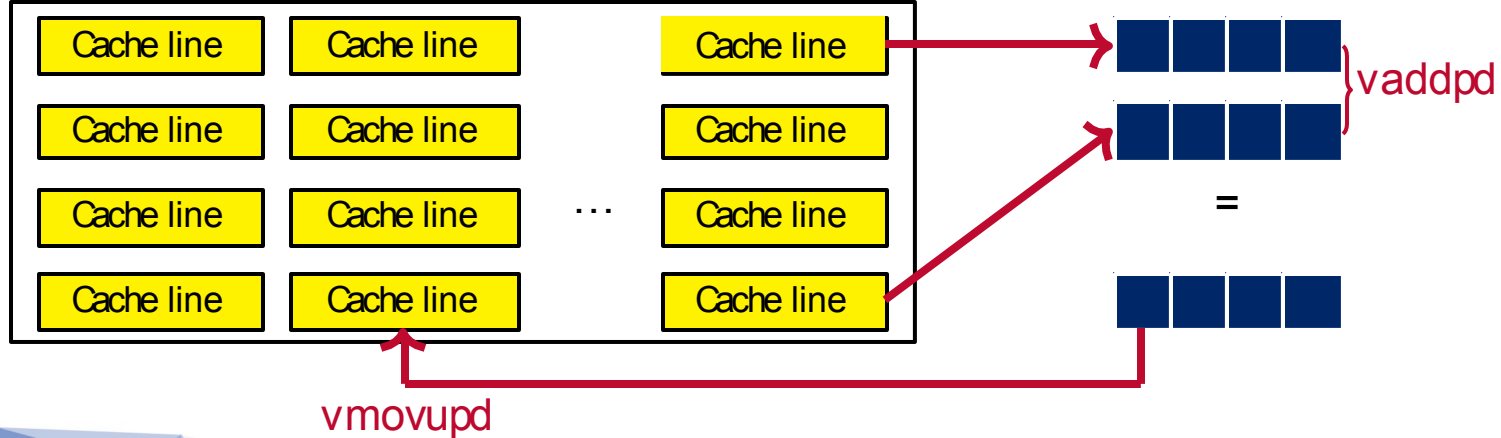
MIC-AVX-512, AVX-512 8DP

8Bytes 8Bytes 8Bytes 8Bytes 8Bytes 8Bytes 8Bytes 8Bytes

Vector Instructions

Vectorized code uses vector instructions

- Vector instructions act on multiple data elements
- Vector instructions exist for moving data and operating on data



Vectorization

Example: vector length=4

```
do i=1, n  
  a(i) = b(i) + c(i)  
enddo
```

Compiler

Start, End, Increment

```
do i=1, n, 4  
  a(i+0) = b(i+0) + c(i+0)  
  a(i+1) = b(i+1) + c(i+1)  
  a(i+2) = b(i+2) + c(i+2)  
  a(i+3) = b(i+3) + c(i+3)  
enddo
```

Compiler unrolls the loop

Vectorization

Example: vector length=4

```
do i=1, n
  a(i) = b(i) + c(i)
  d(i) = e(i) + f(i)
enddo
```

Compiler

```
do i=1, n, 4
  a(i+0) = b(i+0) + c(i+0)
  d(i+0) = e(i+0) + f(i+0)
  a(i+1) = b(i+1) + c(i+1)
  d(i+1) = e(i+1) + f(i+1)
  a(i+2) = b(i+2) + c(i+2)
  d(i+2) = e(i+2) + f(i+2)
  a(i+3) = b(i+3) + c(i+3)
  d(i+4) = e(i+3) + f(i+3)
enddo
```

Compiler unrolls the loop

The compiler can change the order of statements if it's correct to do so

```
do i=1, n, 4
  a(i+0) = b(i+0) + c(i+0)
  a(i+1) = b(i+1) + c(i+1)
  a(i+2) = b(i+2) + c(i+2)
  a(i+3) = b(i+3) + c(i+3)
  d(i+0) = e(i+0) + f(i+0)
  d(i+1) = e(i+1) + f(i+1)
  d(i+2) = e(i+2) + f(i+2)
  d(i+3) = e(i+3) + f(i+3)
enddo
```

Vectorizable code?

```
for ( int i=0; i<n; i++ ) {  
    a[i] = b[i] + c[i];  
}
```

Yes

But why exactly?

Vectorizable code?

```
for ( int i=0; i<n; i++ ) {  
    a[i] = b[i] + c[i];  
}
```

Yes

Loop iterations are independent

Vectorizable code?

```
for ( int i=0; i<n; i++ ) {  
    a[i] = b[i] + c[i];  
}
```

Yes

Loop iterations are independent

```
do i=2, n-1  
    a(i) = a(i-1) + a(i+1)  
end do
```

Vectorizable code?

```
for ( int i=0; i<n; i++ ) {  
    a[i] = b[i] + c[i];  
}
```

Yes

Loop iterations are independent

```
do i=2, n-1  
    a(i) = a(i-1) + a(i+1)  
end do
```

No

Loop iterations are not independent

Vectorizable code?

```
for ( int i=0; i<n; i++ ) {  
    a[i] = b[i] + c[i];  
}
```

Yes

Loop iterations are independent

```
do i=2, n-1  
    a(i) = a(i-1) + a(i+1)  
end do
```

No

Loop iterations are not independent

```
for ( int i=0; i<n; i++ ) {  
    temp = a[i] + 2.;  
    a[i] = b[i];  
    b[i] = temp;  
}
```

How about this one?

There is, sort of, a dependency

Vectorizable code?

```
for ( int i=0; i<n; i++ ) {  
    a[i] = b[i] + c[i];  
}
```

Yes

Loop iterations are independent

```
do i=2, n-1  
    a(i) = a(i-1) + a(i+1)  
end do
```

No

Loop iterations are not independent

```
for ( int i=0; i<n; i++ ) {  
    temp = a[i] + 2.;  
    a[i] = b[i];  
    b[i] = temp;  
}
```

Yes

Compiler resolves dependencies for scalars like temp, and also takes care of unnamed constants

Efficiency of Vectorization

```
do i=1, n  
  a(2*i) = b(2*i) + c(2*i)  
end do
```

Assume double precision

Assume efficient data streaming and pipelining

How many cycles per operation?

(Assuming pipelining)

What is the total bandwidth?

What is the 'effective' total bandwidth?

How many results per cycle?

Efficiency of Vectorization

```
do i=1, n  
  a(2*i) = b(2*i) + c(2*i)  
end do
```

Assume double precision

Assume efficient data streaming and pipelining

How many cycles per operation? 1

What is the total bandwidth? 3×8 wpc

What is the 'effective' total bandwidth? 3×4 wpc

How many results per cycle? 4

Efficiency of Vectorization

```
do i=1, n  
  a(2*i) = b(2*i) + c(2*i)  
end do
```

Assume double precision

Assume efficient data streaming and pipelining

How many cycles per operation? 1

What is the total bandwidth? 3×8 wpc

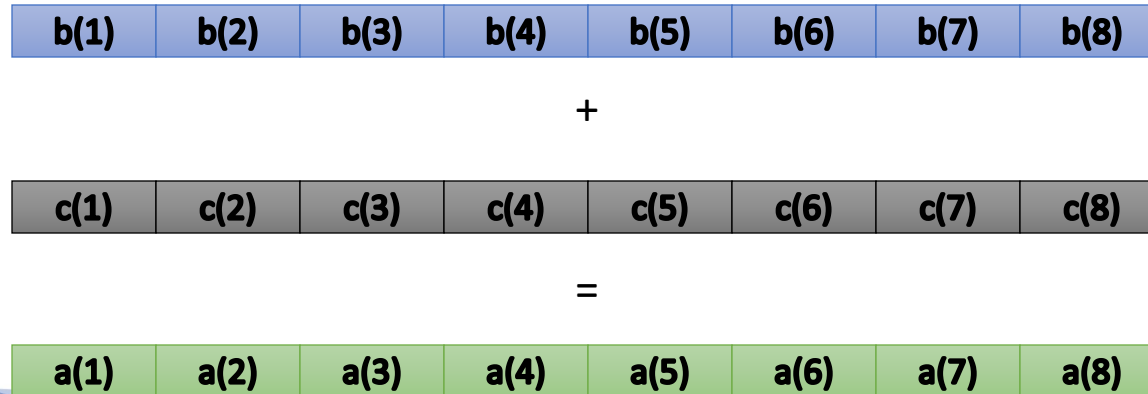
What is the 'effective' total bandwidth? 3×4 wpc

How many results per cycle? 4

So what happens to the 4 elements of the vector unit that we don't need?

Vectorization and Masks

```
do i=1, n  
  a(2*i) = b(2*i) + c(2*i)  
end do
```



Vectorization and Masks

```
do i=1, n  
  a(2*i) = b(2*i) + c(2*i)  
end do
```

b(1)	b(2)	b(3)	b(4)	b(5)	b(6)	b(7)	b(8)
------	------	------	------	------	------	------	------

+

c(1)	c(2)	c(3)	c(4)	c(5)	c(6)	c(7)	c(8)
------	------	------	------	------	------	------	------

0	1	0	1	0	1	0	1
---	---	---	---	---	---	---	---

Mask

=

a(1)	a(2)	a(3)	a(4)	a(5)	a(6)	a(7)	a(8)
------	------	------	------	------	------	------	------

a(1), a(3), a(5), a(7) unchanged

Vectorization and Masks

```
do i=1, n
  a(2*i) = b(2*i) + c(2*i)
end do
```

Complete cache lines are loaded

Unwanted results are either not calculated or not stored back to register

b(1)	b(2)	b(3)	b(4)	b(5)	b(6)	b(7)	b(8)
------	------	------	------	------	------	------	------

+

c(1)	c(2)	c(3)	c(4)	c(5)	c(6)	c(7)	c(8)
------	------	------	------	------	------	------	------

0	1	0	1	0	1	0	1
---	---	---	---	---	---	---	---

Mask

=

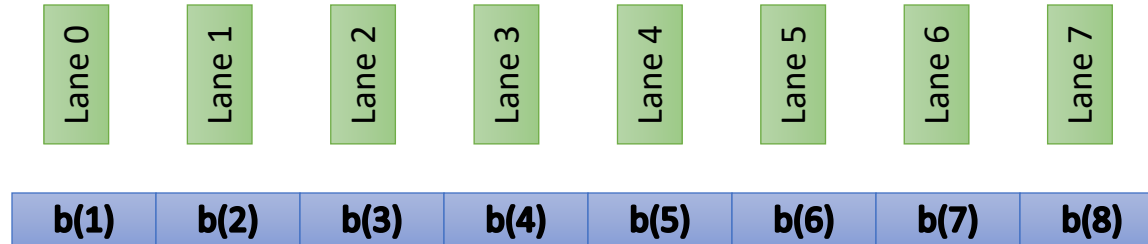
a(1)	a(2)	a(3)	a(4)	a(5)	a(6)	a(7)	a(8)
------	------	------	------	------	------	------	------

a(1), a(3), a(5), a(7) unchanged

Vector Lanes

Double precision: 8 vector lanes
Single precision: 16 vector lanes

Load 8 consecutive elements in memory
to
8 consecutive elements in memory



Vector Lanes

All elements!

```
do i=1, n
  a(i) = b(i) + c(i)
end do
```

Complete cache lines are loaded
Unwanted results are not stored back to register

b(1)	b(2)	b(3)	b(4)	b(5)	b(6)	b(7)	b(8)
------	------	------	------	------	------	------	------

+

c(1)	c(2)	c(3)	c(4)	c(5)	c(6)	c(7)	c(8)
------	------	------	------	------	------	------	------

1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---

=

a(1)	a(2)	a(3)	a(4)	a(5)	a(6)	a(7)	a(8)
------	------	------	------	------	------	------	------

Load b(1:8)
Load c(1:8)
Compute: add
Store a(1:8) – full mask

Total: 4 instructions
8 results

Mask: All results copied back

Vector Lanes

Every other element!

```
do i=1, n
  a(2*i) = b(2*i) + c(2*i)
end do
```

Complete cache lines are loaded

Unwanted results are either not calculated or not stored back to register

b(1)	b(2)	b(3)	b(4)	b(5)	b(6)	b(7)	b(8)
------	------	------	------	------	------	------	------

+

c(1)	c(2)	c(3)	c(4)	c(5)	c(6)	c(7)	c(8)
------	------	------	------	------	------	------	------

0	1	0	1	0	1	0	1
---	---	---	---	---	---	---	---

How many instructions and results?

Mask

=

a(1)	a(2)	a(3)	a(4)	a(5)	a(6)	a(7)	a(8)
------	------	------	------	------	------	------	------

a(1), a(3), a(5), a(7) unchanged

Vector Lanes

Every other element!

```
do i=1, n
  a(2*i) = b(2*i) + c(2*i)
end do
```

Complete cache lines are loaded

Unwanted results are either not calculated or not stored back to register

b(1)	b(2)	b(3)	b(4)	b(5)	b(6)	b(7)	b(8)
------	------	------	------	------	------	------	------

+

c(1)	c(2)	c(3)	c(4)	c(5)	c(6)	c(7)	c(8)
------	------	------	------	------	------	------	------

0	1	0	1	0	1	0	1
---	---	---	---	---	---	---	---

=

a(1)	a(2)	a(3)	a(4)	a(5)	a(6)	a(7)	a(8)
------	------	------	------	------	------	------	------

Load b(1:8)

Load c(1:8)

Compute: add – with mask

Store a(1:8)

Total: 4 instructions

4 results

Mask

a(1), a(3), a(5), a(7) unchanged

Vector Lanes

Extreme case: every 8th element!

```
do i=1, n
  a(8*i) = b(8*i) + c(8*i)
end do
```

Complete cache lines are loaded

Unwanted results are either not calculated or not stored back to register

b(1)	b(2)	b(3)	b(4)	b(5)	b(6)	b(7)	b(8)
------	------	------	------	------	------	------	------

+

c(1)	c(2)	c(3)	c(4)	c(5)	c(6)	c(7)	c(8)
------	------	------	------	------	------	------	------

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

=

a(1)	a(2)	a(3)	a(4)	a(5)	a(6)	a(7)	a(8)
------	------	------	------	------	------	------	------

Load b(1:8)

Load c(1:8)

Compute: add – with mask

Store a(1:8)

Total: 4 instructions

1 results

Mask

a(1), a(2), ... unchanged

Vector Lanes

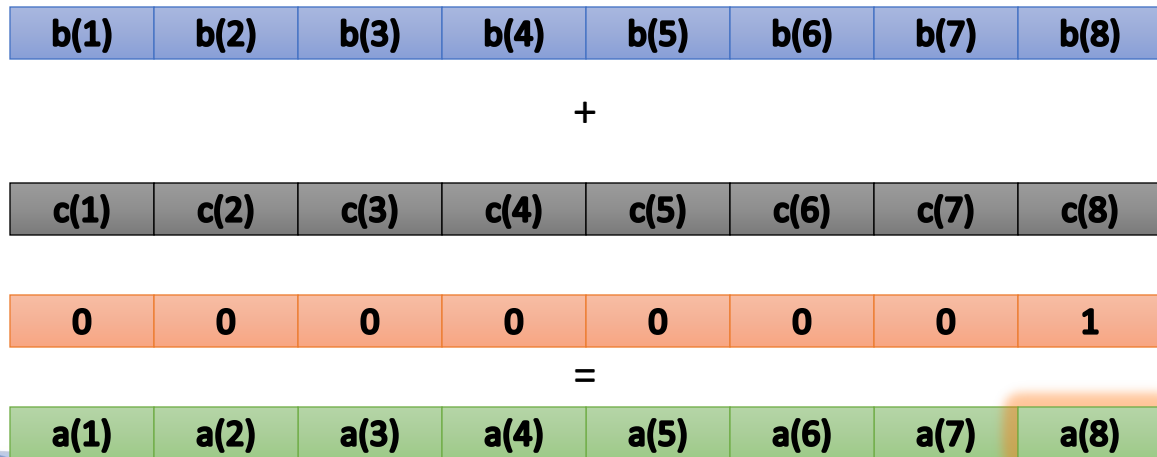
Extreme case: every 8th element!

Why is 'a' also loaded, even if we only write to it?

```
do i=1, n
  a(8*i) = b(8*i) + c(8*i)
end do
```

Complete cache lines are loaded
and written back

Unwanted results are either not calculated or
not stored back to register



Load b(1:8)

Load c(1:8)

Load a(1:8)

Compute: add – with mask
Store a(1:8)

Total: 5 instructions

1 results

a(1), a(2), ... unchanged
a(1) to a(8) is written back

Strided Memory Access

```
do i=1, m  
  a(1*i) = b(1*i) + c(1*i)  
end do
```

Stride 1 access

```
do i=1, m  
  a(2*i) = b(2*i) + c(2*i)  
end do
```

Stride 2 access

```
do i=1, m  
  a(8*i) = b(8*i) + c(8*i)  
end do
```

Stride 8 access

```
do i=1, m  
  a(n*i) = b(n*i) + c(n*i)  
end do
```

Stride 'n>8' access

Which access is best?

Which one is worse?

Strided Memory Access

```
do i=1, m  
  a(1*i) = b(1*i) + c(1*i)  
end do
```

Stride 1 access

```
do i=1, m  
  a(2*i) = b(2*i) + c(2*i)  
end do
```

Stride 2 access

```
do i=1, m  
  a(8*i) = b(8*i) + c(8*i)  
end do
```

Stride 8 access

```
do i=1, m  
  a(n*i) = b(n*i) + c(n*i)  
end do
```

Stride 'n>8' access

Which access is best?

Lower strides are better
Stride-1 is best

Which one is worse?

Higher strides are worse
Stride-8, and stride-n are worst

Lower 'effective' memory bandwidth

Lower number of results per
numerical vector operation

Strided Memory Access

```
do i=1, m  
  a(1*i) = b(1*i) + c(1*i)  
end do
```

Stride 1 access

```
do i=1, m  
  a(2*i) = b(2*i) + c(2*i)  
end do
```

But things are a bit more complicated

Again, we have to do back to 'data streams'

```
do i=1, m  
  a(8*i) = b(8*i) + c(8*i)  
end do
```

Stride 8 access

Which access is best?

Higher strides are worse
Stride-8, and stride-n are worst

```
do i=1, m  
  a(n*i) = b(n*i) + c(n*i)  
end do
```

Stride 'n>8' access

Lower 'effective' memory bandwidth

Lower number of results per
numerical operation