Automated Euclidean Proof

Hasclid Automated Prover

December 19, 2025

1 Theorem Statement

The objective is to prove the following proposition:

$$R\_\{a\}\hat{\ }\{2\} \geq \frac{\{(\backslash beta + \backslash gamma)\}\hat{\ }\{2\} \cdot S\_\{16\}}{4}$$

2 Problem Analysis

The problem is classified as **Mixed**.
It involves 6 variables and 10 constraints.

3 Mathematical Proof

3. Algebraic Reduction:
We reduce the target expression modulo the geometric constraints using a modular F4 basis.
This simplifies the problem into its canonical form:
Reduced Expression:

$$c^4\gamma^2-2b^2c^2\gamma^2+b^4\gamma^2+2\beta c^4\gamma-4b^2\beta c^2\gamma+2b^4\beta\gamma+\beta^2c^4-2b^2\beta^2c^2+b^4\beta^2-2c^2\gamma^2+2b^2\gamma^2+2\beta^2c^2-2b^2\beta^2+\gamma^2-2\beta\gamma+\beta^2$$

4. Positivity Verification:
We observe that the reduced expression is non-negative because it can be written as a sum of squares and proven lemmata.
The expression can be decomposed as:

$$(-4c^2\gamma+2b^2\gamma-2\beta c^2+b^2\beta-2\beta)^2+5*(14/15c^2\gamma-4/5b^2\gamma+1/5\beta c^2-2/15b^2\beta+16/15\gamma+2/5\beta)^2+151/4*(-92/151c^2\gamma+42/151b$$

Conclusion: The inequality holds for all valid configurations.

4 Conclusion

The proposition is **PROVED**.
Proved via compositional Sum-of-Squares