GRAVITATION

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- $F = \frac{G m_1 m_2}{r^2}$

$G = 6.67 \times 10^{-11}\ N\text{-}m^2/Kg^2$

$D(G) = [M^{-1} L^3 T^{-2}]$

* Attractive (always)
* Opposite Direction
* Action - Reaction Pair

- Gravitational Feild ($\vec{E}$) → Gravitional force per unit mass

$M \overset{r}{—} m_0$ $\quad F = \frac{GMm_0}{r^2}$, $E = \frac{GM}{r^2}$

$D(E) = [L^1 T^{-1}]$

- Field at :-

(i) Axis of uniform wire $\longrightarrow \frac{GM}{r(r+l)}$

(ii) Due to rod $\Longrightarrow E_\perp = \frac{G\lambda}{r}(\sin\theta_1 + \sin\theta_2)$ & $E_\parallel = \frac{G\lambda}{r}(\cos\theta_1 + \cos\theta_2)$

(iii) Due to ∞ wire $\Longrightarrow \frac{2G\lambda}{r} = E_\perp$ & $E_\parallel = 0$

(iv) Due to semi∞ wire $\Rightarrow E_\perp = \frac{G\lambda}{r}(1+\sin\theta)$ & $E_\parallel = \frac{G\lambda}{r}(\cos\theta)$

(v) Axis of uniform ring $\Rightarrow \frac{GMx}{(R^2+x^2)^{3/2}}$

(vi) Axis of disc $\Rightarrow 2\pi\sigma G(1-\cos\theta)$

(vii) Due to ∞ sheet $\Rightarrow 2\pi\sigma G$

(viii) Due to hollow Sphere $\Rightarrow \sigma G \Omega$

(ix) Due to Solid Uniform Sphere $\Longrightarrow \frac{GM}{R^2} \times r$

- Potential Energy $\Rightarrow \Sigma -\frac{G m_1 m_2}{r}$

- Gravitational Potential (V) ⇒ Potential energy per unit mass

$V = \frac{U}{m}$, $\frac{J}{kg}$, $D(V) = [LT^{-2}]$

m —— m_0 $\quad U = -\frac{Gmm_0}{r}$, $V = \frac{-GM}{r}$

$V = -\frac{G}{r}\Sigma m$

- Potential at :-

(i) Axis of SL $\Rightarrow -G\lambda \ln\left(\frac{r+l}{r}\right)$

(ii) Axis of ring ⇒ at Centre → $-\frac{GM}{R}$

at dist. x → $-\frac{GM}{\sqrt{R^2+x^2}}$

(iii) Axis of disc → $-2\pi\sigma G(\sqrt{R^2+x^2} - x)$

(iv) Centre of circular arc → $-\frac{GM}{R}$

• Tunnel Problems

↳ It is SHM

↳ $v = \sqrt{\frac{GM}{R}}$

↳ $T = 2\pi\sqrt{\frac{R^3}{GM}}$

• Acceleration Due to gravity.

(i) At Surface, $g = \frac{GM}{R^2}$, $g_{earth} = 9.8 m/s^2$

(ii) At height 'h', $g_{eff} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

(iii) At depth 'd', $g_{eff} = g\left(1-\frac{d}{R}\right)$

(iv) Effect of rotation (ω) => $g_{eff} = g - \omega^2 R\cos^2\theta$

• Escape Speed (V_E)

$V_E = \sqrt{\frac{2GM}{R}}$ & $V_e = 11.2$ km/s

• Orbital velocity (V_o)

$V_o = \sqrt{\frac{GM}{r}}$; $r = h+R$

• $|KE| = |TE| = \frac{1}{2}|PE|$

• Escaping of a sattelite :- $V = \sqrt{2}\, V_o$

• Elliptical Path : $V_p = \sqrt{\frac{GM}{a}\frac{(1-e)}{(1+e)}}$