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Task - 1

$$f(z) = \log_e(1+z)$$

Here, $z = x^T x \quad x \in R^d$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \qquad x^T = \begin{bmatrix} x_1 & x_2 & \cdots & x_d \end{bmatrix}$$

$$\therefore \quad x^T x = \begin{bmatrix} x_1 & x_2 & \cdots & x_d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$$

$$= \left[x_1^2 + x_2^2 + \cdots + x_d^2\right]$$

Using chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz} f(z) \cdot \frac{d}{dx}\left(x^T x\right)$$

$$= \frac{d}{dz}\left(\log_e(1+z)\right) \cdot \frac{d}{dx}\left(\left[x_1^2 + x_2^2 + \cdots + x_d^2\right]\right)$$

$$= \frac{1}{1+z} \cdot (0+1) \cdot 2(x_1 + x_2 + \dots + x_d)$$

$$= \frac{1}{1+x^T x} \cdot 2(x_1 + x_2 + \dots + x_d)$$

$$\boxed{= \frac{1}{(1+x^T x)} \cdot 2 \sum_{i=1}^{d} x_i}$$

The derivative of $f(z) = \log_e (1+z)$.

Task - 2

$$f(z) = e^{-z/2}$$

Here,

$$z = g(y) = y^T S^{-1} y$$

$$y = h(x) = x - \mu$$

$$x, \mu \in R^d, \quad S \in R^{d \times d}$$

Using chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= \frac{d}{dz}(e^{-z/2}) \cdot \frac{d}{dy}(y^T S^{-1} y) \cdot \frac{d}{dx}(x - \mu)$$

$$\Rightarrow \quad \frac{d}{dz}\left(e^{-z/2}\right) = \left(e^{-z/2}\right) \times -\frac{1}{2} \times 1 = -\frac{1}{2}\left(e^{-z/2}\right)$$

$$\Rightarrow \quad \frac{d}{dy}\left(y^T s^{-1} y\right) = \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T+h) s^{-1} (y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{y^T s^{-1} y} + y^T s^{-1} h + h s^{-1} y + h^2 s^{-1} - \cancel{y^T s^{-1} y}}{h}$$

$$= \lim_{h \to 0} \frac{h\left(y^T s^{-1} + s^{-1} y + s^{-1} h\right)}{h}$$

$$= \lim_{h \to 0} \left(y^T s^{-1} + s^{-1} y + s^{-1} h\right)$$

$$= y^T s^{-1} + s^{-1} y$$

$$= \frac{y^T}{S} + \frac{y}{S}$$

$$\Rightarrow \frac{d}{dx}(x-\mu) = 1 - 0 = 1$$

So,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= \left(\frac{-1}{2} e^{-z/2}\right) \cdot \left(\frac{y^T}{S} + \frac{y}{S}\right) \times 1$$

$$\boxed{= -\frac{1}{2} \cdot e^{-z/2} \cdot \left(S^{-1} y^T + S^{-1} y\right)}$$

Which is the derivative.

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