

DIFFERENTIABILITY

• Tangent is the limiting case of chord.

* Prime Condition

- If a function is not Continuous, it won't be differentiable.
- If a function is Continuous, then it may or may not be differentiable.
- * If a function is differentiable, it is definitely continuous.

* CONDITION:

$$\textcircled{1} \text{ LHD} = \text{RHD} = f'(a) \quad \text{OR} \quad f'(a^-) = f'(a^+) = f'(a) \quad \text{OR}$$

$$\textcircled{2} \lim_{x \rightarrow a^-} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow a^+} \frac{f(x) - f(a)}{x - a} = f'(a)$$

* This prime condition check is called Standard method/ Fundamental method

• Methods to check differentiability.

• Fundamental method
 { Discussed above }
 { marked }

* Graphing
 { Self Knowledge }
 { Functions }

* Method of direct differentiation
 { Conditional: Only applicable if f(x) is Continuous }

* MODD


S₁] Check Continuity of given function at doubtful points.

↳ If discontinuity exists → f(x) is not differentiable, don't proceed further.
 ↳ If f(x) is Continuous, goto: S₂.

S₂] $f(x) \rightarrow f'(x)$, equate RHD \rightarrow LHD, if it is equal then differentiable else not differentiable. ①

• Reasons for non-differentiability. {useful for MTCO - Graphing}

1) Discontinuity 2) Corner [Ex. $y = |x|$ at $x = 0$] 3) Vertical Tangent [Ex. $y = x^{1/3}$ at $x = 0$]

4) Cusp \rightarrow Photo:  $y = x^{2/3} \rightarrow$ Cusp \Rightarrow If one of LHD \neq ∞ & other is $-\infty$, then we call it cusp.

* Theorems

$f(x)$	$g(x)$	$f(x) \pm g(x)$	$f(x) \cdot g(x)$	$f(x)/g(x)$
D	D	D	D	D
D	ND	ND	* * D/ND	D/ND
ND	ND	D/ND	D/ND	D/ND

* For * * *

• Let $f(x)$ is diff. at $x = a$ & $g(x)$ is cont but not diff at $x = a$, then

- 1) if $f(a) = 0$, then $f(x)g(x)$ is differentiable at $x = a$
- 2) if $f(a) \neq 0$, then $f(x)g(x)$ is non-differentiable at $x = a$