CONTINUITY

A function is Continuous, If its graph doesn't break anywhere, but this is a graphical Statment, If we talk mathematically, we can Say that,

FOR A FUNCTION TO BE CONTINUOUS, $LHL = RHL = f(a)$, which says that all the right hand limit, left hand limit & $f(a)$ will be equal at 'a'. 'a' is the point where we want to Check Continuity.

- PRIME CONDITION TO CHECK CONTINUITY AT A PARTICULAR 'a'

* $$\lim_{x \to a^-} f(a) = \lim_{x \to a^+} f(a) = f(a) \quad \text{OR} \quad LHL = RHL = f(a)$$

* • DOUBTFUL POINTS : In A Domain Following Would be the Points, where Continuity will be doubtful.

① Where, the defination of function Changes

② If G.I.F or F.P exists, doubtful @ $\mathbb{Z}$

③ If Signum exists, then doubtful at 0

④ All polynomial, modulus, logarathamic, exponential, Trigonometric & Inverse trigonome functions are continuous.
(TF & ITF are only Contiauous in their domains)

* • CONTINUITY OF DIRICHLET FUNCTIONS

$$f(x) \begin{cases} P(x) & x \in Q \text{ (Real)} \\ q(x) & x \in \bar{Q} \text{ (imaginery)} \end{cases}$$

* • JUMP !

If $LHL \neq RHL$ but function is Continuous then a jump is existing. which canbe calculated as $\text{Jump} = |LHL - RHL|$

- INTERMEDIATE VALUE THEOREM

If $f(x)$ is Continuous in $[a, b]$, And $f(a) \cdot f(b) < 0$ then eq^n $f(x) = 0$, will have atleast one root in the interval $[a, b]$

* • THEOREMS

f	g	$f+g$	$f-g$	fg	f/g	$f(g)$
C	C	C	C	C	C	C
C	D	D	D	*** C/D	C/D	C/D
D	D	C/D	C/D	C/D	C/D	C/D

- For ***

Let $f(x)$ be Cont. at a & $g(x)$ be Dcont at $x = a$, then

C_1: If $f(a) \neq 0$, then $f(x)\,g(x)$ willbe dis Cont. at $x = a$.

C_2: If $f(a) = 0$ & LHL & RHL & $g(a)$ are finite, then $f(x)\,g(x)$ willbe cont. at $x = a$

C_3: If $f(a) = 0$, but at least one of LHL or RHL is ∞, then $f(x)\,g(x)$ may or may not be cont. at $x = a$ [can't Comment]