LIMITS

- IF LHL = RHL, THEN LIMIT EXISTS.

- INDETERMINANT FORMS & THEIR APPROACH

1] $\frac{0}{0}$ FORM $\longrightarrow$ Factorisation, Rationalisation, L'Höpital.

2] $\frac{\infty}{\infty}$ FORM $\longrightarrow$ Dominating Term.

3] $0 \times \infty$ FORM $\longrightarrow$ Simplification then std. lims.

4] $\infty \times \infty$ FORM $\longrightarrow$ Same as ~~1~~ 2

* 5] 1^{∞} $\longrightarrow$ Use formula, $\lim\limits_{x \to a} f(x)^{g(x)} = e^{\lim\limits_{x \to a} (f(x)-1)\, g(x)}$

6] ∞^{0} $\longrightarrow$ Simplify & apply std lims

* 7] 0^{0} $\longrightarrow$ Use formula, $\lim\limits_{x \to a} f(x)^{g(x)} = e^{\lim\limits_{x \to a} g(x) \cdot \ln(f(x))}$

* One can also use series expansion or method of substituition to solve.

- * SOME STANDARD LIMITS

1] $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

2] $\lim\limits_{x \to 0} \frac{\tan x}{x} = 1$

3] $\frac{1-\cos x}{x^2} = \frac{1}{2}$

4] $\lim\limits_{x \to 0} \frac{\ln(1+x)}{x} = 1$

5] $\lim\limits_{x \to 0} \frac{a^x - 1}{x} = \ln(a)$

6] $\lim\limits_{x \to a} \frac{x^n - a^n}{x - a} = n(a)^{n-1}$

- * SOME MORE STANDARD LIMITS

1] $\lim\limits_{x \to 0} \frac{\tan x - x}{x^3} = \frac{1}{3}$

2] $\lim\limits_{x \to 0} \frac{x - \sin x}{x^3} = \frac{1}{6}$

3] $\lim\limits_{x \to 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$

4] $\lim\limits_{x \to 0^+} x \ln x = 0$

- * THEOREMS

1] $\lim\limits_{x \to a} f(x) \pm g(x) \Rightarrow \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x)$

2] $\lim\limits_{x \to a} K f(x) \Rightarrow K \cdot \lim\limits_{x \to a} f(x)$

- A FORMULA IN CASE OF SERIES:

* $$\lim_{n \to \infty} \frac{1^r + 2^r + 3^r + \dots + n^r}{n^{r+1}} = \frac{1}{r+1}$$

- * THE SERIES EXPANSIONS

1] $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

2] $\cos x = x - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

3] $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{5} + \dots$

4] $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

5] $a^x = 1 + \frac{x \ln(a)}{1!} + \frac{x^2 \ln^2(a)}{2!} + \dots$

6] $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots$

7] $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 +$

8] $(1+x)^{1/x} = e(1 - x$