

# Schedule --- 'Hardware' segment of the class

## Homework deadline

- 11/3: part 1
- 11/3: part 2

## Class schedule

- 10/18 (today) Quiz review; time for questions regarding the homework
- 10/20 Quiz: 90 minutes, starting at the beginning of normal class time
  - Quiz is designed for 1 hour. 90 minutes will give everybody enough time to complete the task
  - There won't be a zoom session. I will be on slack and will answer your questions. I may drop off after 1 hour
- 10/27 Quiz return

If you can't make the quiz on Thursday

- Let me know in advance. We will find a different date within 7 days

# Recap 'Data Streams'

## Bandwidth

So far we have talked about data streams and how to improve bandwidth

### Recap 'Data streams'

- 300 cycles to move data between main memory to the CPU

Assume no streams for the example:  $a = b + c$

- Average bandwidth = 1 'word' per 200 cycles (0.005 wpc)

- 'a', 'b', and 'c' are 4-byte or 8-byte words

wpc: our unit

- Moving 'b' and 'c' from memory to CPU: 2 words per 300 cycles
- Moving 'a' from CPU to memory: 1 word per 300 cycles
- Average: 3 words per 600 cycles = 0.005 wpc  
round-trip: 600 cycles

# Discussion: Pipelining

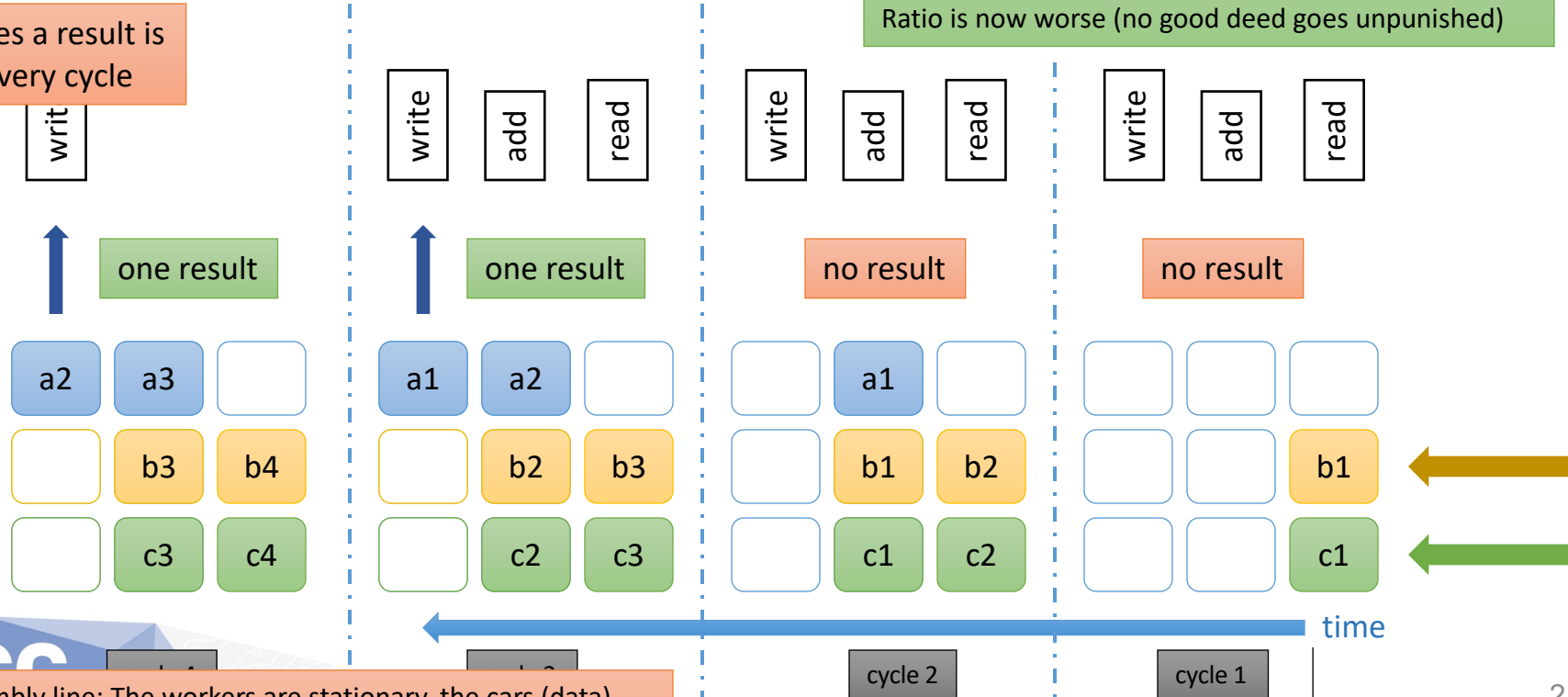
Simple toy model of the implementation of 'add'

'add' takes 3 cycles: 'read', 'add', write'

After 2 cycles a result is produced every cycle

One result per cycle, once the pipeline is filled  
3× performance increase

Operation: 1 cycle  
Data transfer: 300 cycles  
Ratio: 1/300  
Ratio is now worse (no good deed goes unpunished)



Moving assembly line: The workers are stationary, the cars (data)

# Summary

## Part 1

Keep in mind that we have mostly discussed concepts

**Many relevant details of the 3 major hardware features are still missing**  
**We have not discussed the ramifications for code design**

Particularly our discussion of caches is very much incomplete!

(and we will continue with caches in the next section)

### Main bottlenecks

Transfer the data between memory and CPU

Actual computation

### Major technologies to increase **concurrency**

**Streaming** of data between memory and CPU to hide memory latency and increase memory bandwidth

**Pipelining** computation (add/mult) to increase compute throughput

**Caches** to re-use data in order to decrease pressure on the Memory-to-CPU connection  
and to also to hide memory latency and to increase memory bandwidth

### Concurrency

A single action (flops, memory operation, etc.) can only be so fast

Clock speed, power consumption, speed of light

Increasing the '**concurrency**' is the best (maybe only) way to increase performance substantially

# Our Computer: CPU, Cache, Memory, 'Connection'

## CPU

1. Pipelined operation

**System designed to get 1 opc**

## Memory

1. Data streams

**System designed to support 1 wpc (for one row)**

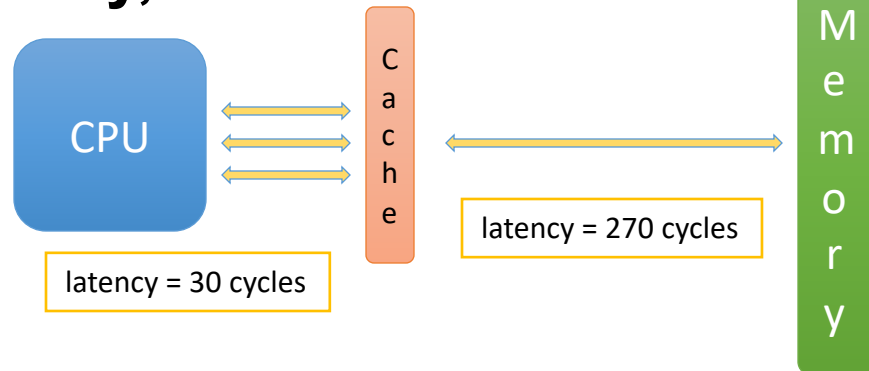
## Caches

1. Managed by run-time
2. Cache size (for stencil update)

**System designed for 'enough' bandwidth to support 2 rows**

**Size: at least  $3 \times n$  words**

Our computer has been somewhat 'hypothetical' so far  
We have designed the specs so that we get 'optimal' performance for a stencil update



Requirement: Size of the cache =  $3 \times n$

$n$  could be any number, any large number

Size of cache in hardware certainly not adjustable

Also differences between chip generations

Hence, cache blocking

# Cache Associativity

Let's make up some address space notation

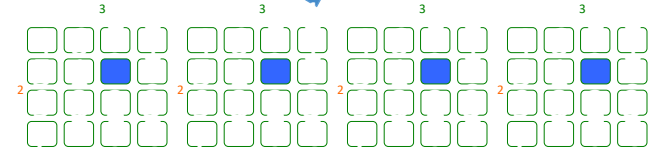
- Address has 6 digits
- Each digit holds a value between 1 and 4

We encode addresses in this order

What if I write it like this

1414 23  
1414 24  
1414 31  
4433 11  
1414 32  
1414 33  
1211 23  
4433 23  
...

Example: Cache with 4-way associativity



An element of data may be cached in 1 of 4 locations  
FIFO: replace the 'oldest' data element (out of the 4)

## Advantages of Caches with Associativity

1. Ineffective mapping alleviated (somewhat)
  2. Compare addresses only for few (4) locations
  3. Eviction policy FIFO or LRU schedule
  4. Keep 'age' for few (4) entries
- (In principle, 'random' eviction could work, too)

# Recap

## Topics that we have addressed so far

Data streams

Pipelining

Caches

- Why?
- Cache blocking (software)
- Address mapping
- Eviction policy (FIFO, LRU, etc.)
- Associativity
  - fully associative, set-associative, direct mapping
- Storage efficiency (for addresses)
  - Cache lines
- Cache coherency (MOSI protocol)
- Shared-memory architecture
- False sharing

## All these topics come with these questions:

- What is it?
- Why do we need it?
- How is it implemented?
- Is there a specific trick?
- How does this increase concurrency?

## Overarching idea

Single transaction is limited in speed

Increasing **concurrency** is our goal

Memory transactions and (floating point) operations

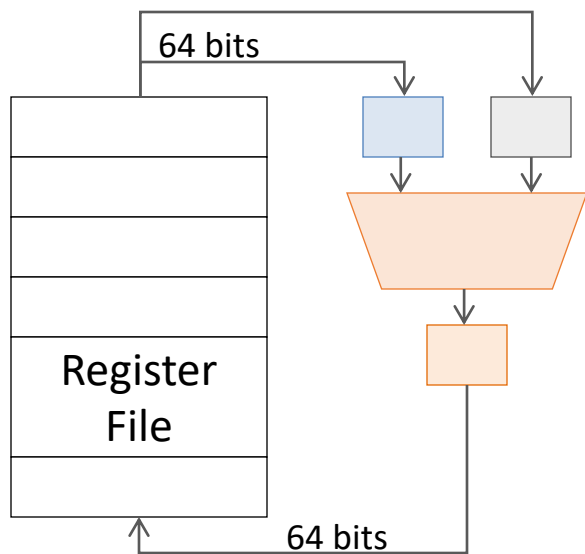
One more hardware feature to cover (unrelated to caches)

And then we will start looking into 'writing fast code'

# Vector Hardware

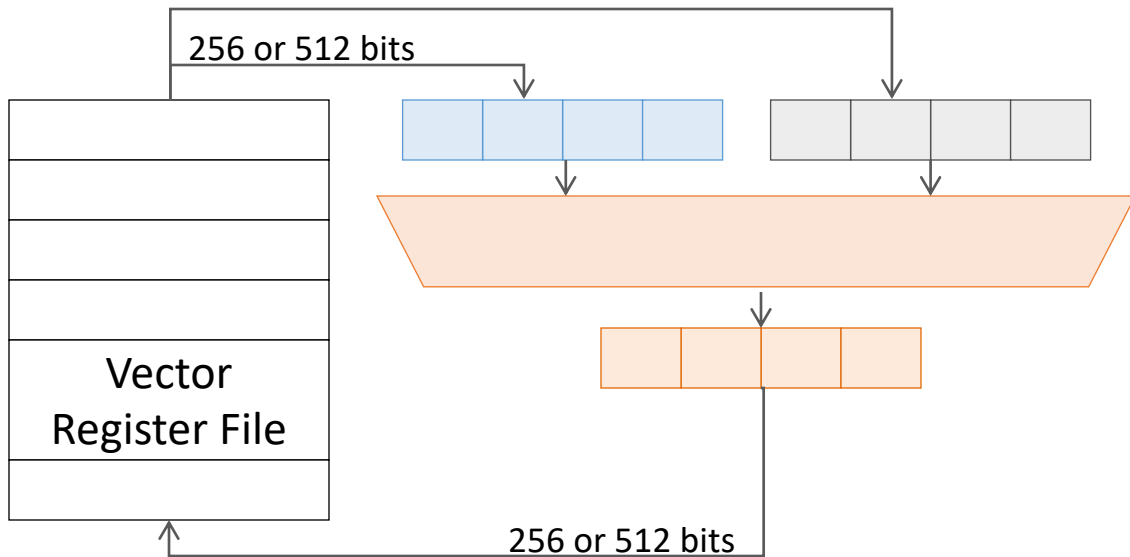
$$a(i) = b(i) + c(i)$$

Example for vector length = 4 words



Scalar Unit

Input: 2 words (single or double precision)  
Output: 1 word  
Operations: 1 operation → 1 result



Vector Unit

Input: 2 cache lines  
Output: 1 cache line  
Operations: 1 operation → 8/16 results (dp/sp)



# Strided Memory Access

```
do i=1, m  
  a(1*i) = b(1*i) + c(1*i)  
end do
```

Stride 1 access

```
do i=1, m  
  a(2*i) = b(2*i) + c(2*i)  
end do
```

Stride 2 access

```
do i=1, m  
  a(8*i) = b(8*i) + c(8*i)  
end do
```

Stride 8 access

```
do i=1, m  
  a(n*i) = b(n*i) + c(n*i)  
end do
```

Stride 'n>8' access

Which access is best?

Lower strides are better  
Stride-1 is best

Which one is worse?

Higher strides are worse  
Stride-8, and stride-n are worst

Lower 'effective' memory bandwidth

Lower number of results per  
numerical vector operation

# Vectorizable code?

```
for ( int i=0; i<n; i++ ) {  
    a[i] = b[i] + c[i];  
}
```

Yes

Loop iterations are independent

```
do i=2, n-1  
    a(i) = a(i-1) + a(i+1)  
end do
```

No

Loop iterations are not independent

```
for ( int i=0; i<n; i++ ) {  
    temp = a[i] + 2.;  
    a[i] = b[i];  
    b[i] = temp;  
}
```

Yes

Compiler resolves dependencies for scalars like **temp**, and also takes care of unnamed constants

# Vector Lanes

All elements!

```
do i=1, n
  a(i) = b(i) + c(i)
end do
```

Complete cache lines are loaded  
Unwanted results are not stored back to register

|      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|
| b(1) | b(2) | b(3) | b(4) | b(5) | b(6) | b(7) | b(8) |
|------|------|------|------|------|------|------|------|

+

|      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|
| c(1) | c(2) | c(3) | c(4) | c(5) | c(6) | c(7) | c(8) |
|------|------|------|------|------|------|------|------|

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|

=

|      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|
| a(1) | a(2) | a(3) | a(4) | a(5) | a(6) | a(7) | a(8) |
|------|------|------|------|------|------|------|------|

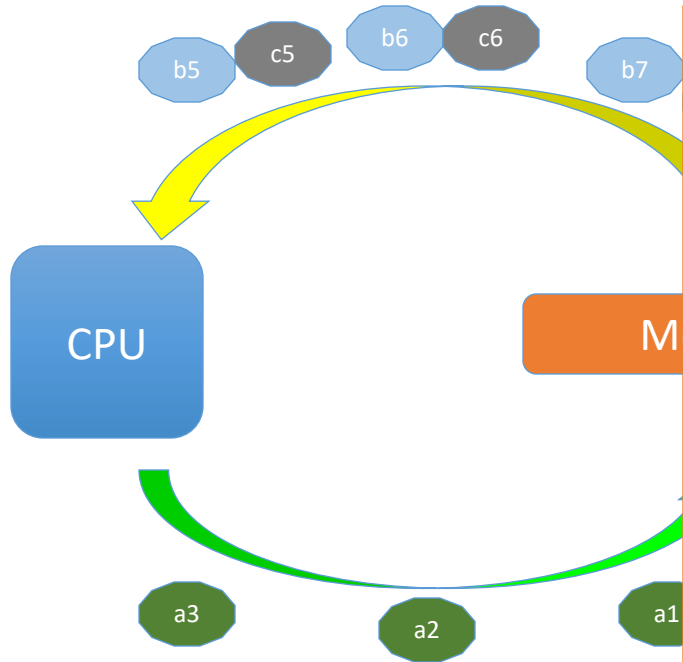
Load b(1:8)  
Load c(1:8)  
Compute: add  
Store a(1:8) – full mask

Total: 4 instructions  
8 results

Mask: All results copied back

# Prefetching

$$a(i) = b(i) + c(i)$$



The computer does not know the code and cannot infer anything from the pattern in the code.

However, it can analyze the pattern from previous memory access. For example, data is requested cache line by cache line.

This is called prefetching

Prefetch instructions are added either

- during execution by the hardware (hardware prefetching)
- or to the assembly code by the compiler (software prefetching)

Typical defaults (x86 architecture):

- Software prefetching is off
- Hardware prefetching is on

Prefetching fills the data streams

Unwanted data (that is not useful) is ignored

# Recap: Vectorization & Prefetching

Vectorization: When, how, why?

Under what circumstances can loops be vectorized?

Vector lanes

Strided data access

Vectorization efficiency

Data transfer efficiency

Prefetching

Multi dimensional arrays: strice-1 v. stride-n

Pointers and 'array overlap'; mostly in C

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operations

# Optimizations in Code

Cost of a division: maybe 60 cycles (depends highly on precision and accuracy)

```
do i=1, n
  a(i) = b(i) + c(i)/x
enddo
```



```
xi = 1. / x
do i=1, n
  a(i) = b(i) + c(i) * xi
enddo
```

Note that the following is not exactly (bit-wise) the same

```
z1 = y / x
```

```
z2 = y * (1./x)
```

Do not assume that z1 is exactly/bit-wise z2

If you change the code, or if you allow the compiler to change the code, you will get a slightly different result (last bits will vary)

Compiler options allow you to control (to some degree)

- Whether operations in code are replaced by cheaper operations
- Whether operations are 'executed' to the fullest accuracy (which is expensive)
  - The default is some 'reasonable' compromise between accuracy and speed

Many optimization techniques will change the rounding bits.

Hence, bit-wise reproducibility is not the goal

# Refresher

```
do i=1, n
  a(i) = a(i) + b(i)
enddo
```

```
do i=1, n
  a(i) = a(i-1) + c(i)
enddo
```

RAW

```
do i=1, n
  a(i) = a(i+1) + c(i)
enddo
```

WAR

```
do i=1, n
  a(i) = b(i) + c(i)
  d(i) = a(i) + e(i)
  a(i) = f(i) + g(i)
enddo
```

WAW

Do you get the same result when running forward and backward?

Can these loops be vectorized?

Definition of 'WAR', 'RAW', and 'WAW' at:  
[cvw.cac.cornell.edu/vector/coding\\_dependencies](http://cvw.cac.cornell.edu/vector/coding_dependencies)