PART (C) : MATHEMATICS

SECTION – I : SINGLE CORRECT ANSWER TYPE
(Maximum Marks : 30)

This section contains 10 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Marking Scheme : +3 for correct answer, 0 if not attempted and -1 in all other cases.

41. The value of k for which the equation $x^2 + 2(k-1)x + k + 5 = 0$ possess at least one positive root is
(A) $[4, \infty)$
(B) $(-\infty, -1) \cup [4, \infty)$
(C) $[-1, 4]$
(D) $(-\infty, -1]$

42. The value of $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + ... + \cos\frac{17\pi}{19}$ is equal to
(A) $1/2$
(B) 0
(C) 1
(D) None of these

43. If $f(\theta) = \sin^4\theta + \cos^2\theta$, then range of $f(\theta)$ is
(A) $\left[\frac{1}{2}, 1\right]$
(B) $\left[\frac{1}{2}, \frac{3}{4}\right]$
(C) $\left[\frac{3}{4}, 1\right]$
(D) None of these

44. The number of positive integral solutions of $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \le 0$ is
(A) four
(B) three
(C) two
(D) only one

45. If $a, b, c \in R$ and $x^2 + (a+b)x + c = 0$ has no real roots then
(A) $c(a+b+c) > 0$
(B) $c + c(a+b+c) > 0$
(C) $c + c(a+b-c) > 0$
(D) $c(a+b-c) > 0$

46. If $0^\circ < x < 90^\circ$ and $\cos x = \frac{3}{\sqrt{10}}$, then the value of $\log_{10} \sin x + \log_{10} \cos x + \log_{10} \tan x$ is

(A) 0
(B) 1
(C) –1
(D) None of these

47. The value of $\tan\frac{\pi}{16} + 2\tan\frac{\pi}{8} + 4$ is equal to

(A) $\cot\frac{\pi}{8}$
(B) $\cot\frac{\pi}{16}$
(C) $\cot\frac{\pi}{16} - 4$
(D) None of these

48. If the roots of the equation $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equation $x^3 - Ax^2 + Bx - C = 0$, where A, B, C, P & Q are constants, then the value of $A + B + C$ is equal to
(A) 18
(B) 19
(C) 20
(D) None of these

49. The value of $\frac{2\cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$ is

(A) $\sin 20^\circ$
(B) $\cos 20^\circ$
(C) 1
(D) $\sqrt{3}$

50. Let α, β, γ are roots of the equations $x^3 + qx + q = 0$, then find the value of $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}$.

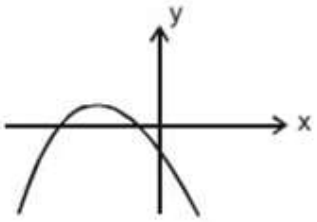
(A) 0
(B) –1
(C) 1
(D) None

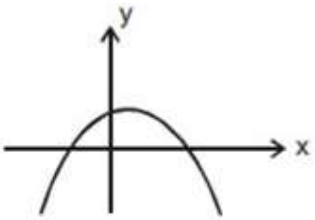
SECTION – II : MULTIPLE CORRECT ANSWER TYPE
(Maximum Marks : 20)

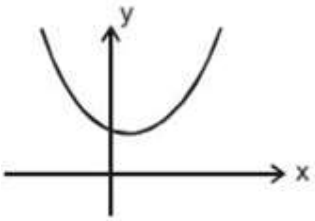
This section contains 5 multiple choice questions. Each question has 4 options (A), (B), (C) and (D) for its answer, out of which ONE OR MORE than ONE option can be correct.

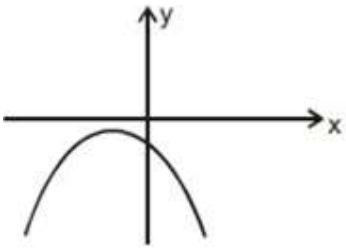
Marking Scheme : +4 for correct answer, +1 Partial Mark, 0 if not attempted and -1 in all other cases.

51. For which of the following graphs of the quadratic expression $y = ax^2 + bx + c$, then product abc is negative.

(A) 

(B) 

(C) 

(D) 

52. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if
(A) $\cos 12x = \cos 14x$
(B) $\sin 13x = 0$
(C) $\sin x = 0$
(D) $\cos x = 0$

53. The equation $4x^2 - 11x + 2k = 0$ and $x^2 - 3x - k = 0$ have a common root α, then
(A) $k = 0$
(B) $k = -17/36$
(C) $\alpha = 0$
(D) $\alpha = 17/6$

54. The equation $|x^2 - x - 6| = x + 2$ has :
(A) two positive roots
(B) two real roots
(C) three real roots
(D) four real roots

55. The expression $\frac{1}{\sqrt{x + 2\sqrt{x-1}}} + \frac{1}{\sqrt{x - 2\sqrt{x-1}}}$ simplifies to :
(A) $\frac{2}{3-x}$ if $1 < x < 2$
(B) $\frac{2}{2-x}$ if $1 < x < 2$
(C) $\frac{2\sqrt{x-1}}{(x-2)}$ if $x > 2$
(D) $\frac{2\sqrt{x-1}}{x+2}$ if $x > 2$

SECTION – III : INTEGER ANSWER TYPE
(Maximum Marks : 10)

This section contains 5 questions. The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 to 9, BOTH INCLUSIVE.**
Marking scheme: +2 for correct answer, 0 if not attempted and 0 in all other cases.

56. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ for $k = 1, 2, 3, \ldots\ldots$. Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to λ, then $24\lambda = ?$.

57. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta;\ \alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta) = -\sqrt{K}$; K is ?

58. The total number of solution of $\sin^4 x + \cos^4 x = \sin x \cdot \cos x$ is $[0, 2\pi]$ is equal to

59. If $f(n,\theta)=(\sec 2^{n-1}\theta)...(\sec 2\theta)(\sec\theta)\sec\left(\frac{\theta}{2}\right)(1+\cos\theta)(1+\cos 2\theta)(1+\cos 2^2\theta)...(1+\cos 2^n\theta)$, $n\in N$, then value of $f\left(3,\frac{2\pi}{17}\right)$ is

60. The number of real solutions of the equation $-x^2+x-1=\sin^4 x$ is