CONTENTS

PHYSICS

MODULE – I

PREFACE

Physics is a science based on experimental observations and mathematical analysis. It is possible to explain the behavior of various physical systems using relatively few fundamental laws and physical quantities. These chapters in this module are common for both JEE Main & Advanced. Here we deal with the categorizations of these physical quantities into two. i.e. one with direction and the other without direction and calculus. Calculus is a mathematical tool which helps us understand and calculate instantaneous values of a physical quantity. Once a teacher was teaching divisions to elementary school students without the help of a board & chalk, he asked the students to distribute ten fruits equally among a group of ten students. Each student was left with one fruit. After making the students practice this with several numbers, he generalised this concept & said if 'the number of students equals to the number of fruits, every student will get one fruit'. A boy got up and asked the teacher, is this true for all numbers? For which the teacher said 'yes'. He then asked the teacher if there are no students and no fruits even then will each student get one fruit?. This boys name is 'Srinivas Ramanujam' who later showed to the world that (0/0) is a singularity and cannot be determined.

This booklet consists of summarized text coupled with sufficient number of solved examples of varying difficulties, which enables the students to develop problem solving ability along with emphasis on physical concept.

The end–of–chapter problems are categorized into four section, namely Exercise – I (objectives where only one of the option is correct) Which emphasizes on JEE (main) pattern, Exercise – II (objectives where more than one option may be correct), Exercise – III (matrix matches and paragraph type questions), Exercise – IV (subjective questions), to help the student assess his understanding of the concept and further improvise on his problem solving skills. Solutions to all the questions in the booklet are available and will be provided to the students (at the discretion of the professor). Every possible attempt has been made to make the booklet flawless. Any suggestions for the improvement of the booklet would be gratefully accepted and acknowledged.

(Dept. of Physics)

IIT –ian's PACE

IIT–JEE SYLLABUS

Scalars and vectors, vector addition and Subtraction, Zero vector, Scalar and vector products, Unit vector, Resolution of a Vector.

CONTENTS

Vector Analysis : Tentative Lecture Flow
(Board Syllabus & Booklet Discussion Included)

Lecture 1	Definition of Scalar & Vector. Representation of Vector, Geometrical representation, Cartisian Co - ordinate system. Position vector, unit vector, coplanar & collinear vector
Lecture 2	Addition & Subtraction of vectors, triangle law, parallelogram law, resolution of vector, polygon law, solved examples involving addition & subtraction of vector.
Lecture 3	Dot product and cross product. Solved examples.

VECTORS

INTRODUCTION

Scalar Quantity:

A physical quantity which can be described completely by its magnitude only and does not require a direction is known as a scalar quantity. It obeys the ordinary rules of algebra.

eg: distance, mass, time, speed, density, volume, temperature, current etc.

Vector Quantity:

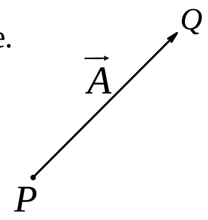
A physical quantity which requires magnitude and a particular direction, when it is expressed is known as vector quantity

eg: displacement, velocity, acceleration, force etc.

A vector is represented by a line headed with an arrow. Its length is proportional to its magnitude, with respect to a suitably chosen scale.

$\vec{A}$ is a vector and $\vec{A}=\overrightarrow{PQ}$

Magnitude of $\vec{A}=|\vec{A}|$ or A



Modulus of a vector:

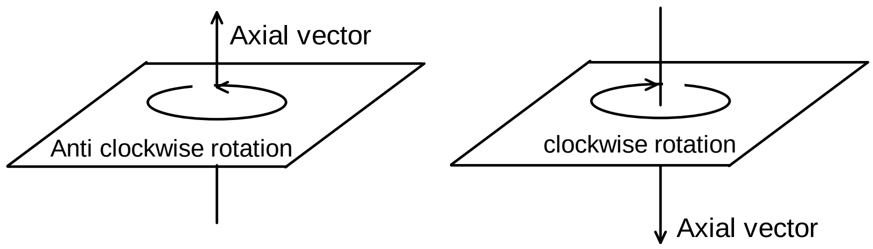
The modulus of a vector means the length of the vector. It is always positive and has no direction.

Modulus of vector $\vec{A}$ is represented as $|\vec{A}|$ or A.

Note:
1. Scalar quantity may be negative e.g. charge, electric current, potential energy, work etc.
2. Scalar quantity are direction independent e.g pressure, electric current, surface tension etc.
3. Vector quantities are direction dependent e.g. force, velocity and displacement.

Axial vector:

A vector which has rotational effect and acts along axis of rotation is called axial vector. Example: angular velocity, torque, angular momentum, angular acceleration etc.

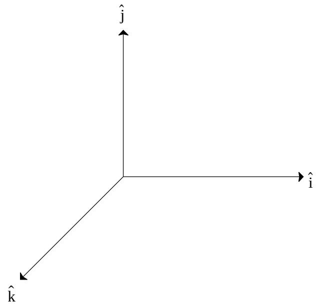


Zero vector or null vector

A vector whose magnitude is zero and has any arbitrary direction is called zero vector. It is represented by $\vec{0}$. The need of a zero arises in the situations:

(i) If $\vec{A}=\vec{B}$, then $\vec{A}-\vec{B}=\vec{0}$ (ii) If $\mu=-\lambda$, then $(\lambda+\mu)\vec{A}=\vec{0}$

Unit vector:



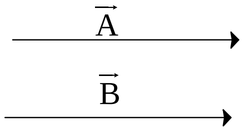
A vector whose magnitude is one unit is called unit vector. A unit vector in the direction of vector $\vec{A}$ is represented by $\hat{A}$, and is given by $\frac{\vec{A}}{A}$

Any vector can be expressed as $\vec{A} = A\hat{A}$.

$\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes.

Equal vectors:

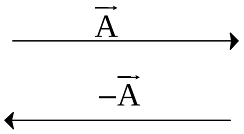
Two vectors are said to be equal if they have same magnitude and same direction.



$\vec{A} = \vec{B}$

Negative of a vector:

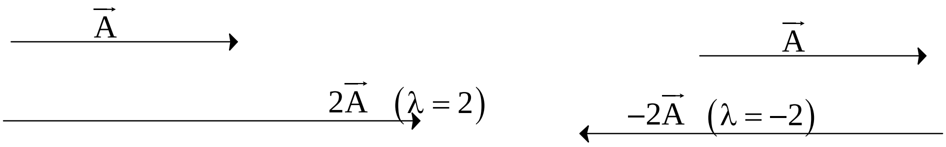
The negative of a vector is defined as vector having same magnitude as that of the vector but has opposite direction.



Multiplication of a vector by a scalar :

When a vector is multiplied by a scalar λ, we get a new vector which is λ times the vector $\vec{A}$ i.e. $\lambda\vec{A}$. The direction of resulting vector is that of $\vec{A}$.

If λ has negative value, then we get a vector whose direction is opposite of $\vec{A}$. The unit of resulting vector is the multiplied units of λ and $\vec{A}$. For example, when mass is multiplied with velocity, we get momentum. The unit of momentum isobtained by multiplying units of mass and velocity.

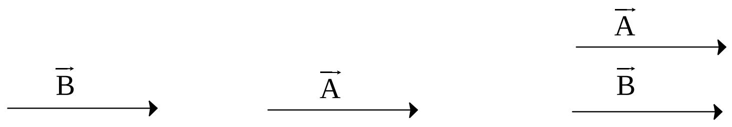


Similarly, we can have vector $\vec{A}$ divided by a scalar λ. The resulting vector becomes $\frac{\vec{A}}{\lambda}$.

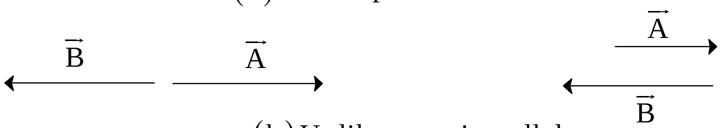
The magnitude of the new vector becomes $\frac{1}{\lambda}$ that of $\vec{A}$ and direction is same as that of $\vec{A}$.

Collinear or parallel vectors :

The vectors which act along the same line or along a parallel line are called collinear vectors.



(a) Like or parallel vectors.

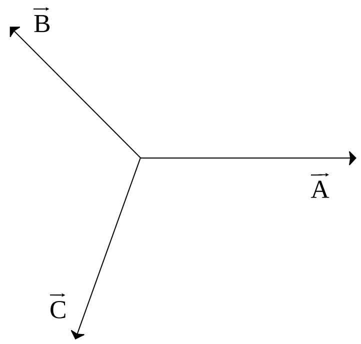


(b) Unlike or antiparallel vectors.

Note: If $\vec{A}$ and $\vec{B}$ be two collinear vectors, then there exists a scalar k such that $\vec{B} = k\vec{A}$, the absolute value of k being the ratio of the length of the two collinear vectors.

Coplanar and concurrent vectors :

Vectors originating from same point are known as concurrent vectors. Vector lying in same plane are called coplanar vectors. In figure, $\vec{A}, \vec{B}$ and $\vec{C}$ are coplanar and concurrent vectors.



VECTOR OPERATIONS

The possible vector operations are:

(i) Addition or subtraction of vectors (ii) Multiplication of vectors

Note: Division of vector by a vector is not defined.

The addition or subtraction of vectors can be done by following two methods:

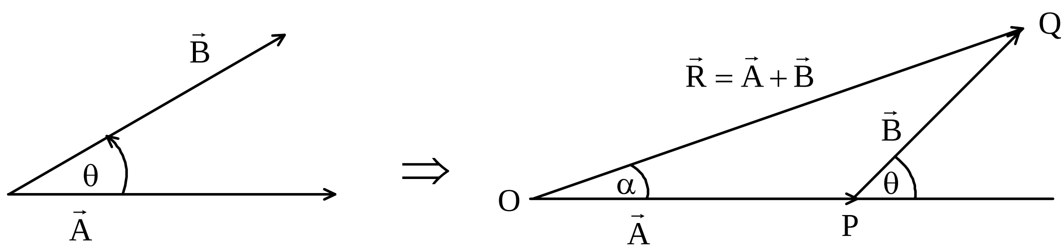
(i) Analytical method (ii) Geometrical method

ADDITION OF TWO VECTORS

Geometrical method

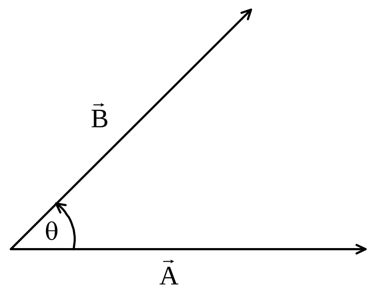
(a) **Triangle law of vector addition :** If two non-zero vectors can be represented by the two sides of a triangle taken in same order, then their resultant is represented by third side of the triangle taken in the opposite order. Consider two vectors $\vec{A}$ and $\vec{B}$ and let angle between them be θ.

Finding $\vec{A} + \vec{B}$: First draw vector $\vec{A}$ $\left(\overline{OP}\right)$ in the given direction. Then draw vector $\vec{B}$ starting from the head of the vector $\vec{A}$. Then close the triangle. $\vec{R}\left(\overline{OQ}\right)$ will be their resultant

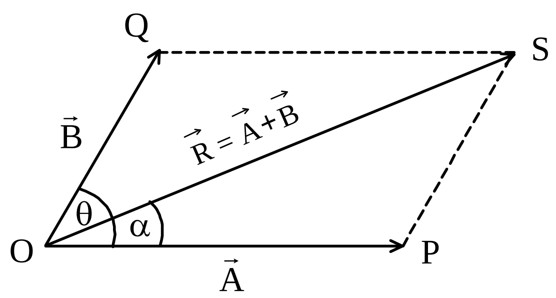


(a) (b)

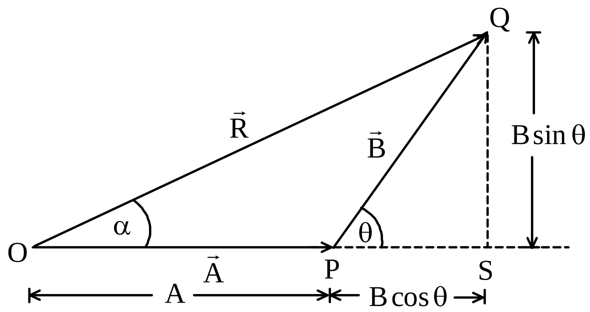
(b) **Parallelogram law of vector addition:** If two non-zero vectors can be represented by the two adjacent sides of a parallelogram, then their resultant is represented by the diagonal of the parallelogram passing though the point of intersection of the vectors. Suppose two vectors $\vec{A}$ and $\vec{B}$ are as shown in figure



Finding $\vec{A}+\vec{B}$: Draw vectors $\vec{A}\left(\overrightarrow{OP}\right)$ and $\vec{B}\left(\overrightarrow{OQ}\right)$ starting from a common point O in the given direction. The diagonal $\overrightarrow{OS}$ of the parallelogram OQSP will represent their resultant.



Analytical method: Finding $\vec{A}+\vec{B}$:



It is clear from the geometry of the figure that resultant of $\vec{A}$ and $\vec{B}$ is equal to the resultant of $\left(\vec{A}+\vec{B}\cos\theta\right)$ and $\vec{B}\sin\theta$. By Pythagoras theorem, we have

$$\therefore\ R^2=\left(A+B\cos\theta\right)^2+\left(B\sin\theta\right)^2=A^2+B^2\cos^2\theta+2AB\cos\theta+B^2\sin^2\theta$$

or $$R=\sqrt{A^2+B^2+2AB\cos\theta}$$

If α is the angle which resultant $\vec{R}$ makes with $\vec{A}$, then:

$$\tan\alpha=\frac{B\sin\theta}{A+B\cos\theta}$$

Special cases :

(i) For $\theta = 0^0$; $R_{max} = \sqrt{A^2 + B^2 + 2AB} = A + B$

(ii) For $\theta = 180^0$; $R_{min} = \sqrt{A^2 + B^2 - 2AB} = A - B$

Thus, $|A - B| \le R \le |A + B|$

(iii) If $A = B$, $R = \sqrt{A^2 + A^2 + 2AA\cos\theta}$

$= \sqrt{2A^2(1 + \cos\theta)}$ $= \sqrt{2A^2 \times 2\cos^2\theta/2}$

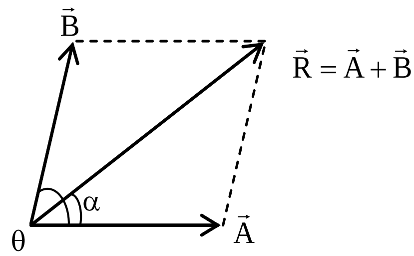
$= 2A\cos\theta/2$ and $\alpha = \dfrac{\theta}{2}$

Illustration 1: Find $\vec{A} + \vec{B}$ in the diagram shown. Given $A = 4$ units and $B = 3$ units and angle between them is 60 degrees.

Solution: $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{16 + 9 + 2.4.3.\cos 60^0}$

$= \sqrt{37}$



$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta} = \dfrac{3\sin 60^0}{4 + 3\cos 60^0}$

$= 0.472$

$\alpha = \tan^{-1}(0.472) = 25.3^0$

Thus, the magnitude of resultant of $\vec{A}$ and $\vec{B}$ is $\sqrt{37}$ units at angle 25.3° from $\vec{A}$ in the direction shown in figure.

Illustration2 : Two forces of equal magnitude are acting at a point. The magnitude of their resultant is equal to magnitude of either. Find the angle between the force vectors.

Solution: Given $R = A = B$. ; Using $R^2 = A^2 + B^2 + 2AB\cos\theta$

$A^2 = A^2 + A^2 + 2AA\cos\theta$; $\Rightarrow \cos\theta = -\dfrac{1}{2}$ $\Rightarrow \theta = 120^o$

SUBTRACTION OF TWO VECTORS

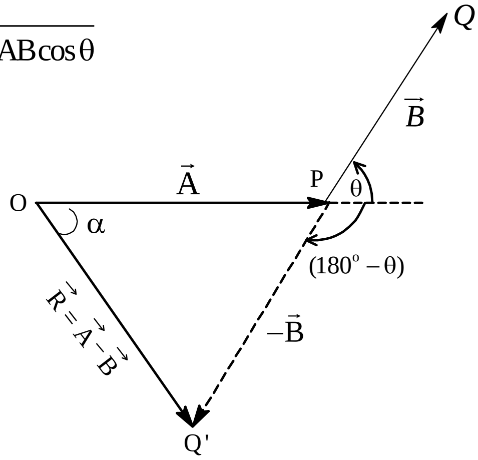
The subtraction of $\vec{B}$ $(\overrightarrow{PQ})$ from $\vec{A}$ $(\overrightarrow{OP})$ means addition of $-\vec{B}$ $(\overrightarrow{PQ'})$ to $\vec{A}$

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

First draw vector $\vec{A}(\overrightarrow{OP})$ in the given direction. Then draw vector $-\vec{B}(\overrightarrow{PQ})$ starting from head of the vector $\vec{A}$. Then close the triangle. $\vec{R}(\overrightarrow{OQ'})$ will be equal to $\vec{A} - \vec{B}$. The angle between $\vec{A}$ and $-\vec{B}$ will be equal to $(180^o - \theta)$.

Therefore, the resultant will be given by:

$$\therefore R=\sqrt{A^2+B^2+2AB\cos\left(180^0-\theta\right)} \qquad =\sqrt{A^2+B^2-2AB\cos\theta}$$



$$\text{and } \tan\alpha=\frac{B\sin\left(180^0-\theta\right)}{A+B\cos\left(180^0-\theta\right)}=\frac{B\sin\theta}{A-B\cos\theta}$$

α is the angle made by $\vec{R}$ with $\vec{A}$

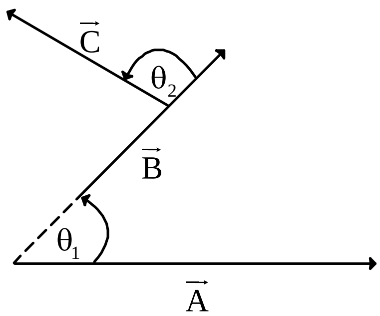
Note: For $\theta=90^0, \left|\vec{A}+\vec{B}\right|=\left|\vec{A}-\vec{B}\right|$

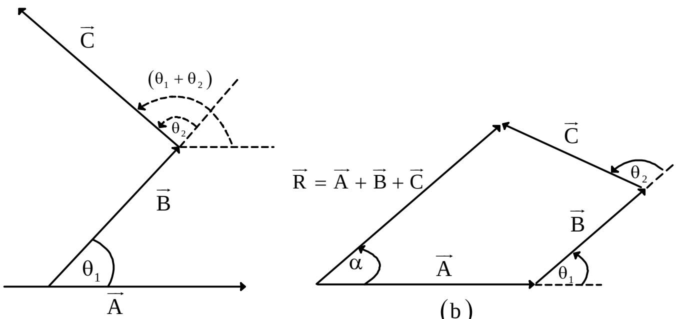
ADDITION OF MORE THAN TWO VECTORS

To find the resultant $\vec{R}=\vec{A}+\vec{B}+\vec{C}$, the polygon law of addition is used.

Polygon law of vector addition :

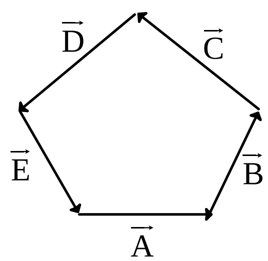
If a number of vectors are represented by the sides of an open polygon taken in the same order, then their resultant is represented by the closing side of the polygon taken in opposite order. Here in the figure, $\vec{R}$ (closing side of polygon) represents the resultant of vectors $\vec{A}, \vec{B}$ and $\vec{C}$





(a) (b)

Note: If n number of vectors makes a closed polygon, their resultant will be zero. Here vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ and $\vec{E}$ make closed polygon.



$$\therefore \vec{A}+\vec{B}+\vec{C}+\vec{D}+\vec{E}=\vec{0}$$

Inchapter Exercise

1. At what angle should the two force vectors 2F and $\sqrt{2}F$ act so that the resultant force is $\sqrt{10}F$?

2. Two forces while acting on a particle in opposite directions, have the resultant of 10N. If they act at right angles to each other, the resultant is found to be 50 N. Find the two forces.

Answer key

1. $\theta = 45^0$ **2.** 30 N, 40 N

RESOLUTION OF VECTORS INTO RECTANGULAR COMPONENTS

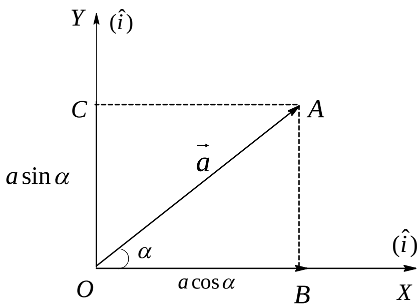
When a vector is split into components which are at right angle to each other then the components are called rectangular or orthogonal components of that vectors.

(i) Let vector $\vec{a} = \overrightarrow{OA}$ in X - Y plane, make angle α with X-axis. Draw perpendiculars AB and **AC** from A on the X-axis and Y-axis respectively.

(ii) The length OB is called projection of $\overrightarrow{OA}$ on X-axis or component of $\overrightarrow{OA}$ along X-axis and is represented by a_x. Similarly OC is the projection of $\overrightarrow{OA}$ on Y-axis and is represented by a_y.

According to law of vector addition $\vec{a} = \overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{OC}$

Thus $\vec{a}$ has been resolved into two parts, one along OX and the other OY, which are mutually perpendicular.



In $\Delta OAB, \dfrac{OB}{OA} = \cos\alpha \quad \Rightarrow OB = OA\cos\alpha \;\Rightarrow a_x = a\cos\alpha$

$\dfrac{AB}{OA} = \sin\alpha \;\Rightarrow\; AB = OA\sin\alpha = OC \;\Rightarrow\; a_y = a\sin\alpha$

If $\hat{i}$ and $\hat{j}$ denote unit vectors along OX and OY respectively then

$\overrightarrow{OB} = a\cos\alpha\;\hat{i}$ and $\overrightarrow{OC} = a\sin\alpha\;\hat{j}$

So according to rule of vector addition $\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{OC} \;\Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j} \Rightarrow \vec{a} = a\cos\alpha\;\hat{i} + a\sin\alpha\;\hat{j}$

RECTANGULAR COMPONENTS OF A VECTOR IN THREE DIMENSIONS

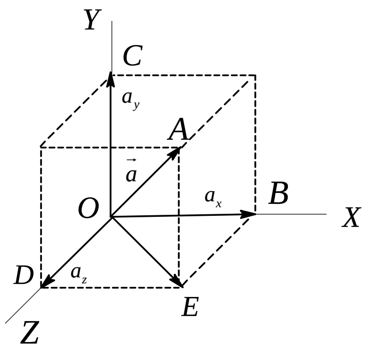
1. Consider a vector $\vec{a}$ represented by $\overrightarrow{OA}$, as shown in figure. Consider O as origin and draw a rectangular parallelopiped with its three edges along the X, Y and Z axes.

2. Vector $\vec{a}$ is the diagonal of the parallelopiped whose projections on x, y and z axis are $\vec{a}_x, \vec{a}_y$ and $\vec{a}_z$ respectively.

These are the three rectangular components of $\vec{A}$.

Using triangle law of vector addition, $\overrightarrow{OA} = \overrightarrow{OE} + \overrightarrow{EA}$

Using parallelogram law of vector addition, $\overrightarrow{OE} = (\overrightarrow{OB} + \overrightarrow{OD})$



$\therefore\; \overrightarrow{OA} = (\overrightarrow{OB} + \overrightarrow{OD}) + \overrightarrow{EA}$

$\because \overrightarrow{EA} = \overrightarrow{OC} \qquad \therefore\; \overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{OD} + \overrightarrow{OC}$

Now, $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = a_x\hat{i}, \overrightarrow{OC} = a_y\hat{j}$ and $\overrightarrow{OD} = a_z\hat{K}$

$\therefore\; \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \qquad$ Also $\quad (OA)^2 = (OE)^2 + (EA)^2$

but $(OE)^2 = (OB)^2 + (OD)^2$

and $EA = OC$ $\therefore\ (OA)^2 = (OB)^2 + (OD)^2 + (OC)^2$

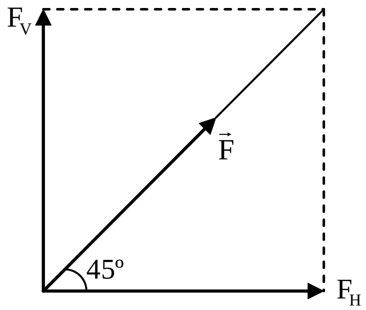
or $a^2 = a_x^2 + a_y^2 + a_z^2 \Rightarrow a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

ADDITION AND SUBTRACTION OF VECTORS IN COMPONENT FORM

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ be two vectors whose addition (or subtraction) has to be done. Then the components in the same direction will be added and new vector will be obtained. For example, in this case,

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

Illustration 3: Resolve horizontally and vertically a force $\vec{F} = 8N$ which makes an angle of 45° with the horizontal.



Solution: Horizontal component of $\vec{F}$ is

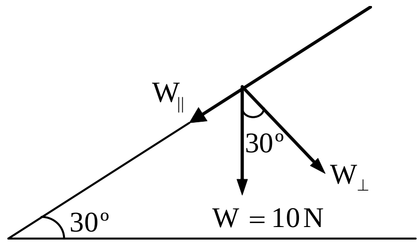
$F_H = F\cos 45° = 8\frac{1}{\sqrt{2}}$

$= 4\sqrt{2}N$

and vertical component of $\vec{F}$ is $F_V = F\sin 45°$

$= (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}N$

Illustration 4: Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.



Solution: Component perpendicular to the plane

$W_{\perp} = W\cos 30^0$

$= (10)\frac{\sqrt{3}}{2}$

$= 5\sqrt{3}N$

and component parallel to the plane

$W_{\parallel} = W\sin 30° = (10)\left(\frac{1}{2}\right) = 5\ N$

Illustration 5: Obtain the magnitude of $2\vec{A} - 3\vec{B}$ if $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$.

Solution: $2\vec{A} - 3\vec{B} = 2(\hat{i} + \hat{j} - 2\hat{k}) - 3(2\hat{i} - \hat{j} + \hat{k}) = -4\hat{i} + 5\hat{j} - 7\hat{k}$

Magnitude of $2\vec{A} - 3\vec{B} = \sqrt{(-4)^2 + (5)^2 + (-7)^2}$

$= \sqrt{16 + 25 + 49} = \sqrt{90}$

Illustration 6: The magnitude of a vector $\vec{A}$ is 10 units and it makes an angle of 30^0 with the X- axis. Find the components of the vector if it lies in the X-Y plane.

Solution: Components of vector $\vec{A}$ lying in the X-Y plane are $A_x = A\cos\theta, A_y = A\sin\theta, A_z = 0$

Thus, $A_x = 10\cos 30^0 = \frac{10\sqrt{3}}{2} = 8.66$ units ; $A_y = 10\sin 30^0 = 10 \times 1/2 = 5$

$A_z = 0$

Illustration 7: Two forces $\vec{F}_1 = 1N$ and $\vec{F}_2 = 2N$ act along the lines x = 0 and y = 0 respectively. Then, find the resultant force.

Solution: x = 0 means y – axis; y = 0 means x – axis; $\therefore$ 1N is acting along y – axis and 2N is acting along x – axis; So, the force $\vec{F} = 2\hat{i} + \hat{j}$

Illustration 8: What vector must be added to the summation of vectors $\hat{i} - 3\hat{j} + 2\hat{k}$ and $3\hat{i} + 6\hat{j} - 7\hat{k}$ so that the resultant vector is a unit vector along the y-axis.

Solution: $\hat{i} - 3\hat{j} + 2\hat{k} + 3\hat{i} + 6\hat{j} - 7\hat{k} = 4\hat{i} + 3\hat{j} - 5\hat{k}$

Now, $(4 + x)\hat{i} + (3 + y)\hat{j} + (-5 + z)\hat{k} = \hat{j}$ So, x = – 4, y = – 2, z = 5

and hence the vector is $-4\hat{i} - 2\hat{j} + 5\hat{k}$

Inchapter Exercise

1. Let $\overrightarrow{AB}$ be a vector in two dimensional plane with magnitude 4 units, and making an angle of 60^0 with x-axis and lying in first quadrant. Find the components of $\overrightarrow{AB}$ along x axis and y axis. Hence represent $\overrightarrow{AB}$ in terms of unit vectors $\hat{i}$ and $\hat{j}$.

2. A 1000 N block is placed on an inclined plane with angle of 30^0. Find the components of the weight (i) parallel (ii) perpendicular to the inclined plane.

3. Let $\vec{A} = 2\hat{i} + \hat{j}, \vec{B} = 3\hat{j} - \hat{k}$, find $2\vec{A}$-$3\vec{B}$.

Answer Key

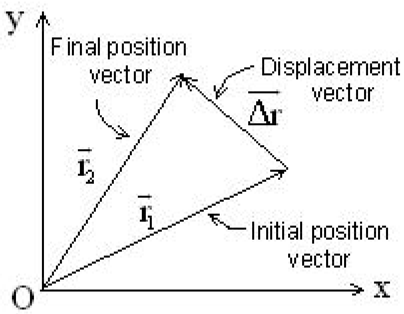
1. Component on x-axis = 2 ; Component along y-axis = $2\sqrt{3}$

   Hence $\overrightarrow{AB} = 2\hat{i} + 2\sqrt{3}\hat{j}$

2. (i) 500 N (ii) 866 N

3. $2\vec{A} - 3\vec{B} = 4\hat{i} - 7\hat{j} + 3\hat{k}$

POSITION VECTOR AND DISPLACEMENT VECTOR



A vector which gives the position of an object with reference to some specified point in a system is called position vector. Displacement vector refers to the change of position vectors. Thus displacement vector = final position vector - initial position vector or $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Illustration 9: **A body is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j} + \hat{k})$ m to the position vector $\vec{r}_2 = (\hat{i} + \hat{j} + \hat{k})$ m.. Find the displacement vector.**

Solution : The body is displaced from $\vec{r}_1$ to $\vec{r}_2$. Therefore, displacement of the body is $\vec{S} = \vec{r}_2 - \vec{r}_1 = (\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k}) = (-\hat{i} - 2\hat{j})$m

PRODUCT OF TWO VECTORS

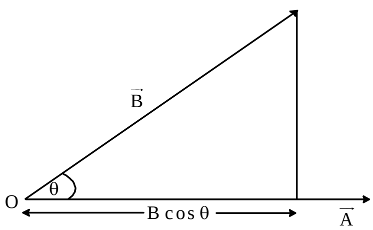
There are two ways in which vectors can be multiplied. They are termed as dot product and cross product.

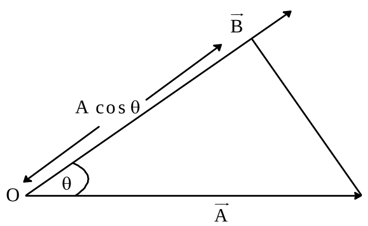
1. SCALAR PRODUCT OR DOT PRODUCT

Dot product of two vectors is defined as the product of magnitude of one of the two vectors & the magnitude of the rectangular component of the second vector in the direction of the first vector.The dot product of two vectors $\vec{A}$ and $\vec{B}$ is the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and cosine of the angle between them. Thus $\vec{A}.\vec{B} = AB\cos\theta$. As A, B and $\cos\theta$ all are scalars, so their product is a scalar quantity. Dot product is also called scalar product.

Geometrical interpretation of scalar product :

We have, $\vec{A}.\vec{B} = AB\cos\theta \quad = A(B\cos\theta)$ or $=(A\cos\theta)B$





(a) $\vec{A}.\vec{B} = A\,(B\cos\theta)$

$= AB\cos\theta$

(b) $\vec{A}.\vec{B} = (A\cos\theta)B$

$= AB\cos\theta$

Properties of scalar product :

(i) The scalar product is commutative i.e $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(ii) The scalar product is distributive over addition i.e $\vec{A}.\left(\vec{B} + \vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(iii) If $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then $\vec{A}.\vec{B} = AB\cos 90^0 = 0$

(iv) If $\vec{A}$ and $\vec{B}$ are parallel having same direction, then $\vec{A}.\vec{B} = AB\cos 0^0 = AB$

(v) If $\vec{A}$ and $\vec{B}$ are antiparallel, then $\vec{A}.\vec{B} = AB\cos 180^0 = -AB$.

(vi) The scalar product of two identical vectors $\vec{A}.\vec{A} = AA\cos 0^0 = A^2$

(vii) If $\hat{i}, \hat{j}$ and $\hat{k}$ are mutually perpendicular unit vectors, then $\hat{i}.\hat{i} = (1)(1)\cos 0^0 = 1$

$\therefore\ \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

$\hat{i}.\hat{j} = (1)(1)\cos 90^0 = 0 \qquad \therefore \hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(viii) If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$, then $\vec{A}.\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$.

Applications of scalar product :

(i) Work done: If a force $\vec{F}$ causes displacement $\vec{s}$, then work done $w = \vec{F}.\vec{s} = Fs\cos\theta$

(ii) Angle between the vectors : For two vectors $\vec{A}$ and $\vec{B}$, we have $\vec{A}.\vec{B} = AB\cos\theta$

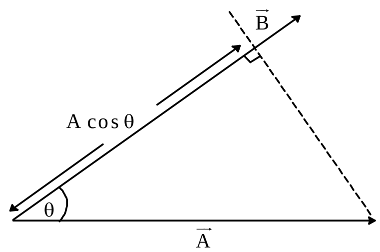
$\therefore \cos\theta = \dfrac{\vec{A}.\vec{B}}{AB}$ or, If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

then $\cos\theta = \dfrac{A_1B_1 + A_2B_2 + A_3B_3}{\sqrt{A_1^2 + A_2^2 + A_3^2}\sqrt{B_1^2 + B_2^2 + B_3^2}}$

(iii) Component or projection of one vector along other vector:

(a) Component of vector $\vec{A}$ along vector $\vec{B}$ is given by

$$A\cos\theta\,\hat{B} = \left(\frac{AB\cos\theta}{B}\right)\hat{B} = \left(\frac{\vec{A}.\vec{B}}{B}\right)\hat{B}$$



(b) Component of vector $\vec{B}$ along vector $\vec{A}$ is given by

$$B\cos\theta\hat{A} = \left(\frac{AB\cos\theta}{A}\right)\hat{A}$$

$$= \left(\frac{\vec{A}.\vec{B}}{A}\right)\hat{A}$$

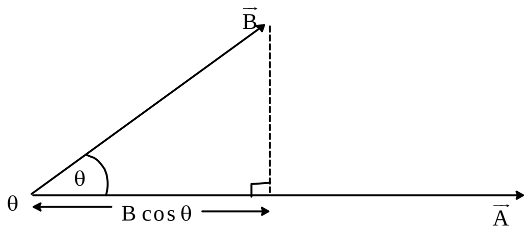


Illustration 10: Work done by a force $\vec{F}$ on a body is $W = \vec{F}.\vec{S}$, where $\vec{S}$ is the displacement of body. Given that under a force $\vec{F} = (2\hat{i} + 3\hat{j} + 4\hat{k})N$ a body is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j} + \hat{k})m$ to the position vector $\vec{r}_2 = (\hat{i} + \hat{j} + \hat{k})m$, find the work done by this force.

Solution: The body is displaced from $\vec{r}_1$ to $\vec{r}_2$. Therefore, displacement of the body is

$\vec{S} = \vec{r}_2 - \vec{r}_1 = (\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k}) = (-\hat{i} - 2\hat{j})$ m

Now, work done by the force is $W = \vec{F}\cdot\vec{S} = (2\hat{i} + 3\hat{j} + 4\hat{k})\cdot(-\hat{i} - 2\hat{j})$

$= (2)(-1) + (3)(-2) = -8J$

Illustration 11: Prove that the vectors $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ are mutually perpendicular.

Solution: $\vec{A}\cdot\vec{B} = (2\hat{i} - 3\hat{j} + \hat{k})\cdot(\hat{i} + \hat{j} + \hat{k}) = (2)(1) + (-3)(1) + (1)(1) = 0 = AB\cos\theta$

$\cos\theta = 0$ or $\theta = 90^\circ$

or the vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular.

Illustration 12: Let for two vectors $\vec{A}$ and $\vec{B}$, their sum $(\vec{A}+\vec{B})$ is perpendicular to the difference $(\vec{A}-\vec{B})$. Find the ratio of their magnitudes (A/B).

Solution: It is given that $(\vec{A}+\vec{B})$ is perpendicular to $(\vec{A}-\vec{B})$. Thus,

$(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = 0$ or $(\vec{A})^2+\vec{B}\cdot\vec{A}-\vec{A}\cdot\vec{B}-(\vec{B})^2=0$

Because of commutative property of dot product

$\vec{A}.\vec{B}=\vec{B}.\vec{A}$ $\therefore A^2-B^2=0$ or $A=B$

Thus $A/B=1$ i.e. the ratio of magnitudes is 1.

Illustration 13: Find the angle between the vectors $3\hat{i}+2\hat{j}+1\hat{k}$ and $5\hat{i}-2\hat{j}-3\hat{k}$

Solution: $|\vec{A}|=\sqrt{3^2+2^2+1^2}=\sqrt{14}$ $|\vec{B}|=\sqrt{(5)^2+(-2)^2+(-3)^2}=\sqrt{38}$

$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z=3\times5+2(-2)+(1)(-3)=8$

$$\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\frac{8}{\sqrt{14\times38}}=0.35 \qquad \theta=\cos^{-1}(0.35)$$

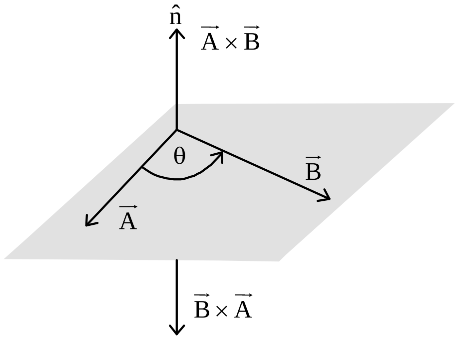
Inchapter Exercise

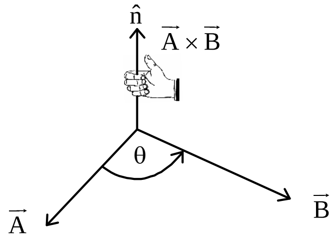
1. Find the angle between two vectors $\vec{A}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{B}=\hat{i}-\hat{k}$.
2. If the vectors $4\hat{i}+\hat{j}-3\hat{k}$ and $2m\hat{i}+6m\hat{j}+\hat{k}$ are mutually perpendicular, find the value of m.
3. What is the angle between $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}$
4. Find the magnitude of component of $3\hat{i}-2\hat{j}+\hat{k}$ along the vector $12\hat{i}+3\hat{j}-4\hat{k}$

Answer Key: **1.** $\theta=30^\circ$ **2.** $m=3/14$ **3.** $\theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **4.** 2

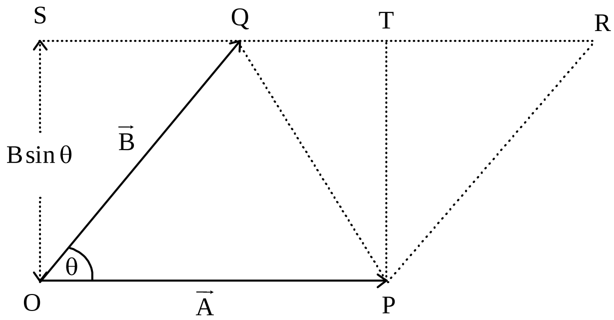
2. VECTOR PRODUCT OR CROSS PRODUCT

The vector product of two vectors is defined as the vector whose magnitude is equal to the product of the magnitudes of two vectors and sine of angle between them and whose direction is perpendicular to the plane of two vectors and is given by right hand thumb rule. Mathematically, if θ is the angle between $\vec{A}$ and $\vec{B}$, then $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the plane of vector $\vec{A}$ and $\vec{B}$. The direction of unit vector $\hat{n}$ is given by the right hand thumb rule. The fingers of right hand should be oriented along the direction of $\vec{A}$ such that they curl towards $\vec{B}$. The direction of outstretched thumb gives direction of $\hat{n}$





Geometrical interpretation of vector product :



Suppose two vectors $\vec{A}$ and $\vec{B}$ are represented by the sides OP and OQ of a parallelogram, as shown in figure. The magnitude of vector product $\vec{A}\times\vec{B}$ is $|\vec{A}\times\vec{B}| = AB\sin\theta$

= $A(B\sin\theta)$
= area of rectangle OPTS
= area of parallelogram OPRQ

Thus the magnitude of the vector product of two vectors is equal to the area of the parallelogram (OPRQ) formed by the two vectors as its adjacent sides.

= $2\times$ area of triangle OPQ.

$\therefore$ Area of triangle OPQ = $\frac{1}{2}$ (area of parallelogram OPRQ).

$$= \frac{1}{2}|\vec{A}\times\vec{B}|.$$

Properties of vector product:

(i) Vector product is not commutative. It is anticommutative i.e.,

$$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A}).$$

(ii) Vector product is distributive over addition i.e.,

$$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}\cdot$$

(iii) Vector product of two parallel or antiparallel vectorsis zero.

$$\vec{A}\times\vec{B} = AB\sin 0^0\,\hat{n} = 0$$
$$\vec{A}\times\vec{B} = AB\sin 180^0\,\hat{n} = 0$$

(iv) Vector product of two identical vectors is also zero.

$$\vec{A}\times\vec{A} = AA\sin 0^o\,\hat{n} = 0\cdot$$

(v) The magnitude of vector product of two mutually perpendicular vectors is equal to the product of their magnitude.

$$|\vec{A}\times\vec{B}| = AB\sin 90^o = AB.$$

(vi) For unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$\hat{i}\times\hat{i} = (1)(1)\sin 0^o\,\hat{n} = 0$$

$\therefore$ $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

and $\hat{i}\times\hat{j} = (1)(1)\sin 90^o\,\hat{k} \quad = \hat{k}$

Similarly $\hat{j}\times\hat{k}=\hat{i}$

and $\hat{k}\times\hat{i}=\hat{j}$

$\therefore\quad \hat{j}\times\hat{i}=-\hat{k},\ \hat{k}\times\hat{j}=-\hat{i},\ \hat{i}\times\hat{k}=-\hat{j}$

(vii) Cross product can be used to find angle between two vectors.

According to definition of vector product of two vectors $\vec{A}\times\vec{B}=AB\sin\theta\hat{n}$

So, $|\vec{A}\times\vec{B}|=AB\sin\theta$ i.e. $\theta=\sin^{-1}\left[\dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}\right]$

However, dot product method is more convenient to find angle between vectors.

Note: To find the cross product of two vectors, the following methods can be used:

Cross Product Method 1:

$$\vec{A}\times\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$\begin{aligned}&=A_xB_x(\hat{i}\times\hat{i})+A_yB_x(\hat{j}\times\hat{i})+A_zB_x(\hat{k}\times\hat{i})\\&\quad+A_xB_y(\hat{i}\times\hat{j})+A_yB_y(\hat{j}\times\hat{j})+A_zB_y(\hat{k}\times\hat{j})\\&\quad+A_xB_z(\hat{i}\times\hat{k})+A_yB_z(\hat{j}\times\hat{k})+A_zB_z(\hat{k}\times\hat{k})\end{aligned}$$

[As $\hat{i}\times\hat{i}=0,\ \hat{j}\times\hat{j}=0,\ \hat{k}\times\hat{k}=0$ and $\hat{i}\times\hat{j}=\hat{k},\ \hat{j}\times\hat{i}=-\hat{k},\hat{k}\times\hat{i}=\hat{j},\hat{i}\times\hat{k}=-\hat{j},\hat{k}\times\hat{j}=-\hat{i}$]

So, we have $\vec{A}\times\vec{B}=A_yB_x(-\hat{k})+A_zB_x\hat{j}+A_xB_y\hat{k}+A_zB_y(-\hat{i})+A_xB_z(-\hat{j})+A_yB_z(\hat{i})$

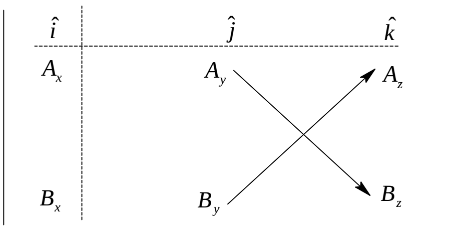
$$=(A_yB_z-A_zB_y)\hat{i}-(A_xB_z-A_zB_x)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$$

Cross Product Method 2:

Cross product of two vectors $\vec{A}$ and $\vec{B}$ can be obtained easily by using the determinant method.

$$\vec{A}\times\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\A_x&A_y&A_z\\B_x&B_y&B_z\end{vmatrix}$$

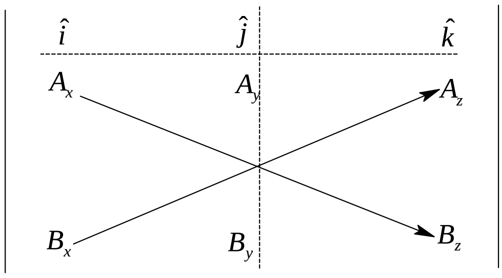


Here, we will use $\hat{i},\hat{j},\hat{k}$ one by one. When $\hat{i}$ is chosen, its corresponding row and column become bound and remaining elements are subtracted after cross multiplication.

So, $(A_yB_z-B_yA_z)$, is the component along $\hat{i}$.

Similarly, in the case of $\hat{j}$, the row and column in which it is present become bound and remaining elements are subtracted after cross multiplication.

So, $\hat{i}(A_yB_z-B_yA_z)-\hat{j}(A_xB_z-B_xA_z)$ is the component along $\hat{i}$ and $\hat{j}$



Same argument will follow for $\hat{k}$ as is for $\hat{i}$ and $\hat{j}$

$$\therefore \vec{A}\times\vec{B} = \hat{i}(A_yB_z - B_yA_z) - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y)$$

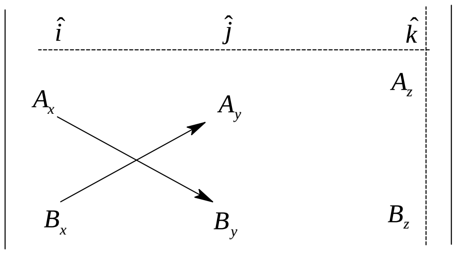


Illustration: 14: If $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

(i) Find the magnitude of $\vec{A}\times\vec{B}$

(ii) Find a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$

(iii) Find the cosine and sine of the angle between the vectors $\vec{A}$ and $\vec{B}$

Solution: (i) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix} = 8\hat{i} - 8\hat{j} - 8\hat{k}$

Magnitude of $\vec{A}\times\vec{B} = |\vec{A}\times\vec{B}| = \sqrt{(8)^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$

(ii) $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \dfrac{8\hat{i} - 8\hat{j} - 8\hat{k}}{8\sqrt{3}} = \dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

There are two unit vectors perpendicular to both $\vec{A}$ and $\vec{B}$, they are $\pm\hat{n} = \pm\dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

(iii) $\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{AB} = \dfrac{8\sqrt{3}}{\sqrt{14}\sqrt{24}} = \dfrac{2}{\sqrt{7}}$ $\qquad \cos = \dfrac{\vec{A}.\vec{B}}{AB} = \dfrac{\sqrt{3}}{\sqrt{7}}$

Illustration15: Let a force $\vec{F}$ be acting on a body free to rotate about a point O and let $\vec{r}$ be the position vector of any point P on the line of action of the force. Then torque of this force about point O is defined as : $\vec{\tau} = \vec{r}\times\vec{F}$

Given, $\vec{F} = (2\hat{i} + 3\hat{j} - \hat{k})$N and $\vec{r} = (\hat{i} - \hat{j} + 6\hat{k})$m, find the torque of this force.

Solution: $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 6 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(1-18) - \hat{j}(1-12) + \hat{k}(3+2)$

$= -17\hat{i} + 11\hat{j} + 5\hat{k}$

Illustration 16: Two vectors, $\vec{A} = 2\hat{i} + 2\hat{j} + p\hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ are given. Find the value of p if (i) the two vectors are perpendicular (ii) the two vectors are parallel.

Solution: (i) $\vec{A}$ and $\vec{B}$ will be perpendicular if $\vec{A} \cdot \vec{B} = 0$

Now $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ = 2 x 1 + 2 x 1 + p x 1 = 4 + p

For $\vec{A}.\vec{B} = 0$, we must have 0 = 4 + p or p = –4

(ii) $\vec{A}$ and $\vec{B}$ will be parallel if $\vec{A} \times \vec{B} = 0$; Now $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & p \\ 1 & 1 & 1 \end{vmatrix}$

$= \hat{i}(2-p) + \hat{j}(p-2) + \hat{k}(2-2)$ $= \hat{i}(2-p) + \hat{j}(p-2)$

For $\vec{A} \times \vec{B} = 0$, we must have each component to be zero. That is 0 = 2 – p, and 0 = p – 2 (both conditions are similar). Thus p = 2

Inchapter exercise

1. Find a unit vector perpendicular to both $\vec{A} = 2\hat{i} + \hat{j}$ and $\vec{B} = \hat{i} + 2\hat{j}$

2. The torque of a force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at the point $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$ is:

(A) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (B) $4\hat{i} + 4\hat{j} + 6\hat{k}$

(C) $-21\hat{i} + 4\hat{j} + 4\hat{k}$ (D) $-14\hat{i} + 34\hat{j} + 16\hat{k}$

3. The magnitude of the vector porduct of two vectors is $\sqrt{3}$ times their scalar porduct. What is the angle between the two vectors?

4. The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$.

If $\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{r} = 4\hat{j} - 3\hat{k}$, then what is the magnitude of $\vec{v}$?

Answer key

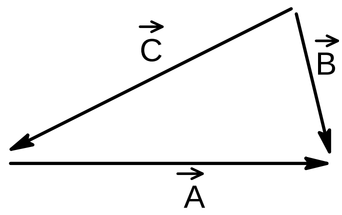
1. $\hat{k}$ **or** - $\hat{k}$ **2.**(A) **3.** $\theta = 60°$ **4.** $\sqrt{29}$ units

JEE MAIN EXERCISE

1. The value of a unit vector in the direction of vector $\vec{A} = 5\hat{i} - 12\hat{j}$, is –

(A) $\hat{i}$ (B) $\hat{j}$ (C) $(\hat{i} + \hat{j})/13$ (D) $(5\hat{i} - 12\hat{j})/13$

2. For the figure –



(A) $\vec{A} + \vec{B} = \vec{C}$ (B) $\vec{B} + \vec{C} = \vec{A}$ (C) $\vec{C} + \vec{A} = \vec{B}$ (D) $\vec{A} + \vec{B} + \vec{C} = 0$

3. Two forces of 4 dyne and 3 dyne act upon a body. The resultant force on the body can only be –
(A) more than 3 dynes (B) more than 4 dynes
(C) between 3 and 4 dynes (D) between 1 and 7 dynes

4. A force of 6 N and another of 8 N can be applied together to produce the effect of a single force of-
(A) 1 N (B) 11 N (C) 15 N (D) 20 N

5. Which of the sets given below may represent the magnitudes of three vectors adding to zero ?
(A) 2, 4, 8 (B) 4, 8, 16 (C) 1, 2, 1 (D) 0.5, 1, 2

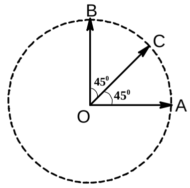
6. A blind person after walking each 10 steps in one direction, each of length 80 cm, turns randomly to the left or to right by 90^0. After walking a total of 40 steps the maximum possible displacement of the person from his starting position could be –

(A) 320 m (B) 32 m (C) $16/\sqrt{2}$ m (D) $16\sqrt{2}$m

7. If the angle between vector **a** and **b** is an acute angle, then the difference **a** – **b** is –
(A) the main diagonal of the parallelogram (B) the minor diagonal of the parallelogram
(C) any of the above (D) none of the above

8. Angle between $(\vec{P} + \vec{Q})$ and $(\vec{P} - \vec{Q})$ will be –
(A) 0^0 only
(B) 90^0 only
(C) 180^0 only
(D) between 0^0 and 180^0 (both the values inclusive)

9. The three vectors **OA**, **OB** and **OC** have the same magnitude R. Then the sum of these vectors have magnitude –



(A) R (B) $\sqrt{2}R$ (C) 3R (D) $\left(1+\sqrt{2}\right)R$

10. Two vectors $\vec{P} = 2\hat{i} + b\hat{j} + 2\hat{k}$ and $\vec{Q} = \hat{i} + \hat{j} + \hat{k}$ are perpendicular to each other if -

(A) b = 0 (B) b = 1 (C) b = 2 (D) b = –4

11. A vector is not changed if –

(A) It is rotated through an arbitrary angle
(B) It is multiplied by an arbitrary scale
(C) It is cross multiplied by a unit vector
(D) It is a slide parallel to it self

12. If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units, the angle between $\vec{A}$ and $\vec{C}$ is–

(A) $\cos^{-1}\left(\frac{3}{5}\right)$ (B) $\cos^{-1}\left(\frac{4}{5}\right)$ (C) $\frac{\pi}{2}$ (D) $\sin^{-1}\left(\frac{3}{4}\right)$

13. The vector $\vec{A}$ and $\vec{B}$ are such that –

$\vec{A} + \vec{B} = \vec{A} - \vec{B}$

(A) $\vec{A} + \vec{B} = 0$ (B) $\vec{A} - \vec{B} = 0$ (C) $\vec{A} = 0$ (D) $\vec{B} = 0$

14. Two forces each of magnitude F have a result of the same magnitude F. The angle between the two forces is –

(A) 45^0 (B) 120^0 (C) 150^0 (D) 60^0

15. Three vectors $\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ are sides of an –

(A) equilateral triangle
(B) right angled triangle
(C) isosceles triangle
(D) none of the above

16. If $\vec{A}, \vec{B}$ and $\vec{C}$ are three vectors, then the wrong relation is –

(A) $\vec{A} + \left(\vec{B} + \vec{C}\right) = \left(\vec{A} + \vec{B}\right) + \vec{C}$
(B) $\vec{A}.\left(\vec{B} + \vec{C}\right) = \left(\vec{A}.\vec{B}\right) + \vec{A}.\vec{C}$
(C) $\vec{A} \times \left(\vec{B} + \vec{C}\right) = \left(\vec{A} \times \vec{B}\right) + \left(\vec{A} \times \vec{C}\right)$
(D) $\left(\vec{A} \times \vec{B}\right).\vec{C} = \vec{A} \times \left(\vec{B}.\vec{C}\right)$

17. Two vector $\vec{A}$ and $\vec{B}$ have equal magnitude and perpendicular to each other. Then the vector $\vec{A} + \vec{B}$ is perpendicular to –

(A) $\vec{A} \times \vec{B}$ (B) $\vec{A} - \vec{B}$ (C) $3\vec{A} - 3\vec{B}$ (D) all of these

18. The angle between two vector $\vec{A}$ and $\vec{B}$ is θ. Then the magnitude of the product $\vec{A}.(\vec{B}\times\vec{A})$ is –

(A) A^2B (B) $A^2B\sin\theta$ (C) $A^2B\sin\theta\cos\theta$ (D) Zero

19. Two force are such that the sum of their magnitudes is 18 N, magnitude of the resultant is 12 N and their resultant is perpendicular to the smaller force. Then the magnitude of the forces are –
(A) 12 N, 6 N (B) 13 N, 5 N (C) 10 N, 8 N (D) 16 N, 2 N

20. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between $\vec{A}$ and $\vec{B}$ is –
(A) π (B) $\pi/3$ (C) $\pi/2$ (D) $\pi/4$

21. If $\vec{A}.\vec{B}=$ magnitude of $\vec{A}\times\vec{B}$, then the angle between vectors $\vec{A}$ and $\vec{B}$ is
(A) 30^0 (B) 45^0 (C) 60^0 (D) 75^0

22. The magnitude of the resultant of $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ is

(A) 2A (B) 2B (C) $\sqrt{A^2+B^2}$ (D) $\sqrt{A^2-B^2}$

23. If $\hat{i},\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively, the angle θ between the vector $\hat{i}$ and $\hat{i}+\hat{j}+\hat{k}$ vector is given by

(A) $\theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\theta=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (C) $\theta=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (D) $\theta=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

24. Given that $0.2\hat{i}+0.6\hat{j}+a\hat{k}$ is a unit vector. What is the value of a ?

(A) $\sqrt{0.3}$ (B) $\sqrt{0.4}$ (C) $\sqrt{0.6}$ (D) $\sqrt{0.8}$

25. Given $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}$. The component of vector $\vec{A}$ along vector $\vec{B}$ is

(A) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (B) $\frac{3}{\sqrt{2}}(\hat{i}+\hat{j})$ (C) $\frac{5}{\sqrt{2}}(\hat{i}+\hat{j})$ (D) $\frac{7}{\sqrt{2}}(\hat{j}+\hat{i})$

EXERCISE - 1

OBJECTIVE PROBLEMS WITH ONE CORRECT ANSWER

1. The sum of magnitudes of two forces acting at a point is 16 N. If their resultant is normal to the smaller force and has a magnitude of 8 N, the forces are :
(A) 6N, 10N (B) 8N, 8N (C) 4N, 12N (D) 2N, 14N

2. The component of a vector is :
(A) always less than its magnitude (B) always greater than its magnitude
(C) always equal to its magnitude (D) none of these

3. A displacement vector, at an angle of 30° with y-axis has a x-component of 10 units. The magnitude of the vector is :
(A) 5.0 (B) 10 (C) 11.5 (D) 20

4. A displacement vector $\vec{r}$ has a magnitude of 25m and makes an angle of 210° with the x-axis. Its y-component is : ($\hat{i}$ and $\hat{j}$ are unit vectors along x & y axes)
(A) 21.7 (B) $-21.7\hat{j}$ (C) $12.5\hat{j}$ (D) $-12.5\hat{j}$

5. Resultant of which of the following may be equal to zero?
(A) 10N; 10N; 30N (B) 10N; 20N; 30N; 40N
(C) 5N; 10N; 20N; 40N (D) none of these

6. The resultant of three vectors of magnitudes 1, 2 and 3 units and whose directions are along the sides of an equilateral triangle taken in the same order is :
(A) 1unit (B) $\sqrt{6}$ units (C) $\sqrt{3}$ units (D) $\sqrt{14}$ units

7. In the above equation the resultant vector makes an angle of :
(A) 210° with the first vector (B) 30° with the first vector
(C) 60° with the first vector (D) 120° with the second vector

8. A body is constrained to move only in Y-direction under the action of a force $\vec{F} = -2\hat{i} + 15\hat{j} + 6\hat{k}$ N. If the body moves a distance of 10m then the work done is :
(A) 150J (B) 190J (C) 163J (D) 185J

9. If $\vec{A} = \hat{i} + \hat{j}$ & $\vec{B} = \hat{i} - \hat{j}$, then a vector $\vec{C}$ perpendicular to both $\vec{A}$ & $\vec{B}$ and having a magnitude equal to 3 is :
(A) $3\hat{k}$ (B) $\sqrt{3}\left(\hat{i} + \hat{j}\right)$ (C) $\sqrt{3}\left(\hat{i} + 2\hat{k}\right)$ (D) $\sqrt{3}\left(\hat{i} - \hat{j}\right)$

10. A vector $\vec{A}$ points vertically downward & $\vec{B}$ points towards east, then the vector product $\vec{A} \times \vec{B}$ is :
(A) along west (B) along east (C) along north (D) along south

11. A particle is moving eastward with a velocity of 5m/s. In 10 seconds the velocity changes to 5m/s northwards. The average acceleration $\frac{\Delta\vec{v}}{\Delta t}$ in this time is :
(A) Zero (B) $1/\sqrt{2}\,m/s^2$ towards north - west
(C) $1/\sqrt{2}\,m/s^2$ towards north - east (D) $1/\sqrt{2}\,m/s^2$ towards north

12. Five equal forces of 10N each are applied at one point and all are lying in one plane. If the angles between any two adjacent forces are equal, the resultant of these forces will be :
(a) zero (B) 10N (C) 20N (D) $10\sqrt{2}$ N

13. At t = 0, a particle at (1,0,0) starts moving towards (4,4,12) with a constant speed of 65 m/s. The position of the particle is measured in metres and time in secs. Assuming constant velocity, the position of the particle at t=2 sec is :

(A) $\left(13\hat{i}-120\hat{j}+40\hat{k}\right)$ m (B) $\left(40\hat{i}+31\hat{j}-120\hat{k}\right)$ m

(C) $\left(13\hat{i}-40\hat{j}+12\hat{k}\right)$ m (D) $\left(31\hat{i}+40\hat{j}+120\hat{k}\right)$ m

14. Forces proportional to AB, BC and 2CA act along the sides of triangle ABC in order. Their resultant represented in magnitude and direction as :
(A) CA (B) AC (C) BC (D) CB

15. The magnitude of resultant of two forces of magnitudes 3P and 2P is R. If the first force is doubled, the magnitude of the resultant is also doubled. The angle between the two forces is :
(A) 30^o (B) 60^o (C) 120^o (D) 150^o

16. Three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are acting at a point in the plane, the angle between $\vec{P}$ and $\vec{Q}$ & $\vec{Q}$ and $\vec{R}$ are 150^o and 120^o respectively. then for equilibrium forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are in the ratio :
(A) 1:2:3 (B) 1:2: $\sqrt{3}$ (C) 3:2:1 (D) $\sqrt{3}$: 2:1

17. The resultant of two vectors $\vec{u}$ and $\vec{v}$ is perpendicular to the vector $\vec{u}$ and its magnitude is equal to half of the magnitude of vector $\vec{v}$, The angle between $\vec{u}$ and $\vec{v}$ in degrees is
(A) 120 (B) 60 (C) 90 (D) 150

18. Choose the wrong statement
(A) Three vectors of different magnitudes may be combined to give zero resultant.
(B) Two vectors of different magnitudes can be combined to give a zero resultant.
(C) The product of a scalar and a vector is a vector quantity.
(D) All of the above statements are wrong.

19. A force of 6 N and another of 8 N can be applied to produce the effect of a single force equal to
(A) 1 N (B) 10 N (C) 16 N (D) 0 N

20. Out of the following pairs of forces, the resultant of which cannot be 4 newton
(A) 2 newton and 2 newton (B) 2 newton and 4 newton
(C) 2 newton and 6 newton (D) 2 newton and 8 newton

21. If $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}+2\hat{k}$. Find the projection vector of $\vec{b}$ on $\vec{a}$.
(A) $\frac{8}{9}\left(\hat{i}+2\hat{j}+2\hat{k}\right)$ (B) $\frac{8}{9}\left(2\hat{i}+\hat{j}+2\hat{k}\right)$ (C) $\frac{9}{8}\left(\hat{i}+2\hat{j}+\hat{k}\right)$ (D) $\frac{9}{8}\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

EXERCISE - 2

OBJECTIVE PROBLEMS WITH MULTIPLE CORRECT ANSWERS

1. Given two vectors $\vec{A}=3\hat{i}+4\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}$. θ is the angle between $\vec{A}$ and $\vec{B}$. Which of the following statements is/are correct?

(A) $|\vec{A}|\cos\theta\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ along $\vec{B}$.

(B) $|\vec{A}|\sin\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.

(C) $|\vec{A}|\cos\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ along $\vec{B}$.

(D) $|\vec{A}|\sin\theta\left(\frac{\hat{j}-\hat{i}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.

2. If $\vec{A}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}=\hat{i}+\hat{j}+\hat{k}$ are two vectors, then the unit vector

(A) perpendicular to $\vec{A}$ is $\left(\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)$ (B) parallel to $\vec{A}$ is $\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$

(C) perpendicular to $\vec{B}$ is $\left(\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)$ (D) parallel to $\vec{A}$ is $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

3. If $(\vec{v}_1+\vec{v}_2)$ is perpendicular to $(\vec{v}_1-\vec{v}_2)$, then

(A) $\vec{v}_1$ is perpendicular to $\vec{v}_2$ (B) $|\vec{v}_1|=|\vec{v}_2|$

(C) $\vec{v}_1$ is null vector (D) the angle between $\vec{v}_1$ and $\vec{v}_2$ can have any value

4. Two vectors $\vec{A}$ and $\vec{B}$ lie in one plane. Vector $\vec{C}$ lies in a different plane. Then, $\vec{A}+\vec{B}+\vec{C}$

(A) cannot be zero

(b) can be zero

(C) lies in the plane of $\vec{A}$ and $\vec{B}$

(D) lies in a plane different from that of any of the three vectors.

5. Which of the following expressions are meaningful?

(A) $\vec{u}.(\vec{v}\times\vec{w})$ (B) $(\vec{u}.\vec{v}).\vec{w}$ (C) $(\vec{u}.\vec{v})\vec{w}$ (D) $\vec{u}\times(\vec{v}.\vec{w})$

6. The magnitude of scalar product of two vectors is 8 and of vector product is $8\sqrt{3}$. The angle between them can be :

(A) 30^o (B) 60^o (C) 120^o (D) 150^o

7. A situation may be described by using different sets of coordinate axes having different orientations. Which of the following do not depend on the orientation of the axes?
(A) The value of a scalar (B) component of a vector
(C) a vector (D) the magnitude of a vector

8. The x-component of the resultant of several vectors
(A) is equal to the sum of the x-components of the vectors
(B) may be smaller than the sum of the magnitudes of the vector
(C) may be greater than the sum of the magnitudes of the vector
(D) may be equal to the sum of the magnitudes of the vector

9. The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ may be
(A) greater than AB (B) equal to AB
(C) less than AB (D) equal to zero

EXERCISE - 3

MATCH THE FOLLOWING

1. $\vec{A} = 4\hat{i} + 4\hat{j}$, $\vec{B} = 4\hat{i} - 4\hat{j}$. Then,

	Column A		Column B
i.	$\lvert\vec{A}+\vec{B}\rvert$	(a)	8
ii.	$\lvert\vec{A}-\vec{B}\rvert$	(b)	4
iii.	$\vec{A}.\vec{B}$	(c)	32
iv.	$\lvert\vec{A}\times\vec{B}\rvert$	(d)	0

2. $\lvert\vec{A}\rvert = 1$, $\lvert\vec{B}\rvert = 2$, Angle between $\vec{A}$ and $\vec{B}$ is 90^0.

	Column A		Column B
i.	$\vec{A}.\vec{B}$	(a)	2
ii.	$\lvert\vec{A}\times\vec{B}\rvert$	(b)	4
iii.	$\lvert\vec{A}+\vec{B}\rvert$	(c)	$\sqrt{5}$
iv.	$\lvert\vec{A}-\vec{B}\rvert$	(d)	0

EXERCISE - 4

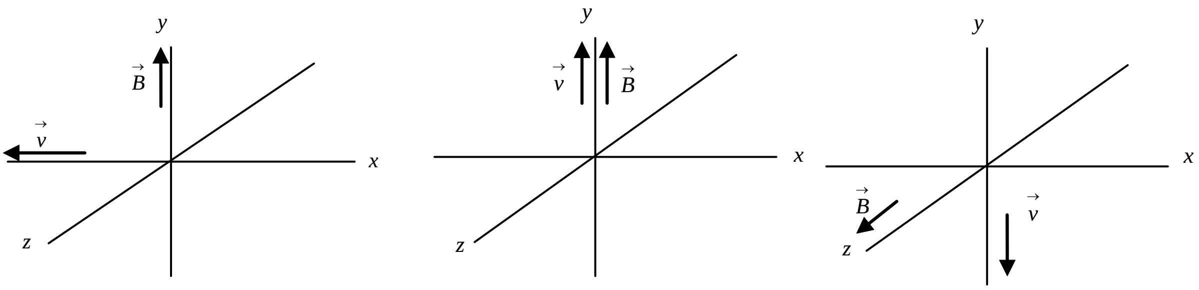
SUBJECTIVE PROBLEMS

1. Which of the following are correct (meaningful) vector expressions? What is wrong with any incorrect expression?

(A) $\vec{A}.\left(\vec{B}.\vec{C}\right)$; **(B)** $\vec{A}\times\left(\vec{B}.\vec{C}\right)$; **(C)** $\vec{A}.\left(\vec{B}\times\vec{C}\right)$; **(D)** $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$;

(E) $\vec{A}+\left(\vec{B}.\vec{C}\right)$; **(F)** $\vec{A}+\left(\vec{B}\times\vec{C}\right)$; **(G)** $5+\vec{A}$; **(H)** $5+\left(\vec{B}.\vec{C}\right)$

2. Suppose that a vector $\vec{F}$ is given by $\vec{F}=q(\vec{V}\times\vec{B})$, where q is a number and $\vec{V}$ and $\vec{B}$ are vectors. What are the directions of $\vec{F}$ in the below three situations, if q is (i) a positive quantity (ii) a negative quantity?



(A) (B) (C)

3. (A) In unit-vector notation, what is

$\vec{r}=\vec{a}-\vec{b}+\vec{c}$ if $\vec{a}=5\hat{i}+4\hat{j}-6\hat{k}$, $\vec{b}=-2\hat{i}+2\hat{j}+3\hat{k}$, and $\vec{c}=4\hat{i}+3\hat{j}+2\hat{k}$

(B) Calculate the angle between $\vec{r}$ and the positive z axis.

(C) What is the magnitude of component of $\vec{a}$ along the direction of $\vec{b}$?

4. Two vectors are given by $\vec{a}=3\hat{i}+5\hat{j}$ and $\vec{b}=2\hat{i}+4\hat{j}$. Find :

(A) $\vec{a}\times\vec{b}$ (B) $\vec{a}.\vec{b}$

(C) $\left(\vec{a}+\vec{b}\right).\vec{b}$ (D) the magnitude of component of $\vec{a}$ along the direction of $\vec{b}$.

5. Three vectors are given $\vec{a}=3\hat{i}+3\hat{j}-2\hat{k}$, $\vec{b}=-1\hat{i}-4\hat{j}+2\hat{k}$ and $\vec{c}=2\hat{i}+2\hat{j}+1\hat{k}$. Find

(a) $\vec{a}.\left(\vec{b}\times\vec{c}\right)$, (b) $\vec{a}.\left(\vec{b}+\vec{c}\right)$, and (c) $\vec{a}\times(\vec{b}+\vec{c})$.

6. A vector with a magnitude of 8m, is added to a vector $\vec{A}$, which lies along an x-axis. The sum of these two vectors is a third vector that lies along the y-axis and has a magnitude that is twice the magnitude of $\vec{A}$. What is the magnitude of $\vec{A}$?

7. A ship standing at point O sets out to sail to a point A 120 km due north of O. An unexpected storm blows the ship to a point B 100 km due east of O. East directon is along $\hat{i}$ and north direction is along $\hat{j}$. Find vector $\overrightarrow{BA}$

8. If $\vec{a}-\vec{b}=2\vec{c},\ \vec{a}+\vec{b}=4\vec{c},\ \vec{c}=3\hat{i}+4\hat{j}$, solve for $\vec{a}$ and $\vec{b}$?

9. Let $\vec{A}$ and $\vec{B}$ be the two vectors of magnitude 10 units each. If they are inclined to the x-axis at angles 30º and 60º respectively, find the resultant of $\vec{A}$ & $\vec{B}$ in the form $p\hat{i}+q\hat{j}$

10. A particle whose speed is 50 m/s moves along the line from $A(2, 1)$ to $B(9, 25)$. Find its velocity vector in the form of $p\hat{i}+q\hat{j}$.

11. Two vectors acting in the opposite directions have a resultant of 17 units. If they act at right angles to each other, then the resultant is 25 units. Calculate the magnitude of two vectors .

12. Answer the following :
(A) Should a quantity having a magnitude & direction be necessarily a vector ?
(B) Can two similar vectors of different magnitude yield a zero resultant ?Can three yield ?
(C) $\vec{R}=\vec{A}+\vec{B}$, Is it possible that $|\vec{R}|<|\vec{A}|$ & $|\vec{R}|<|\vec{B}|$ are both true?
(D) If $\vec{a}+\vec{b}=\vec{c}$ & $|\vec{a}|+|\vec{b}|=|\vec{c}|$, what further information you can have about these vectors ?
(E) If $\vec{a}$ & $\vec{b}$ are two non zero vectors such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$,then what is the angle between $\vec{a}$ & $\vec{b}$?
(F) Time has magnitude as well as direction. Is it a vector ?
(G) When will $\vec{a}\times\vec{b}=\vec{a}\cdot\vec{b}$?($\vec{a}$ & $\vec{b}$ are two non - zero vectors)
(H) Do unit vectors $\hat{i}$, $\hat{j}$ & $\hat{k}$ have units ?

13. Two vectors have magnitude 2m and 3m. The angle between them in degrees is 60. Find
(A) the scalar product of the two vectors.
(B) the magnitude of their vector product.

ANSWER KEY

JEE MAIN EXERCISE

1.	(D)	2.	(C)	3.	(D)	4.	(B)	5.	(C)
6.	(D)	7.	(B)	8.	(D)	9.	(D)	10.	(D)
11.	(D)	12.	(A)	13.	(D)	14.	(B)	15.	(B)
16.	(D)	17.	(D)	18.	(D)	19.	(B)	20.	(A)
21.	(B)	22.	(A)	23.	(A)	24.	(C)	25.	(C)

EXERCISE-1 (OBJECTIVE PROBLEMS WITH ONE CORRECT ANSWER)

1.	(A)	**2.**	(D)	**3.**	(D)	**4.**	(D)	**5.**	(B)
6.	(C)	**7.**	(A)	**8.**	(A)	**9.**	(A)	**10.**	(D)
11.	(B)	**12.**	(A)	**13.**	(D)	**14.**	(A)	**15.**	(C)
16.	(D)	**17.**	(D)	**18.**	(B)	**19.**	(B)	**20.**	(D)
21.	(A)								

EXERCISE-2 (OBJECTIVE PROBLEMS WITH MULTIPLE CORRECT ANSWERS)

1.	(AD)	**2.**	(ABC)	**3.**	(BD)	**4.**	(AD)	**5.**	(AC)
6.	(BC)	**7.**	(ACD)	**8.**	(ABD)	**9.**	(BCD)		

EXERCISE-3 (MATCH THE FOLLOWING)

1. (I) A (II) A (III) D (IV) C **2.** (I) D (II) A (III) C (IV) C

EXERCISE-4 (SUBJECTIVE PROBLEMS)

1. (A) WRONG (B) WRONG (C) RIGHT (D) RIGHT (E) WRONG
(F) RIGHT (G) WRONG (H) RIGHT

2. (I) (1) $-\hat{k}$ (2) ZERO (3) $-\hat{i}$ (II) (1) $\hat{k}$ (2) ZERO (3) $\hat{i}$

3. (A) $11\hat{i}+5\hat{j}-7\hat{k}$ (B) $\cos^{-1}\left(\frac{-7}{\sqrt{195}}\right)$ (C) $\frac{-20}{\sqrt{17}}$

4. (A) $2\hat{k}$ (B) 26 (C) 46 (D) $\frac{13}{\sqrt{5}}$

5. (A) –21 (B) –9 (C) $5\hat{i}-11\hat{j}-9\hat{k}$ **6.** $\frac{8}{\sqrt{5}}$ M **7.** $-100\hat{i}+120\hat{j}$

8. $\vec{a} = 9\hat{i}+12\hat{j}$ $\vec{b} = 3\hat{i}+4\hat{j}$ **9.** $5\left(\sqrt{3}+1\right)\left(\hat{i}+\hat{j}\right)$

10. $14\hat{i}+48\hat{j}$ **11.** 24, 7

12 (A) NO (B) NO, YES (C) YES (D) VECTORS ARE COLLINEAR
(E) 90^{o} (F) NO (G) NEVER (H) NO

13. (A) 3 (B) $3\sqrt{3}$

PREFACE

Calculus is a mathematical tool which helps us understand and calculate instantaneous values of a physical quantity. Once a teacher was teaching divisions to elementary school students without the help of a board & chalk, he asked the students to distribute ten fruits equally among a group of ten students. Each student was left with one fruit. After making the students practice this with several numbers, he generalised this concept & said if 'the number of students equals to the number of fruits, every student will get one fruit'. A boy got up and asked the teacher, is this true for all numbers? For which the teacher said 'yes'. He then asked the teacher if there are no students and no fruits even then will each student get one fruit?. This boys name is 'Srinivas Ramanujam' who showed to the world that (0/0) is a singularity and cannot be determined.

$\text{Velocity} = \frac{\Delta S}{\Delta t}$. To calculate instantaneous velocity we have to substitute $\Delta t = 0$ and $\Delta S = 0$. It is here this mathematical tool 'calculus' comes to our rescue. In this booklet elementary mathematical skills required to calculate instantaneous changes in a physical quantity like the one described above has been dealt with. The second section of this booklet deals with 'Units & Dimension' and 'Errors & Measurement'

The end–of–chapter problems are categorized into four section, namely Exercise – I which emphasizes on JEE (main) pattern, Exercise – II, Exercise – III and Exercise – IV to help the student assess his understanding of the concept and further improvise on his problem solving skills. Solutions to all the questions in the booklet are available and will be provided to the students (at the discretion of the professor). Every possible attempt has been made to make the booklet flawless. Any suggestions for the improvement of the booklet would be gratefully accepted and acknowledged.

(Dept. of Physics)

IIT –ian's PACE

CONTENTS

BASIC MATHEMATICS: TENTATIVE LECTURE FLOW
(Board Syllabus & Booklet Discussion Included)

Lecture 1	Necessity of calculus, definition of instantaneous velocity, general equation of a st.line, parabola, slope of a curve at an instant concept of limits $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$
Lecture 2	Problem based on $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ & Problem based on product rule $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ & Chain rule
Lecture 3	Maxima & minima. Application of derivatives
Lecture 4	Integration and its application.

DIFFERENTIATION

The purpose of differential calculus is to study the nature (i.e., increase or decrease) and the amount of variation in a quantity when another quantity (on which first quantity depends) varies independently.

Quantity: Anything which can be measured is called a quantity.

Constant: A quantity, whose value remains unchanged during mathematical operations, is called a constant quantity. The integers, numbers like π, e, etc are all constants.

Variable: A quantity which can have any numerical value between certain specified limit is called as variable.

Function: A quantity y is called a function of a variable x, if corresponding to any given value of x, there exists a single definite value of y. The phrase 'y is function of x' is represented as $y = f(x)$

For example, consider that y is a function of the variable x which is given by

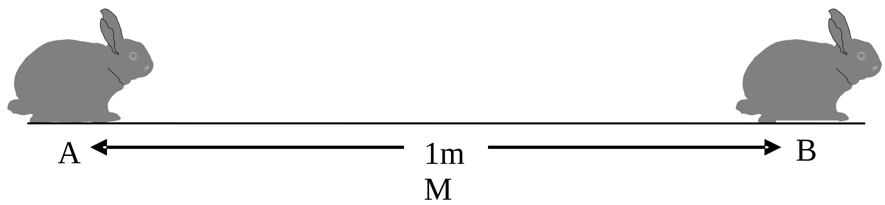
$$y = 3x^2 + 7x + 2$$

If x = 1, then $\quad y = 3(1)^2 + 7(1) + 2 = 12$

and when $x = 2, y = 3(2)^2 + 7(2) + 2 = 28$

Therefore, when the value of variable x is changed, the value of the function y also changes. But corresponding to each value of x, we get a single definite value of y. Hence, $y = 3x^2 + 7x + 2$ represents a function of x.

➢ **Meaning of Limit**



Rabbit A wants to reach rabbit B, which sits stationary. A jumps half the distance remaining between them every second. How soon does the rabbit A reach its goal ?

Never! (Assuming both rabbits to be point objects)

- The gap remaining between them becomes infinitesimal (i.e. very very small) after a long time.
- It is not a number, that can be expressed. It is smaller than the smallest positive number, that you can think of. The gap $\Delta x \to 0$ or it is dx. (dx denotes a very very small change in x.) and is represented as $\lim\limits_{\Delta x \to 0}$ (read as limit of delta x tends to zero)
- Rabbit A tends to rabbit B.
- Rabbit A's limit is rabbit B.

INCHAPTER EXERCISE

1. When x tends to y, is x equal to y ? What is (x-y) equal to ?

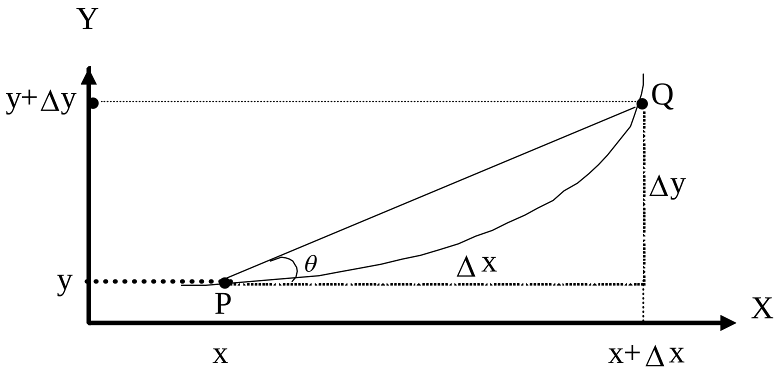
2. Which is smaller, infinitesimal or $10^{-10000000000000000000000000000000000000000000000000000000000000}$?

3. What is 100000000000 x (infinitesimal)?

4. What is (1/infinity) ?

Answer Key : 1) No , dx 2) infinitesimal 3) infinitesimal 4) 0

Slope of secant



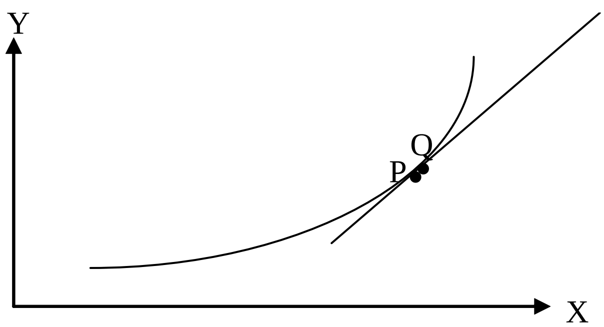
Consider a curve where the variation of a function y = f(x) is plotted with respect to variable x. Let P and Q be two points on the curve. The segment PQ is called as secant.

The slope of the secant: $\text{slope} = \tan\theta = \dfrac{\Delta y}{\Delta x}$

(Slope is defined as tan of angle between line and positive x -axis taken counter - clockwise)

Geometric meaning of derivative

- Let P go closer to Q. When the gap between P and Q becomes infinitesimal (very very small), the secant can be approximated as tangent



$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Hence, $\dfrac{dy}{dx}$ (or $f'(x)$) at any point represents the rate of change of y (or $f(x)$) with respect to x at that point and is also known as derivative of y w.r.t. x.

Physical meaning of $\frac{dy}{dx}$

1. The ratio of change in the function y to change in variable x is called the average rate of change of y w.r.t.x. For example, if a body covers a distance Δs in time Δt, then average velocity of the body, $v_{av} = \frac{\Delta s}{\Delta t}$ Also, if the velocity of a body changes by an amount Δv in small time Δt, then average acceleration of the body, $a_{av} = \frac{\Delta v}{\Delta t}$

2. The differentiation of a function w.r.t. a variable implies the instantaneous rate of change of the function w.r.t. that variable.

Thus, instantaneous velocity of the body, $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

and instantaneous acceleration of the body, $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

***Illustration* 1:** Find $\frac{dy}{dx}$ where $y = x^2$

Solution: $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$ (By definition)

$= \lim_{\Delta x \to 0} \frac{x^2 + 2\Delta x \cdot x + (\Delta x)^2 - x^2}{\Delta x}$; Here $(\Delta x)^2$ can be neglected as Δx itself is very small

$= \lim_{\Delta x \to 0} \frac{2\Delta x \cdot x}{\Delta x} = 2x$ so, $\frac{d(x^2)}{dx} = 2x$

$\frac{dy}{dx}$ of functions and their properties

We have found the derivative of x^2 with respect to x. Like wise we can also find derivatives of other functions.Some standard derivatives are as given in the table

y	$\frac{dy}{dx}$
x^n	nx^{n-1}
sinx	cosx
cosx	-sinx
tanx	$\sec^2 x$
cotx	$-\text{cosec}^2 x$
secx	secx.tanx
cosecx	-cosecx.cotx
e^x	e^x
a^x	$a^x . \ln a$
$\ln x$	$\frac{1}{x}$

Note : In the above table , n and a are constant.

Mathematical operations for derivatives :

(I) $\frac{dK}{dx} = 0$,

(II) $\frac{d(Ku)}{dx} = K\frac{du}{dx}$

(III) $\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

(IV) $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

(V) $\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(Here, K is a constant and u and v are functions of x.)

***Illustration 2* :** If $y = x^5$, then find $\frac{dy}{dx}$.

Solution: Given $y = x^5$

Differentiating both sides w.r.t x, using $\frac{dx^n}{dx} = nx^{n-1}$; $\frac{dy}{dx} = \frac{d}{dx}[x^5] = 5x^{5-1} = 5x^4$

***Illustration 3*:** If $y = x^2 + 5x^{3/2} + \frac{2}{x}$, then find $\frac{dy}{dx}$.

Solution: Differentiating both sides w.r.t. x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^2 + 5x^{3/2} + \frac{2}{x}\right] = \frac{d}{dx}[x^2] + 5\frac{d[x^{3/2}]}{dx} + 2\frac{d}{dx}\left[\frac{1}{x}\right]$$

$$= 2x + 5\frac{3}{2}x^{1/2} + 2(-1)x^{-2}$$

$$= 2x + \frac{15}{2}x^{1/2} - \frac{2}{x^2}$$

***Illustration 4*:** If $y = e^x \ln x$, then find $\frac{dy}{dx}$.

Solution: Using product rule,

Here, $u = e^x, v = \ln x$. Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[e^x \ln x] = e^x \frac{d}{dx}[\ln x] + \ln x \frac{d}{dx}[e^x]$$

$$= e^x \frac{1}{x} + \ln x e^x \qquad = \frac{e^x}{x} + e^x \ln x$$

Illustration 5 **:** If $y = \dfrac{(x^2+2x)}{(3x-4)}$, then find $\dfrac{dy}{dx}$.

Solution: $u = x^2 + 2x, v = 3x - 4.$

Using quotient rule, $\dfrac{dy}{dx} = \dfrac{(3x-4)\dfrac{d(x^2+2x)}{dx} - (x^2+2x)\dfrac{d(3x-4)}{dx}}{(3x-4)^2}$

$$= \frac{(3x-4)(2x+2)-(x^2+2x)3}{(3x-4)^2}$$

$$= \frac{3x^2-8x-8}{(3x-4)^2}$$

INCHAPTER EXERCISE

Differentiate the following functions w.r.t. x

1. $\left(x^2+2\sqrt{x}\right)x$
2. $x^2\cos x$
3. $(4x^2-7x+5).\sec x$
4. $x^4(5\sin x - 3\cos x)$
5. $\dfrac{\sqrt{x}+1}{\sqrt{x}-1}$
6. $\dfrac{x\ln x}{e^x}$

Answer Key

1. $3\left(x^2+\sqrt{x}\right)$
2. $2x\cos x - x^2\sin x$
3. $(8x-7)\sec x + (4x^2-7x+5)\sec x.\tan x$
4. $x^4(5\cos x + 3\sin x) + 4x^3(5\sin x - 3\cos x)$
5. $\dfrac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$
6. $\dfrac{1+\ln x - x\ln x}{e^x}$

Chain Rule

Suppose we have a function given by $y = g(h(x))$. Then its derivative w.r.t. x is given by

$$\frac{dy}{dx} = \frac{d(g(h(x)))}{d(h(x))}\cdot\frac{d(h(x))}{dx}$$

Illustration 6**:** Find the derivative of $y = \ln(\sin^2 x)$

Sol. Here $f(x) = \ln(x)$ $g(x) = \sin^2 x$ $h(x) = \sin x$

$$\frac{dy}{dx}\cdot = \frac{d\log(\sin^2 x)}{d(\sin^2 x)}\cdot\frac{d(\sin x)^2}{d(\sin x)}\cdot\frac{d\sin x}{dx} = \frac{1}{\sin^2 x}\cdot 2\sin x\cdot\cos x = 2\cot x.$$

Illustration 7 : Find the derivative of $y = \sin^2(\ln x)$.

Solution $\dfrac{dy}{dx} = \dfrac{d\left[\sin^2(\ln x)\right]}{d\left[\sin(\ln x)\right]} \times \dfrac{d\left[\sin(\ln x)\right]}{d(\ln x)} \times \dfrac{d(\ln x)}{dx}$

$$2\sin(\ln x).\cos(\ln x).\frac{1}{x}$$

Illustration 8 : The rate of change of radius of a sphere with respect to time is given as 4cm/s. Find the rate of change of volume when the radius is 50cm.

Solution Given rate of change of radius w.r.t. time, $\dfrac{dr}{dt} = 4\text{cm/s}$, $V = \dfrac{4}{3}\pi r^3$ and we have to find $\dfrac{dV}{dt}$.

From chain rule, $\dfrac{dV}{dt} = \dfrac{dV}{dr}.\dfrac{dr}{dt} = \dfrac{d\left(\frac{4}{3}\pi r^3\right)}{dr}.\dfrac{dr}{dt} = 4\pi r^2.\dfrac{dr}{dt}$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2.\frac{dr}{dt} = 4\pi.50^2.4 = 40000\pi \text{ cm}^3/\text{s}$$

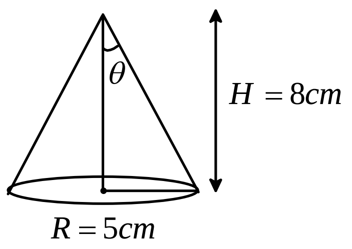
Illustration 9 : The radius of a cone is increasing at a rate of 2 cm/s. If the apex angle does not change, find the rate of change in height when R = 5 cm, H = 8 cm

Solution If the apex angle does not change, for any R and H,

we have $R = H \tan\theta$

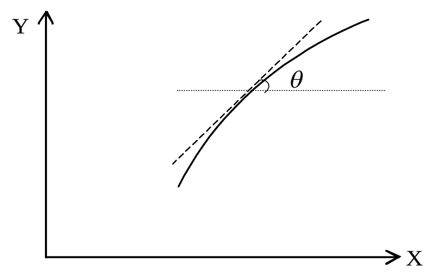
$$\Rightarrow \frac{dR}{dt} = \frac{dH}{dt}\cdot\tan\theta$$

$$\Rightarrow 2 = \frac{dH}{dt}\cdot\frac{5}{8} \qquad \Rightarrow \frac{dH}{dt} = \frac{16}{5}\text{cm/s}$$



Rate of change of height is $\dfrac{16}{5}$ cm/s.

- **Increasing and decreasing functions**

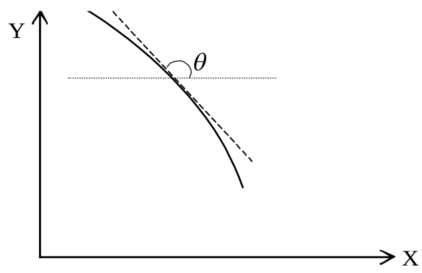


Increasing function

$0 < \theta < \dfrac{\pi}{2} \Rightarrow \tan\theta > 0 \Rightarrow \dfrac{dy}{dx} > 0$

Slope of tangent > 0

Rate of change of y w.r.t. x is positive, therefore the curve is increasing



Decreasing function

$\dfrac{\pi}{2} < \theta < \pi$ *hence* $\tan\theta < 0 \Rightarrow \dfrac{dy}{dx} < 0$

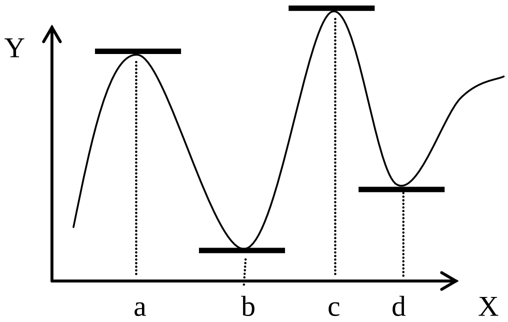
Slope of tangent < 0

Rate of change of y w.r.t. x is negative, therefore the curve is decreasing

Note: A function may not be continuously increasing or decreasing during the entire range (e.g. sin x, cos x).

Then we say function is increasing in the range where $\frac{dy}{dx} > 0$ and decreasing where $\frac{dy}{dx} < 0$.

➢ **Local maxima and local minima**

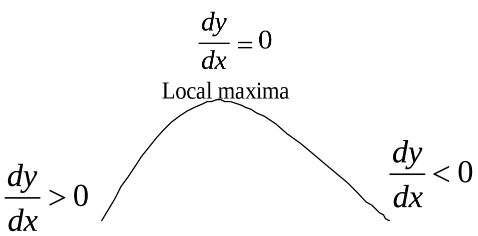


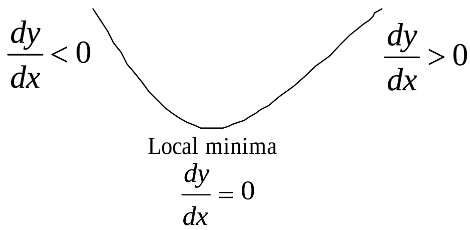
- Function is increasing for x<a,b<x<c,x > d
- Function is decreasing for a<x<b, c<x<d
- Peaks(local maxima) at x = a , c
- Valleys(local mi nima) at x = b , d

For local maxima or minima, slope of the curve where it lies must be zero.

So, $\frac{dy}{dx} = 0$, at the points of local maxima or local minima

➢ **Criterion for local maxima and local minima**





Condition for local maxima
- Slope changes from +ve to -ve
- Rate of change of slope w.r.t. x is -ve
- dy/dx is decreasing with x
- $\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dx} \langle 0$

  $\Rightarrow \frac{d^2y}{dx^2} < 0$

Condition for local minima
- Slope changes from -ve to +ve
- Rate of change of slope w.r.t. x is +ve
- dy/dx is increasing with x
- $\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dx} \rangle 0$

  $\Rightarrow \frac{d^2y}{dx^2} > 0$

Note 1: When we talk of local maxima / minima, we are talking of x; while when we say local maximum value or local minimum value, we are talking of value of y corresponding to that particular local maxima or minima

Note 2: Local maxima/minima does not mean that the function has the highest / lowest value at that point. It only means that the function was increasing before and decreasing after that point in case of local maxima and vice-versa in case of minima.

INCHAPTER EXERCISE

1. Find the points of local maxima or local minima for the following:

(i) $y = \frac{x^3}{3} - x$ (ii) $y = \left(4x^3 - 21x^2 + 18x + 5\right)$

(iii) $y = \sin x$, (for values of $0 \le x \le 2\pi$)

2. Find the local maximum and local minimum value for the following:

(i) $y = x^3 - 3x + 10$ (ii) $y = \frac{x^2}{2} + \frac{1}{x}$

Answer key : 1. (i) $x = \pm 1$ (ii) $x = \frac{1}{2}, 3$ (iii) $x = \frac{\pi}{2}, \frac{3\pi}{2}$

2. (i) Local minima at x = 1, value = 8
Local maxima at x = -1, value = 12

(ii) Local minima at x = 1 , value = 3/2

INTEGRATION

In integral calculus, the differential coefficient of a function is given. We are required to find the function. Thus, integration is the reverse of differentiation.

'$\int$ 'sign is used for integration. If I is integration of f(x) with respect to x then $I = \int f(x)\,dx$ and it is read as integration of f(x) w.r.t. x is I

For example, let us proceed to obtain intergral of x^n w.r.t. x. We already know that $\frac{d}{dx}(x^{n+1}) = (n+1)x^n$

Since the process of integration is the reverse process of differentiation,

$$\int (n+1)x^n dx = x^{n+1} \text{ or } \int x^n dx = \frac{x^{n+1}}{n+1}$$

The above formula holds for all values of n, except n = -1.

It is because, for n = -1, $\int x^n dx = \int x^{-1} dx = \int \frac{1}{x} dx$ $\because \frac{d}{dx}(\log_e x) = \frac{1}{x}$ $\therefore \int \frac{1}{x} dx = \log_e x$

Similarly, the formula for integration of some other functions can be obtained if we know the differential coefficients of various functions

Basic Integration Formulas

1. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x dx = e^x + C$

5. $\int \sin x dx = -\cos x + C$

6. $\int \cos x dx = \sin x + C$

7. $\int \tan x dx = \ln|\sec x| + C$

8. $\int \cot x dx = \ln|\sin x| + C$

9. $\int \sec x dx = \ln|\sec x + \tan x| + C$

10. $\int \operatorname{cosec} x dx = -\ln|\operatorname{cosec} x + \cot x| + C$

11. $\int \sec^2 x dx = \tan x + C$

12. $\int \operatorname{cosec}^2 x dx = -\cot x + C$

13. $\int \sec x \tan x dx = \sec x + C$

14. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$

***Illustration 10*:** Find the following integrals:

1. $\int 6.e^x dx$ **2.** $\int \frac{5}{x} dx$

3. $\int 3\cos x dx$ **4.** $\int [x^3 + 2x^2 + 3x - 4] dx$

Solutions:

1. $\int 6.e^x dx = 6\int e^x dx = 6e^x + c$

2. $\int \frac{5}{x} dx = 5.\int \frac{1}{x} dx = 5.\ln|x| + c$

3. $\int 3\cos x dx = 3\int \cos x dx = 3\sin x + c$

4. $\int [x^3 + 2x^2 + 3x - 4] dx = \int x^3 dx + \int 2x^2 dx + \int 3x dx - \int 4 dx$

$= \int x^3 dx + 2\int x^2 dx + 3\int x dx - 4\int dx$ $= \frac{x^4}{4} + c_1 + \frac{2x^3}{3} + c_2 + 3\frac{x^2}{2} + c_3 - 4x + c_4$

$= \frac{x^4}{4} + \frac{2x^3}{3} + \frac{3x^2}{2} - 4x + c$

INCHAPTER EXERCISE

Find the following integrals:

1. $\int \left[x^5 + \frac{2}{x^2} - \frac{1}{x} - \frac{4}{\sqrt{x}} + 10 \right] dx$

2. $\int \left[7e^x + 4\sin x - \frac{9}{x^3} + e \right] dx$

Answer Key : (i) $\frac{x^6}{6} - \frac{2}{x} - \ln x - 8\sqrt{x} + 10x + c$

(ii) $7e^x - 4\cos x + \frac{9}{2x^2} + ex + c$

Integration by Substitution:

Suppose we represent an integral by $\int f(u)du = F(u) + c$ (i.e., F(u) is the integral of f(u) w.r.t. u).

Now suppose u is a function of x given by $u = g(x)$, then $du = g'(x)dx$ (where $g'(x)$ is $\frac{d(g(x))}{dx}$)

and the above integral can be written as $\int f(g(x)).g'(x)dx = F(g(x)) + c$.

Thus if we have to find the integral given in the form of a function along with its derivative, we can use the above formula to find the integral.

An important result: If $\int f(x)dx = F(x) + c$, then $\int f(ax+b)dx = \frac{F(ax+b)}{a} + c$

Illustration 11: Find $\int x^2\sqrt{x^3+1}dx$

Solution:

$$\int x^2\sqrt{x^3+1}dx$$

$$u = x^3+1$$

$$du = 3x^2dx$$

$$\int x^2\sqrt{x^3+1}dx = \int \frac{1}{3}\sqrt{x^3+1}.3x^2dx$$

$$= \frac{1}{3}\int \sqrt{x^3+1}.3x^2dx$$

$$= \frac{1}{3}\int \sqrt{u}\,du$$

$$= \frac{1}{3}\int u^{\frac{1}{2}}du$$

$$= \frac{1}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{9}u^{\frac{3}{2}} + C$$

$$= \frac{2}{9}(x^3+1)^{\frac{3}{2}} + C$$

$$= \frac{2}{9}\sqrt{(x^3+1)^3} + C$$

INCHAPTER EXERCISE

Find the following integrals:

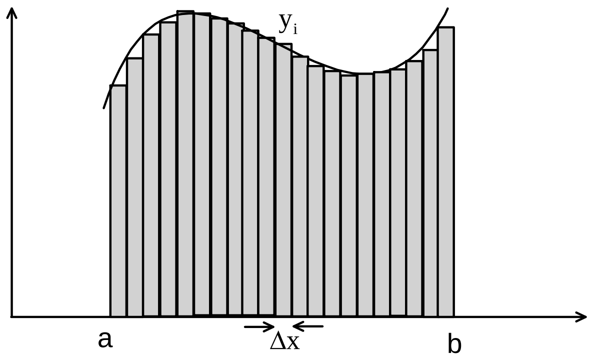
1. $\int x\sin(1+x^2)\ dx$ **2.** $\int \frac{2x}{1+x^2}\ dx$

Answer Key :

1. $-\frac{\cos(1+x^2)}{2} + C$ **2.** $\ln|1+x^2| + C$

Definite Integral

Consider the curve as shown. The area under the curve (the area bounded by the curve and the x-axis) can be found by dividing this area into infinitesimal areas and adding them up.



Consider this area to be divided into n parts, where each part can be assumed as a rectangle if n is very large. The length of each such part at $x = x_i$ will be equal to $y_i = f(x_i)$ while the breadth will be equal to Δx where $\Delta x = \frac{b-a}{n}$

Area of each rectangle = $A_i = y_i.\Delta x$

The total area will be the sum of all these areas and will be given by

$$A = \sum_{i=1}^{i=n} y_i \Delta x$$

If $\Delta x \to 0$, the same area is represented by $\int_{x=a}^{x=b} y\,dx$ or $\int_{x=a}^{x=b} f(x)\,dx$

This integral is known as definite integral of the curve y = f(x) between x = a to x = b, where a and b are known as the lower and upper limits of the integral respectively.

Fundamental theorem of integral calculus:

If $\int f(x)dx = F(x) + C$, then $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

Illustration 12 **:** Integrate $\int_0^{\pi/2} (\sin x + \cos x)dx$

Solution: $\int_0^{\pi/2} (\sin x + \cos x)dx \quad = \int_0^{\pi/2} \sin x dx + \int_0^{\pi/2} \cos x dx$

$= [-\cos x]_0^{\pi/2} + [\sin x]_0^{\pi/2} \quad = -[0-1] + [1-0]$

$= 1 + 1 = 2$

Illustration 13 **:** Integrate $\int_0^1 (x^{3/2}+2e^x)dx$

Solution: $\int_0^1 (x^{3/2}+2e^x)dx = \int_0^1 x^{3/2}dx + 2\int_0^1 e^x dx = \left[\dfrac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_0^1 + \left[2e^x\right]_0^1$

$= \left[\dfrac{x^{5/2}}{5/2}\right]_0^1 + \left[2e^x\right]_0^1 \quad = \left[\dfrac{1}{5/2}-0\right] + \left[2e^1 - 2e^o\right] \quad = \dfrac{2}{5} + 2e - 2 = \dfrac{-8}{5} + 2e$

INCHAPTER EXERCISE

Find the following integrals:

1. $\int_1^5 (3+2t)\,dt$ **2.** $\int_{\pi/4}^{\pi/2} (2\sin x - \cos x)\,dx$

Answer key : **1.** 36 **2.** $\dfrac{3}{\sqrt{2}}-1$

APPLICATION OF CALCULUS IN KINEMATICS

The problems in kinematics can be solved using the differential and integral calculus, in addition to the already known equations which are given as under

$v = \dfrac{dx}{dt}$ **;**

$a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \times \dfrac{dx}{dt} = v\dfrac{dv}{dx}$ **;** $\quad a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d^2x}{dt^2}$

(Where ‘x’ is displacement, ‘v’ is velocity and ‘a’ is acceleration at time t)

Similarly, for circular motion

$\omega = \dfrac{d\theta}{dt}$

$\alpha = \dfrac{d\omega}{dt} = \dfrac{d\omega}{d\theta} \times \dfrac{d\theta}{dt} = \dfrac{\omega d\omega}{d\theta}$ or $\alpha = \dfrac{d\omega}{dt} = \dfrac{d}{dt}\left(\dfrac{d\theta}{dt}\right) = \dfrac{d^2\theta}{dt^2}$

(Where ‘θ’ is angular displacement, ‘ω’ is angular velocity and ‘α’ is angular acceleration at time t)
Also by the definition of integral,

$\Delta x = \int v dt$ $\qquad \Delta v = \int a dt$

$\Delta\theta = \int \omega dt$ $\qquad \Delta\omega = \int \alpha dt$

Note: $\frac{dx}{dt}$ is called as instantaneous velocity, it is the velocity in small time dt. Average velocity over a period of time Δt can be given as $V_{avg} = \frac{\Delta x}{\Delta t}$.

Illustration 14 **:** The speed of a particle is given by $V = (3t^2 + t + 2) m/s$. Find the (a) distance covered by it in first two seconds, (b) acceleration at time t = 1s.

Solution. (a) for calculating distance,

$V = 3t^2 + t + 2$

Also we have $V = \frac{ds}{dt}$ $\Rightarrow$ $\frac{ds}{dt} = 3t^2 + t + 2$ $\Rightarrow$ $ds = (3t^2 + t + 2)dt$

Integrating both the sides, we have $\int_{s=0}^{s=s} ds = \int_{t=0}^{t=2} (3t^2 + t + 2)dt$

Limits are put s = 0 to s = s and t = 0 to t = 2 $\therefore$ $[s]_0^s = \left[t^3 + \frac{t^2}{2} + 2t\right]_0^2$

$[s - 0] = [8 + 2 + 4] - [0 + 0 + 0]$

s = 14 m $\therefore$ Distance covered in first two seconds is 14 m.

(b) For calculating acceleration, $V = 3t^2 + t + 2$

Also we have $a = \frac{dv}{dt}$ $\therefore$ $a = \frac{d(3t^2 + t + 2)}{dt} = 6t + 1$

at t = 1s, acceleration = 6(1) + 1 = 7 m/s^2

INCHAPTER EXERCISE

1. If $s = (2t + 4t^2)m$, find the values of 'v' and 'a' at t = 0s, 2s and 10s.

2. If $v = 3t^2$ m/s, find the values of 's' and 'a' at t = 0s, 2s, & 10s.(assume all quantities to be zero at the start)

3. If v = u + at, derive $s = ut + 0.5at^2$. (where a is a constant)

4. Use $\frac{dv}{ds}\frac{ds}{dt} = a$, to prove $v^2 = u^2 + 2as$. (where a is a constant)

Answer Key :

1. v = 2m/s, 18m/s, 82m/s
a = 8m/s^2, 8m/s^2, 8m/s^2

2. s = 0m, 8m, 1000m
a = 0m/s^2, 12m/s^2, 60m/s^2

EXERCISE # I

SECTION - I

In Problems 1 to 9 differentiate the given functions with respect to x .
(a, b, c are to be treated as constants)

1. $y = (x^2 - 3x + 3)(x^2 + 2x - 1);$

2. $y = \dfrac{x+1}{x-1}$

3. $y = \dfrac{x}{x^2+1}$

4. $y = \dfrac{ax+b}{cx+d}$

5. $z = \dfrac{x^2+1}{3(x^2-1)} + (x^2-1)(1-x)$

6. $y = \dfrac{1-x^3}{1+x^3}.$

7. $y = \dfrac{2}{x^3-1}.$

8. $y = \dfrac{x^2 - x + 1}{a^3 - 3}$

9. $y = \dfrac{1-x^3}{\sqrt{\pi}}.$

SECTION - II

Trigonometric functions :

In problems 10 to 23 differentiate the given functions with respect to x .

10. $y = \sin x + \cos x$

11. $y = \dfrac{x}{1-\cos x}.$

12. $y = \dfrac{\tan x}{x}.$

13. $y = x \sin x + \cos x$

14. $y = \dfrac{\sin x}{x} + \dfrac{x}{\sin x}$

15. $y = \dfrac{\sin x}{1+\cos x}$

16. $y = \dfrac{x}{\sin x + \cos x}.$

17. $y = \dfrac{x \sin x}{1+\tan x}.$

18. $y = \cos^2 x.$

19. $y = \dfrac{1}{4}\tan^4 x.$

20. $y = \cos x - \dfrac{1}{3}\cos^3 x.$

21. $y = 3\sin^2 x - \sin^3 x.$

22. $y = \dfrac{1}{3}\tan^3 x - \tan x + x.$

23. $y = x\sec^2 x - \tan x.$

SECTION - III

Logarithmic functions : (In Problems 24 to 27 differentiate the given functions.)

24. $y = x \ln x$

25. $y = \ln^2 x.$

26. $y = \ln x^2$

27. $y = \sqrt{\ln x}.$

SECTION - IV

In Problems 28 to 32,find the integrals by using the basic table of integrals and simple rules of integration.

28 $\int \sqrt{x}\,dx$

29. $\int \sqrt[m]{x^n}\,dx.$

30. $\int \dfrac{dx}{x^2}$

31. $\int \dfrac{dx}{2\sqrt{x}}.$

32. $\int (1-2u)\,du.$

EXERCISE # II

For problems 1 to 5, differentiate the following w.r.t. x

1. $\dfrac{1}{\sqrt{x}}$

(A) $x^{-\frac{3}{2}}$ (B) $-\dfrac{1}{2}x^{-3/2}$ (C) $\dfrac{1}{\sqrt{x}}$ (D) $\dfrac{1}{2}x^{\frac{1}{2}}$

2. $\dfrac{1}{(ax+b)^2}$

(A) $-2a\,(ax+b)^{-3}$ (B) $-2a(ax+b)^{-1}$ (C) $a(ax+b)^{-1}$ (D) $(ax+b)^{-1}$

3. $x^3+\dfrac{1}{x^3}+8$

(A) $3x^{-2}+8$ (B) $3x^2+3x^3$ (C) $3x^2-3x^{-4}$ (D) $3x^{-2}+3x^2$

4. $\sin x^3$

(A) $3.\cos x^3$ (B) $\cos^3 x$ (C) $\cos^3 x.3x^2$ (D) $\cos x^3.\ 3x^2$

5. $\sqrt{4x^3-5}$

(A) $(4x^2-5)^{\frac{-1}{2}}$ (B) $\sqrt{4x^3-5}$ (C) $6x^2(4x-5)^{\frac{1}{2}}$ (D) $6x^2(4x^3-5)^{-1/2}$

6. $\dfrac{d}{dx}\sin(\ell n\, x):$

(A) $\cos(\,l\,n\,x\,)$ (B) $l\,n(\cos x\,)$ (C) $x\cos(\,l\,n\,x)$ (D) $\dfrac{\cos(\ln x)}{x}$

7. $\dfrac{d}{dx}\sqrt{2x^2+1}$

(A) $2x\,(\,2x^2+1)^{1/2}$ (B) $2x\,(\,2x^2+1)^{-1/2}$ (C) $(2x^2+1\,)^{1/2}$ (D) $(2x^2+1)^{-1/2}$

8. $\dfrac{d}{dx}e^{\sqrt{2x}}$

(A) $\dfrac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (B) $\sqrt{2x}\ e^{\sqrt{2x}}$ (C) $e^{\sqrt{2x}}$ (D) $2^{(2x)^{-1/2}}$

9. $\dfrac{d}{dx}(x^4-2\sin x+3\cos x)$

(A) $4x^3-2\cos x+3\sin x$ (B) $3x^2+2\cos x+3\sin x$

(C) $4x^3+2\cos x-3\sin x$ (D) $4x^3-2\cos x-3\sin x$

10. $\dfrac{d}{dx}\left(x^2\sin x\,\ell n\,x\right)$

(A) $2x\sin x\ \ell nx+x^2\cos x\ \ell nx+x\sin x$ (B) $x^2\sin x\,.\ \ell n\ x+2x\cos x\,\ell n\ x+x\sin x$

(C) $2x.\sin x\ \ell n\ x+x^2\cos x\,\ell n\ x+\sin x$ (D) None of these

11. $\dfrac{d\left(\dfrac{x^2+1}{x+1}\right)}{dx}$

(A) $\dfrac{x^2+2x-1}{(x+1)^2}$ (B) $\dfrac{x^2-2x+1}{(x+1)^2}$ (C) $\dfrac{x^2+2x-1}{x+1}$ (D) $\dfrac{x^2+2x+1}{(x+1)^2}$

12. If $V=\dfrac{4}{3}\pi r^3$, find $\dfrac{dV}{dr}$

(A) $\dfrac{4}{3}\pi r^2$ (B) $4\pi r^3$ (C) $4\pi r$ (D) $4\pi r^2$

13. $xy=c^2$, then $\dfrac{dy}{dx}$

(A) $\dfrac{x}{y}$ (B) $\dfrac{y}{x}$ (C) $-\dfrac{x}{y}$ (D) $-\dfrac{y}{x}$

14. $y=\dfrac{1}{\sqrt{2x+1}}$, find $\dfrac{dy}{dx}$

(A) $\dfrac{1}{(2x+1)^{3/2}}$ (B) $\dfrac{-1}{(2x+1)^{3/2}}$ (C) $\dfrac{1}{(2x+1)}$ (D $\dfrac{-1}{(2x+1)}$

15. $x=at^2$; $y=2at$, then $\dfrac{dy}{dx}=$

(A) t (B) $\dfrac{1}{t}$ (C) 1 (D) None of these

16. $\int \sqrt[5]{x}\,dx$

(A) $\dfrac{5}{6}x^{6/5}+C$ (B) $\dfrac{6}{5}x^2+C$ (C) $\dfrac{6}{3}x^3+C$ (D) None of these

17. $\int \dfrac{1}{(ax+b)^2}dx$

(A) $(ax+b)^{-1}$ (B) $(ax+b)^3$ (C) $-\dfrac{1}{a}\left(\dfrac{1}{ax+b}\right)+C$ (D) None of these

18. $\int$ sinx.cosx dx

(A) $-\dfrac{\cos 2x}{4}+C$ (B) $\dfrac{\sin 2x}{4}+C$ (C) $\cos 2x+C$ (D) None of these

19. $\int \dfrac{x}{x^2+a^2}dx$

(A) $(x^2+a^2)^{\frac{1}{2}}+C$ (B) $\dfrac{1}{2}\log_e(x^2+a^2)+C$ (C) $\log_e(x^2+a^2)$ (D) None of these

20. $\int_{-\pi/2}^{\pi/2} \cos x\, dx$

(A) 0 (B) 1 (C) 2 (D) None of these

21. $\int_{0}^{\pi/2} \sqrt{1+\cos x}\, dx$

(A) 2 (B) 1 (C) 0 (D) None of these

22. $\int (1-x)\sqrt{x}\, dx$:

(A) $\frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} + C$ (B) - $\frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} + C$

(C) - $\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$ (D) $\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$

23. $\int \frac{1}{\sin^2 x.\cos^2 x} dx$

(A) cot x + tan x + C (B) tan x – cot x + C (C) - tan x + cot x + C (D) - tan x – cot x + C

24. $\int \frac{1}{1+e^{-x}} dx$

(A) $ln(1+e^x) + C$ (B) tan x – cot x + C
(C) - tan x + cot x + C (D) - tan x – cot x + C

25. $\int \frac{\operatorname{cosec}^2 x}{1+\cot x}.dx$:

(A) - ln | 1 + cot x| + C (B) ln| 1 + cot x | + C

(C) ln |1 + tan x | + C (D) - ln | 1 + tan x | + C

26. $\int \frac{ln\, x}{x}.dx$

(A) $lnx + C$ (B) $\frac{(ln\, x)^2}{2} + C$ (C) $-\frac{(ln\, x)^2}{2} + C$ (D) $-ln$ x + C

27. $\int_{0}^{\pi/2} (\sin x + \cos x) dx$

(A) 2 (B) 1 (C) 3 (D) 4

28. $\int_{0}^{\infty} e^{-x} dx$

(A) 1 (B) 0 (C) ∞ (D) None of these

EXERCISE # III

1. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. Find the rate at which its area is increasing when radius is 3.2 cm.

2. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

3. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

4. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

5. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one - sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

6. A man of height 2 metres walks at a uniform speed of 5 km/hr away from a lamp post which is 6 metres high. Find the rate at which the length of his shadow increases.

7. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

8. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. Find the size of the square that should be cut off so that the volume of the box is maximum possible.

9. Show that of all the rectangles inscribed in a circle of fixed radius, the square has the maximum area.

10. A body is thrown from earth's surface vertically upwards with velocity v, which varies with time as $v = (29.4 - 9.8t)$ m/s. Find the maximum height reached by the body .

EXERCISE # IV

Application of calculus in kinematics :

1. A point moves in a straight line such that its distance s from the start in time t is equal to

$s = \frac{1}{4}t^4 - 4t^3 + 16t^2$.

(a) At what times was the point at its starting position ?

(b) At what times is its velocity equal to zero ?

2. A body whose mass is 3 kg performs rectilinear motion according to the formula $s = 1 + t + t^2$, where s is measured in centimeters & t in seconds. Determine the kinetic energy $\frac{1}{2}mv^2$ of the body 5 seconds after its start.

3. A body moves in a straight line according to the equation $s = t^3 - 4t^2 - 3t$. Find its acceleration when its velocity is zero.

4. The displacement x of a particle moving in one dimension, under the action of a constant force is related to the time t by the equation $t = \sqrt{x} + 3$, where x is in metres & t is in seconds . Find the displacement of the particle when its velocity is zero .

5. The velocity of a moving point changes according to the equation $v = (3t^2 + 2t + 1)$ m/s . Find the distance covered by the point in 10 seconds from the start.

6. A point moves with velocity $v = (9t^2 - 8t)$ m/s . Find the distance covered by the point during the fourth second from the start.

7. A point moves with velocity $v = (6t^2 + 4)$ m/s . Find the distance covered by the point during 5 seconds from the beginning of motion .

8. Given $s = s_0 + v_0 t + \frac{1}{2}gt^2$ where s_0, v_0 and g are constants. Find $\frac{ds}{dt}$.

9. The motion of a particle in a straight line is defined by the relation $x = t^4 - 12t^2 - 40$ where x is in metres and t is in sec. Determine the position x, velocity v and acceleration a of the particle at t = 2 sec.

ANSWER KEY

EXERCISE # I

SECTION - I

1. $4x^3-3x^2-8x+9$

2. $-\dfrac{2}{(x-1)^2}$ **3.** $\dfrac{1-x^2}{(1+x^2)^2}$ **4.** $\dfrac{ad-bc}{(cx+d)^2}$ **5.** $-\dfrac{4x}{3(x^2-1)^2}+1+2x-3x^2$

6. $-\dfrac{6x^2}{(x^3+1)^2}$ **7.** $-\dfrac{6x^2}{(x^3-1)^2}$ **8.** $\dfrac{2x-1}{a^3-3}$ **9.** $-\dfrac{3x^2}{\sqrt{\pi}}$

SECTION - II

10. $\cos x-\sin x$ **11.** $\dfrac{1-\cos x-x\sin x}{(1-\cos x)^2}$ **12.** $\dfrac{x\sec^2 x-\tan x}{x^2}$

13. $x\cos x$ **14.** $(x\cos x-\sin x)\left(\dfrac{1}{x^2}-\dfrac{1}{\sin^2 x}\right)$ **15.** $\dfrac{1}{1+\cos x}$

16. $\dfrac{\sin x+\cos x+x(\sin x-\cos x)}{1+\sin 2x}$ **17.** $\dfrac{(1+\tan x)(\sin x+x\cos x)-x\sin x\sec^2 x}{(1+\tan x)^2}$

18. $-\sin 2x$ **19.** $\tan^3 x\sec^2 x$ **20.** $-\sin^3 x$

21. $6\sin x\cos x-3\sin^2 x\cos$ **22.** $\tan^4 x$ **23.** $2x\sec^2 x\tan x$

SECTION - III

24. $1+\ln x$ **25.** $\dfrac{2\ln x}{x}$ **26.** $\dfrac{2}{x}$ **27.** $\dfrac{1}{2x\sqrt{\ln x}}$

SECTION - IV

28. $\dfrac{2}{3}\sqrt{x^3}+c$ **29.** $\dfrac{mx^{\frac{n}{m}+1}}{n+m}+c$ **30.** $-\dfrac{1}{\text{x}}+\text{c}$ **31.** $\sqrt{x}+c$

32. $u-u^2+c$

EXERCISE # II

1. **B**	**2.** A	**3.** C	**4.** D	**5.** **D**	**6.** D	**7.** B
8. A	**9.** D	**10.** A	**11.** A	**12.** D	**13.** D	**14.** B
15. B	**16.** A	**17.** C	**18.** A	**19.** B	**20.** C	**21.** A
22. D	**23.** B	**24.** A	**25.** A	**26.** B	**27.** A	**28.** A

EXERCISE # III

1. $0.320\,\pi\text{cm}^2/\text{s}$

2. $900\ \text{cm}^3/\text{s}$

3. $\frac{1}{\pi}\text{cm}/\text{s}$

4. $\frac{8}{3}\text{cm}/\text{s}$

5. $\frac{1}{48\pi}\text{cm}/\text{s}$

6. $\frac{5}{2}\text{km}/\text{h}$

7. $\frac{15}{2},\frac{15}{2}$

8. $3\ \text{cm}$

10. 44.1 m

EXERCISE # IV

1. (a) 0, 8 sec (b) 0, 4, 8 sec

2. 0.01815 joules

3. 10m/s^2

4. 0 m

5. 1110 m

6. 83 m

7. 270 m

8. $v = v_0 + gt$

9. $x = -72$ m , $v = -16$ m/s , $a = 24\ \text{m/s}^2$

CONTENTS

1. ERRORS & MEASUREMENT

- ## Basics SI Quantities:

Base Quantity	Name	Symbol
Length	**metre**	**m**
Mass	**kilogram**	**kg**
Time	**second**	**s**
Electric current	**ampere**	**A**
Thermodynamic Temperature	**kelvin**	**K**
Amount of substance	**mole**	**mol**
Luminous intensity	**candela**	**cd**

Note: Whenever writing the full form of a unit do not use capital letter to start if it is named after a scientist. When writing a symbol use a capital letter to start only if it is named after a scientist.

- ## Units and Dimensions:

All physical quantities are described by parameters such as length, mass and so on, all of which have dimensions. In general we will denote the dimension of length by **L,** of time by **T** and of mass by **M**. The dimension of a quantity **x** will be denoted by square brackets as **[x]**. For example, thedimension of the volume of an object **V** is denoted by **[V]** = $L \times L \times L = L^3$. An equation such as **X + Y = Z** is consistent only if the dimensions of both the sides match, in this case only if all the three quantities **X**, **Y** and **Z** have the same dimensions. Checking that the dimensions agree for an equation is one of the first steps that one should take when analyzing a new set of equations. Dimensional analysis can lead to fairly sophisticated results; however, dimensional analysis can only lead to the general form of the equation, and cannot fully specify the solution. For example, if one has two dimensional quantities **X** and **Y**, and we know that the solution **A** is a dimensionless combination of **X** and **Y**, then we can straight away conclude that the solution must be a function only of the ratio **X/Y**; however, based only on dimensional considerations the solution can be either **A = 1 + X/Y** or equivalently **A= 100-Y/X**, and hence the complete answer for **A** cannot simply be obtained from dimensional analysis. The **units** that one uses for the various dimensional quantities are specific choices of what is one unit of length, time and so on. Units are arbitrary, and any set of consistent units can be used. For example, we can use meters or equivalently inches to measure length, grams or pounds to measure mass, and so on. Of course one can always convert from one set of units to another, and is analogous to converting from one currency to another. We will use the SI (System International)units, which previously was known as the MKS system of units. In these units distance is measured in metres (m), mass is measured in terms of kilogram (kg), and time is measured in seconds (s). Hence the dimension of velocity v is given by $[v] = LT^{-1}$, and in SI units we have $[v] = ms^{-1}$.

• Application of Dimensional Analysis

(i) To find the unit of a physical quantity in a given system of units :
Writing the definition or formula for the physical quantity we find its dimensions. Now in the dimensional formula replacing *M*, *L* and *T* by the fundamental units of the required system we get the unit of physical quantity. However, sometimes to this unit we further assign a specific name, *e.g.*, Work = Force × Displacement So, $[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$
So its units in C.G.S. system will be gcm^2/s^2 which is called *erg* while in M.K.S. system will be $kg\ m^2/s^2$ which is called *joule*.

Sample problems

Problem 1. The equation $\left(P + \frac{a}{v^2}\right)(V - b) = $ constant. The units of a is

(a) $\text{Dyne} \times \text{cm}^5$ (b) $\text{Dyne} \times \text{cm}^4$ (c) Dyne/cm^3 (d) Dyne/cm^2

Solution : (b) According to the principle of dimensional homogeneity $[P] = \left[\frac{a}{V^2}\right]$

$\Rightarrow [a] = [P][V^2] = [ML^{-1}T^{-2}][L^6] = [ML^5T^{-2}]$ or unit of

$a = \text{gm} \times \text{cm}^5 \times \text{sec}^{-2} = \text{Dyne} \times \text{cm}^4$

Problem 2. If $x = at + bt^2$, where x is the distance travelled by the body in kilometre while t the time in seconds, then the units of b are

(a) km/s (b) km-s (c) km/s^2 (d) $km\text{-}s^2$

Solution :(c) From the principle of dimensional homogeneity $[x] = [bt^2] \Rightarrow [b] = \left[\frac{x}{t^2}\right]$

∴ Unit of b= km/s^2.

Problem 3. Unit of Stefan's constant is

(a) Js^{-1} (b) $Jm^{-2}s^{-1}K^{-4}$ (c) Jm^{-2} (d) Js

Solution : (b) Stefan's formula $\frac{Q}{At} = \sigma T^4 \therefore \sigma = \frac{Q}{AtT^4}$ Unit of $\sigma = \frac{\text{Joule}}{m^2 \times \text{sec} \times K^4} = Jm^{-2}s^{-1}K^{-4}$

Problem 4. The unit of surface tension in SI system is

(a) Dyne/cm^2 (b) Newton/m (c) Dyne/cm (d) Newton/m^2

Solution : (b) From the formula of surface tension $T = \frac{F}{l}$

Problem 5. A suitable unit for gravitational constant is

(a) kgmetre sec^{-1} (b) Newton metre^{-1} sec
(c) Newton metre^2 kg^{-2} (d) kg metre sec^{-1}

Solution : (c)

Problem 6. The SI unit of universal gas constant (R) is

(a) Watt $K^{-1}mol^{-1}$ (b) Newton $K^{-1}mol^{-1}$ (c) Joule $K^{-1}mol^{-1}$ (d) Erg $K^{-1}mol^{-1}$

Solution : (c) Ideal gas equation PV = nRT

$$\therefore [R]\frac{[P][V]}{[nT]}=\frac{[ML^{-1}T^{-2}][L^3]}{[mole][K]}=\frac{[ML^2T^{-2}]}{[mole]\times[K]}$$

So the unit will be Joule $K^{-1}mol^{-1}$.

(ii) To find dimensions of physical constant or coefficients :

As dimensions of a physical quantity are unique, we write any formula or equation incorporating the given constant and then by substituting the dimensional formulae of all other quantities, we can find the dimensions of the required constant or coefficient.

(a) Gravitational constant : According to Newton's law of gravitation $F=G\frac{m_1m_2}{r^2}$ or $G=\frac{Fr^2}{m_1m_2}$

Substituting the dimensions of all physical quantities $[G]=\frac{[MLT^{-2}][L^2]}{[M][M]}=[M^{-1}L^3T^{-2}]$

(b) Plank constant : According to Planck $E = h\,\upsilon$ or $h=\frac{E}{v}$

Substituting the dimensions of all physical quantities $[h]=\frac{[ML^2T^{-2}]}{[T^{-1}]}=[ML^2T^{-1}]$

(c) Coefficient of viscosity : According to Poiseuille's formula $\frac{dV}{dt}=\frac{\pi pr^4}{8\eta l}$ or $\eta=\frac{\pi pr^4}{8l(dV/dt)}$

Substituting the dimensions of all physical quantities $[\eta]=\frac{[ML^{-1}T^{-2}][L^4]}{[L][L^3/T]}=[ML^{-1}T^{-1}]$

Note : The exponent always has to be dimensionless.

e.g. 5^x, x must be dimensionless. Anythinginside logarithm must be dimensionless.

Sample problems based on dimension finding

Problem 7. $x=3YZ^2$ find dimension of Y in (MKSA) system, if x and Z are the dimension of capacitance and magnetic field respectively. $[x]=[M^{-1}L^{-2}T^4A^2],[Z]=[MT^{-2}A^{-1}]$

(a) $M^{-3}L^{-2}T^{-4}A^{-1}$ (b) ML^{-2} (c) $M^{-3}L^{-2}T^4A^4$ (d) $M^{-3}L^{-2}T^8A^4$

Solution : (d) $x=3YZ^2$ $\therefore [Y]=\frac{[X]}{[Z^2]}=\frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2}$.

Problem 8. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$ where symbols have their usual meaning, are (Given that velocity of light $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$)

(a) $[LT^{-1}]$ (b) $[L^{-1}T]$ (c) $[L^{-2}T^2]$ (d) $[L^2T^{-2}]$

Solution : (d)We know that velocity of light $C=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\therefore \frac{1}{\mu_0\varepsilon_0}=C^2$

So $\left[\frac{1}{\mu_0\varepsilon_0}\right]=[LT^{-1}]^2=[L^2T^{-2}]$.

Problem 9. A force F is given by $F=at+bt^2$, where t is time. What are the dimensions of a and b

(a) MLT^{-3} and ML^2T^{-4} (b) MLT^{-3} and MLT^{-4}
(c) MLT^{-1} and MLT^0 (d) MLT^{-4} and MLT^1

Solution :(b) From the principle of dimensional homogeneity [F] = [at]

$$\therefore [a]=\left[\frac{F}{t}\right]=\left[\frac{MLT^{-2}}{T}\right]=[MLT^{-3}] \text{ Similarly } [F]=[bt^2]$$

$$\therefore [b]=\left[\frac{F}{t^2}\right]=\left[\frac{MLT^{-2}}{T^3}\right]=[MLT^{-4}]$$

Problem 10. The position of a particle at time t is given by the relation $x(t)=\left(\frac{v_0}{\alpha}\right)(a-e^{-\alpha t})$, where v_0 is a constant and $\alpha>0$. The dimensions of v_0 and α are respectively

(a) $M^0L^1T^{-1}$ and T^{-1} (b) $M^0L^1T^0$ and T^{-1}
(c) $M^0L^1T^{-1}$ and LT^{-2} (d) $M^0L^1T^{-1}$ and T

Solution : (a) From the principle of dimensional homogeneity $[\alpha t]$ = dimensionless

$$\therefore [\alpha]=\left[\frac{1}{t}\right]=[T^{-1}]. \text{ Similarly } [x]=\frac{[v_0]}{[\alpha]} \therefore [v_0]=[x][\alpha]=[L][T^{-1}]=[LT^{-1}].$$

Problem 11. The dimensions of physical quantity X in the equation Force $=\frac{X}{\text{Density}}$ is given by

(a) $M^1L^4T^{-2}$ (b) $M^2L^{-2}T^{-1}$ (c) $M^2L^{-2}T^{-2}$ (d) $M^1L^{-2}T^{-1}$

Solution : (c) [X] = [Force] × [Density] $=[MLT^{-2}]\times[ML^{-3}]=[M^2L^{-2}T^{-2}]$.

Problem 12.E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively, then the dimension of $\frac{El^2}{m^5G^2}$ are [angular momentum] = mL^2T^{-1}

$[G]=m^{-1}L^3T^{-2}$

(a) Angle (b) Length (c) Mass (d) Time

Solution : (a) The required dimension works out to be dimensionless. Hence it will be dimension of angle.

Problem 13. The equation of a wave is given by $Y = A\sin\omega\left(\frac{x}{v} - k\right)$ where ω is the angular velocity and v is the linear velocity. x is the displacement along x – axis. The dimension of k is $\left([\omega] = T^{-1}\right)$.

(a) L T (b) T (c) T^{-1} (d) T^2

Solution : (b)

Problem 14. The potential energy of a particle varies with distance x from a fixed origin as $U = \frac{A\sqrt{x}}{x^2 + B}$ where A and B are dimensional constants then dimensional formula for ABis :

(a) $ML^{7/2}T^{-2}$ (b) $ML^{11/2}T^{-2}$ (c) $M^2L^{9/2}T^{-2}$ (d) $ML^{13/2}T^{-3}$

Solution : (b)

Problem 15. The equation of the stationary wave is $y = 2a\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$, which of the following statements is wrong

(a) The unit of ct is same as that of λ

(b) The unit of x is same as that of λ

(c) The unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda t$

(d) The unit of c/λ is same as that of x/λ

Solution : (d)

(iii) To convert a physical quantity from one system to the other :

The measure of a physical quantity is nu = constant

If a physical quantity X has dimensional formula $[M^aL^bT^c]$ and if (derived) units of that physical quantity in two systems are $\left[M_1^a L_1^b T_1^c\right]$ and $\left[M_2^a L_2^b T_2^c\right]$ respectively and n_1 and n_2 be the numerical values in the two systems respectively, then $n_1[u_1] = n_2[u_2]$

$$\Rightarrow n_1\left[M_1^a L_1^b T_1^c\right] = n_2\left[M_2^a L_2^b T_2^c\right] \qquad \Rightarrow n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Where M_1, L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system. Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

(a) conversion of *Newton* into *Dyne*.

The Newton is the S.I. unit of force and has dimensional formula $[MLT^{-2}]$.

So 1 N = 1 $kg\text{-}m/sec^2$

By using

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = \left[\frac{kg}{gm}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 1\left[\frac{10^3\,gm}{gm}\right]^1\left[\frac{10^2\,cm}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 10^5$$

$\therefore$ 1 N = 10^5 *Dyne*

(b) Conversion of gravitational constant (G) from C.G.S. to M.K.S. system

The value of G in C.G.S. system is 6.67 $\times 10^{-8}$ C.G.S. units while its dimensional formula is $[M^{-1}L^3T^{-2}]$ So $G = 6.67 \times 10^{-8} cm^3/g\, s^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 6.67\times10^{-8}\left[\frac{gm}{kg}\right]^{-1}\left[\frac{cm}{m}\right]^3\left[\frac{sec}{sec}\right]^{-2}$

$$= 6.67\times10^{-8}\left[\frac{gm}{10^3\,gm}\right]^{-1}\left[\frac{cm}{10^2\,cm}\right]^3\left[\frac{sec}{sec}\right]^{-2} = 6.67\times10^{-11}$$

$G = 6.67\times 10^{-11}$ M.K.S. units

Sample problems based on conversion :

Problem 16. A physical quantity is measured and its value is found to be nu where n = numerical value and u = unit. Then which of the following relations is true

(a) $n \propto u^2$ (b) $n \propto u$ (c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$

Solution : (d) We know $P = nu = \text{constant} \therefore n_1u_1 = n_2u_2$ or $n \propto \frac{1}{u}$.

Problem 17. In C.G.S. system the magnitude of the force is 100 dynes. In another system where the fundamental physical quantities are kilogram, metre and minute, the magnitude of the force is

(a) 0.036 (b) 0.36 (c) 3.6 (d) 36

Solution: (c) $n_1 = 100, M_1 = g, L_1 = cm, T_1 = sec$ and $M_2 kg, L_2 = meter, T_2 = minute, x = 1, y = 1,$ $z = -2$ By substituting these values in the following conversion formula

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^2; n_2 = 100\left[\frac{gm}{kg}\right]^1\left[\frac{cm}{meter}\right]^1\left[\frac{sec}{minute}\right]^{-2};$$

$$n_2 = 100\left[\frac{gm}{10^3 gm}\right]^1\left[\frac{cm}{10^2\,cm}\right]^1\left[\frac{sec}{60\,sec}\right]^{-2} = 3.6$$

Problem 18. The temperature of a body on Kelvin scale is found to be X K. When it is measured by a Fahrenheit thermometer, it is found to be XF. Then X is

(a) 301.25 (b) 574.25 (c) 313 (d) 40

Solution : (b) Relation between kelvin and Fahrenheit $\frac{K-273}{5} = \frac{F-32}{9}$

Problem 19. To determine the Young's modulus of a wire, the formula is $Y = \frac{F}{A}.\frac{L}{\Delta l};$

whereL= length, A= area of cross- section of the wire, $\Delta L =$ Change in length of the wire when stretched with a force F. The conversion factor to change it from CGS to MKS system is :

(a) 1 (b) 10 (c) 0.1 (d) 0.01

Solution : (c)

Problem 20. Conversion of 1 MW power on a new system having basic units of mass, length and time as 10kg, 1dm and 1 minute respectively is

(a) 2.16×10^{12} unit (b) 1.26×10^{12} unit (c) 2.16×10^{10} unit (d) 2×10^{14} unit

Solution : (a)

Problem 21. In two systems of relations among velocity, acceleration and force are respectively $v_2 = \frac{\alpha_2}{\beta} v_1, a_2 = \alpha\beta a_1$ and $F_2 = \frac{F_1}{\alpha\beta}$. If α and β are constants then relations among mass, length and time in two systems are

(a) $M_2 = \frac{\alpha}{\beta} M_1, L_2 = \frac{\alpha^2}{\beta^2} L_1 \, T_2 = \frac{\alpha^3 T_1}{\beta}$ (b) $M_2 = \frac{1}{\alpha^2\beta^2} M_1, L_2 = \frac{\alpha^3}{\beta^3} L_1 \, T_2 = T_1 \frac{\alpha}{\beta^2}$

(c) $M_2 = \frac{\alpha^3}{\beta^3} M_1, L_2 = \frac{\alpha^2}{\beta^2} L_1 \, T_2 = \frac{\alpha}{\beta} T_1$ (d) $M_2 = \frac{\alpha^2}{\beta^2} M_1, L_2 = \frac{\alpha}{\beta^2} L_1 \, T_2 = \frac{\alpha^3}{\beta^3} T_1$

Solution : (b)

Problem 22. If the present units of length, time and mass (m, s, kg) are changed to 100m, 100s, and $\frac{1}{10}$ kg then:

(a) The new unit of velocity is increased 10 times

(b) The new unit of force is decreased $\frac{1}{1000}$ times

(c) The new unit of energy is increased 10 times

(d) The new unit of pressure is increased 1000 times

Solution : (b)

Problem 23. Suppose we employ a system in which the unit of mass equals 100 kg, the unit of length equals 1 km and the unit of time 100 s and call the unit of energy eluoj (joule written in reverse order), then

(a) 1 eluoj = 10^4 joule (b) 1 eluoj = 10^{-3} joule

(c) 1 eluoj = 10^{-4} joule (d) 1 joule = 10^3 eluoj

Solution : (a)

Problem 24. If 1gmcms^{-1} = xNs, then number x is equivalent to

(a) 1×10^{-1} (b) 3×10^{-2} (c) 6×10^{-4} (d) 1×10^{-5} 1×10^{-1}

Solution : (d)

(iv) To check the dimensional correctness of a given physical relation:

This is based on the '*principle of homogeneity*'. According to this principle the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$ X=, then according to principle of homogeneity

$[X] = [A] = [(BC)^2] = \left[\sqrt{DEF}\right]$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

(a) $F = mv^2/r^2$, By substituting dimension of the physical quantities in the above relation

$\left[MLT^{-2}\right] = [M]\left[LT^{-1}\right]^2$; *i.e.* $\left[MLT^{-2}\right] = \left[MT^{-2}\right]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally, so can never be physically.

(b) $S = ut + at^2$

By substituting dimension of the physical quantities in the above relation –

$[L] = [LT^{-1}][T] + [LT^{-2}][T^2]$; *i.e.* $[L] = [L] + [L]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct. However, from equations of motion we know that $s = ut + (1/2)at^2$

Sample problems based on formulae checking

Problem 25. From the dimensional consideration, which of the following equation is correct $\left([G] = m^{-1}L^3T^{-2}\right)$

(a) $T = 2\pi\sqrt{\frac{R^3}{GM}}$ (b) $T = 2\pi\sqrt{\frac{GM}{R^3}}$ (c) $T = 2\pi\sqrt{\frac{GM}{R^2}}$ (d) $T = 2\pi\sqrt{\frac{R^2}{GM}}$

Solution : (a)

Problem 26. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and of low elastic modulus η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn block A executes small oscillations. The time period will be : (Given that $[\eta] = \left[ML^{-1}T^{-2}\right]$)

(a) $2\pi\sqrt{\frac{M\eta}{L}}$ (b) $2\pi\sqrt{\frac{L}{M\eta}}$ (c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

Solution : (d)

Problem 27. A small steel ball of radius r is allowed to fall under gravity through a column of a viscous liquid of coefficient of viscosity η. After some time the velocity of the ball attains a constant value known as terminal velocity v_T. The terminal velocity depends on (i) the mass of the ball. (ii) η (iii) r and (iv) acceleration due to gravity g. which of the following relations is dimensionally correct $[\eta] = \left[ML^{-1}T^{-1}\right]$.

(a) $v_T \propto \frac{mg}{\eta r}$ (b) $v_T \propto \frac{\eta r}{mg}$ (c) $v_T \propto \eta rmg$ (d) $v_T \propto \frac{mgr}{\eta}$

Solution : (a)

Problem 28. A dimensionally consistent relation for the volume V of a liquid of coefficient of viscosity η flowing per second through a tube of radius r and length l and having a pressure difference p across its end, is $\left([\eta] = \left[ML^{-1}T^{-1}\right]\right)$

(a) $V = \frac{\pi pr^4}{8\eta l}$ (b) $V = \frac{\pi\eta l}{8pr^4}$ (c) $V = \frac{8p\eta l}{\pi r^4}$ (d) $V = \frac{\pi p\eta}{8lr^4}$

Solution : (a)

Problem 29. With the usual notations, the following equation $S_t = u + \frac{1}{2}a(2t-1)$ is

(a) Only numerically correct
(b) Only dimensionally correct
(c) Both numerically and dimensionally correct
(d) Neither numerically nor dimensionally correct

Solution : (c)

Problem 30. If velocity v, acceleration A and force F are chosen as fundamental quantities, then the dimensional formula of angular momentum in terms of v, A and F would be [Angular momentum] = mL^2T^{-1}

(a) $FA^{-1}v$ (b) Fv^3A^{-2} (c) Fv^2A^{-1} (d) $F^2v^2A^{-1}$

Solution : (b)

Problem 31. The largest mass (m) that can be moved by a flowing river depends on velocity (v), density (ρ) of river water and acceleration due to gravity (g). The correct relation is

(a) $m \propto \frac{\rho^2 v^4}{g^2}$ (b) $m \propto \frac{\rho v^6}{g^2}$ (c) $m \propto \frac{\rho v^4}{g^3}$ (d) $m \propto \frac{\rho v^6}{g^3}$

Solution : (d)

(v) As a research tool to derive new relations :
If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.

(a) Time period of a simple pendulum. Let time period of a simple pendulum is a function of mass of the bob (*m*), effective length (*l*), acceleration due to gravity (*g*) then assuming the function to be product of power function of *m*, *l* and *g* *i.e.*, $T = Km^x l^y g^z$; where *K* = dimensionless constant. If the above relation is dimensionally correct then by substituting the dimensions of quantities –$[T] = [M]^x [L]^y [LT^{-2}]^z$ or $[M^0L^0T^1] = [M^xL^{y+z}T^{-2z}]$
Equating the exponents of similar quantities $x = 0$, $y = 1/2$ and $z = -1/2$

So the required physical relation becomes $T = K\sqrt{\frac{l}{g}}$ .The value of dimensionless constant is found (2π) through experiments so $T = 2\pi\sqrt{\frac{l}{g}}$

(b) Stoke's law : When a small sphere moves at low speed through a fluid, the viscous force *F*, opposing the motion, is found experimentally to depend on the radius *r*, the velocity of the sphere *v* and the viscosity η of the fluid.
So $F = f(\eta, r, v)$
If the function is product of power functions of η, r and v, $F = K\eta^x r^y v^z$; where *K* is dimensionless constant. If the above relation is dimensionally correct

$$\left[MLT^{-2}\right] = \left[ML^{-1}T^{-1}\right]^x \left[L\right]^y \left[LT^{-1}\right]^z \text{ or } \left[MLT^2\right] = \left[M^xL^{-x+y+z}T^{-x-z}\right]$$

Equating the exponents of similar quantities $x = 1$; $-x + y + z = 1$ and $-x - z = -2$
Solving these for *x*, *y* and *z*, we get $x = y = z = 1$

So eq^n (i) becomes $F = K\eta rv$
On experimental grounds, $K = 6\pi$; so $F = 6\ \pi\eta rv$. This is the famous Stoke's law.

Sample problem based on formulae derivation

Problem 32. If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, then the dimensions of mass in new system is
$[G] = m^{-1}L^3T^{-2}\,[h] = mL^2T^{-1}$.
(a) $c^{1/2}\,G^{1/2}\,h^{1/2}$ (b) $c^{1/2}\,G^{1/2}\,h^{-1/2}$ (c) $c^{1/2}\,G^{-1/2}\,h^{1/2}$ (d) $c^{-1/2}\,G^{1/2}\,h^{1/2}$

Solution : (c)

Problem 33. If the time period (T) of vibration of a liquid drop depends on surface tension (S), radius (r) of the drop and density (ρ) of the liquid, then the expression of T is
(a) $T = K\sqrt{\rho r^3/S}$ (b) $T = K\sqrt{\rho^{1/2}\,r^3/S}$ (c) $T = K\sqrt{\rho r^3/S^{1/2}}$ (d) None of these

Solution : (a)

Problem 34. If P represents radiation pressure, C represents speed of light and Q represents radiation energy striking a unit area per second, then non-zero integers x, y and z such that $P^xQ^yC^z$ dimensionless, are
(a) x = 1, y = 1, z = – 1 (b) x = 1, y = – 1, z = 1
(c) x = - 1, y = 1, z = 1 (d) x = 1, y = 1, z = 1

Solution : (b)

Problem 35. If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be
(a) $V^{-2}F^0E$ (b) V^0FE^2 (c) $VF^{-2}E^0$ (d) $V^{-2}F^0E$

Solution : (d)

Limitations of Dimensional Analysis

Although dimensional analysis is very useful it cannot lead us too far as :

(1) If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example if the dimensional formula of a physical quantity is $\left[ML^2T^{-2}\right]$ it may be work or energy or torque.

(2) Numerical constant having no dimensions [*K*] such as (1/2), 1 or 2π*etc*. cannot be deduced by the methods of dimensions.

(3) The method of dimensions can not be used to derive relations other than product of power functions. For example,

$$s = ut + (1/2)at^2 \text{ or } y = a\sin\omega t$$

cannot be derived by using this theory . However, the dimensional correctness of these can be checked.

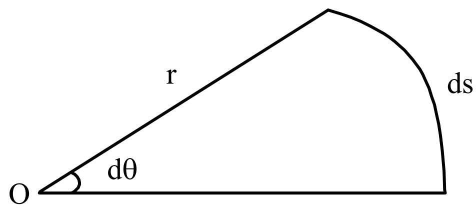
(4) The method of dimensions cannot be applied to derive formula. In mechanics a physicalquantity may be dependent on more than 3 physical quantities. This implies number ofequations are less than the number of variables. However still we can check correctness of

thegiven equation dimensionally. For example $T = 2\pi\sqrt{I/mgl}$ can not be derived by theory ofdimensions but its dimensional correctness can be checked.

(5) Even if a physical quantity depends on 3 physical quantities, out of which two have samedimensions, the formula cannot be derived by theory of dimensions, *e.g.*, formula for the frequency of a tuning fork $f = \left(d/L^2\right)v$ cannot be derived by theory of dimensions but can be checked.

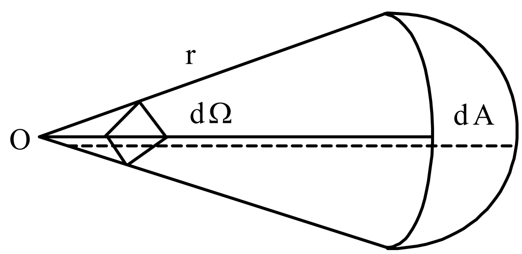
- **ANGULAR MEASUREMENTS**

In two dimension we use radians to measure plane angle $d\theta$



$d\theta = ds/r$ *radian*

Note: The angular dimension radian generally isn't mentioned so sin 2 means sine of 2radian.
In three dimensions we use steradian to measure the solid angle $d\Omega$.



$d\Omega = dA/r^2$ steradian

Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

(1) All non-zero digits are significant.

Example : 42.3 has three significant figures. ; 243.4 has four significant figures.
24.123 has five significant figures.

(2) A zero becomes significant figure if it appears between two non-zero digits.

Example : 5.03 has three significant figures. ; 5.604 has four significant figures.
4.004 has four significant figures.

(3) Leading zeros or the zeros placed to the left of the number are never significant.

Example : 0.543 has three significant figures. ; 0.045 has two significant figures.
0.006 has one significant figure.

(4) Trailing zeros or the zeros placed to the right of the number are significant.

Example : 4.330 has four significant figures. ; 433.00 has five significant figures.
343.000 has six significant figures.

(5) In exponential notation, the numerical portion gives the number of significant figures.

Example : 1.32×10^{-2} has three significant figures. ; 1.32×10^{4} has three significant figures.
Food for Thought : How many significant figures are there in 0.0 ?

Rounding Off :

While rounding off measurements, we use the following rules by convention:

(1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example : x = 7.82 is rounded off to 7.8, again x = 3.94 is rounded off to 3.9.

(2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.

Example : x = 6.87 is rounded off to 6.9, again x = 12.78 is rounded off to 12.8.

(3) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.

Example : x = 16.351 is rounded off to 16.4, again x = 6.758 is rounded off to 6.8.

(4) If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.

Example: x = 3.250 becomes 3.2 on rounding off, again x = 12.650 becomes 12.6 on rounding off.

(5) If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.

Example : x = 3.750 is rounded off to 3.8, again x = 16.150 is rounded off to 16.2.

- ## Significant Figures in Calculation

In most of the experiments, the observations of various measurements are to be combined mathematically, *i.e.*, added, subtracted, multiplied or divided as to achieve the final result. Since, all the observations in measurements do not have the same precision, it is natural that the final result cannot be more precise than the least precise measurement. The following two rules should be followed to obtain the proper number of significant figures in any calculation.

(i) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places. The rule is illustrated by the following examples :

(a) 33.3 ←(has only one decimal place)
3.11
+ 0.313
36.723←(answer should be reported to one decimal place) Answer = 36.7

(b) 3.1421
0.241
+ 0.09 ← (has 2 decimal places)
3.4731 ← (answer should be reported to 2 decimal places) Answer = 3.47

(c) 62.831 ← (has 3 decimal places)
– 24.5492
38.2818 ←(answer should be reported to 3 decimal places after rounding off)
Answer = 38.282

(ii) The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation. The rule is illustrated by the following examples:

(a) 142.06
×0.23←(two significant figures)
32.6738 ←(answer should have two significant figures) Answer = 33

(b) 51.028
×1.31 ← (three significant figures)
66.84668 Answer = 66.8

(c) Answer = 0.21

• Order of Magnitude

In scientific notation the numbers are expressed as, Number $= M \times 10^{x}$. where M is a number lying between 1 and 10 and *x* is integer. Order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example,

(1) Speed of light in vacuum $= 3 \times 10^{8} ms^{-1} \approx 10^{8}$ m/s (ignoring 3 < 5)

(2) Mass of electron $= 9.1 \times 10^{-31}$ kg $\approx 10^{-30}$ kg (as 9.1 > 5).

Sample problems based on significant figures

Problem 1. Each side a cube is measured to be 7.203 m. The volume of the cube up to appropriate significant figures is

(a) 373.714 (b) 373.71 (c) 373.7 (d) 373

Solution : (c) Volume $= a^{3} = (7.023)^{3} = 373.715 \, m^{3}$

In significant figures volume of cube will be $373.7 m^{3}$ because its side has four significant figures.

Problem 2. The number of significant figures in 0.007 m^{2} is

(a) 1 (b) 2 (c) 3 (d) 4

Solution : **(a)**

Problem 3. The length, breadth and thickness of a block are measured as 125.5 cm, 5.0 cm and 0.32 cm respectively. Which one of the following measurements is most accurate
(a) Length (b) Breadth (c) Thickness (d) Height

Solution : (**a**) Relative error in measurement of length is minimum, so this measurement is most accurate.

Problem 4. The mass of a box is 2.3 kg. Two marbles of masses 2.15 g and 12.39 g are added to it. The total mass of the box to the correct number of significant figures is
(a) 2.340 kg (b) 2.3145 kg. (c) 2.3 kg (d) 2.31 kg

Solution : **(c)** Total mass = 2.3 + 0.00215 + 0.01239 = 2.31 kg
Total mass in appropriate significant figures be 2.3 kg.

Problem 5. The length of a rectangular sheet is 1.5 cm and breadth is 1.203 cm. The area of the face of rectangular sheet to the correct number of significant figures is :
(a) 1.8045 cm^2 (b) 1.804 cm^2 (c) 1.805 cm^2 (d) 1.8 cm^2

Solution : (**d**) Area $1.5\times1.203=1.8045\,cm^2=1.8\,cm^2$ (Upto correct number of significant figure).

Problem 6. Each side of a cube is measured to be 5.402 cm. The total surface area and the volume of the cube in appropriate significant figures are :
(a) $175.1\,cm^2, 157\,cm^3$ (b) $175.1\,cm^2, 157.6\,cm^3$
(c) $175.1\,cm^2, 157\,cm^3$ (d) $175.08\,cm^2, 157.639\,cm^3$

Solution : **(b)** Total surface area $=6\times(5.402)^2=175.09\,cm^2=175.1\,cm^2$
(Upto correct number of significant figure)
Total volume $=(5.402)^3=157.64\,cm^3=157.6$
(Upto correct number of significant figure)

Problem 7. Taking into account the significant figures, what is the value of 9.99 m + 0.0099 m
(a) 10.00 m (b) 10 m (c) 9.9999 m (d) 10.0 m

Solution : **(b)** 9.99 has three significant figures, 0.0099 has two significant figures so the answer must contain atmost two significant figures, hence b is the correct answer.

Problem 8. The value of the multiplication 3.124×4.576 correct to three significant figures is
(a) 14.295 (b) 14.3 (c) 14.295424 (d) 14.305

Solution : **(b)** $3.124\times4.576=14.295=14.3$ (Correct to three significant figures).

Problem 9. The number of the significant figures in $11.118\times10^{-6}\,V$ is
(a) 3 (b) 4 (c) 5 (d) 6

Solution : **(c)** The number of significant figure is 5 as 10^{-6} does not affect this number.

Problem 10. If the value of resistance is 10.845 ohms and the value of current is 3.23 amperes, the potential difference is 35.02935 volts. Its value in significant number would be
(a) 35 V (b) 35.0 V (c) 35.03 V (d) 35.025 V

Solution : **(b)** Value of current (3.23 A) has minimum significant figure (3) so the value of potential difference V (= IR) shouldhave only 3 significant figure. Hence its value is 35.0 V.

• Errors of Measurement

The measuring process is essentially a process of comparison. Inspite of our best efforts, the measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

(i) **Absolute error :** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times. Let the measured value be a_1, a_2, a_3, …..a_n.

The arithmeticmean of these value is $a_m = \frac{a_1 + a_2 + ...a_n}{n}$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots\ldots\ldots\ldots$$
$$\Delta a_n = a_m - a_n$$

(ii) **Mean absolute error :** It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ...|\Delta a_n|}{n}$$

Hence the final result of measurement may be written as $a = a_m \pm \overline{\Delta a}$

This implies that any measurement of the quantity is likely to lie between $\left(a_m + \overline{\Delta a}\right)$ and $\left(a_m - \overline{\Delta a}\right)$

(iii) **Relative error or Fractional error :** The relative error or fractional error of measurement is defined as the ratio of mean absolute error to the mean value of the quantity measured.

Thus, Relative error or Fractional error $= \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\overline{\Delta a}}{a_m}$

(iv) **Percentage error :** When the relative/fractional error is expressed in percentage, we call it percentage error. Thus, Percentage error $= \frac{\overline{\Delta a}}{a_m} \times 100\%$

• Propagation of Errors

(i) **Error in sum of the quantities :** Suppose $x = a + b$

Let Δa= absolute error in measurement of a ; Δb = absolute error in measurement of b

Δx= absolute error in calculation of x *i.e.* sum of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \frac{(\Delta a + \Delta b)}{a + b} \times 100\%$

(ii) **Error in difference of the quantities :** Suppose $x = a - b$

Let Δa= absolute error in measurement of a, ; Δb= absolute error in measurement of b

Δx= absolute error in calculation of x i.e. difference of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \dfrac{(\Delta a + \Delta b)}{a - b} \times 100\%$

(iii) **Error in product of quantities :** Suppose $x = a \times b$

Let Δa= absolute error in measurement of a, ; Δb= absolute error in measurement of b

Δx= absolute error in calculation of x i.e. product of a and b.

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

Percentage error in the value of x= (Percentage error in value of a)+ (Percentage error in value of b)

(iv) **Error in division of quantities :** Suppose $x = \dfrac{a}{b}$

Let Δa= absolute error in measurement of a ; Δb= absolute error in measurement of b

Δx= absolute error in calculation of x i.e. division of a and b.

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

(v) **Error in quantity raised to some power :** Suppose $x = \dfrac{a^n}{b^m}$

Let Δa= absolute error in measurement of a,

Δb= absolute error in measurement of b Δx= absolute error in calculation of x

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(n\dfrac{\Delta a}{a} + m\dfrac{\Delta b}{b}\right)$ (here m, n ≥ 0)

Percentage error in the value of x = n (Percentage error in value of a) + m (Percentage error in value of b)

Solved Examples :

Problem 1. A physical parameter a can be determined by measuring the parameters b, c, d and e using the relation $a = b^\alpha c^\beta / d^\gamma e^\delta$. If the maximum errors in the measurement of b, c, d and e are b_1%, c_1%, d_1% and e_1%, then the maximum error in the value of a determined by the experiment is

(a) $(b_1 + c_1 + d_1 + e_1)\%$ (b) $(b_1 + c_1 - d_1 - e_1)\%$

(c) $(\alpha b_1 + \beta c_1 - \gamma d_1 - \delta e_1)\%$ (d) $(\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$

Solution : (d) $a = b^\alpha c^\beta / d^\gamma e^\delta$

So maximum error in a is given by

$$\left(\frac{\Delta a}{a} \times 100\right)_{\max} = \alpha.\frac{\Delta b}{b} \times 100 + \beta.\frac{\Delta c}{c} \times 100 + \gamma.\frac{\Delta d}{d} \times 100 + \delta.\frac{\Delta e}{e} \times 100 = (\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$$

Problem 2. The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in the measurement of force and length are respectively 4% and 2%, The maximum error in the measurement of pressure is

(a) 1% (b) 2% (c) 6% (d) 8%

Solution : (d)

Problem 3. The resistance $R = \frac{V}{i}$ where V= 100 ± 5 volts and i = 10 $\pm$ 0.2 amperes. What is the total error in R :

(a) 5% (b) 7% (c) 5.2% (d) 3%

Solution : (b)

Problem 4. The period of oscillation of a simple pendulum in the experiment is recorded as 2.63s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. The average absolute error is

(a) 0.1 s (b) 0.11 s (c) 0.01 s (d) 1.0 s

Solution : (b) Average value $= \frac{2.63+2.56+2.42+2.71+2.80}{5} = 2.62\,\text{sec}$

Now $|\Delta T_1| = 2.63 - 2.62 = 0.01$; $|\Delta T_2| = 2.62 - 2.56 = 0.06$

$|\Delta T_3| = 2.62 - 2.42 = 0.20$; $|\Delta T_4| = 2.71 - 2.62 = 0.06$; $|\Delta T_5| = 2.80 - 2.62 = 0.18$;

Mean absolute error $\Delta T = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4| + |\Delta T_5|}{5} = \frac{0.54}{5} = 0.108 = 0.11\,\text{sec}$

Problem 5. The length of a cylinder is measured with a meter rod having least count 0.1 cm. Its diameter is measured with venier calipers having least count 0.01 cm. Given that length is 5.0 cm. and radius is 2.0 cm. The percentage error in the calculated value of the volume will be

(a) 1% (b) 2% (c) 3% (d) 4%

Solution : (c)

Problem 6. If there is a positive error of 50% in the measurement of velocity of a body, then the error in the measurement of kinetic energy is

(a) 25% (b) 50% (c) 100% (d) 125%

Solution : (d) % error in measuring kinetic energy $= \frac{\frac{1}{2}m(1.5V)^2 - \frac{1}{2}mv^2}{\frac{1}{2}mv^2} \times 100 = 125\%$

We can't apply $\frac{\Delta k}{k} = 2\frac{\Delta V}{V}$, as error is large in this case.

EXERCISE – I

1. Which one of the following is *not* a unit of length?
(A) angstrom (B) light year (C) fermi (D) radian

2. The unit of impulse is the same as that of
(A) moment of force (B) linear momentum
(C) rate of change of linear momentum (D) force

3. Which pair of quantities has dimensions different from the other three pairs?
(A) Impulse and linear momentum
(B) Planck's constant and angular momentum
(C) Moment of inertia and moment of force
(D) Young's modulus and pressure

4. The dimensions of the coefficient of viscosity are
(A) ML^2T^{-2} (B) MLT^{-1} (C) $ML^{-1}T^{-1}$ (D) $ML^{-1}T^{-2}$

5. The dimensions of surface tension are
(A) ML^0T^{-2} (B) MLT^{-2} (C) $ML^{-1}T^{-2}$ (D) $ML^{-2}T^{-2}$

6. The SI unit of the universal gravitational constant G is
(A) $Nm\ kg^{-2}$ (B) $Nm^2\ kg^{-2}$ (C) $Nm^2\ kg^{-1}$ (D) $Nm\ kg^{-1}$

7. The dimensions of the coefficient of thermal conductivity are
(A) $MLT^{-3}K^{-1}$ (B) $MLT^{-2}K^{-1}$ (C) $MLT^{-1}K^{-1}$ (D) $MLT^{-2}K^{-2}$

8. The SI unit of Stefan's constant is
(A) $W s^{-1} m^{-2} K^{-4}$ (B) $J s m^{-2} K^{-4}$ (C) $J s^{-1} m^{-2} K^{-1}$ (D) $W m^{-2} K^{-4}$

9. The SI unit of magnetic permeability is
(A) $A\ m^{-1}$ (B) A m
(C) $H\ m^{-1}$ (D) No unit; it is a dimensionless number

10. The quantities *L/R* and *RC* (where *L, C* and *R* stand for inductance, capacitance and resistance respectively) have the same dimensions as those of
(A) velocity (B) acceleration (C) time (D) force

11. The dimensions of entropy are
(A) $M^0L^{-1}T^0K$ (B) $M^0L^{-2}T^0K^2$ (C) $MLT^{-2}K$ (D) $ML^2T^{-2}K^{-1}$

12. What is the physical quantity whose dimensions are ML^2T^{-2}?
(A) kinetic energy (B) pressure (C) viscosity (D) power

13. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?
(A) torque (B) surface tension (C) viscosity (D) stress

14. The dimensions of angular momentum are
(A) MLT^{-1} (B) ML^2T^{-1} (C) $ML^{-1}T$ (D) ML^0T^{-2}

15. The gravitational force F between two masses m_1 and m_2 separated by a distance r is given by $F=\frac{Gm_1m_2}{r^2}$ where G is universal gravitational constant. What are the dimensions of G?
(A) $M^{-1}L^3T^{-2}$ (B) ML^3T^{-2} (C) ML^2T^{-3} (D) $M^{-1}L^2T^{-3}$

16. The equation of state of a real gas can be expressed as $\left(P+\frac{a}{V^2}\right)(V-b)=cT$ where P is thepressure, V the volume, T the absolute temperature and a, b and c are constant. What are the dimensions of a?
(A) $M^0L^3T^{-2}$ (B) ML^5T^{-2} (C) $M^0L^3T^0$ (D) $ML^{-2}T^5$

17. The equation of state for *n* moles of an ideal gas is $PV = nRT$ where *R* is the universal gas constant and *P, V* and *T* have the usual meanings. What are the dimensions of *R*?
(A) $M^0LT^{-2}K^{-1}\ mol^{-1}$ (B) $ML^2T^{-2}K^{-1}\ mol^{-1}$ (C) $M^0L^2T^{-2}K^{-1}\ mol^{-1}$ (D) $ML^{-2}T^2K^{-1}mol^{-1}$

18. The SI unit of the universal gas constant *R* is
(A) erg $K^{-1}\ mol^{-1}$ (B) watt $K^{-1}\ mol^{-1}$ (C) newton $K^{-1}\ mol^{-1}$ (D) joule $K^{-1}\ mol^{-1}$

19. According to the quantum theory, the energy *E* of a photon of frequency *v* is given by $E = hv$where *h* is Planck's constant. What is the dimensional formula for *h?*
(A) ML^2T^{-2} (B) ML^2T^{-1} (C) ML^2T (D) ML^2T^2

20. What is the SI unit of Planck's constant?
(A) watt second (B) watt per second (C) joule second (D) joule per second

21. The dimensions of Planck's constant are the same as those of
(A) energy (B) power (C) angular frequency (D) angular momentum

22. Time period T of a simple pendulum may depend on *m,* the mass of the bob, *1,* the length of the string and g, the acceleration due to gravity, i.e. $T\propto m^a\, l^b\, g^c$
What are the values of a, b and c?
(A) $0,\frac{1}{2},-\frac{1}{2}$ (B) $0,-\frac{1}{2},\frac{1}{2}$ (C) $\frac{1}{2},0,-\frac{1}{2}$ (D) $-\frac{1}{2},0,\frac{1}{2}$

23. The volume *V* of water passing any point of a uniform tube during *t*seconds is related to the cross-sectional area *A* of the tube and velocity *u* of water by the relation $V\propto A^\alpha u^\beta t^\gamma$ Which one of the following will be true?
(A) $\alpha=\beta=\gamma$ (B) $\alpha\neq\beta=\gamma$ (C) $\alpha=\beta\neq\gamma$ (D) $\alpha\neq\beta\neq\gamma$

24. Which one of the following relations is dimensionally consistent where *h* is height to which a liquid of density ρ rises in a capillary tube of radius, *r,* T is the surface tension of the liquid, θ the angle of contact and g the acceleration due to gravity?
(A) $h=\frac{2T\cos\theta}{r\,\rho g}$ (B) $h=\frac{2Tr}{\rho g\cos\theta}$ (C) $h=\frac{2\rho g\cos\theta}{2T\ r}$ (D) $h=\frac{2Tr\rho g}{\cos\theta}$

25. The frequency n of vibrations of uniform string of length l and stretched with a force F is given by $n = \frac{P}{2l}\sqrt{\frac{F}{m}}$ where p is the number of segments of the vibrating string and m is a constant of the string. What are the dimensions of *m?*
(A) $ML^{-1}T^{-1}$ (B) $ML^{-3}T^{0}$ (C) $ML^{-2}T^{0}$ (D) $ML^{-1}T^{0}$

26. When a wave traverses a medium, the displacement of a particle located at x at time t is given by $y = a \sin(bt - cx)$ where a, b and c are constants of the wave. The dimensions of b are the same as those of
(A) wave velocity (B) amplitude (C) wavelength (D) wave frequency

27. In Q.26, the dimensions of $\frac{b}{c}$ are the same as those of
(A) wave velocity (B) wavelength (C) wave amplitude (D) wave frequency

28. The van der Waal equation for n moles of a real gas is $\left(P + \frac{a}{V^2}\right)(V - b) = nRT$ where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are van der Waal constants. The dimensions of a are the same as those of
(A) PV (B) PV^2 (C) P^2V (D) P/V

29. In Q.28, the dimensions of b are the same as those of
(A) P (B) V (C) PV (D) nRT

30. In Q.28, the dimensions of nRT are the same as those of
(A) energy (B) force (C) pressure (D) specific heat

31. In Q. 28, the dimensional formula for *ab*is
(A) ML^2T^{-2} (B) ML^4T^{-2} (C) ML^6T^{-2} (D) ML^8T^{-2}

32. If velocity *(V),* acceleration *(A)*and force *(F)* are taken as fundamental quantities instead of mass (M), length (L) and time (T), the dimensions of Young's modulus would be
(A) FA^2V^{-2} (B) FA^2V^{-3} (C) FA^2V^{-4} (D) FA^2V^{-5}

33. The dimensions of permittivity (ε_0) of vacuum are
(A) $M^{-1}L^{-3}T^4A^2$ (B) $ML^{-3}T^2A^2$ (C) $M^{-1}L^3T^4A^2$ (D) $ML^3T^2A^2$

34. What are the dimensions of permeability (μ_0) of vacuum?
(A) $MLT^{-2}A^2$ (B) MLT^2A^{-2} (C) $ML^{-1}T^{-2}A^2$ (D) $ML^{-1}T^{-2}A^{-2}$

35. The dimensions of $1/\sqrt{\mu_0\varepsilon_0}$ are the same as those of
(A) velocity (B) acceleration (C) force (D) energy

36. The dimensions of specific heat are
(A) $MLT^{-2}K^{-1}$ (B) $ML^2T^{-2}K^{-1}$ (C) $M^0L^2T^{-2}K^{-1}$ (D) $M^0LT^{-2}K^{-1}$

37. What are the dimensions of latent heat?
(A) ML^2T^{-2} (B) $ML^{-2}T^{-2}$ (C) M^0LT^{-2} (D) $M^0L^2T^{-2}$

38. What are the dimensions of Boltzmann's constant?
(A) $MLT^{-2}K^{-1}$ (B) $ML^2T^{-2}K^{-1}$ (C) $M^0LT^{-2}K^{-1}$ (D) $M^0L^2T^{-2}K^{-1}$

39. The dimensions of potential difference are
(A) $ML^2T^{-3}A^{-1}$ (B) $MLT^{-2}A^{-1}$ (C) $ML^2T^{-2}A$ (D) $MLT^{-2}A$

40. What are the dimensions of electrical resistance?
(A) $ML^2T^{-2}A^2$ (B) $ML^2T^{-3}A^{-2}$ (C) $ML^2T^{-3}A^2$ (D) $ML^2T^{-2}A^{-2}$

41. The dimensions of electric field are
(A) $MLT^{-3}A^{-1}$ (B) $MLT^{-2}A^{-1}$ (C) $MLT^{-1}A^{-1}$ (D) MLT^0A^{-1}

42. The dimensions of magnetic induction field are
(A) $ML^0T^{-1}A^{-1}$ (B) $M^0LT^{-1}A^{-1}$ (C) $MLT^{-2}A^{-1}$ (D) $ML^0T^{-2}A^{-1}$

43. What are the dimensions of magnetic flux?
(A) $ML^2T^{-2}A^{-1}$ (B) $ML^2T^{-2}A^{-2}$ (C) $ML^{-2}T^{-2}A^{-1}$ (D) $ML^{-2}T^{-2}A^{-2}$

44. The dimensions of self inductance are
(A) $ML^2T^{-2}A^{-1}$ (B) $ML^2T^{-2}A^{-2}$ (C) $ML^{-2}T^{-2}A^{-1}$ (D) $ML^{-2}T^{-2}A^{-2}$

45. The dimensions of capacitance are
(A) $M^{-1}L^{-2}TA^2$ (B) $M^{-1}L^{-2}T^2A^2$ (C) $M^{-1}L^{-2}T^3A^2$ (D) $M^{-1}L^{-2}T^4A^2$

46. If velocity (*V*), force *(F)* and energy (E) are taken as fundamental units, then dimensional formula for mass will be
(A) $V^{-2}F^0E^{-1}$ (B) V^0FE^2 (C) $VF^{-2}E^0$ (D) $VF^{-2}E^0$

47. Frequency *(n)* of a tuning fork depends upon length *(1)* of its prongs, density (ρ) and Young's modulus (Y) of its material. Then frequency and Young's modulus will be related as
(A) $n \propto \sqrt{Y}$ (B) $n \propto Y$ (C) $n \propto \frac{1}{\sqrt{Y}}$ (D) $n \propto \frac{1}{Y}$

48. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ ($\varepsilon_0 =$ permittivity of free space and E = electric field) are
(A) MLT^{-1} (B) ML^2T^{-2} (C) $ML^{-1}T^{-2}$ (D) ML^2T^{-1}

49. Of the following quantities, which one has dimensions different from the remaining three
(A) Energy per unit volume
(B) Force per unit area
(C) Product of voltage and charge per unit volume
(D) Angular momentum

50. If the time period t of a drop of liquid of density d, radius r, vibrating surface tension s is given by the formula $t=\sqrt{d^a r^b s^c}$ and if $a=1, c=1$, then b is
(A) 1 (B) 2 (C) 3 (D) 4

51. A pair of physical quantities having the same dimensional formula is
(A) angular momentum and torque (B) torque and energy
(C) entropy and power (D) power and angular momentum

52. In the measurement of a physical quantity $X=\frac{A^2B}{C^{1/3}D^3}$. The percentage errors introduced in the measurements of the quantities A,B,C and D 2%, 2%, 4% and respectively. Then the minimum amount of percentage of error in the measurement of X is contributed by:
(A) A (B) B (C) C (D) D

53. Which of the following has the dimensions $ML^{-1}T^{-1}$?
(A) Surface tension (B) Coefficient of viscosity
(C) Bulk modulus (D) Angular momentum

54. Pressure gradient dp/px is the rate of change of pressure with distance. What are the dimensions of dp/px?
(A) $ML^{-1}T^{-1}$ (B) $ML^{-2}T^{-2}$ (C) $ML^{-1}T^{-2}$ (D) $ML^{-2}T^{-1}$

55. If E, M, J and G respectively denote energy, mass, angular momentum and gravitational constant, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of
(A) length (B) angle (C) mass (D) time

56. If e, ε_0, h and c respectively represent electronic charge, permittivity of free space, Planck's constant and speed of light, then $\frac{e^2}{\varepsilon_0 hc}$ has the dimensions of
(A) current (B) pressure
(C) angular momentum (D) angle

57. If L, R, C and V respectively represent inductance, resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as those of
(A) current (B) $\frac{1}{\text{current}}$ (C) charge (D) $\frac{1}{\text{charge}}$

58. If E and B respectively represent electric field and magnetic induction field, then the ratio $\frac{E}{B}$ has the dimensions of
(A) displacement (B) velocity (C) acceleration (D) angle

59. If C and V respectively represent the capacitance of a capacitor and the potential difference between its plates, then the dimensions of CV^2 are
(A) ML^2T^{-2} (B) $ML^3T^{-2}I^{-1}$ (C) $ML^2T^{-1}I^{-1}$ (D) $M^0L^0T^0$

60. If h and e respectively represent Planck's constant and electronic charge, then the dimensions of $\left(\frac{h}{e}\right)$ are the same as those of
(A) magnetic field (B) electric field (C) magnetic flux (D) electric flux

61. If energy E, velocity V and time T are chosen as the fundamental units, the dimensional formula for surface tension will be
(A) EV^2T^{-2} (B) $EV^{-1}T^{-2}$ (C) $EV^{-2}T^{-2}$ (D) $E^2V^{-1}T^{-2}$

62. The number of particles crossing a unit area perpendicular to the x- axis in a unit time is given by $n = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$ where n_1 and n_2 are the number of particles per unit volume at $x = x_1$ and $x = x_2$ respectively and D is the diffusion constant. The dimensions of D are
(A) M^0LT^{-2} (B) $M^0L^2T^{-4}$ (C) M^0LT^{-3} (D) $M^0L^2T^{-1}$

63. A gas bubble from an explosion under water oscillates with a period proportional to $P^a d^b E^c$ where P is the static pressure, d is the density of water and E is the energy of explosion. Then a, b and c respectively are
(A) $\frac{-5}{6}, \frac{1}{2}, \frac{1}{3}$ (B) $\frac{1}{2}, \frac{-5}{6}, \frac{1}{3}$ (C) $\frac{1}{3}, \frac{1}{2}, \frac{-5}{6}$ (D) 1, 1, 1

64. In a system of units in which the unit of mass is a kg unit of length is b metre and the unit of time is c second, the magnitude of a calorie is
(A) $\frac{4.2\,c}{ab^2}$ (B) $\frac{4.2\,c^2}{ab^2}$ (C) $\frac{abc}{4.2}$ (D) $\frac{4.2}{abc}$

65. The error in the measurement of the radius of a sphere is 1%. The error in the measurement of the volume is
(A) 1% (B) 3% (C) 5% (D) 8%

66. If the error in the measurement of the volume of a sphere is 6% , then the error in the measurement of its surface area will be
(A) 2% (B) 3% (C) 4% (D) 7.5%

67. A physical quantity X is represented by $X = \left(M^xL^yT^{-z}\right)$. The maximum percentage errors in the measurement of M,L and T respectively are a%, b% and c% . The maximum percentage error in the measurement of X will be
(A) $(ax + by - cz)$ percent (B) $(ax + by + cz)$ percent
(C) $(ax - by + cz)$ percent (D) $(ax - by - cz)$ percent

68. The percentage errors in the measurements of the length of a simple pendulum and its time period are 2% and 3% respectively. The maximum error in the value of the acceleration due to gravity obtained from these measurements is
(A) 5% (B) 1% (C) 8% (D) 10%

69. The moment of inertia of a body rotating about a given axis is $6.0\,\text{kg}\,\text{m}^2$ in the SI system. What is the value of the moment of intertia in a system of units in which the unit of length is 5 cm and the unit of mass is 10 g?
(A) 2.4×10^3 (B) 2.4×10^5 (C) 6.0×10^3 (D) 6.0×10^5

70. A quantity X is given by $\varepsilon_0 L\dfrac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of free space, L is length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of
(A) resistance (B) charge (C) voltage (D) current

71. The coefficient of viscosity (η) of a liquid by the method of flow through a capillary tube is given by the formula $\eta = \dfrac{\pi}{8}\dfrac{R^4}{l}\dfrac{P}{Q}$
where $R =$ radius of the capillary tube,
$l =$ length of the tubs and
$P =$ Pressure difference between its ends, and
Q = volume of liquid flowing per second.
Which quantity must be measured most accurately?
(A) R (B) l (C) P (D) Q

72. The mass m of the heaviest stone that can be moved by the water flowing in a river depends on υ, the speed of water, density (D) of water and the acceleration due to gravity (g). Then m is proportional to
(A) υ^2 (B) υ^4 (C) υ^6 (D) υ^8

73. The speed (υ) of ripples depends upon their wavelength (λ), density (ρ) and surface tension (σ) of water. Then υ is proportional to
(A) $\sqrt{\lambda}$ (B) λ (C) $\dfrac{1}{\lambda}$ (D) $\dfrac{1}{\sqrt{\lambda}}$

74. The period of revolution (T) of a planet moving round the sun in a circular orbit depends upon the radius (r) of the orbit, mass (M) of the sun and the gravitation constant (G). Then T is proportional to
(A) $r^{1/2}$ (B) r (C) $r^{3/2}$ (D) r^2

75. If energy (E), momentum (ρ) and force (F) are chosen as fundamental units, the dimensions of mass in the new system will be
(A) $E^{-1}P^2F^0$ (B) $E^1P^{-2}F^0$ (C) $E^{-1}P^2F^{-2}$ (D) $E^{-2}P^1F^2$

76. If the velocity of light (c), gravitational constant (G) and planck's constant (G) and planck's constant (h) are chosen as fundamental units, the dimensions of time in the new system will be
(A) $C^{-5/2}G^2h^{-1/2}$ (B) $C^{-3/2}G^{-2}h^2$ (C) $C^2G^{-2}h^{1/2}$ (D) $C^{-5/2}G^{1/2}h^{1/2}$

77. The amplitude of a damped oscillator of mass m varies with time t as $A = A_0 e^{(-at/m)}$. The dimensions of a are
(A) ML^0T^{-1} (B) M^0LT^{-1} (C) MLT^{-1} (D) $ML^{-1}T$

78. A student measures the value of g with the help of a simple pendulum using the formula

$$g = \frac{4\pi^2 L}{T^2}$$

The errors in the measurements of L and T are ΔL and ΔT respectively. In which of the following cases is the error in the value of g the minimum?

(A) $\Delta L = 0.5\,cm, \Delta T = 0.5\,s$ (B) $\Delta L = 0.2\,cm, \Delta T = 0.2\,s$

(C) $\Delta L = 0.1\,cm, \Delta T = 1.0\,s$ (D) $\Delta L = 0.1\,cm, \Delta T = 0.1\,s$

79. A student performs an experiment to determine the Young's modulus of a wire, exactly 2m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of $\pm 0.05\,mm$ at a load of exactly $1.0\,kg$. The student also measures the diameter of the wire to be $0.4\,mm$ with a uncertainty of $\pm 0.01\,mm$. Take $g = 9.8\,m/s^2$ (exact). The Young's modulus obtained from the reading is

(A) $(2.0 \pm 0.3) \times 10^{11}\ N/m^2$ (B) $(2.0 \pm 0.2) \times 10^{11}\ N/m^2$

(C) $(2.0 \pm 0.1) \times 10^{11}\ N/m^2$ (D) $(2.0 \pm 0.05) \times 10^{11}\ N/m^2$

80. In a vernier callipers, one main scale division is $x\,cm$ and n divisions of the vernier scale coincide with $(n-1)$ divisions of the main scale. The least count (in cm) of the callipers is

(A) $\left(\frac{n-1}{n}\right)x$ (B) $\frac{nx}{(n-1)}$ (C) $\frac{x}{n}$ (D) $\frac{x}{(n-1)}$

EXERCISE – II

PASSAGE – I

The dimensional method is a very convenient way of finding the dependence of a physical quantity on other physical quantities of a given system. This method has its own limitations. In a complicated situation, it is often not easy to guess the factors on which a physical quantity will depend. Secondly, this method gives no information about the dimensionless proportionality constant. Thirdly, this method is used only if a physical quantity depends on the product of other physical quantities. Fourthly, this method will not work if a physical quantity depends on another quantity as a trigonometric or exponential function. Finally, this method does not given complete information in cases where a physical quantity depends on more than three quantities in problem in mechanics.

1. The dimensional method cannot be used to obtain dependence of

(A) the height to which a liquid rises in a capillary tube on the angle of contact

(B) speed of sound in an elastic medium on the modulus of electricity.

(C) height to which a body, projected upwards with a certain velocity, will rise on time t.

(D) the decrease in energy of a damped oscillator on time t.

2. In dimensional method, the dimensionless proportionally constant is to be determined

(A) experimentally

(B) by a detailed mathematical derivation

(C) by using the principle of dimensional homogeneity.

(D) by equating the powers of M,L and T

PASSAGE – II

In the study of physics, we often have to measure the physical quantities. The numerical value of a measured quantity can only be approximate as it depends upon the least count of the measuring instrument used. The number of significant figures in any measurement indicates the degree of precision of that measurement. The importance of significant figures lies in calculation. A mathematical calculation cannot increase the precision of a physical measurement. Therefore, the number of significant figures in the sum or product of a group of number cannot be greater than the number that has the least number of significant figure. A chain cannot be stronger than its weakest link.

3. A bee of mass $0.000087\,kg$ site on a flower of mass $0.0123\,kg$. What is the total mass supported by the stem of the flower upto appropriate significant figures?

(A) $0.012387\,kg$ (B) $0.01239\,kg$ (C) $0.0124\,kg$ (D) $0.012\,kg$

4. The radius of a uniform wire is $r = 0.021\,cm$. The value π is given to be 3.142. What is the area of cross-section of the wire upto appropriate significant figures?

(A) $0.0014\,cm^2$ (B) $0.00139cm^2$ (C) $0.001386\,cm^2$ (D) $0.0013856cm^2$

5. A man runs $100.5\,m$ in $10.3\,s$. His average speed upto appropriate significant figures is

(A) $9.76\,ms^{-1}$ (B) $9.757\,ms^{-1}$ (C) $9.7573\,ms^{-1}$ (D) $9.8\,ms^{-1}$

WINDOW TO JEE MAIN

1. Identify the pair whose dimensions are equal. **[AIEEE 2002]**

(A) Torque and work (B) Stress and energy
(C) Force and stress (D) Force and work

2. The physical quantities not having same dimensions are **[AIEEE 2003]**

(A) torque and work (B) momentum and Planck's constant
(C) stress and Young's modulus (D) speed and $(\mu_0\varepsilon_0)^{-1/2}$

3. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are **[AIEEE 2003]**

(A) $[L^{-1}T]$ (B) $[L^2T^2]$ (C) $[L^2T^{-2}]$ (D) $[LT^{-1}]$

4. Which one of the following represents the correct dimensions of the coefficient of viscosity? **[AIEEE 2004]**

(A) $[ML^{-1}T^{-2}]$ (B) $[MLT^{-1}]$ (C) $[ML^{-1}T^{-1}]$ (D) $[ML^{-2}T^{-2}]$

5. Out of the following pairs, which one does not have identical dimensions? **[AIEEE 2005]**

(A) Angular momentum and Planck's constant
(B) Impulse and momentum
(C) Moment of inertia and moment of a force
(D) Work and torque

6. The 'rad' is the correct unit used to report the measurement of **[AIEEE 2006]**
(A) the ability of a beam of gamma ray photons to produce ions in a target
(B) the energy delivered by radiation to a target
(C) the biological effect of radiation
(D) the rate of decay of a radioactive source

7. Which of the following units denotes the dimensions $[ML^2/Q^2]$, where Qdenotes the electric charge? **[AIEEE 2006]**
(A) Wb/m^2 (B) henry (H) (C) H/m^2 (D) weber (Wb)

8. A body of mass $m = 3.513$kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as **[AIEEE 2008]**
(A) 17.6 kg ms^{-1} (B) $17.565 \text{ kg ms}^{-1}$ (C) 17.56 kg ms^{-1} (D) 17.57 kg ms^{-1}

9. The dimensions of magnetic field in M, L, T and C (coulomb) is given as **[AIEEE 2008]**
(A) $[MLT^{-1}C^{1}]$ (B) $[MT^2C^{-2}]$ (C) $[MT^{-1}C^{-1}]$ (D) $[MT^2C^{-1}]$

10. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half – a – degree (= 0.5°), then the least count of the instrument is **[AIEEE 2009]**
(A) one minute (B) half minute (C) one degree (D) half degree

11. The respective number of significant figures for the numbers 23.023, 0.0003, 2.1×10^{-3} are **[AIEEE 2010]**
(A) 5, 1, 2 (B) 5, 1, 5 (C) 5, 5, 2 (D) 4, 4, 2

12. The dimensions of angular momentum, latent heat and capacitance are, respectively. **[JEE MAIN ONLINE 2013]**
(A) $ML^2T^1A^2, L^2T^{-2}, M^{-1}L^{-2}T^2$ (B) $ML^2T^{-2}, L^2T^2, M^{-1}L^{-2}T^4A^2$
(C) $ML^2T^{-1}, L^2T^{-2}, ML^2TA^2$ (D) $ML^2T^{-1}, L^2T^{-2}, M^{-1}L^{-2}T^4A^2$

13. A metal sample carrying a current along X – axis with density J_x is subjected to a magnetic field B_z (along z - axis). The electrical field E_y developed along Y – axis is directly proportional to J_x as well as B_z. The constant of proportionality has SI unit. **[JEE MAIN ONLINE 2013]**
(A) $\frac{m^3}{As}$ (B) $\frac{m^2}{A}$ (C) $\frac{As}{m^3}$ (D) $\frac{m^2}{A}$

14. If the time period t of the oscillation of a drop of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{r^{2b}\ S^c\ d^{a/2}}$. It is observed that the time period is directly proportional to $\sqrt{\frac{d}{s}}$. The value of b should therefore be: **[JEE MAIN ONLINE 2013]**
(A) $\frac{3}{4}$ (B) $\sqrt{3}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

15. Let $[e_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then : **[JEE MAIN OFFLINE 2013]**
(A) $[e_0] = [M^{-1}L^{-3}T^4A^2]$ (B) $[e_0] = [M^{-1}L^2T^1A^{-2}]$
(C) $[e_0] = [M^{-1}L^2T^1A^1]$ (D) $[e_0] = [M^{-1}L^{-3}T^2A^1]$

16. An experiment is performed to obtain the value of acceleration due to gravity g by using a simple pendulum of length L. In this experiment time for 100 oscillations is measured by using a watch of 1 second least count and the value is 90.0 seconds. The length L is measured by using a meter scale of least count 1 mm and the value is 20.0 cm. The error in the determination of g would be :

[JEE MAIN ONLINE 2014]

(A) 1.7% (B) 2.7% (C) 4.4% (D) 2.27%

17. From the following combinations of physical constants (expressed through their usual symbols) the only combination, that would have the same value in difference systems of units, is :

[JEE MAIN ONLINE 2014]

(A) $\frac{ch}{2\pi \epsilon_O^2}$ (B) $\frac{e^2}{2\pi \epsilon_O Gm_e^2}\left(m_e = \text{mass of electron}\right)$

(C) $\frac{\mu_o \epsilon_o}{c^2}\frac{G}{he^2}$ (D) $\frac{2\pi\sqrt{\mu_o \epsilon_o}}{ce^2}\frac{h}{G}$

18. The current voltage relation of diode is given by I = ($e^{1000V/T}$–1) mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring $\pm$ 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

[JEE MAIN OFFLINE 2014]

(A) 0.02 mA (B) 0.5 mA (C) 0.05 mA (D) 0.2 mA

19. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it? **[JEE MAIN OFFLINE 2014]**

(A) A vernier caliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(B) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
(C) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.
(D) A meter scale.

20. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of g is:

[JEE MAIN OFFLINE 2015]

(A) 2% (B) 3% (C) 1% (D) 5%

21. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation AD = C *ln*(*BD*) holds true. Then which of the combination(s) is not a meaningful quantity? **[JEE MAIN ONLINE 2016]**

(i) $A^2 - B^2C^2$ **(ii)** $\frac{(A-C)}{D}$ **(iii)** $\frac{A}{B} - C$ **(iv)** $\frac{C}{BD} - \frac{AD^2}{C}$

(A) all (B) i, ii & iv (C) ii & iv (D) only ii

22. A student measures the time period of 100 oscillations of a simple pendulum for times. The data set is 90s, 91s, 95 s and 92 s. If the minimum division in the measuring clock is 1s, then the reported mean time should be : **[JEE MAIN OFFLINE 2016]**

(A) $92 \pm 5.0\,s$ (B) $92 \pm 1.8\,s$ (C) $92 \pm 3\,s$ (D) $92 \pm 2\,s$

23. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminum. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45^{th} division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25^{th} division coincides with the main scale line?

[JEE MAIN OFFLINE 2016]

(A) 0.80 mm (B) 0.70 mm (C) 0.50 mm (D) 0.75 mm

24. In the following 'I' refers to current and other symbols have their usual meaning. Choose the option that corresponds to the dimensions of electrical conductivity: **[JEE MAIN ONLINE 2016]**

(A) $ML^{-3}T^{-3}I^2$ (B) $M^{-1}L^3T^3I$ (C) $M^{-1}L^{-3}T^3I^2$ (D) $M^{-1}L^{-3}T^3I$

25. Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be :

[JEE MAIN ONLINE 2017]

(A) $[M] = [T^{-1}C^{-2}h]$ (B) $[M] = [T^{-1}C^2h]$

(C) $[M] = [T^{-1}C^{-2}h^{-1}]$ (D) $[M] = [TC^{-2}h]$

26. A physical quantity P is described by the relation

$$P = a^{1/2}b^2c^3d^{-4}$$

If the relative errors in the measurement of a, b, c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be : **[JEE MAIN ONLINE 2017]**

(A) 8% (B) 12% (C) 32% (D) 25%

27. The following observations were taken for determining surface tension T of water by capillary method: diameter of capillary, $D = 1.25\times10^{-2}\,m$ rise of water, $h = 1.45\times10^{-2}\,m$

Using g = 9.80 m/s^2 and the simplified relation $T = \frac{rhg}{2}\times10^3\,N/m$, the possible error in surface tension is closest to: **[JEE MAIN OFFLINE 2017]**

(A) 1.5% (B) 2.4% (C) 10% (D) 0.15%

WINDOWS TO JEE ADVANCED

1. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is **[2011]**

(A) 0.9% (B) 2.4% (C) 3.1% (D) 4.2%

2. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length L = 2 and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension l = 0.25 mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

[2012]

(A) due to the errors in the measurement of d and l are the same

(B) due to error in the measurement of d is twice that due to the error in the measurement of l.

(C) due to the error in the measurement of *l* is twice that due to the error in the measurement of *d*.
(D) due to the error in the measurement of *d* is four times that due to the error in the measurement of *l*.

3. The diameter of a cylinder is measured using a Vernier calipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is **[2013]**
(A) 5.112 cm (B) 5.124 cm (C) 5.136 cm (D) 5.148 cm

4. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale division. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is **[2014]**

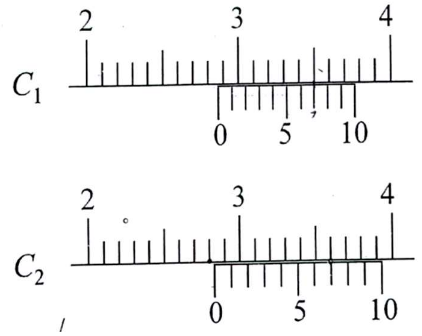
5. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then:
(A) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.
(B) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.
(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.
(D) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm. **[2015]**

6. Planck's constant *h*, speed of light *c* and gravitational constant G are used to form a unit of length *L* and a unit of mass *M*. Then the correct option(s) is(are) **[2015]**
(A) $M \propto \sqrt{c}$ (B) $M \propto \sqrt{G}$ (C) $L \propto \sqrt{h}$ (D) $L \propto \sqrt{G}$

7. In terms of potential difference *V*, electric current *I*, permittivity ε_0, permeability μ_0 and speed of light *c*, the dimensionally correct equations) is(are) **[2015]**
(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$ (C) $I = \varepsilon_0 c V$ (D) $\mu_0 c I = \varepsilon_0 V$

8. A length-scale (*l*) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression (s) for *l* is (are) dimensionally correct? **[2016]**
(A) $l = \sqrt{\left(\dfrac{nq^2}{\varepsilon k_B T}\right)}$ (B) $l = \sqrt{\left(\dfrac{\varepsilon k_B T}{nq^2}\right)}$ (C) $l = \sqrt{\left(\dfrac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D) $l = \sqrt{\left(\dfrac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

9. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are **[2016]**



(A) 2.87 and 2.86 (B) 2.87 and 2.87 (C) 2.85 and 2.82 (D) 2.87 and 2.83

10. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true? **[2016]**

(A) The error in the measurement of r is 10%
(B) The error in the measurement of T is 3.57%
(C) The error in the measurement of T is 2% .
(D) The error in the determined value of g is 11%

ANSWER KEY

EXERCISE – I

1.	(D)	2.	(B)	3.	(C)	4.	(C)	5.	(A)
6.	(B)	7.	(A)	8.	(D)	9.	(C)	10.	(C)
11.	(D)	12.	(A)	13.	(D)	14.	(B)	15.	(A)
16.	(B)	17.	(B)	18.	(D)	19.	(B)	20.	(C)
21.	(D)	22.	(A)	23.	(B)	24.	(A)	25.	(D)
26.	(D)	27.	(A)	28.	(B)	29.	(B)	30.	(A)
31.	(D)	32.	(C)	33.	(A)	34.	(B)	35.	(A)
36.	(C)	37.	(D)	38.	(B)	39.	(A)	40.	(B)
41.	(A)	42.	(D)	43.	(A)	44.	(B)	45.	(D)
46.	(D)	47.	(A)	48.	(C)	49.	(D)	50.	(C)
51.	(B)	52.	(C)	53.	(B)	54.	(B)	55.	(B)
56.	(D)	57.	(B)	58.	(B)	59.	(A)	60.	(C)
61.	(C)	62.	(D)	63.	(A)	64.	(B)	65.	(B)
66.	(C)	67.	(B)	68.	(C)	69.	(B)	70.	(D)
71.	(A)	72.	(C)	73.	(D)	74.	(C)	75.	(A)
76.	(D)	77.	(A)	78.	(D)	79.	(B)	80.	(C)

EXERCISE – II

PASSAGE – I :	1.	(1, 3, 4)	2.	(1, 2)			
PASSAGE – II :	3.	(C)	4.	(A)	5.	(A)	

WINDOW TO JEE MAIN

1.	(A)	2.	(B)	3.	(C)	4.	(C)	5.	(C)
6.	(C)	7.	(B)	8.	(A)	9.	(C)	10.	(A)
11.	(A)	12.	(D)	13.	(A)	14.	(C)	15.	(A)
16.	(B)	17.	(B)	18.	(D)	19.	(A)	20.	(B)
21.	(C)	22.	(D)	23.	(A)	24.	(C)	25.	(A)
26.	(C)	27.	(A)						

WINDOW TO JEE ADVANCED

1.	(C)	2.	(A)	3.	(B)	4.	(8)	5.	(BC)
6.	(ACD)	7.	(AC)	8.	(BD)	9.	(D)	10.	(ABD)

PREFACE

As a first step in studying classical mechanics, we describe the motion of an object while ignoring the interaction with external agents that might be causing or modifying that motion. This portion of classical mechanics is called kinematics.

To facilitate the learning process for the students we have split kinematics in to two parts. Kinematics – I deals with motion in one and two dimensions involving uniform acceleration. This chapter consists of summarized text coupled with sufficient number of solved examples of varying difficulties. Kinematics – II is the second chapter in this booklet &deals with calculus based kinematics and non uniform motion. This booklet consists of summarized text coupled with sufficient number of solved examples of varying difficulties, which enables the students to develop problem solving ability along with emphasis on physical concept.

The end–of–chapter problems are categorized into four section, namely Exercise–I (objectives where only one of the option is correct), Exercise–II (objectives where more than one option may be correct), Exercise–III (matrix matches and paragraph type questions), Exercise–IV (subjective questions), to help the student assess his understanding of the concept and further improvise on his problem solving skills. Solutions to all the questions in the booklet are available and will be provided to the students. Every possible attempt has been made to make the booklet flawless. Any suggestions for the improvement of the booklet would be gratefully accepted and acknowledged.

(Dept. of Physics)

IIT–ian’s PACE

IIT–JEE SYLLABUS

Frame of reference, Motion in a straight line: Position – time graph, speed and velocity. Uniform and non – uniform motion, average speed and instantaneous velocity Uniformly accelerated motion, velocity – time, position – time graphs, relations for uniformly accelerated motion. Relative velocity, Motion in a plane, Projectile motion, Uniform circular Motion.

CONTENTS

Kinematics – I Analysis : Tentative Lecture Flow (Board Syllabus & Booklet Discussion Included)

Lecture 1	Definition of position, distance, displacement, average speed, average velocity, uniform motion, uniform acceleration
Lecture 2	Motion of a vertically projected body, (one dimensional motion) solved examples
Lecture 3	Two dimensional motion (Projectile motion) range of a projectile, time of light, general equation of projectile, solved examples.
Lecture 4	Relative velocity concept (One dimension) relative velocity concept (two dimension i.e. rain man problem). Solved examples

KINEMATICS

1. Kinematics

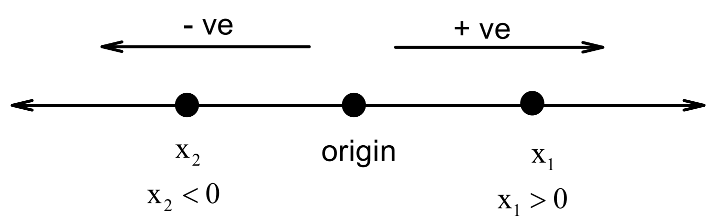
- Kinematics is the study of motion of physical bodies without going into the cause of the motion.
- Kinematics deals with physical quantities like distance, displacement, speed, velocity, acceleration etc.

1.1 Motion and Rest

- Motion is a combined property of the object under study and observer.
- If the position of the object under study changes with time, as seen by the observer, the object is said to be in motion from the frame of reference of the observer.
- If position of the object does not change with time, as seen by observer, object is said to be at rest from the frame of reference of the observer.
- Rest and motion of an object under study depend on the frame of reference of the observer. For eg. A book kept on a table may be at rest for all students sitting in the class. But the same book will be in motion, as seen by an observer on a moving bus. Thus absolute rest and absolute motion are meaningless.
- In most cases, if attributes of motion of an object are given without specifying the frame of the observer, it is to be assumed that the object under consideration is being observed by an observer who is at rest with respect to the earth.

1.2 Position

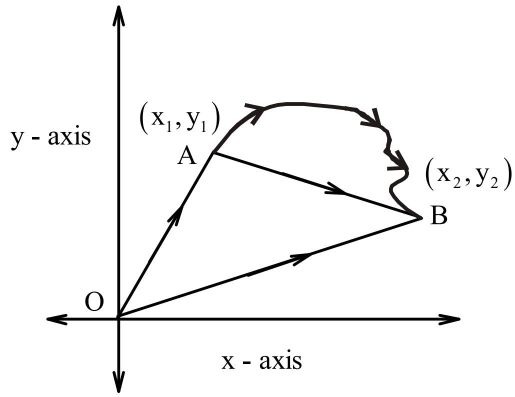
- For a particle moving along a straight line, position of the particle can be specified with only one coordinate. A coordinate system is chosen by choosing some reference point as the origin. The origin is assigned the number zero. Most situations can be analysed by setting up an appropriate coordinate system. In order to do so, following are the essential requirements:
- Choice of origin
- Choice of coordinate axis
- Choice of positive direction of axis. All these parameters constitute a reference frame. In any physics problem, the reference frame must be specified.
- In the figure below, point O is the chosen origin, X – axis is the chosen coordinate axis and rightward direction is chosen as the positive direction.



- Similarly, if the motion of a particle is 2 – dimensional or 3 – dimensional, the coordinate axes will comprise of x, y and z axes and position will include x, y, and z coordinates.

1.3 Displacement and distance :

- Displacement is a vector quantity. It is the change in position vector. Distance is the total length of the actual path covered. Distance is a scalar quantity.
- Suppose a particle travels from point A to point B as shown in the fig below along a zig – zag path, in a finite time interval.



Coordinates of A are (x_1, y_1) and that of B are (x_2, y_2). Position vector of A is $\vec{r}_A = x_1\hat{i} + y_1\hat{j}$, position vector of B is $\vec{r}_B = x_2\hat{i} + y_2\hat{j}$. Distance will be equal to the total length of the actual path covered by the particle. Displacement will be $\vec{S} = \vec{r}_B - \vec{r}_A = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

- The distance covered will always be greater than or equal to the magnitude of the displacement.
- Displacement and distance are equal in magnitude in case the particle is travelling along a straight line without change in direction.
- SI unit of distance and displacement is meters.
- In simple language, displacement can be said to be the shortest line joining the initial and final positions of a body in motion, irrespective of path followed and it is directed from initial position to final position.
- Change in position vector is displacement and change in displacement vector is also displacement.

1.4 Average speed

- Average speed $= \dfrac{\text{Total Distance travelled}}{\text{Total time taken}}$, We define average speed of a particle as the ratio of the total distance travelled to the total time taken.

- SI units of speed is m/s

1.5 Instantaneous speed

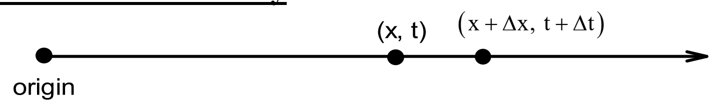
- Speed of a particle at a particular instant is called instantaneous speed.

- The speedometer of a vehicle indicates the instantaneous speed. The speedometer reading on a crowded city road continuously changes, indicating instantaneous speed is continuously changing.

1.6 Velocity

- Velocity is defined as rate of change of displacement with time. Velocity is a vector quantity. SI unit of velocity is m/s.
- Average velocity $= \dfrac{\text{Total Displacement}}{\text{Total time}}$, Average velocity is defined as the ratio of the total displacement covered to the total time taken.
- Just as distance is always greater than or equal to the magnitude of displacement, average speed is greater than or equal to the magnitude of average velocity. Average speed and the magnitude of average velocity are equal when particle is travelling in a straight line without change in direction.

1.7 Instantaneous velocity



- Suppose a particle moves from position x at time t to position $x+\Delta x$ at time $t+\Delta t$. Then, the average velocity of the particle over time interval Δt is $\frac{\Delta x}{\Delta t}$.

Making Δt infinitely small, $\frac{\Delta x}{\Delta t}$ gives the velocity of the particle at instant t and can be written as $v = \lim\limits_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$, where v is the instantaneous velocity of the particle at time instant t.

- The magnitudes of instantaneous velocity and instantaneous speed are always equal.

1.8 Uniform motion

- Motion of a body in a straight line with uniform velocity is called uniform motion.
- $\frac{ds}{dt} = v$, but v is constant in uniform motion.

$\therefore \int ds = \int v\,dt = v\int dt = vt$

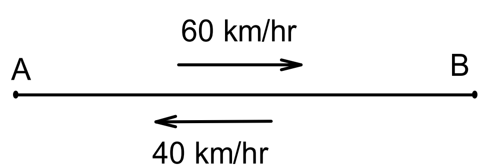
$\therefore s = vt$, where s is the displacement, t is the time interval, v is uniform velocity.

- Uniform motion can also be said to be motion in which equal displacements are covered in equal intervals of time, however small the time intervals may be.

SOLVED EXAMPLES (Based on uniform Motion)

Illustration 1.A particle travels from point A to B in a straight line with uniform speed of 60 km/hr. It immediately returns back from B to A with uniform speed of 40 km/hr. Find average velocity and average speed of particle over the whole journey.

Solution:



Let AB = x

Displacement = 0. $\therefore$ Average velocity $= \frac{\text{Total Displacement}}{\text{Total time}}$

$\therefore$ Average velocity = 0

Total distance = 2 x

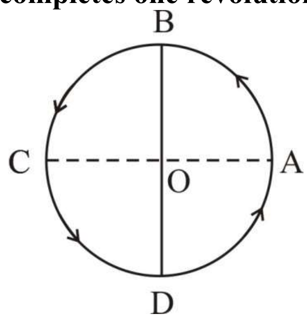
$t_{AB} = \frac{x}{60}$, $t_{BA} = \frac{x}{40}$, Total time $= t_{AB} + t_{BA}$

$\text{Total time} = \frac{x}{60} + \frac{x}{40}$

$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2x}{\frac{x}{60}+\frac{x}{40}}$

$\therefore$ Average speed = 48 km/hr.

Illustration 2. **As shown in fig, a particle moves along a circular path of radius r. It starts from point A and moves anticlockwise. Find the magnitude of displacement and distance travelled by the particle as it**
(i) moves from A to B (ii) moves from A to C (iii) moves from A to D
(iv) completes one revolution



Solution:

(i) From A to B. Distance covered $= \frac{1}{4} \times (2\pi r)$

$= \frac{\pi r}{2}$

Displacement $|\overrightarrow{AB}| = \sqrt{OA^2 + OB^2} = r\sqrt{2}$

(ii) From A to C Distance covered $= \frac{1}{2} \times 2\pi r$

$= \pi r$

Displacement $= |\overrightarrow{AC}| = 2r$

(iii) From A to D, Distance covered $= \frac{3}{4} \times 2\pi r = \frac{3\pi r}{2}$

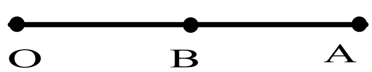
Displacement $= |\overrightarrow{AD}| = r\sqrt{2}$

(iv) From A to A, distance covered $= 2\pi r$

Displacement = 0, because the initial position coincides with final position.

Illustration 3. **An athlete runs 150 m in 15 seconds, then turns around and jogs 50 m back towards starting point in 25 s. What is his average speed and average velocity for total time? Assume he travels along the same straight line always.**

Solution:



O is the origin and OA = 150 m, AB = 50 m
Total distance travelled = OA + AB = 150 + 50 = 200 m
Total time taken = 40 s.

Average speed $= \frac{200\,m}{40\,s} = 5\,m/s$

Total displacement = 100 m (OB)

Average velocity $= \frac{100\,m}{40\,s} = 2.5\,m/s$

Illustration 4. A body covers one – third of its journey with speed V_o. The remaining portion of the distance was covered with velocity V_1, for half the remaining time and with velocity V_2 for the other half of the remaining time. Assuming the body always travels in a straight line, find the average velocity of body over the whole journey.

Solution: Let x be the total distance and T be the total time. We require average velocity $=\frac{x}{T}$

Now time required to cover 1/3rd of total distance $=\frac{x}{3\,Vo}$

As per data

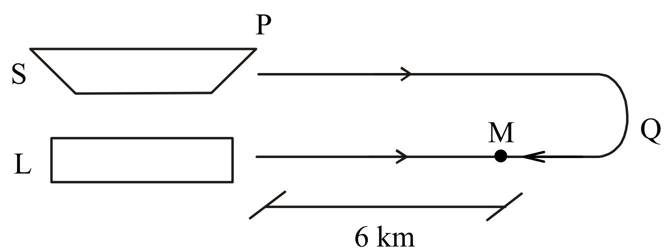
$$\frac{2x}{3}=\frac{1}{2}\times\left[T-\frac{x}{3\,Vo}\right]\times V_1+\frac{1}{2}\times\left[T-\frac{x}{3\,Vo}\right]\times V_2$$

$$x\times\left[\frac{2}{3}+\frac{V_1}{6\,Vo}+\frac{V_2}{6\,Vo}\right]=T\times\left[\frac{V_1}{2}+\frac{V_2}{2}\right]$$

$\therefore$ Average velocity $=\frac{x}{T}=\frac{3\,V_o\left(V_1+V_2\right)}{\left(V_1+V_2+4Vo\right)}$

Illustration 5. A steamer going downstream overcomes a wooden log at point P. 1 hour later, the steamer turns backs and after some time passes the wooden log at a distance 6 km from point P. Find speed of river.

Solution:



Let V_s be speed of steamer in still water.

Let V_r be speed of river.

Let steamer meet the log in a time t further after it has turned.

$PQ=\left(V_s+V_r\right)\times 1$ (Motion of steamer downstream) … Eq. (I)

$QM=\left(V_s-V_r\right)\times t$ (Motion of steamer upstream) … Eq. (II)

$PM=V_r\times\left(t+1\right)$ (Motion of log) … Eq. (III)

$PQ = PM + QM$

$\left(V_s+V_r\right)=V_st-V_rt+V_r+V_rt$ $\quad t = 1$ hour

Substitute in Eq. III

$6=V_r\left(2\right)$ $\quad\therefore\quad$ $V_r=3\,km/hr$

Inchapter Exercise (Based on uniform motion)

1. A car travels first half distance between two places with uniform speed of 60 km / hr. What should be its uniform speed for the second half of the distance so that its average speed over the entire journey becomes 90 km / hr.

2. A body covers one – third of its journey with speed u, next third with speed v and the last third with speed w. Calculate the average speed of the body over the entire journey.

3. A 10 hour trip is made at an average speed of 40 km / hr. If during the first half of the distance, average speed of the bus was 30 km / hr, what was the average speed for the second half of trip?

4. A body, moving in a straight line, covers a certain distance in the following four separate and independent ways.
(a) half the time is covered with speed V_1 and the other half of the time with speed V_2.
(b) half the distance is covered with speed V_1 and next half distance is covered with speed V_2.
(c) $1/3^{rd}$ of the distance is covered with speed V_1 and for the remaining distance, the first $1/4^{th}$ of the remaining time is covered with speed V_2 and $3/4^{th}$ of the remaining time is covered with speed V_3.
(d) $1/5^{th}$ of the time is covered with speed V_1 and in the remaining time, $3/4^{th}$ of remaining distance is covered with speed V_2 and $1/4^{th}$ of the remaining distance is covered with speed V_3.
In each of the above four cases find average speed over the entire journey.

5. A steamer travelling in a river, moves from P to Q (downstream) in 2.5 hours and from Q to P (upstream) in 5 hours. If due to monsoon, speed of the river flow gets doubled, find the new time taken by the same steamer to go from
(a) P to Q (downstream) (b) Q to P (upstream)

ANSWER KEY

1. **Ans. 180 km / hr** 2. **Ans.** $\frac{3\,u\,v\,w}{u\,v+v\,w+u\,w}$ 3. **Ans. 60 km / hr**

4. **Ans:** (a) $\frac{V_1+V_2}{2}$ (b) $\frac{2V_1\,V_2}{V_1+V_2}$ (c) $\frac{3\,V_1\,V_2+9\,V_1\,V_3}{8\,V_1+V_2+3\,V_3}$ (d) $\frac{16\,V_2\,V_3+3\,V_1\,V_3+V_1\,V_2}{15\,V_3+5\,V_2}$

5. **Ans.** (a) 2 hours (b) 10 hours

1.9 Acceleration

- Acceleration is defined as the rate of change of velocity with time.
- Acceleration is a vector quantity.
- SI unit of acceleration is m/s^2.

1.10 Average acceleration

- Average acceleration is defined as the ratio of change in velocity over a time interval to the time interval.
- If a particle moving along a straight line has velocity V_1 at an instant t_1 and velocity V_2 at instant t_2 $(t_2 > t_1)$, then average acceleration during time interval $(t_2 - t_1)$ is given by $a_{av} = \frac{\Delta v}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1}$

1.11 Summary of equations for uniformly accelerated motion.

$v = u + at$ … Eq. I

$s = ut + \frac{1}{2}at^2$ … Eq. II

$x - x_0 = ut + \frac{1}{2}at^2$ … Eq. III

$v^2 = u^2 + 2as$ … Eq. IV

$v^2 = u^2 + 2a(x - x_0)$ … Eq. V

$s = \frac{(u+v)}{2} \times t$ …Eq. VI

$S_{n^{th}} = u + a\left[n - \frac{1}{2}\right]$ … Eq. VII

Where

u – Initial velocity or instantaneous velocity at time t = 0

v – Instantaneous velocity at time instant t

a – uniform acceleration

s – Displacement at time t

t – Time instant

x_0 – Initial position or position at t = 0.

x – Position at time t

$S_{n^{th}}$ - Displacement in n^{th} second

- Equations for uniformly accelerated motion in vector form.

$\vec{v} = \vec{u} + \vec{a}t$; $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$; $\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s}$; $\vec{s} = \frac{(\vec{u}+\vec{v})}{2} \times t$

SOLVED EXAMPLES (Uniformly accelerated motion)

Illustration 6. An automobile manufacturer claims that his sports car will accelerate from rest to a speed of 42 m/s in 8 seconds. Assuming that acceleration is constant.
(a)Determine acceleration of car
(b)Distance travelled by car in 8 s.
(c)Distance travelled by car in 8^{th} second.

Solution:

(a) $v = u + at$ $\therefore$ $a = \frac{v-u}{t} = \frac{42-0}{8} = 5.25\,m/s^2$

(b) Distance travelled in 8 s $s = 0 \times t + \frac{1}{2} \times 5.25 \times 8^2 = 168$ m

(c) $S_{n^{th}} = u + a\left(n - \frac{1}{2}\right) = 0 + 5.25\left[8 - \frac{1}{2}\right] = 39.375$ m

Illustration 7.A particle starts form rest and under uniform acceleration covers a distance x in t sec. Find the distance it will cover in the next t sec.

Solution: $s = ut + \frac{1}{2}at^2$

$x = \frac{1}{2}at^2$ … (I)

Now if it travels additional distance y in next t sec, total distance travelled in 2t s will be

$x + y = 2at^2$ …(II)

Dividing Eq. II by Eq. I

$\frac{x+y}{x} = 4$ $\quad \therefore \quad$ $y = 3x$

Illustration 8.In a car race, car A takes time t less than car B and passes the finishing point with velocity v more than car B. Both cars start from rest and travel with uniform accelerations a_A and a_B **respectively. Show that** $\frac{v}{t} = \sqrt{a_A a_B}$ **.**

Solution: Let s be distance covered by each car. Let times taken by the 2 cars to complete journey be t_1 & t_2, and velocities at finishing point be v_1 & v_2.

$\therefore \quad v_1 - v_2 = v, \qquad t_2 - t_1 = t$

$$\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\sqrt{2a_A s} - \sqrt{2a_B s}}{\sqrt{\frac{2s}{a_B}} - \sqrt{\frac{2s}{a_A}}} \quad \therefore \quad \frac{v}{t} = \sqrt{a_A a_B}$$

Illustration 9. Two particles P and Q starts simultaneously from point A with velocities 15 m/s and 20 m/s respectively. They move in the same direction with different uniform accelerations. When P over takes Q at B, velocity of P is 30 m/s. Find velocity of Q at B.

Solution: For P $\quad s = \left(\frac{15+30}{2}\right) \times t$ … (I)

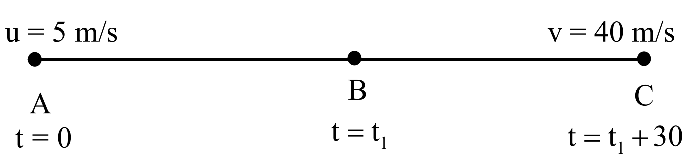
For Q $\quad s = \left(\frac{20+v}{2}\right) \times t$ … (II).

Equating Eq. I and Eq. II

V = 25 m/s

Illustration 10.A particle moves along a straight path ABC with uniform acceleration of $0.5\,m/s^2$**. It crosses A with 5 m/s velocity. It reaches C with a velocity of 40 m/s, 30 seconds after it has crossed B in its path. Find AB.**

Solution:



Consider AC, u = 5 m/s

v = 40 m/s

a = $0.5\,m/s^2$

$$t = \frac{v-u}{a} = \frac{40-5}{0.5} = 70\,s$$

For AB, u = 5 m/s, $a = 0.5 m/s^2$
t = 70 – 30 = 40 s

$AB = s = ut + \frac{1}{2}at^2 = 5 \times 40 + \frac{1}{2} \times 0.5 \times 40^2$

= 600 m

Illustration 11. A particle moving with uniform acceleration in a straight line covers 3 m in the 8th second and 5 m in the 16th second of its motion. What is the displacement of the particle from beginning of 6th second to the end of 15th second?

Solution: $S_{n^{th}} = u + a\left(n - \frac{1}{2}\right)$

$3 = u + a\left[8 - \frac{1}{2}\right]$ … Eq. I

$5 = u + a\left[16 - \frac{1}{2}\right]$ … Eq. II

From Eq. I and Eq. II, $a = \frac{1}{4} m/s^2$

Putting $a = \frac{1}{4} m/s^2$ in Eq. I

We get $u = \frac{9}{8} m/s$

Now, velocity at end of 5 s (velocity at beginning of 6th second), $v_1 = u + 5a$. Velocity after 15 s = $v_2 = u + 15a$. $S = \left(\frac{v_1 + v_2}{2}\right) \times t = \left(\frac{2u + 20a}{2}\right) \times 10 = 36.25$ m

Inchapter Exercise

6. A race car accelerates on a straight road from rest to 180 km/hr in 25s. Assuming uniform acceleration, find the distance covered in this time.

7. A car moving along a straight highway with 126 km/hr speed is brought to rest in 200 m. What is the uniform retardation of the car and how long does it take for the car to stop.

8. A driver takes 0.2s reaction time to apply brakes after he sees the need for it. If he is driving the car at a speed of 54 km/hr, and the brakes cause uniform retardation of $6 m/s^2$, find the distance travelled by the car after he sees the need to apply brakes.

9. A body moving with uniform acceleration, covers 20 m in the 7th second and 24 m in the 9th second of its motion. Find the distance it will cover in the 15th second of its motion.

10. A body moving with uniform acceleration covers 12 m in the 2nd second and 20 m in the 4th second of its motion. How much distance will it cover in 4 seconds after the 5th second?

11. An object is moving along the x – axis with uniform acceleration of $4 m/s^2$. At time t = 0, x = 5 m and v = 3 m/s.
 (a) What will be the velocity and the position of the object at time t = 2s.
 (b) What will be the position of the object when it has velocity of 5 m/s.

12. On a foggy day, drivers of 2 trains travelling in opposite direction on the same railway track spot each other when they are 80 m apart travelling at 72 km/hr and 60 km/hr respectively. Both apply brakes simultaneously retarding their engines at uniform rate of $5\,m/s^2$. Determine the distance by which collision is averted.

ANSWER KEY

6. 625 m	**7.** $3.06\,m/s^2$, 11.43 s	**8.** 21.75 m	**9.** 36 m
10. 136 m	**11.** (a) 11 m/s and 19 m	(b) 7 m	**12.** 12.22 m

1.12 Motion under gravity

- A body thrown vertically upwards or vertically downwards or dropped from a height will move in a straight vertical line.
- If air resistance is ignored, the body will be subjected to acceleration due to gravitational force exerted by the earth, which is denoted by g. The value of g on the earth is $9.8\,m/s^2$ in the downward direction.
- For small heights, the value of g is constant, we can use equations of uniformly accelerated motion.
- We shall take upward direction as positive & down direction as negative, as our convention.

1.13 Motion of a particle projected vertically upward from the ground.

- Consider a particle projected vertically upward from the ground with velocity u.
- Taking upward direction positive u = u, a = – g

$\therefore$ At any time t, velocity v = u – g t and displacement $s = u\,t - \frac{1}{2}gt^2$

- To find time of ascent (t_a), apply v = u + at between the point of projection and the highest point.

  v = 0, u = u, a = – g.

  $\therefore \quad t_a = \frac{u}{g}$

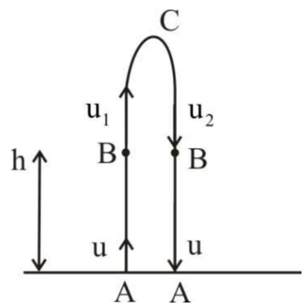
- To find total time of flight (T), apply $s = ut + \frac{1}{2}at^2$ between the point of projection and the time instant when the particle is again at point of projection $s = 0,\ u = u,\ a = -g$

  $\therefore \quad T = \frac{2u}{g}$

- Time of descent (t_d) between the highest point and back to the point of projection is also $\frac{u}{g}$

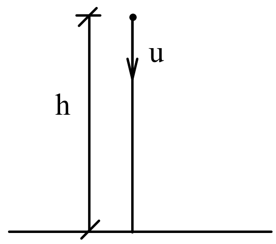
  $\therefore t_d = \frac{u}{g}$

- For maximum height attained (h_{max}) apply $v^2 = u^2 + 2as$ between the point of projection and the topmost point, $v = 0, u = u, a = -g \ \therefore h_{max} = \frac{u^2}{2g}$
- The particle will return back to the point of projection with same speed as the speed of projection but in the opposite direction.
- Motion under gravity is symmetric



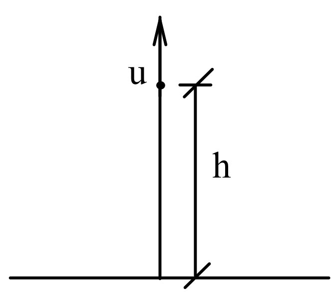
Consider a particle projected from A. B is a point at height h and C is the topmost point.
In the above case speeds u_1 and u_2 are equal, $t_{BC} = t_{CB}$, $t_{AB} = t_{BA}$.

1.14 Motion of a particle projected downwards from height h above surface of earth



Suppose a particle is projected downwards from height h above the surface of the earth with speed u. To find the time taken by it to strike the surface of the earth, taking upward direction as positive, $u = -u,\ a = -g,\ s = -h$, apply $s = ut + \frac{1}{2}at^2$, solve the quadratic and get the positive value of t.

1.15 Motion of a particle projected vertically upwards from height h above surface of earth.

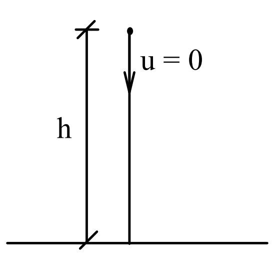


$-h = ut - \frac{1}{2}gt^2$, solve the quadratic and get the positive value of t.

1.16 Motion of a particle dropped from a height h above surface of earth.

Solve using $v^2 = u^2 + 2as$ and $s = ut + \frac{1}{2}at^2$, taking u = 0,

Velocity with which it strikes the surface will be $\sqrt{2gh}$ and the time it will take to strike the surface will be $\sqrt{\frac{2h}{g}}$



SOLVED EXAMPLES (Motion under gravity)

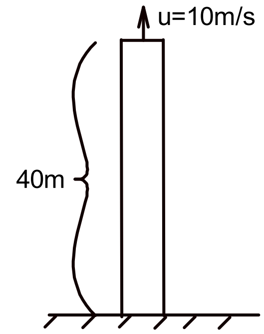


Illustration 12.A particle is projected upwards from top of a tower of height 40m with a speed of 10m/s. Find time it will take to strike ground (g =$10m/s^2$)

Solution: Take upward direction positive and apply $s = ut + \frac{1}{2}at^2$

$s = -40, u = 10, a = -10$ $\quad \therefore -40 = 10t - \frac{1}{2}\times 10\times t^2$

$\therefore t = 4s \;\; or \;\; t = -2s$ $\quad \therefore t = 4s$

Illustration 13. A particle is projected vertically upwards. Let t_1 *and* t_2 be the times at which it is at height h while ascending and descending respectively. Find h and velocity of projection.

Solution: Take upward direction positive

$h = ut_1 - \frac{1}{2}gt_1^2$ $\quad h = ut_2 - \frac{1}{2}gt_2^2$

We get on solving for u and h,

$u = \frac{1}{2}g(t_1 - t_2)$ $\quad h = \frac{1}{2}g\,t_1t_2$

Illustration 14.A parachutist bails out form an aero plane and after dropping through a distance of 40m, he opens parachute and decelerates at 2 m/s^2. If he reaches ground with speed of 2 m/s, how long is he in air? At what height did he bail out? $\left(g = 9.8m/s^2\right)$

Solution: After falling 40 m, he attains speed $v^2 = 0^2 + 2\times 9.8\times 40$, or $v = 28m/s$

Time taken for this $40 = 0 + \frac{1}{2}\times 9.8\times t_1^2$ $\quad t_1 = 2.86s$

When parachutist decelerates uniformly,

$u = 28m/s,\quad v = 2m/s,\quad a = -2m/s^2$,

apply, $v^2 = u^2 + 2as$ we gets = 195 m

For time of this motion

$v = u - at_2$ $\quad$ or $\quad 28 = 2 - 2t_2$

$t_2 = 13s$

$\therefore$ Total time = 15.86s

Height at which parachutist bailed out= 195 + 40 = 235m

Illustration 15.A stone A is dropped from rest from height h above ground. A second stone B is simultaneously thrown vertically up with velocity v. Find v so that B meets A midway between their initial positions.

Solution: Time of travel of each stone = t

Distance travelled by each stone = $\frac{h}{2}$

For stone A, $\frac{h}{2} = \frac{1}{2}g\,t^2$, $t = \sqrt{\frac{h}{g}}$

For stone B, $\frac{h}{2} = vt - \frac{1}{2}gt^2$ $\quad$ Put $t = \sqrt{\frac{h}{g}}$, $\quad \therefore v = \sqrt{gh}$

Illustration 16.A body is dropped from rest from height h. It covers $\frac{9h}{25}$ distance in last second of fall. Find h$\left(g=10m/s^2\right)$.

Solution: $h=\frac{1}{2}gt^2, \left(1-\frac{9}{25}\right)h=\frac{1}{2}g(t-1)^2$ $\therefore t=5\,secs,\ h=125m$

Illustration 17. A stone is dropped from rest form top of a cliff. A 2nd stone is thrown vertically down form same point with 30 m/s velocity 2s later. At what distance form top of the hill, will the two meet. $g=10m/s^2$

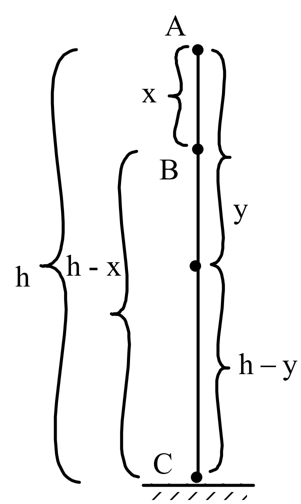
Solution: Let the 2 stones meet at a distance S form from top of cliff t seconds after first stone is dropped.

For 1st stone $s=\frac{1}{2}gt^2=5t^2$

For 2nd stone $s=30(t-2)+\frac{1}{2}\times10\times(t-2)^2$

$\therefore t=4s$ and $s=80m$.

Illustration 18.A stone falling from top of a vertical tower has descended x m when another is let fall from a height y m, below the top. If they fall form rest and reach ground together, show that height of tower is $\frac{(x+y)^2}{4x}$ m.



Solution: Time of 2nd particle = $\sqrt{\frac{2(h-y)}{g}}$

Velocity of 1st particle at B = $\sqrt{2gx}$

Apply $s=ut+\frac{1}{2}at^2$ from B to C for the first particle

$$h-x=\sqrt{2gx}\sqrt{\frac{2(h-y)}{g}}+\frac{1}{2}g\times\frac{2(h-y)}{g}$$

$$y-x=2\sqrt{x}\left(\sqrt{h-y}\right)$$

$$\frac{(y-x)^2}{4x}=h-y \qquad \therefore h=\frac{(x+y)^2}{4x}$$

Illustration 19.A body projected vertically upwards from the top of a tower reaches the ground in t_1 seconds. If it is projected vertically downwards form the same point with same speed, it reaches ground in t_2 seconds. If it is just dropped from top, it reaches ground in t seconds. Prove that $t=\sqrt{t_1t_2}$

Solution: Let h be height of tower

For 1st body

$$-h=ut_1-\frac{1}{2}gt_1^2 \qquad \ldots eq\ (1)$$

For 2nd body

$$h=ut_2+\frac{1}{2}gt_2^2 \qquad \ldots eq\ (2)$$

For 3rd body

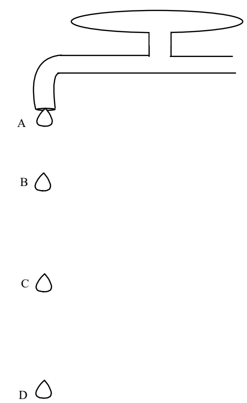
$h = \frac{1}{2}gt^2$ ...eq (3)

Form eq 1 and 2, eliminating u,

$h = \frac{1}{2}gt_1t_2$ ...eq(4)

Form eq 3 and eq 4, we get $t = \sqrt{t_1t_2}$

Illustration 20.Water drops fall at regular interval from a tap. At an instant, when the 4th drop is about to leave tap, find ratio of separation between 3 successive drops below tap.



Solution: At the given instant 1st drop is at D having travelled for 3t time, 2nd drop is at C having travelled for 2t time, 3rd drop is at B, having travelled for t time, 4th drop is at A, on verge of falling form tap.

$AB = \frac{1}{2}gt^2$

$AC = \frac{1}{2}g(2t)^2 = 2gt^2.$

$AD = \frac{1}{2}g(3t)^2 = 4.5gt^2.$

$BC = AC - AB = 1.5t^2.$

$CD = AD - AC = 4.5gt^2 - 2gt^2$

$CD = 2.5gt^2.$

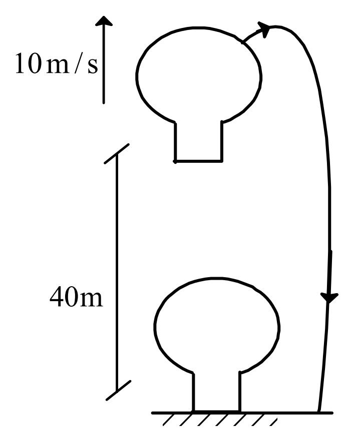
$AB : BC : CD = 0.5gt^2 : 1.5t^2 : 2.5gt^2 = 1:3:5$

Illustration 21.A ball is thrown upwards from ground with initial speed u. Ball is at height of 80 m at 2 time instants, time interval being 6s. Find u. $g = 10m/s^2$.

Solution: $\frac{2v}{g} = 6s$ (motion from B to B) $v = 30m/s$

$30^2 = u^2 - 2\times10\times80$ (motion from A to B) $\therefore$ $u = 50m/s.$

Illustration 22.A balloon starts from ground with 1.25 m/s^2 **acceleration. After 8 s, a stone is released from balloon. Find time in which stone will strike ground. Find distance covered by stone and its displacement form point from where it was released. Find also height of balloon when stone strikes ground.** $g = 10m/s^2$.



Solution: For balloon, $s = 0\times8 + \frac{1}{2}\times1.25\times8^2 = 40m$

$v = 0 + 1.25\times8 = 10m/s$

For stone, $-40 = 10\times t - \frac{1}{2}\times10\times t^2$

$t = -2$ or $4s$ $\therefore t = 4s.$

Distance covered by stone $= 40 + \frac{2\times10^2}{2\times10} = 50m$

Height of balloon = Displacement of balloon in 12s.

$s = \frac{1}{2}\times1.25\times12^2 = 90m$

Inchapter Exercise

13. A body is thrown vertically upwards from the ground with velocity of 98 m/s. Calculate
    (i) The maximum height reached
    (ii) Time taken to reach the highest point.
    (iii) Velocity at height of 196m from the point of projection.
    (iv) Velocity with which it returns to the ground.
    (v) Time taken to reach the ground $\left(g = 9.8 m/s^2\right)$

14. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower strikes the ground in 6s. Find the height of the tower. $\left(g = 9.8 m/s^2\right)$

15. A ball thrown vertically up is caught by the person who threw the ball after 4s. How high did it go and with what velocity was it thrown? How far was it below the highest point, 3s after it was thrown? $\left(g = 9.8 m/s^2\right)$

16. A balloon ascending at constant velocity of 9.8m/s is at a height of 39.2m above ground when a food packet is dropped from the balloon. After how much time and with what velocity does the packet reach the ground. $\left(g = 9.8 m/s^2\right)$

17. Two bodies A and B are thrown simultaneously. A is projected vertically upwards with 20 m/s speed from the ground and B is projected vertically downwards from a height of 40 m with the same speed and along the same line of motion. At what point do the 2 bodies collide? $\left(g = 9.8 m/s^2\right)$

18. A stone falls form a cliff and travels 24.5 m in the last second of its motion before it reaches the ground at the foot of a cliff. Find the height of the cliff. $\left(g = 9.8 m/s^2\right)$

19. A stone is dropped from the top of a tall tower and after one second another stone is dropped from a balcony 20m below the top. If both stones reach the ground at the same instant, calculate the height of the tower. $\left(g = 10 m/s^2\right)$

20. A stone is dropped into a well of 45 m depth. Sound of splash due to the stone striking the water surface in the well is heard after 3.125s. Find the velocity of sound in air $\left(g = 10 m/s^2\right)$

21. Acceleration due to gravity on a planet is $\frac{1}{5}^{\text{th}}$ that on earth. If it is safe to jump from a height of 2m on earth, what will be corresponding safe height on this planet? (Assume safety depends on the velocity of reaching ground)

22. A stone is dropped from a point 0.4m above the top of a 0.5m high window. Find the time taken by the stone to pass the span of the window. $\left(g = 9.8 m/s^2\right)$

ANSWER KEY

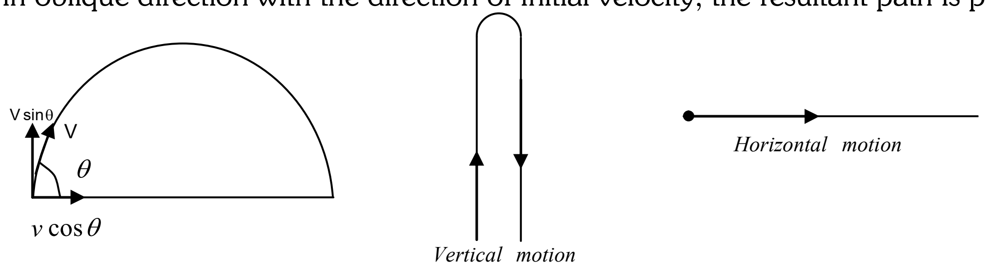
13.	(i) 490 m (ii) 10 s (iii) 75.9 m/s (iv) 98 m/s (v) 20s	**14.**	58.8m
15.	19.6m, 19.6 m/s , 4.9m	**16.**	4s, 29.4 m/s
17.	15.1m from ground after 1s	**18.**	44.1m
19.	31.25m	**20.**	360m/s
21.	10m	**22.**	0.14s

2.1 Basic concept

(I) Any particle which is thrown into space or air such that it moves under the influence of an external force (e.g. gravity, electric forces etc.) is called a projectile. The motion of such a particle is referred to as projectile motion.

(II) It is an example of two dimensional motion with constant acceleration.

(III) If the force acting on the projectile is constant, then acceleration is constant. When the force is in oblique direction with the direction of initial velocity, the resultant path is parabolic.

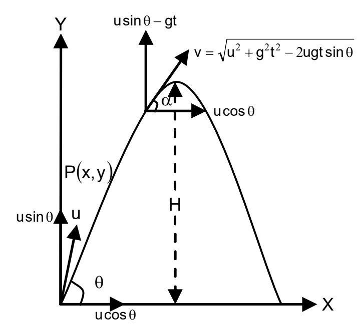


Parabolic motion = vertical motion + horizontal motion

(IV) Projectile motion can be considered to be two simultaneous motions in mutually perpendicular directions which are completely independent of each other i.e. horizontal motion and vertical motion.

2.2 Ground to ground projectile

Consider a projectile thrown from horizontal ground with a velocity u making an angle θ with the horizontal. Take the point of projection as origin O and the path of the projectile in the first quadrant of xy – plane, as shown in the figure. The initial velocity u is resolved in the horizontal and vertical directions i.e.



$$\boxed{u_x = u\cos\theta \qquad u_y = u\sin\theta}$$

Since gravity is the only force acting on the projectile in vertically downward direction, (ignoring air resistance)

$$\boxed{a_x = 0\ ,\ a_y = -g}$$

Analyzing the motion of the projectile in horizontal and vertical directions:

Horizontal direction	Vertical direction
(a) Initial velocity $u_x = u\cos\theta$	(a) Initial velocity $u_y = u\sin\theta$
(b) Acceleration $a_x = 0$	(b) Acceleration $a_y = -g$
(c) Velocity after time t, $v_x = u\cos\theta$	(c) Velocity after time t, $v_y = u\sin\theta - gt$

- The position vector of the projectile after time t is $\vec{r} = x\hat{i} + y\hat{j} = (u\cos\theta.t)\hat{i} + \left(u\sin\theta t - \frac{1}{2}gt^2\right)\hat{j}$;
- Velocity after time t is $\vec{v} = v_x\hat{i} + v_y\hat{j} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$;
- Acceleration is constant, $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$
- **Trajectory equation:** The path traced by the projectile is called the trajectory of the projectile.

For displacement in the horizontal direction, $x = u_x . t$

$$x = u\cos\theta . t \qquad \dots (1)$$

For displacement in the vertical direction, $y = u_y . t - \frac{1}{2} g t^2$

$$y = u\sin\theta\, t - \frac{1}{2} g t^2 \qquad \dots (2)$$

Substituting the value of t from eqn. (1) into eqn. (2), we get

$$y = u\sin\theta . \frac{x}{u\cos\theta} - \frac{1}{2} g . \left(\frac{x}{u\cos\theta}\right)^2$$

$\Rightarrow y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \Rightarrow y = x\tan\theta\left[1 - \frac{x}{R}\right]$ (R is the horizontal range covered by the projectile)

The equation of trajectory of the projectile is that of a parabola because the projectile covers a parabolic path.

- **Time of flight:** The displacement along vertical direction is zero for ground to ground projectile.

$$(u\sin\theta)T - \tfrac{1}{2} g T^2 = 0 \qquad \boxed{T = \frac{2u\sin\theta}{g}}$$

- **Horizontal range:** The horizontal displacement of the projectile from the point of projection to the point it strikes the ground is called the horizontal range of the projectile.

$$R = u_x . T$$

$$R = u\cos\theta . \frac{2u\sin\theta}{g} \qquad \boxed{R = \frac{u^2\sin 2\theta}{g}}$$

- **Maximum height:**

Applying $v^2 = u^2 + 2as$ in the vertical direction between the point of projection and the topmost point, we get $0^2 = u^2\sin^2\theta - 2gH$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

- **Resultant velocity, at any instant t:**

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = u\cos\theta \hat{i} + (u\sin\theta - gt)\hat{j}$$

$$|\vec{v}| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2} \ \& \ \tan\alpha = v_y / v_x = \frac{u\sin\theta - gt}{u\cos\theta}$$

α is the angle made by the velocity vector of the projectile with the horizontal at any time instant t.

- **General result:**

(i) For maximum range $\theta = 45°$ $\quad R_{max} = \frac{u^2}{g}$

In this situation $H_{max} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g}$, $H_{max} = \frac{R_{max}}{4}$

(ii) We get the same range for two angles of projection α and $(90 - \alpha)$. But in each of the two cases, maximum height attained by the particle is different.

$$R = \frac{2u^2 \sin\alpha\cos\alpha}{g} = \frac{2u^2 \sin(90-\alpha)\cos(90-\alpha)}{g}$$

(iii) If R = H i.e. $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$, $\frac{2u^2 \sin\theta\cos\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$

$\therefore \tan\theta = 4$; $\theta = \tan^{-1} 4$

(iv) Range can also be expressed as $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u\sin\theta . u\cos\theta}{g} = \frac{2u_x u_y}{g}$

(v) Change in momentum

(i) Initial velocity $\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

(ii) Final velocity $\vec{u}_f = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

Change in velocity from the point of projection to the point where the projectile strikes the ground. $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -2u\sin\theta\,\hat{j}$

(iii) Change in momentum from the point of projection to the point where the projectile strikes the ground. $\Delta\vec{P} = \vec{P}_f - \vec{P}_i = m\left(\vec{u}_f - \vec{u}_i\right) = m(-2u\sin\theta)\hat{j}$ $= -2\,mu\sin\theta\,\hat{j}$,

where m is the mass of the projectile

(iv) Velocity at the highest point of the projectile is $u\cos\theta\,\hat{i}$. Change in momentum from the point of projection to the highest point $= -mu\sin\theta\,\hat{j}$

Solved examples (ground to ground projectile)

Illustration 23: A body is projected with a velocity of 30 ms^{-1} at an angle of 30° with the vertical. Find the maximum height, time of flight and the horizontal range.

Solution: Here $30\,ms^{-1}$, Angle of projection, $\theta = 90^0 - 30^0 = 60^0$

Maximum height, $H = \frac{u^2\sin^2\theta}{2g} = \frac{30^2\sin^2 60^o}{2x9.8} = 34.44\,m$

Time of flight, $T = \frac{2u\sin\theta}{g} = \frac{2x30\sin 60^o}{9.8} = 5.3s$

Horizontal range, $R = \frac{u^2\sin 2\theta}{g} = \frac{30^2\sin 120^o}{9.8} = \frac{30^2\sin 60^o}{9.8} = 79.53\,m.$

Illustration 24. Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum horizontal range.

Solution: For $\theta = 45^\circ$, the horizontal range is maximum and is given by, $R_{max} = \frac{u^2}{g}$

Maximum height attained, $H_{max} = \frac{u^2 \sin^2 45^0}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$ or $R_{max} = 4H_{max}$

Illustration 25. Show that a given gun will shoot the bullet three times as high when elevated at an angle of 60° as when fired at angle of 30° but will carry the same distance on a horizontal plane.

Solution: The vertical height attained by a projectile is given by,

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

When $\theta = 60^\circ$, $H_1 = \frac{u^2 \sin^2 60^0}{2g} = \frac{u^2}{2g}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3u^2}{8g}$

When $\theta = 30^\circ$, $H_2 = \frac{u^2 \sin^2 30^0}{2g} = \frac{u^2}{2g}\left(\frac{1}{2}\right)^2 = \frac{u^2}{8g}$

$\therefore$ $H_1 : H_2 = \frac{3u^2}{8g} \times \frac{8g}{u^2} = 3:1$

Thus the same gun will shoot three times as high when elevated at an angle of 60° as when fired at an angle of 30°.

Horizontal range of a projectile, $R = \frac{u^2 \sin 2\theta}{g}$

When $\theta = 60^\circ$, $R_1 = \frac{u^2 \sin 120^0}{g} = \frac{\sqrt{3}u^2}{2g}$

When $\theta = 30^\circ$, $R_2 = \frac{u^2 \sin 60^0}{g} = \frac{\sqrt{3}u^2}{2g}$

Thus $R_1 = R_2$, i.e. the horizontal distance covered will be same in both cases.

Illustration 26. Show that there are two values of time for which a projectile is at the same height. Also show that the sum of these two times is equal to the time of flight.

Solution: For vertically upward motion of a projectile,

$$y = u\sin\theta.t - \frac{1}{2}gt^2 \quad \text{Or} \quad \frac{1}{2}gt^2 - u\sin\theta.t + y = 0$$

This is a quadratic equation in t. Its roots are

$$t_1 = \frac{u\sin\theta - \sqrt{u^2\sin^2\theta - 2gy}}{g} \quad \text{(Lower value)}$$

$$t_2 = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta - 2gy}}{g} \quad \text{(Higher value)}$$

These are two values of time for which the vertical height y is same, first while going up and second while going down.

Now $\quad t_1 + t_2 = \dfrac{u\sin\theta - \sqrt{u^2\sin^2\theta - 2gy}}{g} + \dfrac{u\sin\theta + \sqrt{u^2\sin^2\theta - 2gy}}{g}$

or $\quad t_1 + t_2 = \dfrac{2u\sin\theta}{g} = T$, the time of flight

Illustration 27. Two projectile are thrown with different velocities and at different angles so as to cover the same maximum height. Show that the sum of the times taken by each to reach the highest point is equal to the total time taken by either of the projectiles.

Solution: If the two projectile are thrown with velocities u_1 and u_2 at angle θ_1 and θ_2 with horizontal, then their maximum height will be,

$$H_1 = \frac{u_1^2\sin^2\theta_1}{2g} \text{ and } H_2 = \frac{u_2^2\sin^2\theta_2}{2g}$$

But, $\quad H_1 = H_2$,

$\therefore \quad \dfrac{u_1^2\sin^2\theta_1}{2g} = \dfrac{u_2^2\sin^2\theta_2}{2g}$

or $\quad u_1\sin\theta_1 = u_2\sin\theta_2 \quad \ldots (1)$

Times of flight for two projectile are

$$T_1 = \frac{2u_1\sin\theta_1}{g} \text{ and } T_2 = \frac{2u_2\sin\theta_2}{g}$$

Making use of equation (1) we get

$$T_1 = T_2 = \frac{2u_1\sin\theta_1}{g} = \frac{2u_2\sin\theta_2}{g}$$

Times taken to reach the highest point in two cases will be,

$$t_1 = \frac{u_1\sin\theta_1}{g} \text{ and } t_2 = \frac{u_2\sin\theta_2}{g}$$

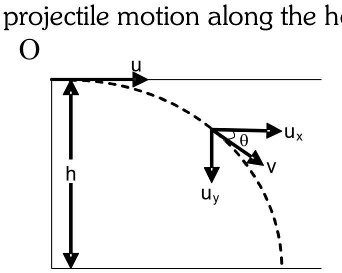
$\therefore \quad t_1 + t_2 = \dfrac{u_1\sin\theta}{g} + \dfrac{u_2\sin\theta_2}{g} = \dfrac{2u_1\sin\theta_1}{g} + \dfrac{2u_2\sin\theta_2}{g} \quad$ [Using (1)]

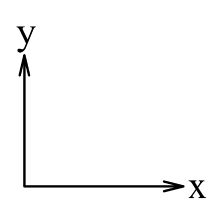
or $\quad t_1 + t_2 =$ Time of flight of either projectile

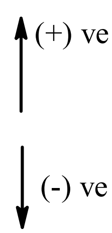
2.3 Projectile thrown horizontally from an elevated point

Consider a projectile thrown from point O at some height h from the ground with a velocity u, in the horizontal direction.

Analyzing the projectile motion along the horizontal and vertical directions







Horizontal direction
(i) Initial velocity $u_x = u$
(ii) Acceleration $a_x = 0$

Vertical direction
(i) Initial velocity $u_y = 0$
(ii) Acceleration $a_y = -g$

Trajectory Equation: The path traced by the projectile is called the trajectory.

After time t, $x = ut$ … (1)

$$y = -\frac{1}{2}gt^2 \quad \dots (2)$$

From eqn. (1) $t = x/u$

Put value of t in eqn. (2) $y = -\frac{1}{2}g\left(\frac{x}{u}\right)^2, \quad y = -\frac{gx^2}{2u^2}$

This is the equation of trajectory

Velocity at a general point P (x, y) after time t

Here horizontal velocity of the projectile after time t, $v_x = u$

Velocity of projectile in vertical direction after time t

$$v_y = 0 - gt = -gt$$

$$\therefore \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = u\hat{i} - gt\hat{j}$$

Displacement: The displacement of the particle after time t, is expressed by

$$S = x\hat{i} + y\hat{j}, \text{ where } x = ut, \; y = -\frac{1}{2}gt^2, \qquad \therefore \; s = ut\hat{i} - \frac{1}{2}gt^2\hat{j}$$

Time of flight: Time taken by the projectile to strike the ground, is the time of flight.

Apply $S = ut + \frac{1}{2}at^2$, along vertical direction

$u = 0$ in vertical direction, $y = -h$

$$-h = -\frac{1}{2}gt^2 \qquad t = \sqrt{\frac{2h}{g}}$$

Horizontal range: Distance covered by the projectile along the horizontal direction between the point of projection to the point where it strikes the ground. $R = u_x t = u\sqrt{\frac{2h}{g}}$

Solved examples (Horizontally thrown projectile)

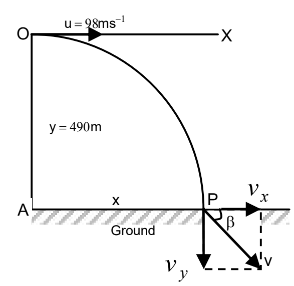


Illustration 28. A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a 490m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the projectile hits the ground.

Solution: (i) The projectile is fired from the top O of a hill with velocity $u = 98ms^{-1}$ along the horizontal. It reaches the target P in time t. The initial velocity in the downward direction = 0

vertical distance OA = y = 490m.

As, $y = \frac{1}{2}gt^2$

$\therefore \quad 490 = \frac{1}{2} \times 9.8\, t^2 \qquad$ or $\qquad t = \sqrt{100} = 10\,s$

(ii) Distance of the target from the hill is given by,

AP = x = u x t = 98 x 10 = 980m.

(iii) The horizontal and vertical components of velocity v of the projectile at point P are

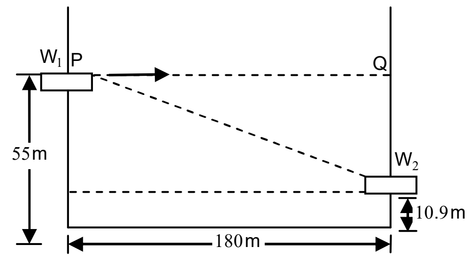
$v_x = u = 98\ ms^{-1} \qquad v_y = u_x + gt = 0 + 9.8 \times 10 = 98ms^{-1}$(downwards)

$$\therefore \quad V = \sqrt{V_x^2 + V_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2} = 138.59ms^{-1}$$

Now if the resultant velocity v makes angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \qquad \therefore \beta = 45^o$$

Illustration 29. Two tall buildings face each other and are at a distance of 180m from each other. With what velocity must a ball be thrown horizontally from a window 55m above the ground in one building, so that it enters a window 10.9m above the ground in the second building.



Solution: In figure P and Q are two tall buildings which are 180m apart. W_1 and W_2 are the two windows in P and Q respectively. Vertical downward distance to be covered by the ball

= Height of W_1 – Height of W_2

= 55 – 10.9 = 44.1 m

Initial vertical velocity of ball, $u_y = 0$

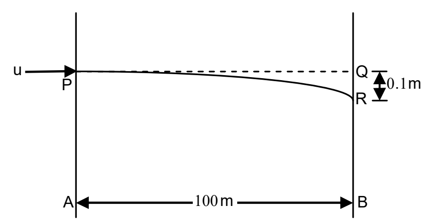
As $y = u_y t + ½ gt^2$

$\therefore$ $44.1 = 0 + ½ \times 9.8t^2$

or $t^2 = \dfrac{44.1 \times 2}{9.8} = 9$ or t = 3sec

Required horizontal velocity $= \dfrac{\text{Horizontal distance}}{\text{Time}} = \dfrac{180 \text{ m}}{3 \text{ s}} = 60 \text{ ms}^{-1}$

Illustration 30. Two paper screens A and B are separated by a distance of 100m. A bullet pierces A and then B. The hole in B in 10cm below the hole in A. If the bullet is traveling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect the resistance of paper and air.



Solution: Assume that the bullet hits the screen with velocity u and pierces the screen after time t.

$\therefore$ Horizontal distance, PQ = ut = 100m

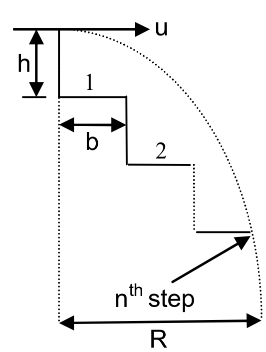
or $t = \dfrac{100}{u}$

vertical distance, $QR = ½ gt^2 = 0.1$

$$\frac{1}{2} g \times \frac{10000}{u^2} = 0.1$$

or $u^2 = \dfrac{9.8 \times (100)^2}{2 \times 0.1} = 49 \times (100)^2$ or $u = 700 \text{ ms}^{-1}$

Illustration 31. A ball rolls off top of a stairway with a horizontal velocity u m/s. If the steps are h m high and b meters wide, the ball will just hit the edge of the n^{th} step, Find n



Solution: If the ball hits the nth step, the horizontal and vertical distances traversed are nb and nh respectively. Let t be the time taken by the ball for these horizontal and vertical displacement. Then velocity along horizontal direction remains constant = u

initial vertical velocity is zero.

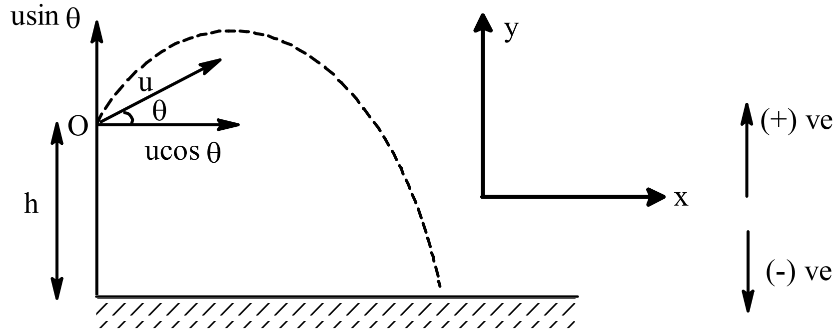
$\therefore$ nb = ut ..... (1)

$nh = 0 + (1/2)gt^2$ ..... (2)

From (1) & (2), we get by eliminating t,

$$nh = (1/2)g(nb/u)^2 \qquad n = \frac{2hu^2}{gb^2}$$

2.4 Projectile fired from the top of a tower in upward inclined direction.



Consider a projectile thrown from point O at height h from the ground with speed u at an angle θ with the horizontal in the upward direction.

Horizontal direction

(i) Initial velocity $u_x = u\cos\theta$

(ii) Acceleration $a_x = 0$

Vertical direction

(i) Initial velocity $u_y = u\sin\theta$

(ii) Acceleration $a_y = -g$

- Equation of trajectory

  At any time t

  $$y = u\sin\theta\, t - \frac{1}{2}g\, t^2 \quad ...(1)$$

  $$x = u\cos\theta \times t \quad ...(2)$$

  Eliminating t between (1) & (2)

  $$y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$$

- Velocity at any time t

  At any time *t*

  $$V_x = u\cos\theta \qquad V_y = u\sin\theta - g t$$

  $$\vec{V} = V_x\hat{i} + V_y\hat{j} = u\cos\theta\,\hat{i} + (u\sin\theta - g t)\hat{j}$$

- Displacement at any time t

  At any time t

  $$y = u\sin\theta\, t - \frac{1}{2}g\, t^2 \qquad x = u\cos\theta\, t$$

  $$\vec{S} = x\hat{i} + y\hat{j} = (u\cos\theta\, t)\,\hat{i} + (u\sin\theta\, t - \frac{1}{2}g\, t^2)\hat{j}$$

- Time of flight

  $$y = u\sin\theta\, t - \frac{1}{2}gt^2 \quad \text{(for displacement in vertical direction)}$$

  $$y = -h, \quad -h = u\sin\theta\, t - \frac{1}{2}gt^2$$

Solving the above quadratic and accepting the positive value of t, we get the time of flight.

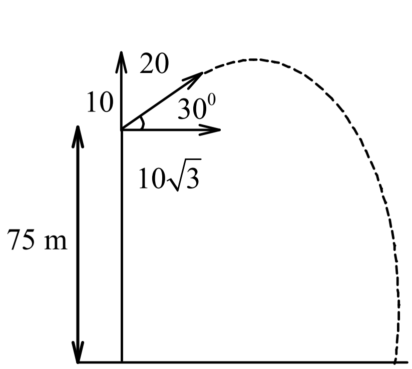
- Horizontal range – Distance covered by projectile along horizontal direction between point of projection and the point where it strikes the ground.

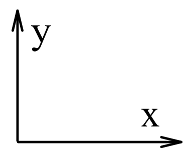
$R = u_x\, t = u\cos\theta \times \text{time of flight}.$

Solved example

Illustration 32. From top of cliff $75\ m$ high, a body is projected with $20\,\text{m/s}$ velocity at angle 30^0 with horizontal in upward inclined direction Determine time of flight, horizontal range covered & velocity with which it strikes the ground $(g = 10\,\text{m/s}^2)$

Solution:





For vertical motion

$-75 = 10t - \frac{1}{2} \times 10 \times t^2$ $\quad$ $t = 5s$ or $t = -3s$ $\quad$ $\therefore$ time of flight $= 5s$

Horizontal range $R = 10\sqrt{3} \times 5 = 50\sqrt{3}\,m$

$v_x = 10\sqrt{3},\ v_y = 10 - 10 \times 5$

$= -40$

$\therefore$ Velocity of striking ground $= v_x\,\hat{i} + v_y\,\hat{j} = 10\sqrt{3}\,\hat{i} - 40\,\hat{j}$

Inchapter Exercise

23. A body is thrown horizontally from the top of a tower & strikes the ground after 3 seconds at angle of 45^0 with horizontal. Find the height of tower &the speed with which the body was projected $(g = 9.8\,\text{m/s}^2)$

24. A bomb is dropped from an aeroplane when it is directly above a target at height of 1000 m. The aeroplane is moving horizontally with a speed of $500\,\text{km/hr}$. By how much distance will the bomb miss the target $(g = 9.8\,m/s^2)$

25. A body is projected horizontally from the top of a cliff with velocity of $9.8\ \text{m/s}$. Find time elapsed before the magnitudes of horizontal & vertical components of the velocities of the bodybecome equal. $(g = 9.8\,\text{m/s}^2)$

26. A body is projected from the ground with velocity of $30\,\text{m/s}$, making an angle 30^0 with the vertical. Find maximum height attained, time of flight & horizontal rangecovered by the body. $(g = 9.8\,\text{m/s}^2)$

27. A cricketer can throw a ball to a maximum horizontal distance of $100\,m$. How high above the ground can the cricketer throw the same ball, assuming the speed of projection of the ball to be the same in both the cases. $\left(g=10m/s^2\right)$

28. The ceiling of a long hall is $25\,m$ high. What is the horizontal distance that a ball thrown from a point on the ground with a speed of $40\,m/s$ will cover, given that the ball just scraps the ceiling. $(g = 9.8\ m/s^2)$

29. A bullet fired from a point on horizontal ground at angle 30^0 with the horizontal hits the ground $3\,km$ away. By adjusting the angle of projection, can one hope to hit a target $5\,km$ away

30. A projectile fired from horizontal ground has a range of $50\,m$ & reaches a maximum height of 10m. Calculate the angle with the horizontal, at which the projectile is fired. $\left(g=10m/s^2\right)$

31. A ball is projected from a point on the horizontal ground at horizontal distance of 39.2 m from the foot of a building. When it is at the highest point of its trajectory, it passes through a window of the building, which is at a vertical height of 19.6 m above ground. Calculate the speed of projection $(g=9.8\,m/s^2)$

32. A ball is thrown at angle θ with the horizontal &another ball is thrown at an angle $(90 - \theta)$ with the horizontal from the same point on horizontal ground with the same speed of 39.2 m/s. The second ball attains a maximum height of 50m more as compared to the maximum height attained by the first ball. Find the maximum height, attained by each of the ball. $(g=9.8\,m/s^2)$

33. Show that there are two angles of projection for which horizontal range is same for a projectile fired from horizontal ground. Also show that the sum of the maximum heights attained by the projectile when fired at the two angles is independent of the angle of projection.

34. A ball is kicked from horizontal ground at an angle 30^0 with vertical. If the horizontal component of velocity is 19.6 m/s, find the maximum height attained and the horizontal range covered by the ball. $\left(g=9.8\,m/s^2\right)$

35. A fighter plane flying horizontally at an altitude of 1.5 km with a speed of 720km/hr passes directly overhead an anti – aircraft gun. The gun fires a shell with muzzle speed of 600 m/s at a certain angle with the horizontal at the instant the plane is vertically above the gun. If the shell hits the plane, find angle made by the shell with the horizontal, at the instant it was fired. Find the minimum altitude at which pilot should fly the plane to avoid being hit, if the shell was fired with the same speed &at the same angle as before.

36. A bomber, flying upwards at angle 53^0 with vertical, releases a bomb at $800m$ altitude. The bomb strikes the ground $20\ s$ after its release. Find
(i) Velocity of bomber at time of release of bomb
(ii) Maximum height attained by bomb.
(iii) Horizontal distance covered by the bomb before it strikes the ground .
(iv) Velocity of the bomb when it strikes the ground. $(g=10\,m/s^2)$

37. A particle of mass $100\,g$ is fired from a point on horizontal ground with a speed of $20m/s$ making angle 30^0 with horizontal. Find the magnitude of change in momentum of the particle between the highest point of its trajectory and the point of projection.

38. A body of mass m is projected with velocity v at angle of 45^0 with the horizontal on a horizontal plane. Find the magnitude of change in momentum between the point of projection and the point where the body strikes the ground.

39. In the above problem, find the change in kinetic energy between the point of projection and the point where the body strikes the ground.

40. For a projectile launched from a point on horizontal ground, the speed when it is at the greatest height is $\sqrt{\frac{2}{5}}$ times the speed when it is at half of its greatest height. Determine the angle of projection.

ANSWER KEY

23.	$44.1\,m, 29.4\,m/s$	**24.**	1984.13 m	**25.**	1 s
26.	34.44 m, 5.3s, 79.53 m	**27.**	50 m	**28.**	150. 7 m
29.	No, maximum range is $2\sqrt{3}\,km$				
30.	$\tan^{-1}(0.8)$	**31.**	$19.6\sqrt{2}$ m/s.	**32.**	14.2 m, 64.2 m
34.	$58.8\,m, 135.8\,m$	**35.**	$\cos^{-1}\left(\frac{1}{3}\right), 16\,km$		

36. (i) $100\,m/s$ (ii) $980\,m$ (iii) $1600\,m$ (iv) $V_x = 80\,m/s, V_y = 140\,m/s$)

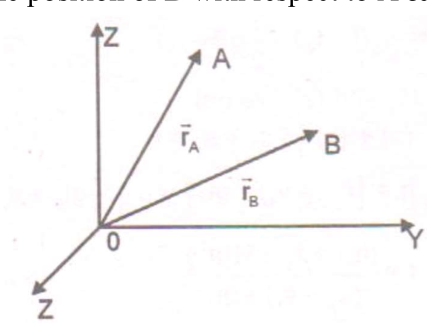
37. $1\,kg.m/s$ **38.** $(mv\sqrt{2})$ **39.** 0 **40.** 60^0

Relative Velocity

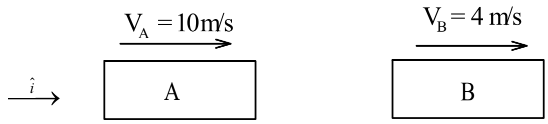
Let two particles A & B be placed at two points as shown in the figure. The position vectors of the particles with respect to the inertial reference frame are $\vec{r}_A$ & $\vec{r}_B$ respectively. The relative separation between the particles is given by $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$. Differentiating both sides w.r.t. time, we obtain.

$$\frac{d\vec{r}_{BA}}{dt} = \frac{d\vec{r}_B}{dt} - \frac{d\vec{r}_A}{dt} \Rightarrow \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$\overrightarrow{r_{BA}}$ is the position of B with respect to A & $\vec{v}_{BA}$ is the velocity of B relative to A.



- Consider 2 cars A and B moving along same line in the same direction with constant velocity $V_A = 10\,m/s$ and $V_B = 4\,m/s$



- Observer on A cannot register motion of A. The observer on A notices Car B to be moving towards him with speed 10 - 4 = $6\,m/s$. This is velocity of B with respect to A

  $\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}$ = velocity of B with respect to A

  $\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A} = 4\hat{i} - 10\hat{i} = -6\hat{i}$ (Rightward positive)

  This means observer on A will register that B is moving with 6 m/s in negative x direction or towards the left, i.e B is approaching him with 6 m/s.

- Similarly observer on B cannot register motion of B. Observer on B notices car A to approach him with $(10-4) =$ 6 m/s. This is velocity of A with respect to B.

  $\overrightarrow{V_{AB}} = \overrightarrow{V_A} - \overrightarrow{V_B} = 10\hat{i} - 4\hat{i} = 6\hat{i}$ (Rightward positive)

  This means observer on B will register that A is moving with 6 m/s in positive x-direction or towards the right i.e A is approaching B with 6 m/s

Solved Examples (Relative Velocity in 1D)

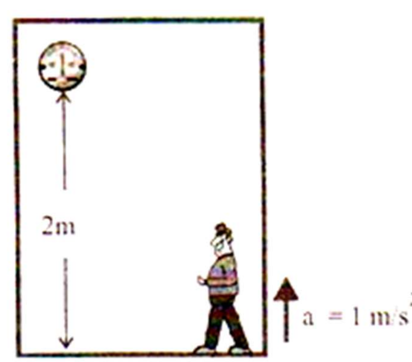
Illustration 33. An elevator, in which a man is standing, is moving upward with a constant acceleration of $1\,m/s^2$. At some instant when speed of elevator is 10 m/s, the man drops a coin from a height of 2 m. Find the time taken by the coin to reach the floor. $(g = 9.8 m/s^2)$

Solution: Analyses the motion of coin with respect to the observer standing in the elevator. As the coin releases from rest inside elevator, its velocity with respect to ground is equal to the velocity of elevator, 10 m/s.

$\therefore \quad \vec{V}_{coin.elevator} = \vec{V}_{coin.ground} - \vec{V}_{elevator.ground}$

or $\vec{V}_{CE} = \vec{V}_{CG} - \vec{V}_{EG}$

$= 10 - 10 = 0$



To find acceleration of coin with respect to the observer in the elevator:

$\vec{a}_{CE} = \vec{a}_{CG} - \vec{a}_{EG} = -g - a$

Now using second equation for relative motion

$$\vec{s}_{CE} = \vec{u}_{CE}\, t + \frac{1}{2}\vec{a}_{CE} t^2$$

$$2 = 0 + \frac{1}{2}(g+a)t^2 \text{ or } 2 = \frac{1}{2}(9.8+1)t^2 \text{ or } t = \sqrt{\frac{2\times 2}{10.8}} = 0.61\,s$$

Illustration 34. The engineer of a train moving at a speed v_1 sights a freight train a distance 'd' ahead of him on the same track moving in the same direction with a slower speed v_2. He puts on the brakes and gives his train a constant

deceleration α. Find the minimum value of d at which brakes are applied so as to avoid collision.

Solution: Collision will be avoided if speed of train v_1 become equal to v_2 in traveling a relative distance d. Therefore final relative speed of trains become zero. The initial relative speed $v_{12} = v_1 - v_2$. By third equation of motion, we have

$$v_{12}^2 = u_{12}^2 - 2a_{12}s \qquad 0 = (v_1 - v_2)^2 - 2\,\alpha\,d \text{ or } d = \frac{(v_1 - v_2)^2}{2\alpha}$$

The collision can be avoided if $d \geq \frac{(v_1 - v_2)^2}{2\alpha}$

Illustration 35. Two parallel rail tracks run north south. Train A moves north with a speed of 54 km/h and train B moves south with a speed of 90 km/h. What is the
(i) relative velocity of B with respect to A?
(ii) relative velocity of ground with respect to B?
(iii) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km/h with respect to the train A) as observed by a man standing on the ground?

Solution: (i) Let v_A = 54 km/h then v_B = -90 km/h (Taking Northward positive)

$\therefore$ Relative velocity of B with respect to A

$\vec{v}_{BA} = -90 - 54$ = - 144 km/h Ans.

The velocity of train B with respect to train A appears 144 km/h along south.

(ii) Relative velocity of ground with respect to train B

$v_g - v_B = 0 + 90 = 90\,\text{km/hr}$

To train B, the ground appears to move with a speed of 90 km/h along North

(iii) The velocity of monkey with respect to train A

$= V_{\text{monkey,A}} = V_{\text{monkey,ground}} - V_{\text{A,ground}}$,

$\therefore$ $V_{\text{monkey,ground}} = V_{\text{monkey,A}} + V_{\text{A,ground}} = -18 + 54$

= 36 km/h Ans.

To an observer on ground, monkey appears to travel with a speed of 36 km/hr towards North.

Illustration 36. A police van moving on a highway with a speed of 30 km/h fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km/h. If the muzzle speed of the bullet is 150 m/s, with what speed does the bullet hit the thief's car?

Solution: Speed of police van = $30 \times \frac{5}{18} = \frac{25}{3}$ m/s

The muzzle velocity, that is velocity of bullet with respect to van

$V_{bullet.van} = V_{bullet.ground} - V_{van.ground}$ or $V_{bullet.ground} = V_{bullet.van} - V_{van.ground}$

$= 150 + \frac{25}{3} \quad = \frac{475}{3} m/s$

Speed of thief's car = $192 \times \frac{5}{18} = \frac{160}{3}$ m/s

Now velocity of bullet with respect to the thief's car

$V_{bullet.car} = V_{bullet.ground} - V_{car.ground} \quad = \frac{475}{3} - \frac{160}{3} \quad = 105 m/s$

Hence the speed of the bullet with which it hits the thief's car = 105 m/s.

Illustration 37. **Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 km/h in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?**

Solution: Let speed of each bus = v km/h

The distance between the nearest buses plying on either side = vT km ….. (i)

For buses going from town A to B.

Relative speed of bus in the direction of motion of man, = (v – 20)

Buses plying in this direction go past the cyclist after every 18 min. Therefore

separation between the buses = $(v-20)\times\frac{18}{60}$

From equation (i), $(v-20)\times\frac{18}{60} = vT$ ….. (ii)

For buses coming from B to A:

The relative velocity of bus with respect to man = (v + 20)

Buses coming from town B go past the cyclist after every 6 min therefore

$(v+20)\times\frac{6}{60} = vT$ … (iii)

Solving equations (ii) and (iii), we get v = 40 km/h and T = $\frac{3}{20}$ h Ans.

Illustration 38. **Two trains each having a speed of 30 km/h are headed at each other on the same straight track. A bird that can fly at 60 km/h flies off from one train when they are 60 km apart and heads directly for the other train. On reaching the other train it flies directly back to the first, and so forth**

(a) What is the total distance the bird travels?

(b) How many trips can the bird make from one train to the other before they crash?

Solution: (a) The velocity of approach of trains = 30 – (-30) = 60 km/h

The time taken by trains before crashing = $\frac{60}{60} = 1 h$

The bird continues to fly for 1 hour with constant speed of 60 km/h. therefore distance travelled by bird in this duration = 60×1 = 60 km

(b) For first trip : s_1 = 60 km

The relative velocity of approach of bird towards the incoming train = 60 + 30 = 90 km/h

Time taken by bird to reach the incoming train $= \frac{60}{90} = \frac{2}{3}\text{h}$

Separation between the trains after $\frac{2}{3}\text{h}, = 60 - 60 \times \frac{2}{3} = 20\,\text{km}$

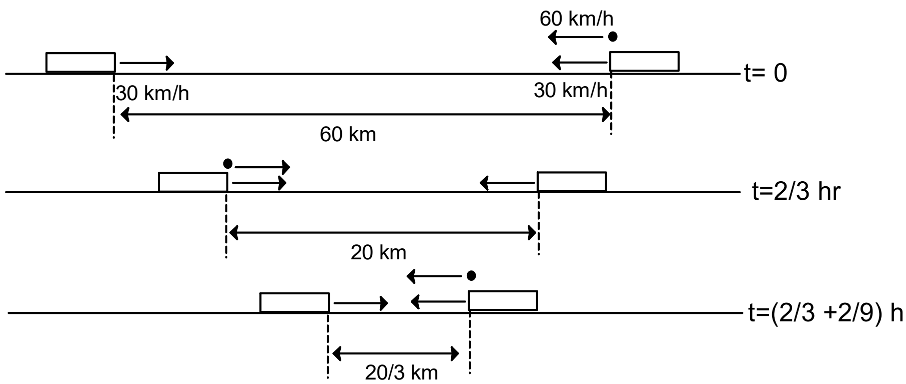
The bird returns to the first train in time $= \frac{20}{90} = \frac{2}{9}\text{h}$

In this duration trains approach by $= 60 \times \frac{2}{9} = \frac{40}{3}\text{km}$

$\therefore$ The separation between them after first round trip $= 20 - \frac{40}{3} = \frac{20}{3}\text{km} = 60\left(\frac{1}{3}\right)^2 \text{km}$

For second trip: $s_2 = \frac{20}{3}\text{km}$

Time taken to reach the incoming train $= \frac{20/3}{90} = \frac{2}{27}\text{h}$



Separation between train after $\frac{2}{27}\text{h}$ $= \frac{20}{3} - 60 \times \frac{2}{27} = \frac{20}{9}\text{km}$

The bird return to first train in time $= \frac{20/9}{90} = \frac{2}{81}\text{h}$

In this duration trains approach by $= 60 \times \frac{2}{81} = \frac{120}{81}\text{km}$

$\therefore$ The separation between the trains after second round trip $= \frac{20}{9} - \frac{120}{81}$

$$= \frac{60}{81}\text{km} = 60\left(\frac{1}{3}\right)^4 \text{km}$$

In a similar way one can get the separation after n^{th} round trip $s_n = 60\left(\frac{1}{3}\right)^{2n}$

The trains to be crashed $s_n \to 0$

$\therefore$ $0 = 60\left(\frac{1}{3}\right)^{2n}$ Which gives n = ∞

$\therefore$ Bird makes infinite trips before the two trains crash.

Inchapter Exercise

41. Car A moving at $10\,m/s$ on a straight road, is ahead of car B moving in the same direction at $6\,m/s$. Find velocity of A relative to B & velocity of B relative to A

42. Two trains $120\,m$ & $80\,m$ in length are running in opposite directions with velocities $42\,km/hr$ & $30\,km/hr$ and approach each other. In what time will they completely cross each other .

43. Two trains A & B of length $400\,m$ each are moving on two parallel tracks with uniform speeds of $72\ km/hr$ in the same direction with A head of B. Driver of B decides to overtake A & accelerates by $1\,m/s^2$. If after $50\ s$, the guard of B just brushes past the driver of A, what was original distance between them .

44. On a two lane road, car A is traveling with speed of $36\,km/hr$. Two cars B and C approach car A in opposite directions with speed of $54\,km/hr$ each. Car B is travelling in the same direction as car A while car C is travelling opposite to the direction of car A. At a certain instant, when the distance AB is equal to AC, both being $1\,km$, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

45. A man is $9\,m$ behind the door of a train when it starts from rest with a uniform acceleration of $2\,m/s^2$. The man runs at constant speed. How far does he have to run & after how much time does he get into train, assuming that the man and the train have the same velocity when the man boards the train? What is the speed with which the man was running?

46. Two buses start simultaneously towards each other from towns A & B which are $480\,km$ apart. First bus takes 8 hours to travel from A to B while the second bus takes 12 hours to travel from B to A, both moving with uniform velocity. Determine when & where the buses will meet.

47. Two trains A & B, each of length 100 m, are running on parallel tracks .If one were to completely overtake the other, travelling in the same direction, it would take 20s.It would take 10 s for one to completely cross the other, if the trains are travelling in the opposite direction. Calculate the velocities of trains A & B.

48. Particle A is at top of a tower of height 100 m. Particle B is at the bottom of the same tower. Find the time taken by the particles to meet if they are thrown as given below for three separate cases.
(a) A is dropped & B is projected vertically upwards with a speed of $50\,m/s$.
(b) A is projected downwards with a speed of $25\,m/s$ & B is projected upwards with a speed of $50\,m/s$
(c) A is projected upwards with a speed of $25\,m/s$ & B is projected upwards with a speed of $50\,m/s$

49. Particles A, B and C are in motion. Motion of particle A as seen by particle B is with speed V in North East direction. Motion of B as seen by C is with speed V in north west direction. What will be the direction of motion of C as seen by A

50. A particle moving with velocity of magnitude V changes its direction of motion by angle θ without change in speed. Find the
(a) Magnitude of change of velocity. (b) Change in magnitude of velocity.

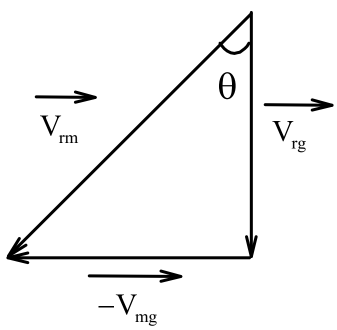
ANSWER KEY

41.	4 m/s, - 4 m/s	**42**	10 s	**43:**	1250 m	**44**	1 m/s^2.
45	18m, 3s, 6 m/s.	**46:**	4.8 hours, 288 km from A			**47:**	$(15\,m/s, 5\,m/s)$
48:	(a) 2 s (b) $\frac{4}{3}$ s (c) 4 s	**49:**	South	**50:**	(a) $2v\sin\frac{\theta}{2}$, (b) 0		

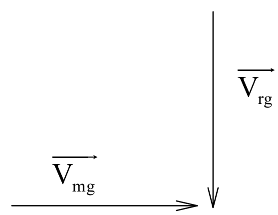
Rain - man problems

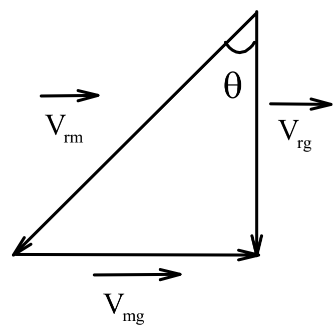
- $\overrightarrow{V_{rm}}$ = velocity of rain w.r.t. man = $\overrightarrow{V_{rg}} + \left(-\overrightarrow{V_{mg}}\right)$

  $\overrightarrow{V_{rg}}$ = velocity of rain w.r.t. ground, $\overrightarrow{V_{mg}}$ = velocity of man w.r.t. ground.
- Suppose rain is falling vertically with respect to the ground& man is walking on a horizontal road towards right.



$\overrightarrow{V_{rm}} = \overrightarrow{V_{rg}} + \left(-\overrightarrow{V_{mg}}\right)$





$\overrightarrow{V_{rm}} + \overrightarrow{V_{mg}} = \overrightarrow{V_{rg}}$

θ is the angle made by V_{rm} with the vertical

$$\tan\theta = \frac{V_{mg}}{V_{rg}}$$

In case wind is blowing

$\overrightarrow{V_{rg}} = \overrightarrow{V_{rw}} + \overrightarrow{V_{wg}}$

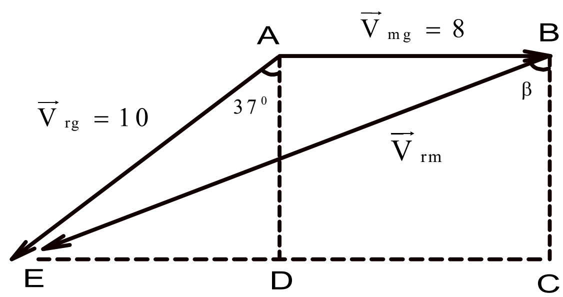
$\overrightarrow{V_{rg}}$ = velocity of rain w.r.t. ground.

$\overrightarrow{V_{rw}}$ = velocity of rain w.r.t wind.

$\overrightarrow{V_{wg}}$ = velocity of wind w.r.t. ground.

Solved Examples

Illustration 42. Rain pouring at 37^0 with vertical has $10\,m/s$ speed. A person runs against rain with $8\,m/s$ &sees rain making angle β with vertical. Find B.



$\overrightarrow{V_{mg}} + \overrightarrow{V_{rm}} = \overrightarrow{V_{rg}}$

From fig $\tan\beta = \dfrac{CE}{BC}$

$BC = AD = 10\cos 37 = 8$

$CE = CD + DE = 8 + 10\sin 37 = 14$

$\therefore \quad \tan\beta = \dfrac{14}{8} = \dfrac{7}{4},$

$\therefore \quad \beta = \tan^{-1}\dfrac{7}{4}$

Inchapter Exercises

51. In the absence of wind, rain is falling vertically with a speed of $4\,m/s$. After some time, wind starts blowing with 3 m/s speed in North to South direction. Find the angle with the vertical that a man standing on horizontal ground must hold his umbrella to keep himself dry.

52. In the absence of wind, rain is falling vertically with $20\,m/s$ speed. A person is running in the rain at a speed of 5 m/s on a straight horizontal road. Wind starts blowing with $15\,m/s$ speed. Directions of both wind & person is from West to East. Find the angle with the vertical that a person should hold his umbrella, to keep the rain away.

53. A man standing on a horizontal road has to hold his umbrella at 30^0 to the vertical to keep rain away. When he walks on the road at $10\,km/hr$, he has to hold the umbrella vertically to keep himself dry. Find the speed of the rain drops, with respect to the ground.

54. For a man running on horizontal road at $8\,km/hr$, rain appears to fall vertically. He increases his speed to $12\,km/hr$. Now he finds rain hitting him at 30^0 with the vertical. Find the speed of the rain drops with respect to the ground.

55. For a man moving on a horizontal road at a uniform speed of $6\,km/hr$, rain appears to fall vertically downwards. When he increases his speed to $12\,km/hr$, rain appears to make 30^0 with vertical. Find the magnitude & direction of rain w.r.t ground.

56. When a man walks with a speed V on a horizontal road, rain appears to fall vertically. When he doubles his speed, rain appears to fall at 45^0 to the vertical. Find the direction of rain as it appears to the man when he triples his speed.

57. Rain is falling vertically with respect to the ground. For a man walking on a horizontal road at a speed of 5m/s, rain appears to fall at 45^0 to the vertical. When the man reduces his speed, rain appears to fall at 30^0 to the vertical. Find the reduced speed of the man.

ANSWERS KEY

51. 37^0	52. $\tan^{-1}\left(\frac{1}{2}\right)$	53. 20 km/hr
54. $4\sqrt{7}$ km/hr	55. 12 km/hr at 30^0 to vertical	56. $\tan^{-1} 2$ with vertical
57. $\frac{5}{\sqrt{3}}$ m/s		

EXERCISE – 1

Single Option Correct :

1. For motion in a straight line, which of the following is incorrect?
(a) Acceleration at some instant may be zero even if the velocity is not zero
(b) Velocity at some instant may be zero even if the acceleration is not zero
(c) Speed is zero but velocity may not be zero
(d) The direction of velocity and acceleration, at any instant, may be different.

2. A particle moving in a straight line starts from a point and comes back to the same point after some time, then:
(a) The speed of the particle, must have reduced to zero at some instant
(b) The direction of the velocity must have reversed at some instant
(c) Both the above
(d) None of these

3. Choose the correct statement:
(a) If the average velocity of a particle moving along a straight line in a given interval of time is zero, instantaneous speed within that interval is never zero.
(b) If the direction of displacement of a particle moving in a straight line is opposite to the direction of acceleration, the particle is necessarily moving towards the point from where it started.
(c) If the direction of velocity for a particle moving in a straight line is opposite to that of acceleration then the particle is heading towards the point from where it started.
(d) None of these

4. Choose the correct statement.
(a) The speed of a particle may never be zero even though the average speed within a time interval is zero
(b) The average velocity of a particle moving along a straight line is zero in a time interval. It is possible that the instantaneous speed is never zero in that interval
(c) The magnitude of the instantaneous velocity is equal to the instantaneous speed
(d) The magnitude of the average velocity in a time interval is equal to its average speed in that interval.

5. A particle moving in a straight line covers half the distance with a speed of 3 m/s. The other half of the distance is covered in two equal time intervals with uniform speeds of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is
(a) 1 m/s (b) 2 m/s (c) 3 m/s (d) 4 m/s

6. A bullet fired into a thick block loses half of its velocity after penetrating 3 cm. The total length penetrated by the bullet before it comes to rest inside the block is
(a) 3 cm (b) 5 cm (c) 4 cm (d) 6 cm

7. A particle starts from rest and travels a distance of 6 m along a straight line in two parts, the first part of the journey at a constant acceleration of 2 m/s^2 and the second part at a constant retardation of 4 m/s^2. The particle comes to rest at the end of the second part of the motion. The total time during which the particle is in motion is
(a) 2s (b) 1s (c) 3 s (d) $\sqrt{3}$ s

8. A train moves in a straight line with a uniform acceleration. If x and y be the velocities with which the front end and the rear end of the train respectively cross a fixed electric pole, which is adjacent to the railway track, then the velocity with which the middle of the train crosses the pole is :
(a) $\frac{x+y}{2}$ (b) $\frac{2xy}{x+y}$ (c) $\sqrt{\frac{1}{2}\left(x^2+y^2\right)}$ (d) $\sqrt{\frac{1}{2}\left(y^2-x^2\right)}$

9. A body leaving a certain point O moves with an acceleration which is constant in magnitude and direction. At the end of the first second its velocity is 1.5 m/s. At the end of the sixth second the body stops momentarily and begins to move backwards. The distance traversed by the body before it stops momentarily is:
(a) 9 m (b) 18 m (c) 5.4 m (d) 36 m

10. A particle moving with an initial velocity $\hat{i}-4\hat{j}+10\hat{k}$ has acceleration $\hat{i}+\hat{j}-2\hat{k}$. Its velocity at the end of 2 seconds, is directed along the unit vector:
(a) $\frac{1}{7}\left(-3\hat{j}+2\hat{j}-6\hat{k}\right)$ (b) $\frac{1}{\sqrt{382}}\left(3\hat{i}-7\hat{j}+18\hat{k}\right)$
(c) $\frac{1}{7}\left(3\hat{j}-2\hat{j}+6\hat{k}\right)$ (d) $\frac{1}{\sqrt{77}}\left(2\hat{j}-3\hat{j}+8\hat{k}\right)$

11. A bus starts from rest and moves with an acceleration of $1 m/s^2$. A boy who is 48m behind the bus starts running with constant speed of 10m/s. The earliest time in which the boy can catch the bus is
(A) 8 sec. (B) 10 sec. (C) 12 sec. (D) 14 sec.

12. A person is travelling from the ground floor to the first floor in a mall using an escalator. Consider the following three separate cases. When the person is standing on the moving escalator it takes one minute for him to reach the first floor. If the escalator does not move it takes him 3 minutes to walk up the stationary escalator to reach the top. How long will it take for the person to reach the top if he walks up the moving escalator?
(A) 30 sec (B) 45 sec (C) 40 sec (D) 35 sec

13. A particle takes t second less and acquires a velocity of v ms^{-1} more in falling through a certain height (starting from rest) on a planet where the acceleration due to gravity is 8 g when compared with freely falling motion of a particle on another planet, where the particle is released from rest at the same height as the previous planet. The acceleration due to gravity on the second planet is 2g, then

(A) v = 2gt (B) v = 4gt (C) v = 5 gt (D) v = 16 gt

14. A ball is thrown vertically down with velocity of 5m/s from the top of a tower. With what velocity should another ball be thrown vertically down after 2 seconds from the top of the same tower, so that it can hit the first ball in a further 2 seconds(Take g=10 m/s^2)

(A) 40 m/s (B) 55 m/s (C) 15 m/s (D) 25 m/s

15. A particle is projected vertically upwards from a point A on the ground. It takes t_1 time to reach a point B but it still continues to move up. If it takes further t_2 time to reach the ground from point B, then height of point B from the ground is

(A) $\frac{1}{2}g\left(t_1+t_2\right)^2$ (B) $g\,t_1\,t_2$ (C) $\frac{1}{8}g\left(t_1+t_2\right)^2$ (D) $\frac{1}{2}g\,t_1\,t_2$

16. A juggler throws balls vertically upwards in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height attained by each ball is 5 m, the number of balls thrown per minute will be (Take g=10 m/s^2)

(A) 40 (B) 50 (C) 60 (D) 120

17. Four particles are fired with the same speed at angles $25^0, 40^0, 55^0$ and 70^0 with the horizontal, from a point on horizontal ground. The horizontal range covered will be maximum for the particle projected at angle

(A) 25^0 (B) 55^0 (C) 40^0 (D) 70^0

18. Two stones are projected from horizontal ground with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of the first stone is $\pi/3$ and the maximum height attained by the first stone is y_1, then the maximum height attained by the second stone will be-

(A) $3y_1$ (B) $2y_1$ (C) $y_1/2$ (D) $y_1/3$

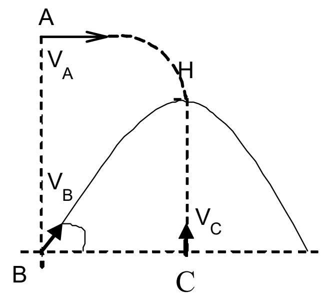
19. A ball is projected upwards from the top of a tower with a velocity 50m/s making an angle 30° with the horizontal. The height of the tower is 70m.After how many seconds from the instant of projection, will the ball reach the ground- (Take g=10 m/s^2).

(A) 2 s (B) 5 s (C) 7 s (D) 9 s

20. A cannon ball has a range R, when fired from the ground level, on a horizontal plane. If h and h' are the greatest heights attained by the cannon ball in the two paths for which the Range R is possible, then-

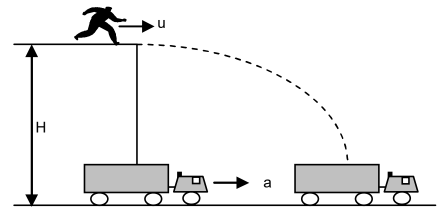
(A) $R = 4\sqrt{(hh')}$ (B) $R = \frac{4h}{h'}$ (C) $R = 4h\,h'$ (D) $R = \sqrt{hh'}$

21. Three particle A, B and C are thrown with speeds v_A, v_B, and v_C respectively. A is projected horizontal, B is projected at an angle of 30° with the horizontal and C is projected vertically in such a manner that they collide simultaneously at H, the highest point of the parabolic path of B, as shown in the figure. If the acceleration due to gravity is g, then the possible ratio of the speeds $v_A : v_B : v_C$ is



(A) 1 : 1 : 1 (B) 1 : 2 : $\sqrt{3}$
(C) $\sqrt{3}$: 1 : 1 (D) $\sqrt{3}$: 2 : 1

22. A stunt performer is to run and dive off a tall platform and land in a net in the back of a truck below. Originally the truck is directly under the platform. It starts forward with a constant acceleration a at the same instant that the performer leaves the platform. If the platform is H above the net in the truck, then the horizontal velocity u that the performer must have as he leaves the platform is

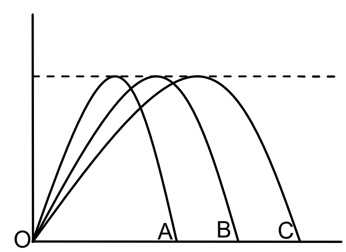


(A) $a\sqrt{2H/g}$ (B) $a\sqrt{H/2g}$ (C) $\sqrt{g/2H}$ (D) None of these

23. An object is thrown at an angle α to the horizontal (0° $<\alpha<$ 90°) with a certain velocity on horizontal ground. Then during ascent (ignoring air drag)
(A) Acceleration with which the object moves is equal at all points
(B) Magnitude of acceleration tangential to the path decreases
(C) Magnitude of acceleration normal to the path increases, becoming equal to g at the highest point
(D) all of the above

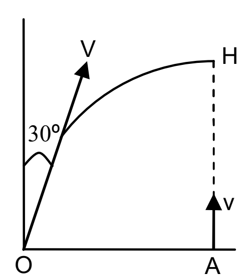
24. A bullet is fired in the horizontal direction from the top of a tower while a stone is simultaneously dropped form the same point then -
(A) The bullet and the stone will reach the ground simultaneously
(B) The stone will reach earlier
(C) The bullet will reach earlier
(D) Nothing can be predicted

25. Three projectile A, B and C are thrown from the same point in the same plane. Their trajectories are shown in the figure. Then which of the following statement is true –



(A) The time of flight is the same for all the three
(B) The launch speed is greatest for particle C
(C) The horizontal velocity component is greatest for particle C
(D) All of the above

26. A particle is projected with a speed V from a point O making an angle of 30° with the vertical. At the same instant, a second particle is thrown vertically upwards form a point A with speed v. Refer the figure. The two particles reach point H, the highest point of the parabolic path of the first particle, simultaneously. The ratio $\frac{V}{v}$ is –



(A) $3\sqrt{2}$ (B) $2\sqrt{3}$ (C) $2/\sqrt{3}$ (D) $\sqrt{3}/2$

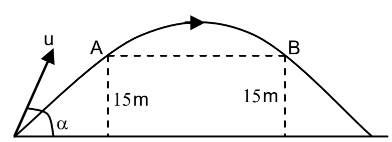
27. R is the horizontal range of a projectile, fired with a certain speed at a certain angle with the horizontal, on a horizontal plane and h is the maximum height attained by it. Then the maximum horizontal range that can be attained with the same speed of projection as before is -
(A) 2h (B) $\frac{R^2}{8h}$ (C) $2R+\frac{h^2}{8R}$ (D) $2h+\frac{R^2}{8h}$

28. A particle is thrown over a scalene triangle from one end of the horizontal base of the triangle. It grazes the vertex and falls on the other end of the base. If α and β be the base angles of the triangle and θbe the angle of projection of the particle with the horizontal, then the relation between θ,α and β is -
(A) $\tan\theta = \tan\alpha - \tan\beta$ (B) $\tan\theta = \tan\alpha + \tan\beta$
(C) $\tan\beta = \tan\theta - \tan\alpha$ (D) None of these

29. A bomber is flying horizontally with a constant speed of 150 m/s at a height of 78.4 m. The pilot has to drop a bomb at the enemy target. At what horizontal distance from the target should he release the bomb so that it hits the target-$\left(g = 9.8\,m/s^2\right)$
(A) Zero (B) 300 m (C) 600 m (D) 750 m

30. An aircraft dives towards a stationary target which is at sea level. When it is at a height of 1390 m above sea level it launches a missile towards the target. The initial velocity of the missile is 410m/s in a direction making an angle θ below the horizontal where $\tan\theta = 9/40$. Then the time of flight of the missile from the instant it was launched until it reaches sea level is nearly -
(A) 10 sec (B) 15 sec (C) 20 sec (D) 25 sec

31. A golfer standing on level ground hits a ball with a velocity of u = 52m/s at an angle α above the horizontal. If $\tan \alpha = 5/12$, then the time for which the ball is at least 15m above the ground(i.e. between A and B) will be (take g = 10 m/s²) –



(A) 1 sec (B) 2 sec (C) 3 sec (D) 4 sec

32. A projectile is thrown with a velocity of 20m/s, at an angle of 60° with the horizontal. After how much time will the velocity vector make an angle of 45° with the horizontal (take g = 10m/s²) -
(A) $\sqrt{3}$ sec (B) $1\sqrt{3}$ sec (C) $(\sqrt{3} + 1)$ sec (D) $(\sqrt{3} - 1)$ sec

33. If T be the total time of flight of a ground to ground projectile on a horizontal plane and H be the maximum height attained by it from the point of projection, then H/T will be- (u = projectile velocity, θ = projectile angle with the horizontal)
(A) (1/2) u sin θ (B) (1/4) u sin θ (C) u sin θ (D) 2 u sin θ

34. A hunter aims his gun and fires a bullet directly at a monkey hanging on the branch of a tree. At the instant the bullet leaves the gun, monkey leaves the branch and drops freely, the bullet:
(A) hits the monkey (B) misses to hit the monkey
(C) can not be said (D) None of these

35. A projectile can have the same range R for two angles of projections. If t_1 and t_2 be the times of flight in the two cases, then the product of the times of flight will be-
(A) $t_1t_2 \propto R$ (B) $t_1t_2 \propto R^2$ (C) $t_1t_2 \propto 1/R$ (D) $t_1t_2 \propto 1/R^2$

36. If a baseball player can throw a ball from the ground level so that it covers a maximum distance of d over a horizontal ground, the maximum vertical height to which he can throw it, will be – (The speed of projection of the ball is the same in both the cases)
(A) d/2 (B) d (C) 2d (D) d/4

37. A projectile is launched from the ground level with a speed u at an angle θ with the horizontal. What is its average velocity between the two instants that it crosses half the maximum height.
(A) $u \sin \theta$ (B) $u \cos \theta$ (C) $u \tan \theta$ (D)u

38. A boy throws a ball with a velocity V_0 at an angle α to the horizontal. At the same instant, he starts running with uniform velocity to catch the ball before it hits the ground. To achieve this, he should run with a velocity of –
(A) $V_0 \cos \alpha$ (B) $V_0 \sin \alpha$ (C) $V_0 \tan \alpha$ (D) $\sqrt{V_0^2 \tan \alpha}$

39. An artillery piece which consistently shoots its shells with the same muzzle speed has a maximum range of R. To hit a target which is R/2 from the gun and on the same level, at what elevation angle should the gun be pointed-
(A) 30° (B) 45° (C) 60° (D) 75°

40. A large number of bullets are fired in all direction with the same speed v. What is the maximum area on the ground on which these bullets will spread-
(A) $\frac{\pi v^2}{g}$ (B) $\frac{\pi v^4}{g^2}$ (C) $\frac{\pi^2 v^4}{g^2}$ (D) $\frac{\pi^2 v^2}{g^2}$

41. A stone is thrown from a bridge at an angle of 30° with the horizontal in the downward direction with a speed of 25m/s. If the stone strikes the water after 2.5sec, then calculate the height of the bridge from the water surface - $\left(g = 9.8\, m/s^2\right)$
(A) 61.9m (B) 35m (C) 70m (D) None

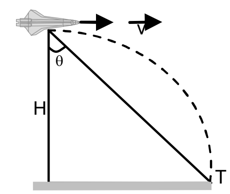
42. An aeroplane is flying horizontally with a velocity of 720km/h at an altitude of 490m. When it is just vertically above the target a bomb is dropped from it. How far horizontally does it miss the target? $\left(g = 9.8\, m/s^2\right)$
(A) 1000m (B) 2000m (C) 100m (D) 200m

43. An aeroplane is flying in the horizontal direction with a velocity 600 km/hr, at a height 1960m. When it is vertically above a point A, on the ground, a body is dropped from it. The body strikes the ground at point B. The distance AB is equal to: $\left(g = 9.8\, m/s^2\right)$
(A) 3.333km (B) 33.33km (C) 333.3km (D) 33.33m

44. An aeroplane is moving with a horizontal velocity u at a height h above the ground. If a packet is dropped from it the speed of the packet when it reaches the ground will be –
(A) $\sqrt{u^2 + 2gh}$ (B) $\sqrt{2gh}$ (C) $\sqrt{u^2 - 2gh}$ (D) 2gh

45. From the top of a tower A of height h a body of mass m is projected in the horizontal direction with a velocity v. It falls on the horizontal ground at a distance x from the foot of the tower A.If a body of mass 2m is projected from the top of another tower B of height 2h in the horizontal direction so that if falls on the ground at a distance 2x from the foot of the tower B, the horizontal velocity of the second body is –
(A) 2v (B) $\sqrt{2}V$ (C) $V/2$ (D) $V/\sqrt{2}$

46. A bomber is moving in horizontal direction with a velocity v (m/s) at a height of H meter from the ground. The bomber releases a bomb to hit a target T when the sighting angle with the vertical is θ as shown in the figure. Then the relation between θ, H and v is –



(A) $\theta = \tan^{-1} V\sqrt{2Hg}$
(B) $\theta = \tan^{-1} V\sqrt{2/gH}$
(C) $\theta = \tan^{-1} V\sqrt{H/2g}$
(D) None of these

EXERCISE – 2

One or More Option may be Correct:

1. A particle has initial velocity 10 m/s. It moves against a constant retarding force along the line of velocity which produces a retardation of 5 m/s². Then
(A) the maximum displacement in the direction of the initial velocity is 10 m
(B) the distance travelled in the first 3 seconds is 7.5 m
(C) the distance travelled in the first 3 seconds is 12.5 m
(D) the distance travelled in the first 3 seconds is 17.5 m.

2. Mark the correct statements for a particle going on a straight line
(A) if the velocity is zero at any instant, the acceleration should also be zero at that instant
(B) if the average velocity is zero for a given time interval, the average speed must also be zero within the same time interval.
(C) if the velocity and acceleration have opposite sign, the object is slowing down
(D) if the position and velocity have opposite sign, the particle is moving towards the origin

3. An observer moves with a constant speed along the line joining two stationary objects. He will observe that the two objects
(A) have the same speed
(B) have the same velocity
(C) move in the same direction
(D) move in opposite directions

4. For a body in one–dimensional motion
(A) Speed must decrease when acceleration is negative
(B) Speed must increase when acceleration is positive
(C) Speed will increase when both velocity and acceleration are positive
(D) Speed will decrease when velocity is negative and acceleration is positive

5. Which of the following statements are true for a moving body?
(A) If its speed changes, its velocity must change and it must have some acceleration
(B) If its velocity changes, its speed must change and it must have some acceleration
(C) If its velocity changes, its speed may or may not change, and it must have some acceleration
(D) If its speed changes but direction of motion does not change, its velocity may remain constant

6. Let ‘v’ and ‘a’ denote the instantaneous velocity and the instantaneous acceleration respectively of a body, then
(A) ‘a’ can be non zero when ‘v’ = 0
(B) ‘a’ must be zero when ‘v’ = 0
(C) ‘a’ may be zero when $'v' \neq 0$
(D) The direction of ‘a’ must have some correlation with the direction of ‘v’

7. An object may have
(A) varying speed without having varying velocity
(B) varying velocity without having varying speed
(C) non zero acceleration without having varying velocity
(D) non zero acceleration without having varying speed

8. A body moving with uniform acceleration in a straight line describes $25\,\text{m}$ in the 5^{th} second & $33\,\text{m}$ in 7^{th} second.
(A) Initial velocity is $6\,\text{m/s}$
(B) Initial velocity is $7\,\text{m/s}$
(C) Acceleration is $2\,\text{m/s}^2$
(D) acceleration is $4\,\text{m/s}^2$

9. A particle starts with an initial velocity of $4\,\text{m/s}$ &an acceleration of $2\,\text{m/s}^2$
(A) The distance described in the 5^{th} second is $13\,\text{m}$.
(B)Time taken to attain velocity of $50\,\text{m/s}$ is 23 s.
(C) Time taken in travelling the first $60\,\text{m}$ is 6 s.
(D) Velocity it attains when it has described $45\,\text{m}$ is $14\,\text{m/s}$.

10. A motorist is travelling at $54\,\text{km/hr}$ when he observes a traffic signal at a distance of $280\,\text{m}$ ahead turning red at time t = 0. If he decides to decelerate & then accelerate by the same magnitude, so as to pass the signal at $54\,\text{km/hr}$ at t = 28 seconds, when the signal turns green, then
(A) Magnitude of deceleration is $\frac{5}{7}\,\text{m/s}^2$
(B) Magnitude of deceleration is $1\,\text{m/s}^2$
(C) Minimum speed reached is $5\,\text{m/s}$
(D) Minimum speed reached is $10\,\text{m/s}$

11. A particle moving with constant acceleration along a straight line covers distance between 2 points $80\,\text{m}$ apart in 10 seconds. Its velocity as it passed the second point is $18\,\text{m/s}$.
(A) Its velocity at the first point is $-2\,\text{m/s}$
(B) Its acceleration is $2\,\text{m/s}^2$
(C) The total distance travelled during these 10 seconds is $82\,\text{m}$
(D) All above are false.

12. A projectile is projected with an initial velocity of $(6\hat{i}+8\hat{j})\,\text{m/s}$. Then $(g = 10\,\text{m/s}^2)$.
(A) Its horizontal range is $9.6\,\text{m}$
(B) Maximum height attained is $3.2\,\text{m}$
(C) Time of flight is $1.6\,\text{s}$
(D) All above are false.

13. An enemy ship is at a horizontal distance of $180\sqrt{3}\,\text{m}$ from a security cannon having a muzzle velocity of $60\,\text{m/s}$ $(g = 10\,\text{m/s}^2)$

(A) Angle of elevation of cannon to hit ship is 30^0 or 60^0
(B) Time of flight can be 6 s
(C) Time of flight can be 10.4 s
(D) Distance that the ship should be moved away from its initial position so that it becomes beyond the range of the cannon is $48.6\,\text{m}$.

14. A dive bomber, diving at an angle of 53^0 with the vertical, releases a bomb at an altitude of $2400\,\text{ft}$. The bomb hits the ground $5\,\text{s}$ after being released $(g = 32\,\text{ft/s}^2)$.

(A) Speed of the bomber is $667\,\text{ft/s}$
(B) Horizontal distance travelled during the flight is $2667\,\text{ft}$.
(C) Horizontal component of the velocity of the bomb just before striking the ground is $534\,\text{ft/s}$
(D) Vertical component of the velocity of the bomb just before striking the ground is $560\,\text{ft/s}$

EXERCISE – 3

Comprehension – 1

Two balls A and B are thrown with the same speed from the top of a tower. Ball A is thrown vertically upwards and the ball B is thrown vertically downwards. $\left(g = 10\,\text{m/s}^2\right)$

1. Choose the correct statement

(A) Ball B reaches the ground with greater velocity
(B) Ball A reaches the ground with greater velocity
(C) Both the balls reach the ground with same velocity
(D) Cannot be interpreted

2. If t_A and t_B are the times taken by the balls A and B respectively to reach the ground, then identify the correct statement

(A) $t_A > t_B$ (B) $t_A = t_B$ (C) $t_A < t_B$ (D) Cannot be interpreted

3. If $t_A = 6$ s and $t_B = 2$ s, then the height of the tower is

(A) 80 m (B) 60 m (C) 45 m (D) none of these

4. The speed with which each ball was thrown is

(A) 10 ms^{-1} (B) 15 ms^{-1} (C) 20 ms^{-1} (D) none of these

5. If a ball C is thrown with the same speed as A and B, but in the horizontal direction from the top of the tower, then it will reach the ground in time

(A) 4 s (B) 3.46 s (C) 4.2 s (D) none of these

Match List - I

In column I, the description of the change in the velocity of a body moving with constant acceleration (or retardation) in a 10 s interval is given , while column II gives the information about the acceleration in corresponding time interval. Match the entries of column I with the entries of column II.

Column – I	**Column – II**
(A) At the beginning of the interval the body is moving towards the right along the x – axis at 5 m/s and at the end of the interval it is moving towards right at 20 m/s	(p) magnitude of acceleration is 1.5 m/s^2
(B) At the beginning , the body is moving towards right at 20 m/s and at the end it is moving towards right at 5 m/s.	(q) average acceleration is towards right
(C) At the beginning, the body is moving towards left at 5m/s and at the end it is moving towards left at 20m/s.	(r) average acceleration is towards towards left
(D) At the beginning the body is moving towards right at 5 m/s and at the end it is moving towards left at 10 m/s.	(s) body is decelerated for atleast some part of its motion

Match List – II

For a particle moving along x -axis with constant acceleration towards negative x - axis, match the entries of column (I) with entries of column (II) .

Column – I	**Column – II**
(A) Initial velocity > 0	(p) Particle may move in positive x – direction with increasing speed.
(B) Initial velocity < 0	(q) Particle may move in positive x – direction with decreasing speed.
(C) displacement > 0	(r) Particle may move in negative x – direction with increasing speed.
(D) displacement < 0	(s) Particle may move in negative x – direction with decreasing speed.

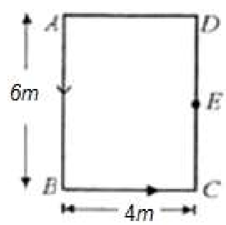
EXERCISE – 4

1. A particle starts from a point A with a velocity of 200 cm/sec towards east and moves in a straight line with a constant acceleration of 10 cm/sec^2 towards west. Find the time it takes to reach a point B which is at a distance of 1500 cm east of A.

2. A particle starts from a point A with an initial velocity of 10 m/sec and moves along a straight line with a constant acceleration. At the instant the particle attains velocity of 50 m/sec, the direction of its acceleration gets reversed, magnitude of the acceleration remaining the same. Find the velocity of the particle when it reaches the point A again.

3. A bullet enters a thick wooden block with a certain velocity and loses $\frac{1}{20}$ of its velocity when it emerges from the opposite face of the block. What is the minimum number of such identical blocks that must be placed in series (in contact one after the other) so that the bullet fired into the first of the blocks with the same velocity as before will come to rest inside the last of the blocks. Assume that all the blocks offer same uniform retardation to the bullet.

4. A particle starts moving from point A, along a rectangular plot via the path shown in Figure. If time taken to reach point E (where E is the mid point of DC) is 0.5 sec, then find the average speed and average velocity of the particle between points A & E.



5. A parachutist drops from a stationary helicopter and falls freely for 10 sec. At this instant, the parachute opens out after which he decends with a net deceleration of 2m/sec^2. If he reaches the ground with a velocity of 8m/sec, find the height above the ground at which he dropped out of the helicopter.(g = 10m/sec^2)

6. A particle is projected vertically upwards from the ground at time t = 0. The particle is at height h at t = $t_1$seconds and is again at height h at t = t_2 seconds. Prove that h = (1/2)gt_1t_2. Also find the initial velocity of projection.

7. From the foot of a tower 90m high, a stone is thrown vertically upwards with velocity of $30\sqrt{2}\,\text{m/s}$. Two seconds later another stone is dropped from the top of the tower. This stone collides with the first stone after a certain time interval. Find when and where the two stones meet. (g = 10m/sec^2)

8. From an elevated point A, a stone is projected vertically upwards. In the course of its motion, velocity of the stone at a distance *h* below A, is double of its velocity at a height *h* above A. Show that the greatest height attained by the stone above point A is (5/3)*h*.

9. A person sitting on the top of a tall building is dropping balls at regular intervals of one second. Find the positions of the 3rd, 4th and 5th balls with respect to the top of the building, when the 6th ball is being dropped. (g = 10m/sec^2)

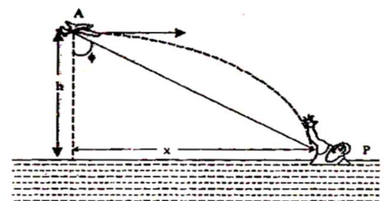
10. A steamer moving in a river takes m seconds to go 'a' meters downstream and n seconds to go 'a' meters upstream. Find the speed of the river water and the speed of the steamer relative to the river water in m/sec.

11. A particle projected vertically upwards from the top of a tower with a certain speed strikes the ground in 9 seconds. The same particle thrown vertically downwards from the top of the same tower with the same speed as before strikes the ground in 4 seconds. Find the speed of projection and the height of the tower $\left(g = 10\,m/s^2\right)$

12. A particle is projected from horizontal ground with a speed of 20m/s at a certain angle with the horizontal, so that it just clears two vertical walls of equal heights 10m which are at a horizontal distance of 20m from each other. Find the time taken by the particle to pass between the two walls.

13. A stone is thrown horizontally from an elevated point. After 0.5 seconds, the magnitude of its velocity is 1.5 times the magnitude of its initial velocity. Find the initial speed of stone.

14. A relief aeroplane is flying horizontally at a constant height of 1960m above the water level with speed600km/hr towards a point directly over a person struggling in flood water (see figure). At what angle of sight with the vertical ϕ , should the pilot release a survival kit if it is to reach the person in water. (g = 9.8 m/s^2)



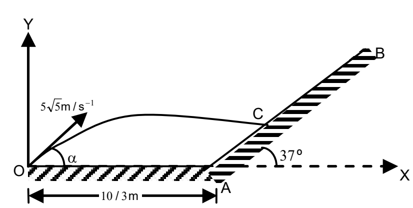
15. An aeroplane is flying at a constant vertical height of 1960 meter with a constant horizontal velocity of 100m/s. When it is vertically above an object M on the ground it drops a bomb. If the bomb reaches the ground at the point N, then calculate the time taken by the bomb to reach the ground and also find the distance MN. (g = 9.8m/sec^2)

16. Show that a bullet fired from a gun on horizontal ground, will attain three times the maximum height, when elevated at an angle of 60° with the horizontal, as compared to the maximum height attained when fired at an angle of 30° with the horizontal. Also show that the horizontal distance covered in the two cases will be equal. Assume that the muzzle velocity is the same in both the cases.

17. Two balls are projected from the same point on horizontal ground at angles of projection 60° and 30° to the horizontal respectively. If they attain the same maximum height, what is the ratio of their speeds of projection? If they are now projected with new speeds, what is the ratio of their speeds of projection if they attain the same horizontal range, the angles of projection being the same as before?

18. A batsman hits a pitched ball at a height 4.0ft above the ground so that the balls leaves the bat at 45° with the horizontal and its horizontal range is 350 ft. The ball approaches a 24 ft high vertical fence which is located at a horizontal distance of 320 ft from the batsman. Will the ball clear the fence? (Take g = 32 ft /sec^2)

19. A particle is projected from point O on the ground with velocity $u = 5\sqrt{5}\,m/s$ at angle α = tan^{-1} (0.5) with the horizontal. It strikes a point C on a fixed smooth plane AB having inclination of37° with horizontal as shown in the figure. Calculate the coordinates of point C in reference to coordinate system shown in figure.



20. A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60^0 to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

ANSWER KEY

EXERCISE – 1

1. (C)	2. (C)	3. (D)	4.(C)	5. (D)	6. (C)	7. (C)
8. (C)	9. (C)	10. (C)	11. (A)	12. (B)	13. (B)	14. (A)
15. (D)	16. (C)	17. (C)	18. (D)	19. (C)	20. (A)	21. (D)
22. (B)	23. (D)	24. (A)	25. (D)	26. (C)	27. (D)	28. (B)
29. (C)	30. (A)	31. (B)	32. (D)	33. (B)	34. (A)	35. (A)
36. (A)	37. (B)	38. (A)	39. (D)	40. (B)	41. (A)	42. (B)
43. (A)	44. (A)	45. (B)	46. (B)			

EXERCISE – 2

1. (A, C)	2. (C, D)	3. (A, B, C)	4. (C, D)	5. (A, C)	6. (A, C)
7. (B. D)	8. (B,D)	9. (A, B, C, D)	10. (A, C)	11. (A, B, C)	12. (A, B, C)

13. (A, B, C, D) 14. (A, B, C, D)

EXERCISE – 3

COMPREHENSION – 1

1. (C) 2. (A) 3. (B) 4. (C) 5. (B)

MATCH LIST – I

A – p, q B – p, r, s C – p, r D – p, r, s

MATCH LIST – II

A –q, r B –r C – q, r D – q, r

EXERCISE – 4

1. 10s, 30s 2. 70 m/s 3. 11 4. 26 m/s, 10 m/s 5. 2984 m

6. $u = \frac{g}{2}(t_1 + t_2)$ 7. 3.12 s, 6.3 m from tower top 9. 45 m, 20 m, 5 m

10. $\frac{a}{2}\left(\frac{1}{m}-\frac{1}{n}\right), \frac{a}{2}\left(\frac{1}{m}+\frac{1}{n}\right)$ 11. 25m/s, 180m 12. 2 sec 13. 4.5 m/s

14. $\tan^{-1}(1.7)$ 15. 20 s, 2000 m 17. $1:\sqrt{3}, 1:1$

18. Yes 19. (5m, 1.25 m) 20. 5

IIT–JEE SYLLABUS

Frame of reference, Motion in a straight line: Position – time graph, speed and velocity. Uniform and non – uniform motion, average speed and instantaneous velocity Uniformly accelerated motion, velocity – time, position – time graphs, relations for uniformly accelerated motion. Relative velocity, Motion in a plane, Projectile motion, Uniform circular Motion.

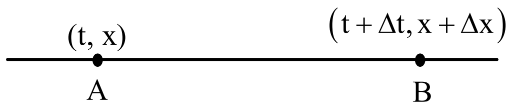
CONTENTS

Kinematics – II Analysis : Tentative Lecture Flow (Board Syllabus & Booklet Discussion Included)

Lecture 1	Instantaneous velocities, acceleration, s - t, v - t, a - t graphs, v - s and a - s graphs. Solved examples
Lecture 2	Projectile on an inclined plane. Range, Time of flight, max range. Solved examples
Lecture 3	Relative velocity concept in river boat problems. Solved examples
Lecture 4	Relative velocities concept in two dimensions (like two projectiles, two bodies approaching shortest distance of approach)

KINEMATICS – II

1. a Instantaneous velocity

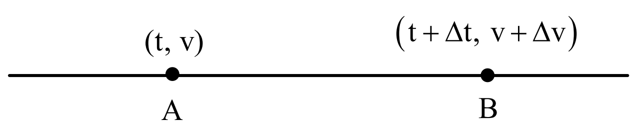


Suppose a particle moving in a straight line is at point A at time t. The coordinate of A is x. Now the particle moves to point B at time $t+\Delta t$. The coordinate of B is $x+\Delta x$. The average velocity over the time interval Δt can be written as $\frac{\Delta x}{\Delta t}$

- To get instantaneous velocity at time t, $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
- Thus, instantaneous velocity at any time t is $v = \frac{dx}{dt} = \frac{ds}{dt}$

Here,
v = instantaneous velocity at time t
s = instantaneous displacement at time t
x = instantaneous position at time t

1. b Instantaneous acceleration



Suppose a particle moving in a straight line has velocity v at time t at position A and velocity $v+\Delta v$ at time $t+\Delta t$ at position B, the average acceleration over time interval Δt can be written as $a = \frac{\Delta v}{\Delta t}$.

- To get instantaneous acceleration at time t, $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$.
- $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$
- $a = \frac{dv}{dt} = \frac{dv}{dx}\,\frac{dx}{dt} = v\left(\frac{dv}{dx}\right)$

$\therefore \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\left(\frac{dv}{dx}\right)$

Solved Examples

Illustration 1. **A particle has position x, varying with time t as** $x=\sin t-\sqrt{3}\cos t$**. Find**
(a) instants when particle is at origin
(b) velocity at $t=\frac{\pi}{4}$
(c) maximum magnitude of velocity

Solution: (a) $x=0 \quad \therefore \quad 0=\sin t-\sqrt{3}\cos t$

$\therefore \quad \tan t=\sqrt{3}$

$\therefore \quad t=\frac{\pi}{3}+n\pi, \qquad n=1,2,3...$

(b) $v=\frac{dx}{dt}=\cos t+\sqrt{3}\sin t$

$$v\left(t=\frac{\pi}{4}\right)=\cos\frac{\pi}{4}+\sqrt{3}\sin\frac{\pi}{4}=\frac{1+\sqrt{3}}{\sqrt{2}}$$

(c) $v=\cos t+\sqrt{3}\sin t$

For maximum velocity $\frac{dv}{dt}=0$

$\therefore \quad -\sin t+\sqrt{3}\cos t=0 \quad \therefore \quad t=\frac{\pi}{3}$

$$\text{at } t=\frac{\pi}{3}, v=\cos\frac{\pi}{3}+\sqrt{3}\times\sin\frac{\pi}{3}=2$$

Illustration 2. **A particle starts moving such that displacement with respect to origin O varies with time t as** $s=t^2(t-1)$**.**
(a) Find velocity and acceleration when particle is at origin O
(b) Displacement when velocity is zero.
(c) Displacement when acceleration is zero.

Solution: (a) $S=t^3-t^2, \quad v=\frac{ds}{dt}=3t^2-2t, \quad a=\frac{dv}{dt}=6t-2$

When particle is at origin O

$S=0, \quad \therefore \quad t^2(t-1)=0 \quad \therefore \quad t=0 \text{ or } t=1s$

There are 2 instants t = 0 and t = 1s when particle is at O.

$\therefore \quad t=0, v=0, \ a=-2$

and $\quad t=1, v=1, \ a=4$

(b) $v=0 \quad \therefore \quad 3t^2-2t=0, t=0 \text{ or } t=\frac{2}{3}$

$$\therefore \quad S=0 \text{ or } S=\left(\frac{2}{3}\right)^3-\left(\frac{2}{3}\right)^2=-\frac{4}{27}$$

$\therefore$ $S = 0$ at $t = 0$

and $S = -\dfrac{4}{27}$ at $t = \dfrac{2}{3}$

(c) $a = 0$ $\therefore$ $6t - 2 = 0$ $\therefore$ $t = \dfrac{1}{3}$

$$S = \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 = \frac{-2}{27}$$

Illustration 3. The instantaneous velocity of a particle varies with position x as v = 2x + 1. Assuming that particle was at origin at t = 0, find the relation between x and t.

Solution: $v = 2x + 1$

$$\frac{dx}{dt} = 2x + 1, \quad \int_{x=0}^{x=x} \frac{dx}{2x+1} = \int_{t=0}^{t=t} dt$$

$$\frac{1}{2}\ln[2x+1]_0^x = [t]_0^t$$

$$\frac{1}{2}\ln[2x+1] = t$$

$\therefore$ $\ln(2x + 1) = 2t$

$\therefore$ $2x + 1 = e^{2t}$

$\therefore$ $x = \dfrac{e^{2t} - 1}{2}$

Inchapter Exercise

1. Displacement x of a particle varies with time t as $x = 4t^2 - 15t + 25$.
   (a) Find position, velocity and acceleration of particle at t = 0.
   (b) When will velocity of particle become zero?
   (c) Is the body uniformly accelerated?

2. Velocity of a particle is given by the equation $v = 2t^2 + 5$ cm/s. Find
   (a) Average acceleration during the time interval $t_1 = 2s$ & $t_2 = 4s$.
   (b) Instantaneous acceleration at $t_2 = 4s$.

3. Position x of a particle moving in one dimension is related to time t as $t = \sqrt{x} + 3$, where x is in meters and t is in seconds. Find position of particle when its velocity is zero.

4. Position x of a particle varies with time t as $x = 2 - 5t + 6t^2$. Find initial velocity of particle.

5. Position x of a particle is related to time t as $x = 3 + 8t + 7t^2$. Find its velocity and acceleration at t = 2s. x is in m, t is in second.

6. Position x of a particle moving in a straight line is related to time t as $x = 180t + 50t^2$. Find
   (a) Initial velocity of particle (b) Acceleration of particle
   (c) Velocity after 4 s

Answer Key

1. (a) $x = 25m,\ v = -15m/s,\ a = 8\,m/s^2$ (b) $t = 1.875$ s
(c) yes as acceleration does not depend on time.

2. (a) $12\,cm/s^2$ (b) $16\,cm/s^2$ **3.** 0 meters

4. – 5 units. **5.** 36 m/s, $14\ m/s^2$

6. (a) 180 m/s (b) $100\ m/s^2$ (c) 580 m /s

2. Uniformly accelerated motion –

- A particle moving in a straight line is said to be uniformly accelerated, if it makes equal changes in velocities in equal intervals of time, however small the time interval may be.
- In uniformly accelerated motion, instantaneous acceleration is constant and is independent of time.
- **Derivation of equations for uniformly accelerated motion.**

(a) $a = \frac{dv}{dt}$

$$\int_{v=u}^{v=v} dv = a\int_{t=0}^{t=t} dt \qquad [v]_u^v = a[t]_0^t$$

$$\boxed{\therefore v = u + at} \qquad \text{… Eq. (I)}$$

(b) $v = \frac{ds}{dt}$

From Eq. (I) $v = u + at$

$$\frac{ds}{dt} = u + at$$

$$\int_{s=0}^{s=s} ds = \int_{t=0}^{t=t} (u + at)dt$$

$$\int_{s=0}^{s=s} ds = \int_{t=0}^{t=t} u\,dt + \int_{t=0}^{t=t} at\,dt$$

$$\boxed{s = ut + \frac{1}{2}at^2} \qquad \text{… Eq. (II)}$$

(c) Let at $t = 0,\quad x = x_0$

$t = t,\quad x = x$

$$v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = u + at$$

$$\int_{x=x_0}^{x=x} dx = \int_{t=0}^{t=t} (u+at)\,dt$$

$$\boxed{x = x_0 + ut + \frac{1}{2}at^2}$$

Or $\quad x - x_0 = ut + \frac{1}{2}a\,t^2$ … Eq. (III)

(Here $s = x - x_0$ = displacement or change in position)

(d) $a = v\left(\frac{dv}{ds}\right)$

$$\int_{v=u}^{v=v} vdv = a\int_{s=0}^{s} ds$$

$$\boxed{v^2 = u^2 + 2as}$$ … Eq. (IV)

(e) $a = v\left(\frac{dv}{dx}\right)$

$$\int_{v=u}^{v=v} vdv = a\int_{x=x_0}^{x=x} dx$$

$$\boxed{v^2 = u^2 + 2a(x - x_0]}$$ …Eq. (V)

(f) Eliminating t between Eq. (I) and Eq. (II), we get

$$\boxed{s = \frac{(u+v)}{2}\times t}$$ … Eq. (VI)

(g) Displacement in n^{th} second for a uniformly accelerated body.

Let total displacement of body in n seconds be S_n.

$$\therefore \quad S_n = un + \frac{1}{2}an^2$$

Let total displacement of body in (n – 1) seconds be S_{n-1}.

$$\therefore \quad S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

Then displacement of body in n^{th} second will be $S_{n^{th}} = S_n - S_{n-1}$

$$S_{n^{th}} = un + \frac{1}{2}an^2 - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$\boxed{S_{n^{th}} = u + a\left[n - \frac{1}{2}\right]}$$ … Eq. VII

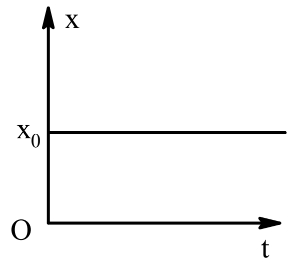
3. Graphs describing motion of a particle along a straight line.

3.1 Position versus time graph (x – t graph)

- Slope of tangent drawn at any point on the curve of the x – t graph gives instantaneous velocity and magnitude of slope gives instantaneous speed

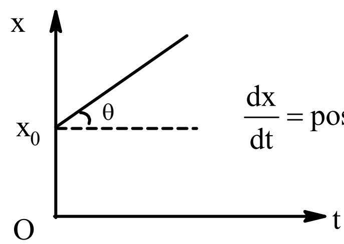
Description of motion using x – t graph.

(A) Particle at rest



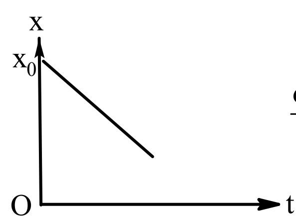
$\frac{dx}{dt} = 0$, Particle is at rest

(B) Particle moving with uniform velocity in the positive direction.



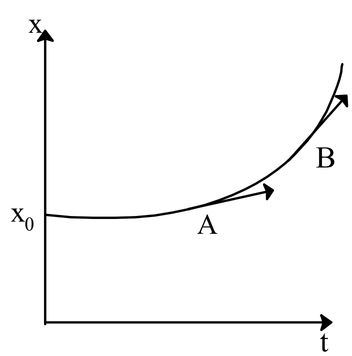
$\frac{dx}{dt} =$ positive constant

(C) Particle moving with uniform velocity in negative direction.



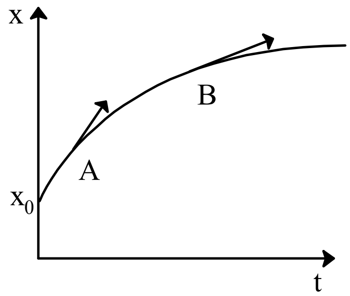
$\frac{dx}{dt} =$ negative constant

(D) Particle moving with uniform acceleration.



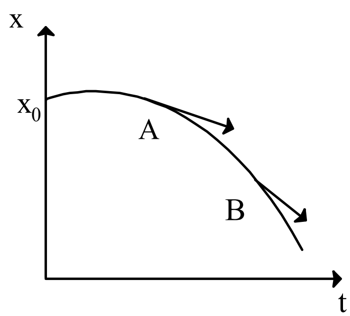
Slope at B is greater than slope at A, implying acceleration

(E) Particle moving with uniform retardation.



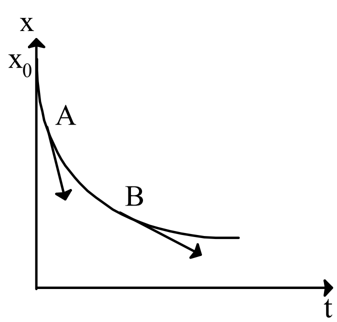
Slope at B is less than slope at A, implying retardation.

(F) Particle moving with uniform acceleration in negative direction



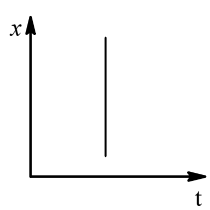
Magnitude of slope at B is greater than that at A, but motion is in negative direction

(G) Particle moving with uniform retardation in negative direction.



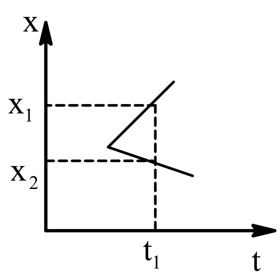
Slope at B is less than that of A. Motion is in the negative direction.

(H) The following x-t graphs are not possible.



Slope is infinite, implying infinite velocity, which is not possible

Particle is at two positions x_1 and x_2 at a given time instant t_1 which is not possible.



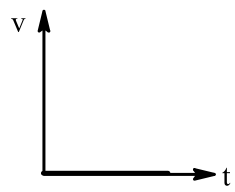
3.2 Velocity versus time graph

- $\frac{dv}{dt} = a$, Slope of v – t graph gives instantaneous acceleration
- $\frac{ds}{dt} = v, \quad \therefore \int ds = \int vdt \qquad \therefore \int vdt = s,$

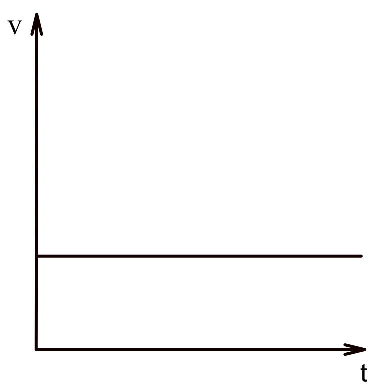
  Area under v –t graph gives displacement.
- Thus v – t graph gives us instantaneous velocity, instantaneous acceleration as well as displacement covered. Hence v – t graphs can be effectively utilized in solving problems.

Description of motion with v – t graph

(A) Particle at rest

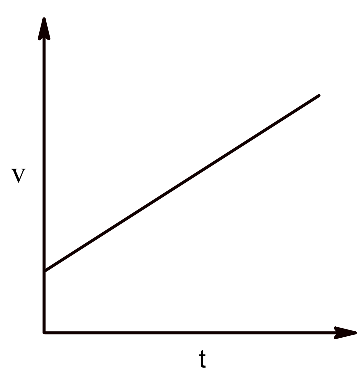


(B) Particle moving with uniform velocity



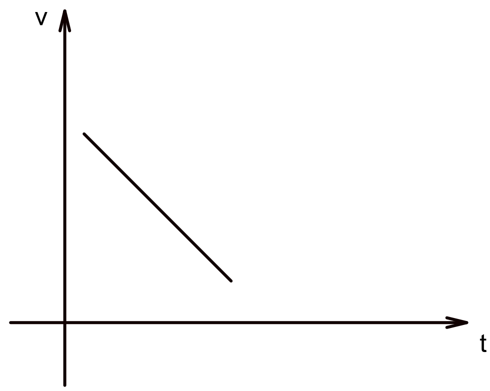
$\frac{dv}{dt} = 0$, acceleration is zero.

(C) Particle moving with uniform acceleration



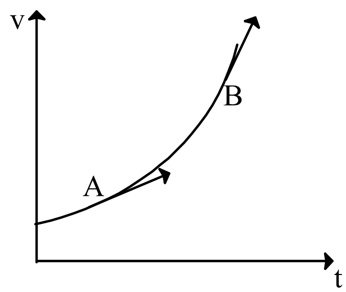
Slope of above graph is constant and positive, implying uniform acceleration.

(D) Particle moving with uniform retardation



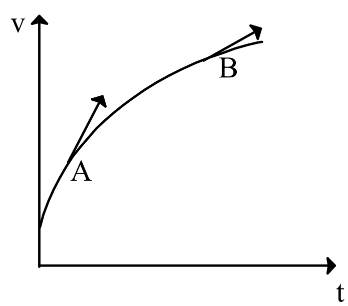
Slope of above graph is constant and negative, implying uniform retardation.

(E) Particle moving with increasing acceleration.



Slope at A is less, slope at B is more, implying increasing acceleration.

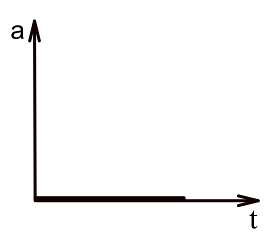
(F) Particle moving with decreasing acceleration.



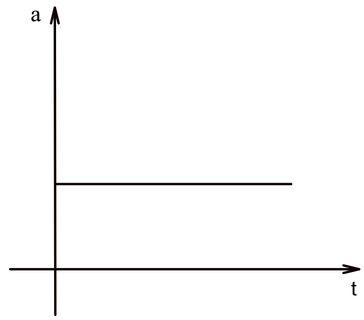
Slope at B is less than that at A implying decreasing acceleration.

3.3 <u>Acceleration time graph</u>

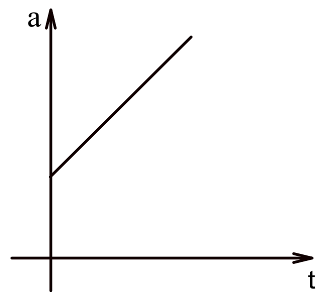
(A) Particle with zero acceleration. (rest or uniform velocity)



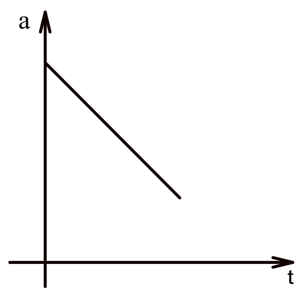
(B) Particle moving with uniform acceleration



(C) Particle moving with increasing acceleration



(D) Particle moving with decreasing acceleration



4. Area under various graphs

- $\frac{dv}{dt} = a \quad \therefore \int_u^v dv = \int_0^t a dt \quad \therefore v - u = \int a dt$

  $\therefore$ Area under a – t graph gives change in velocity.
- $a = v\left(\frac{dv}{ds}\right) \quad \therefore \int a ds = \int_u^v v\, dv$

  $\therefore \int a ds = \frac{v^2 - u^2}{2}$

  Area under a – s graph $= \frac{v^2 - u^2}{2}$

  Where, v is instantaneous velocity and u is initial velocity.
- $\frac{ds}{dt} = v$

  $\int ds = \int v dt \quad \therefore \; s = \int v\, dt$

  $\therefore$ Area under v – t graph gives displacement.

Solved Examples

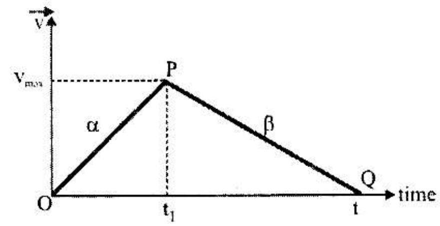
Illustration 4. **A car accelerates from rest at a constant rate α for some time, after which it declerates at a constant rate of β to come to rest. If the total time elapsed is t second, then calculate;**

(i) The maximum velocity attained by the car, and
(ii) The total displacement travelled by the car in terms of α, β and t.

Solution: Let v_{max} be the maximum velocity attained and t_1 be the time at which maximum velocity will occur. The velocity vs time graph can be drawn as follows:

The slope of line OP, $\alpha = \frac{v_{max}}{t_1}$

$\Rightarrow \quad v_{max} = \alpha t_1$ ..... (i)



The slope of line PQ, $\beta = \frac{v_{max}}{t - t_1}$

$v_{max} = \beta(t - t_1)$ ..... (ii)

From equations (i) and (ii), we get $\alpha t_1 = \beta(t - t_1)$

which gives $t_1 = \frac{\beta t}{\alpha + \beta}$ ..... (iii)

Substituting value of t_1 in equation (i), we get

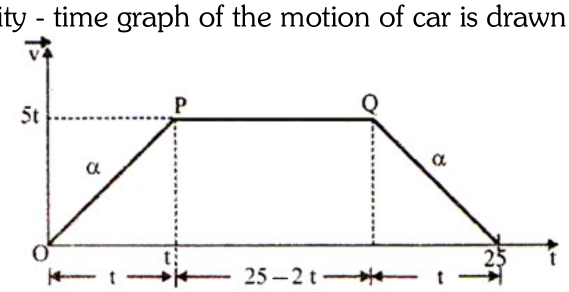
(i) $v_{max} = \frac{\alpha\beta t}{\alpha + \beta}$ Ans.

(ii) Total displacement $s = \text{area of } v - t \text{ graph}$

$$= \frac{1}{2} \times v_{max} \times t = \frac{1}{2} \times \frac{\alpha\beta t}{\alpha + \beta} \times t = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$$ Ans.

Illustration 5. **A car starts moving rectilinearly, first with acceleration $\alpha = 5 m/s^2$ (the initial velocity is equal to zero), then uniformly, and finally, decelerating at the same rate α comes to a stop. The total time of motion equals $\tau = 25 s$. The average velocity during that time is equal to $< v >$ = 72 km/h. How long does the car move uniformly?**

Solution: Let t be the time duration for which car accelerates or decelerates. The maximum velocity attained in this duration is 5 t. The time during which car move uniformly = 25 - 2t. The velocity - time graph of the motion of car is drawn as follows;



Given the average velocity in whole time of motion $v_{av} = \frac{72 \times 5}{18} = 20\,m/s$

The average velocity from the graph can be obtained as $v_{av} = \frac{\text{Total displacement}}{\text{Total time}}$

$= \frac{\text{Area of } v - t \text{ graph}}{\text{Total time}}$ $\therefore \quad 20 = \frac{\frac{1}{2} \times [25 + (25 - 2t)] \times 5t}{25}$

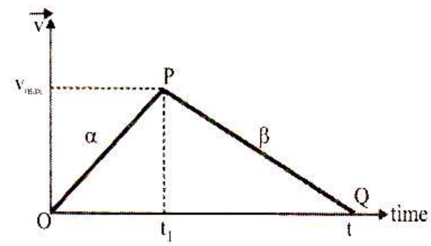
$= \frac{\frac{1}{2} \times [50 - 2t)] \times 5t}{25}$ or $200 = 50t - 2t^2$

or $t^2 - 25t + 100 = 0$ $(t - 20)(t - 5) = 0$ $t = 5$ s or 25s

But t = 25 is not possible t = 5s

The time for which car moves uniformly = 25 - 2t = 25 - 2 x 5 = 15 s Ans.

Illustration 6. **The distance (s) between two stations is to be covered in minimum time. The maximum value of acceleration or retardation of a car can not exceed** α and β **respectively. Find the time of motion.**



Solution: To cover the distance in minimum time the car must get the maximum possible acceleration α and then retard to maximum possible value β. Let t_1 be the time up to which car accelerates and t is the required time of motion. The velocity-time graph of motion of car can be drawn as follows:

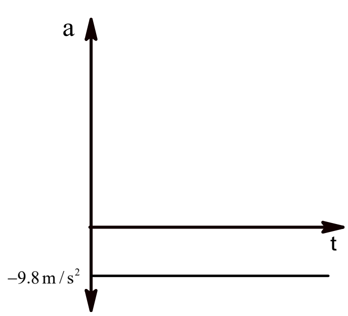
We have already calculated that $s = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$ (Refer illustration 4)

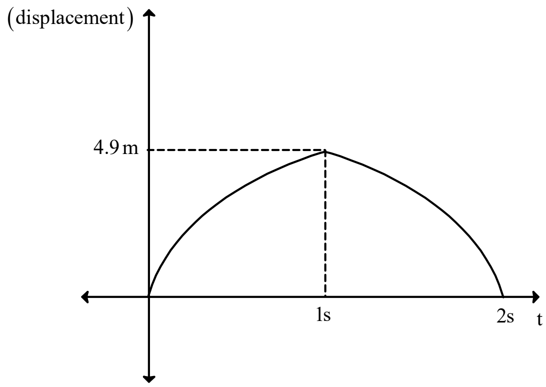
Solved above equation for t, we have $t = \sqrt{\frac{2s(\alpha + \beta)}{\alpha\beta}} = \sqrt{2s\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)}$ Ans.

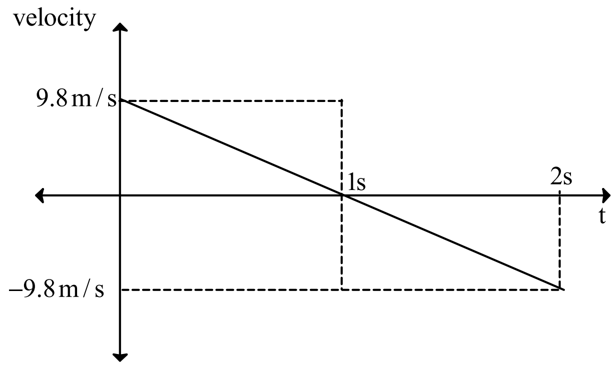
Illustration 7. **A body is thrown vertically upwards with 9.8 m/s. Take origin as point of projection and upward direction positive, plot displacement –time, distance-time, velocity-time, speed-time, acceleration-time graph.** $\left(g = 9.8\,m/s^2\right)$

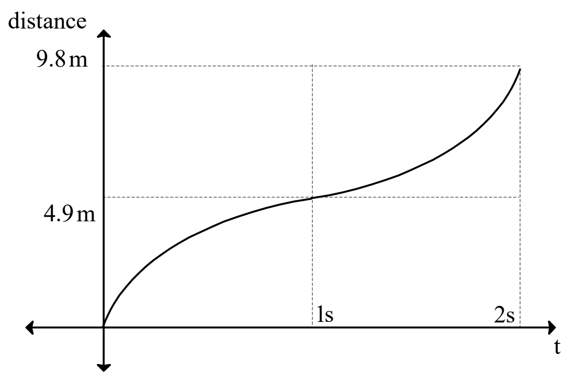
Solution : Time of ascent = time of descent = $\frac{u}{g} = 1s$

$$h_{max} = \frac{u^2}{2g} = \frac{9.8^2}{2 \times 9.8} = 4.9m$$




(displacement)
4.9 m
1s
2s
t


velocity
9.8 m / s
1s
2s
t
–9.8 m / s


distance
9.8 m
4.9 m
1s
2s
t

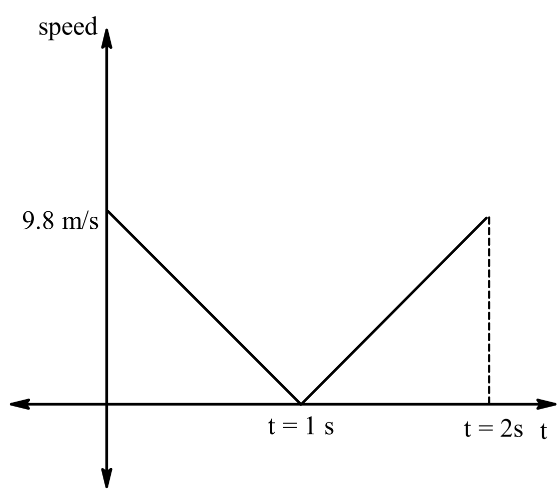
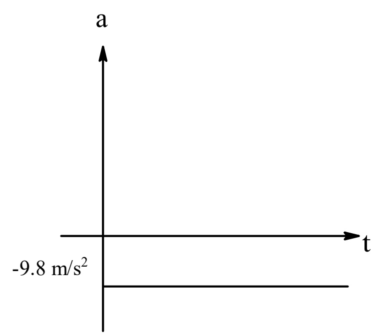

speed
9.8 m/s
t = 1 s
t = 2s
t

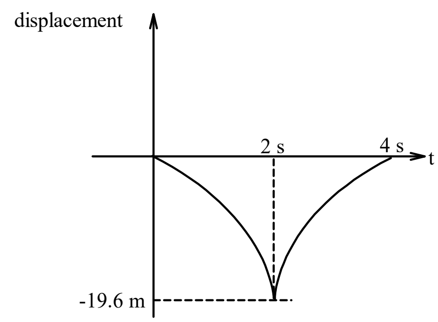
Illustration 8 **A body is dropped from a height of 19.6m. It strikes the ground and rebonds back to the same height of 19.6m. Taking point of dropping as origin and upwards positive, plot displacement – time, velocity-time, distance-time, speed-time, acceleration time graph.** $\left(g = 9.8 m/s^2\right)$

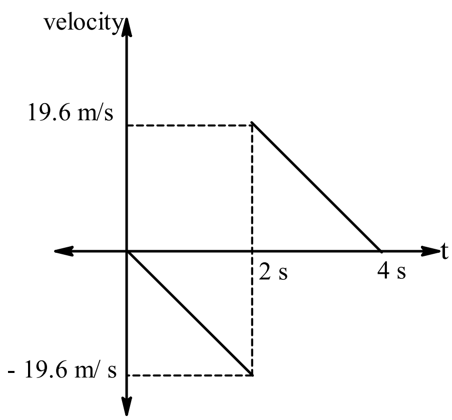
Solution:

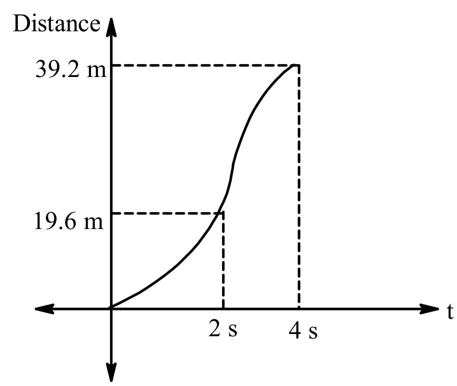
Time to reach ground $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 19.6}{9.8}} = 2s$

Velocity of striking ground $= \sqrt{2gh} = \sqrt{2 \times 9.8 \times 19.6} = 19.6 m/s$









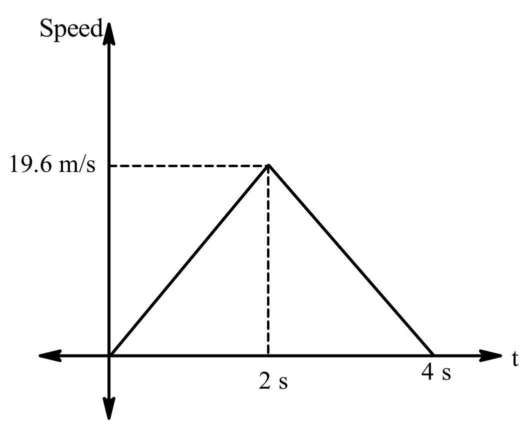
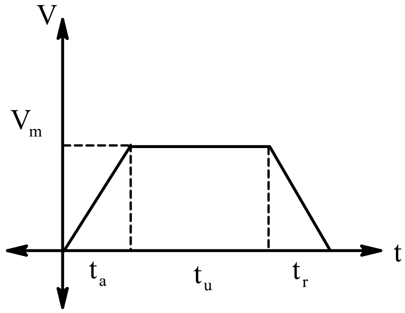


Illustration 9. **A car starts from rest and moves in a straight line with uniform acceleration of $5m/s^2$, then with uniform velocity and finally uniform retardation of $5m/s^2$, before coming to rest. Total time of motion is equal to 25s. Average velocity during entire motion is 20m/s. Find the time, for which car moves uniformly.**

Solution:



$t_a + t_u + t_r = 25s$ …eq 1

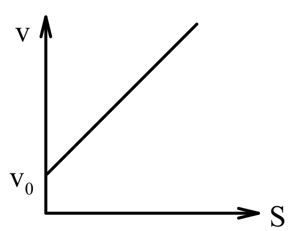
$\frac{V_m}{t_a} = \frac{V_m}{t_r} = 5$ …eq 2 and eq 3

$$\frac{\left(\frac{2t_u + t_a + t_r}{2}\right) \times V_m}{25} = 20$$ …eq 4

Solving we get

$t_a = t_r = 5s, t_u = 15s$

Illustration 10.



Convert $\vec{v} - \vec{s}$ graph into a – s graph

Solution:

Equation of given line can be written as

$v = ms + v_0$

$$a = \frac{vdv}{ds} = (ms + v_o)(m) = m^2s + mv_o$$

$a = m^2s + mv_o$

This is the equation of a straight line with slope m^2 and y intercept mv_0

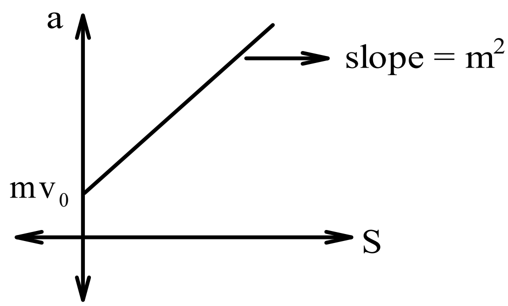
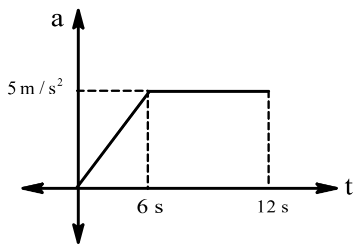


Illustration 11. **a – t graph is given. The particle starts from rest at origin at t = 0. Find displacement at t =12s.**



Solution:

from t = 0s, to t = 6 s, $a = \frac{5}{6}t$, $\frac{dv}{dt} = \frac{5t}{6}$

$\int_{v=0}^{v=v} dv = \frac{5}{6}\int_{t=0}^{t=t} t\,dt$ $\quad\therefore\quad v = \frac{5}{6} \times \frac{t^2}{2} = \frac{5t^2}{12}$

$v = \frac{5t^2}{12}$ $\qquad \frac{ds}{dt} = \frac{5t^2}{12}$

$\int_{s=0}^{s=s} ds = \int_{t=0}^{t=t} \frac{5t^2}{12}dt$ $\qquad s = \frac{5t^3}{36}$

S at t = 6 s = $\frac{5}{36} \times 6^3 = 30\,m$

v at t = 6 s = $\frac{5}{12} \times 36 = 15\,m/s$

From t = 6s to t = 12s

Uniformly accelerated motion

$S = 15 \times 6 + \frac{1}{2} \times 5 \times 6^2 = 180$ m $\quad\therefore\quad$ Total distance = 180 m + 30 m = 210 m

Inchapter Exercise

7. A hundred meter sprinter increases his speed from rest uniformly, at the rate of $1 m/s^2$ upto three quarters of the total run and covers the last quarter with uniform speed. How much time is taken to cover the first half and second half of the run.

8. A motor car starts from rest & accelerates uniformly for 10 s to a velocity of 20 m/s. It then runs at a constant speed & finally is brought to rest in 40 m with constant retardation. Total distance covered is 640 m. Find value of acceleration, retardation & total time taken.

9. An athlete runs a distance of 1500 m in the following manner.
   Starting from rest, he accelerates himself uniformly at 2 m/s^2 till he covers 900 m
   He, then runs remaining distance of $600\,m$ at uniform speed. Calculate time taken by athlete to cover the 2 parts of distance covered. Also find time taken by him to reach the center of track.

10. A vehicle starting from rest can accelerate uniformly at rate of $10\,cm/s^2$ & can retard uniformly at rate of 20cm/s^2. Find the least time in which it can complete journey of 5 km, if maximum velocity attained by body is 72 km/hr.

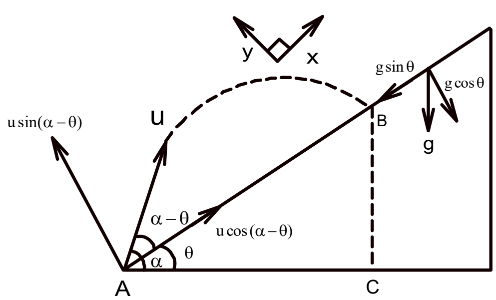
11. A train starting from rest accelerates uniformly for 100s, runs at a constant speed for 5 minutes & then comes to rest with uniform retardation in next 150 seconds. During motion, it covers a distance of 4.25 km. Find (i) the constant speed (ii) acceleration (iii) retardation

Answer Key

7. 10 s, 4.24 s **8.** 2 m/s^2, 5 m/s^2, 39 s **9.** 30s, 10s, 27.4s
10. 400 s. **11.** $10 \text{m/s}, 0.1 \text{m/s}^2, 0.067 \text{m/s}^2$

4. Projectile on an inclined plane

4.1 Projectile fired from bottom of inclined plane



Consider a particle fired with speed u from bottom of an inclined plane. The angle of projection with the horizontal is α & angle made by incline with horizontal is θ.

As shown in figure, x & y direction will be along incline & perpendicular to incline, respectively.

Point of projection is A & point where the particle strikes incline is B

Displacement is AB along x - axis

Displacement in y direction is zero.

- **Time of flight**

  Apply $S = ut + \frac{1}{2} a t^2$ along y - axis

  $$0 = u \sin(\alpha - \theta) T - \frac{1}{2} g \cos\theta T^2$$

  $$\therefore \text{Time of flight } T = \frac{2u \sin(\alpha - \theta)}{g \cos\theta}$$

- **Range along incline**

  Length AB is the range

  Horizontal displacement in time T

  $$AC = u \cos\alpha\ T = \frac{2u \sin(\alpha - \theta) \times u \cos\alpha}{g \cos\theta}$$

  $$\text{Range along incline} = AB = \frac{AC}{\cos\theta}$$

  $$AB = \frac{2u^2 \cos\alpha \sin(\alpha - \theta)}{g \cos^2\theta}$$

  $$\text{or } AB = \frac{u^2\left[\sin(2\alpha - \theta) - \sin\theta\right]}{g \cos^2\theta}$$

- **Alternative method for range**

  To find AB, apply $s = ut + \frac{1}{2} a t^2$ along x direction

  $$AB = u \cos(\alpha - \theta) T - \frac{1}{2} g \sin\theta\ T^2$$

  Put $T = \frac{2u \sin(\alpha - \theta)}{g \cos\theta}$ & simplifying

  We get $AB = \frac{2u^2 \sin(\alpha - \theta) \cos\alpha}{g \cos^2\theta}$

- **Value of α for maximum range**

  $$\text{Range} = AB = \frac{u^2\left(\sin(2\alpha - \theta) - \sin\theta\right)}{g \cos^2\theta}$$

  For a given u & θ, for maximum range

  $\text{Sin}(2\alpha - \theta) = 1$, implies $2\alpha - \theta = \frac{\pi}{2}$

  Or $\alpha = \frac{\pi}{4} + \frac{\theta}{2}$

$$R_{max} = \frac{u^2(1-\sin\theta)}{g\cos^2\theta} = \frac{u^2}{g(1+\sin\theta)}$$

- **The two values of α to get the some range along inclined plane (u = constant)**

Range = AB $= \frac{u^2(\sin(2\alpha-\theta)-\sin\theta)}{g\cos^2\theta}$

Let α_1 & α_2 be the 2 angles to give the same range.

$\therefore \sin(2\alpha_1-\theta) = \sin(\pi-(2\alpha_2-\theta)),\ 2\alpha_1-\theta = \pi-2\alpha_2+\theta$

$\therefore \quad \alpha_1+\alpha_2 = \frac{\pi}{2}+\theta$

$\therefore$ There are two angles of projection for which we obtain the same range along incline for the same speed of projection such that $\alpha_1+\alpha_2 = \frac{\pi}{2}+\theta$

- **Summary of various results obtained for projectile fired from bottom of inclined plane.**

(a) Range AB $= \frac{2u^2\cos\alpha\sin(\alpha-\theta)}{g\cos^2\theta}$

(b) Condition for maximum range $\alpha = \frac{\pi}{4}+\frac{\theta}{2}$

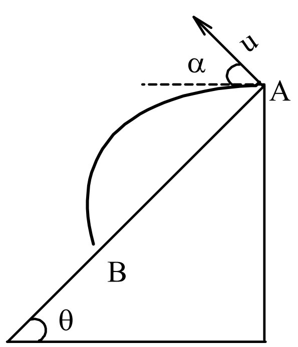
(c) Value of maximum range. $= \frac{u^2}{g(1+\sin\theta)}$

(d) Relation between the 2 angles as projection to get same range.

$\alpha_1+\alpha_2 = \frac{\pi}{2}+\theta$, keeping u constant

(e) Time of flight $= \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$

4.2 <u>Projectile fired from top of inclined plane.</u>



The results in this case are the same as results for projectile projected from the bottom of the incline, except that θ is to be replaced by $-\theta$

- **Summary of various results for projectile fired from top of inclined plane.**

  (a) Range $AB = \dfrac{2\,u^2 \cos\alpha \sin(\alpha+\theta)}{g\cos^2\theta}$

  (b) Condition for maximum range $\alpha = \dfrac{\pi}{4} - \dfrac{\theta}{2}$

  (c) Value of maximum range $= \dfrac{u^2}{g\,(1-\sin\theta)}$

  (d) Relation between the 2 angles of projection to get same range. (keeping u constant)

  $\alpha_1 + \alpha_2 = \dfrac{\pi}{2} - \theta$

  (e) Time of flight $= \dfrac{2u\sin(\alpha+\theta)}{g\cos\theta}$

4.3 PROJECTILE WITH VARIABLE ACCELERATION

Suppose a projectile moves in the two dimensional plane with velocity $v = a\hat{i} + bx\hat{j}$ where a and b are constant. Initially consider the particle to be situated at origin i.e. at t = 0, x = 0 & y = 0. Now let us first find out the equation of trajectory of the projectile. So,

$$v = a\hat{i} + bx\hat{j}$$

$\therefore$ $\quad v_x = a$ and $v_y = bx$

But $\quad v_x = \dfrac{dx}{dt},\ v_y = \dfrac{dy}{dt}$

i.e. $\quad \dfrac{dx}{dt} = a$ and $\dfrac{dy}{dt} = bx$,

$\therefore$ $\quad dx = a\,dt$,

$$\int_{x=0}^{x=x} dx = \int_{t=0}^{t=t} a\,dt, \qquad [x]_0^x = a[t]_0^t,$$

$$x = at \qquad \ldots(1)$$

$$dy = bx\,dt$$

On substituting value of x we have

$$dy = b.at\,dt, \quad \int_{y=0}^{y=y} dy = \int_{t=0}^{t=t} b\,a\,t\,d\,t \qquad \therefore [y]_0^y = ba\left[\frac{t^2}{2}\right]_0^t$$

On integrating $\quad y = \dfrac{abt^2}{2} \qquad \ldots(2)$

Eliminating t between equation 1 and equation 2

$$\Rightarrow \quad y = \frac{ab}{2}\left(\frac{x}{a}\right)^2 \qquad \Rightarrow \quad y = \frac{bx^2}{2a}$$

This is the equation of trajectory of the projectile.

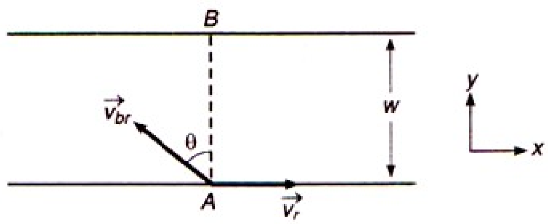
5. River-Boat Problems

In river-boat problems we come across the following three terms:

$\vec{v}_r$ = absolute velocity of river

$\vec{v}_{br}$ = velocity of boat with respect to river or velocity of boat in still water and

$\vec{v}_b$ = absolute velocity of boat.



Here, it is important to note that $\vec{v}_{br}$ is the velocity of boat with which it is steered and $\vec{v}_b$ is the actual velocity of boatman relative to ground. Further, $\vec{v}_b = \vec{v}_{br} + \vec{v}_r$

Now, let us derive some standard results and their special cases. A boat starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in Figure. River is flowing along positive x-direction with velocity $\vec{v}_r$. Width of the river is w. Then $\vec{v}_b = \vec{v}_r + \vec{v}_{br}$

Therefore, $\quad v_{bx} = v_{rx} + v_{brx} = v_r - v_{br}\sin\theta$

and $\quad v_{by} = v_{ry} + v_{bry} = 0 + v_{br}\cos\theta = v_{br}\cos\theta$

now, time taken by the boat to cross the river is:

$$t = \frac{w}{v_{by}} = \frac{w}{v_{br}\cos\theta} \text{ or } t = \frac{w}{v_{br}\cos\theta} \qquad \ldots \text{(i)}$$

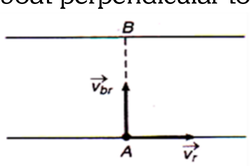
Further, displacement along x-axis when he reaches on the other bank (also called drift) is

$$x = v_{bx}t = (v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} \text{ or } x = (v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} \qquad \ldots \text{(ii)}$$

Two special cases are:

(i) Condition when the boatman crosses the river in shortest interval of time

From Eq. (i) we can see that time (t) will be minimum when $\theta = 0^\circ$, i.e., the boatman should steer his boat perpendicular to the river current.



Also, $\quad t_{min} = \dfrac{w}{v_{br}}$ as $\cos\theta = 1$

(ii) Condition when the boatman wants to reach point B, i.e., at a point just opposite from, the point he started. In this case, the drift (x) should be zero.

$\therefore \quad x = 0 \quad$ from eq. (ii) $\quad$ or $\quad (v_r - v_{br}\sin\theta)\dfrac{w}{v_{br}\cos\theta} = 0 \quad$ or $\quad v_r = v_{br}\sin\theta$

or $\quad \sin\theta = \dfrac{v_r}{v_{br}} \quad$ or $\quad \theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$

Hence, to reach point B, the boatman should row at an angle $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$

Further, since $\sin\theta \not> 1$, so, if $v_r \geq v_{br}$, the boatman can never reach at point B. If $v_r = v_{br}$, then $\theta = 90^0$, $v_b = 0$. If $v_r \geq v_{br}$, then $\sin\theta$ needs to be greater than 1, which is not possible

(iii) Shortest path

Path length travelled by the boatman when he reaches the opposite shore is $s = \sqrt{w^2 + x^2}$

Here, w = width of river is constant. So for s to be minimum modulus of x (drift) should be minimum. Now two cases are possible.

When $v_r < v_{br}$: In this case x = 0, when $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$ or $s_{min} = w$ at $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$.

This is zero drift condition.

When $v_r > v_{br}$: In this case x is minimum, where $\dfrac{dx}{d\theta} = 0$

Note that in this case zero drift is not possible. So we can only minimize drift.

or $\quad \dfrac{d}{d\theta}\left\{\dfrac{w}{v_{br}\cos\theta}(v_r - v_{br}\sin\theta)\right\} = 0 \quad$ or $\quad -v_{br}\cos^2\theta - (v_r - v_{br}\sin\theta)(-\sin\theta) = 0$

or $\quad -v_{br} + v_r\sin\theta = 0 \quad\quad$ or $\quad \theta = \sin^{-1}\left(\dfrac{v_{br}}{v_r}\right)$

Now, at this angle, we can find x_{min}

Put $\sin\theta = \dfrac{v_{br}}{v_r}$ in formula for x, we get $x_{min} = \dfrac{w}{v_{br}}\sqrt{{v_r}^2 - {v_{br}}^2}$

$$S_{min} = \sqrt{w^2 + {x_{min}}^2} = \frac{w\,v_r}{v_{br}}$$

Illustration 12. A swimmer starts from point A on the bank of 200 m wide river, crosses the river to reach opposite bank. He returns to the point B on the original bank such that AB = 300m in the downstream direction. Find the magnitude and the direction of the velocity of the swimmer relative to the bank if his velocity w.r.t.to river is always perpendicular to the bank.

Solution: Time to cross the river = 5 min The displacement perpendicular to flow = 200 m

$\therefore \quad$ Velocity of swimmer $v_{sy} = \dfrac{200}{5\times 60} = \dfrac{2}{3}$ m/s

The displacement covered in the direction of flow = 300 m in 10 min.

∴ The velocity of river flow

The velocity of swimmer in the direction of flow $v_{sx} = \frac{300}{10 \times 60} = \frac{1}{2} m/s$

His velocity with respect to bank $v_s = \sqrt{{v_{sy}}^2 + {v_{sx}}^2} = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \frac{5}{6}$m/s $= 3$ km/h **Ans.**

The velocity $\vec{v}_s$ makes an angle θ with the bank, then $\tan\theta = \frac{v_{sy}}{v_{sx}} = \frac{2/3}{1/2}$

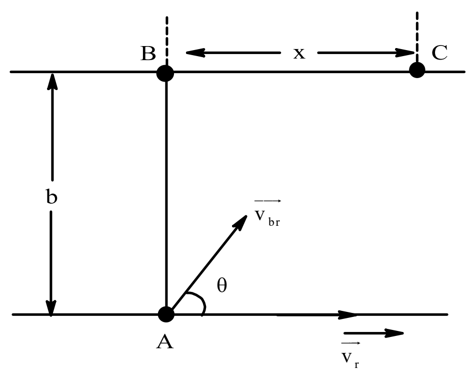
$= \frac{4}{3}$ or $\theta = \tan^{-1}(4/3)$ **Ans.**

Illustration 13. A boat moves relative to water with a velocity which is η = 2.0 times less than the river flow velocity. At what angle to the stream direction must the boat move to minimize drifting? Find the minimum drift achieved.

Solution: Suppose velocity of river flow v_r = v, then velocity of boat relative to water $v_{br} = \frac{v}{2}$.

Let boat moves at an angle θ with the direction of stream, then

Time to cross the stream $t = \frac{b}{v_{br}\sin\theta}$, where $v_{br}\sin\theta$ is the velocity of boat along AB.



The velocity of boat along the direction of flow, $v_{bx} = (v_r + v_{br}\text{Cos}\theta)$ and drift in the direction of flow, $x = v_{bx} \times t$

$$= (v_r + v_{br}\cos\theta) \times \frac{b}{v_{br}\sin\theta} \quad = \left(v + \frac{v}{2}\cos\theta\right) \times \left(\frac{b}{\frac{v}{2}\sin\theta}\right) = b\left(\frac{2+\cos\theta}{\sin\theta}\right)$$

Where b is the width of the river.

For the drift to be minimum, $\frac{dx}{d\theta} = 0$ or $\frac{d}{d\theta}\left[b\left(\frac{2+\cos\theta}{\sin\theta}\right)\right] = 0$

or $\sin\theta \times (-\sin\theta) - (2+\cos\theta) \times (\cos\theta) = 0$ or $\sin^2\theta + 2\cos\theta + \cos^2\theta = 0$

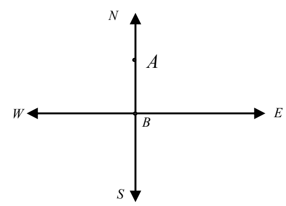
or $2\cos\theta = -(\sin^2\theta + \cos^2\theta)$ or $2\cos\theta = -1$ ∴ $\cos\theta = -\frac{1}{2}$ or $\theta = 120^o$

Hence to minimize drifting boat should move at an angle 120° with the direction of stream.

Thus $x_{min} = b\left(\dfrac{2+\cos 120^o}{\sin 120^o}\right) \qquad = b\left(\dfrac{2-1/2}{\sqrt{3}/2}\right) = b\sqrt{3}$ Ans.

Illustration 14. Two ships A and B are 10 km apart on a line running south to north. Ship A further north is streaming west at 20 km/hr and ship B streaming north at 20 km/hr. What is their distance of closest approach and how long do they take to reach it?

Solution: Consider a rectangular coordinate axes system with y axis from south to north and x –axis from west to east. The positions of the ships A and B are given by the co – ordinates



(0 , 10 km) and (0 , 0) respectively. Velocity of ship

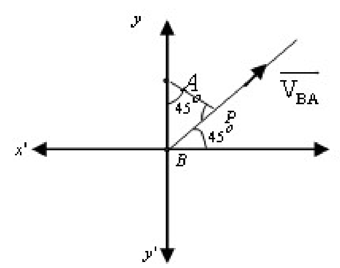
A = $\vec{V}_A = (-20 km/hr)\hat{i}$

Velocity of ship B = $\vec{V}_B = (20 km/hr)\hat{j}$

Relative velocity of ship B w.r.t. ship A = $\vec{V}_{BA}$

$\vec{V}_B - \vec{V}_A = (20 km/hr)\hat{j} + (20 km/hr)\hat{i}$

Angle made by $\vec{V}_{BA}$ with x axis is



$$\tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{20}{20}\right) = \frac{\pi}{4}$$

$\therefore$ $\vec{V}_{BA}$ is in north east direction. Hence relative to A ,
B is moving in north east direction
Since the perpendicular distances is the shortest distance , the minimum distance between A and B is AP as shown in above figure.
ΔBAP is an isosceles right angled triangle with right angle at P.

From ΔBAP, AP = BA cos 45° = $\dfrac{10}{\sqrt{2}} km = 5\sqrt{2} km$

Time taken to attain the minimum distance $= \dfrac{BP}{|\vec{V}_{BA}|} = \dfrac{BA \sin 45^0}{|\vec{V}_{BA}|} = \dfrac{5\sqrt{2}}{20\sqrt{2}} = \dfrac{1}{4} hr = 0.25 hr$

Inchapter Exercise

12. A man swims across a river & reaches a point directly opposite in time t_1. He swims an equal distance downstream & covers distance in time t_2. If speed of man in still water is V and speed of river water is U, find $\dfrac{t_1}{t_2}$.

13. An aeroplane has to go from point A to another point B, 500 km away due 30^0 east of North. Wind is blowing due north at speed of 20 m/s. Air speed of plane (speed of plane in still air) is 150 m/s. Find the direction in which pilot should head plane w.r.t line AB to reach point B

14. A river 400 m wide is flowing with a speed of 2m/s. A boat is sailing with $10\,\text{m/s}$ speed w.r.t. the river water in direction such that the boat reaches the opposite bank in minimum time.
(a) Find the time taken by the boat to reach the opposite bank.

(b) Find the drift suffered by the boat.

15. A swimmer wishes to cross a 500 m wide river flowing at $5\,\text{km/hr}$. His speed in still water is $3\,\text{km/hr}$

(a) If he heads in direction making angle θ with flow, find time he takes to cross river.

(b) Find shortest possible time to cross.

16. Consider a river 500 m wide. Speed of river water is 5 km/hr. Swimmer's speed in still water is 3 km/hr. If swimmer wants to reach a point directly opposite the point from where he started to swim, find minimum distance he has to walk.

Answer key

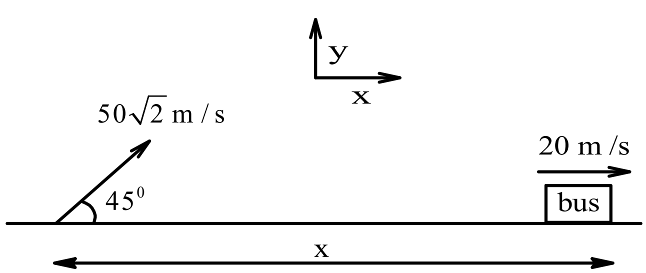
12. $\sqrt{\dfrac{V+U}{V-U}}$ **13.** $\sin^{-1}\left(\dfrac{1}{15}\right)$ **14.** 40 s, 80 m

15. Ans: (a) $\dfrac{10}{\sin\theta}$ minutes (b) 10 minutes when $\theta=90^0$ **16.** $\dfrac{2}{3}km$

Solved Examples (Relative motion in projectile)

Illustration 15. A shell is fired from a cannon on ground with speed $50\sqrt{2}\text{ m/s}$, at angle 45^0 with horizontal. There is a bus moving away from the shell at a constant speed of 20 m/s. If the shell strikes the bus, find the distance of the bus from the cannon at the instant the shell was fired.

Solution:



Relative motion of shell w.r.t. bus

$\overrightarrow{u_{sb}} = 30\hat{i} + 50\hat{j}$

$\overrightarrow{a_{sb}} = -10\hat{j}$

$\overrightarrow{s_{sb}} = x\hat{i} + 0\hat{j}$

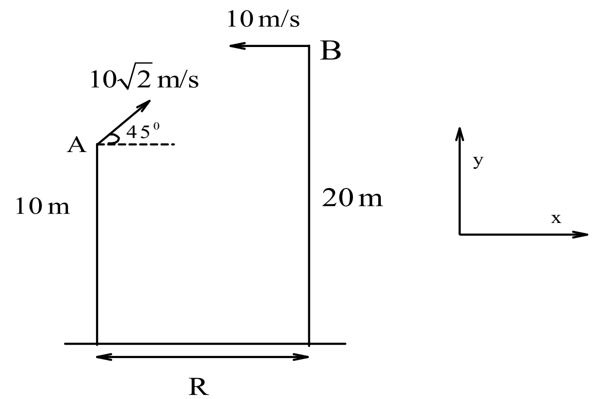
$\overrightarrow{s_{sb}} = \overrightarrow{u_{sb}}\,t + \dfrac{1}{2}\overrightarrow{a_{sb}}\,t^2$

$x\hat{i}+0\hat{j}=30t\hat{i}+50t\hat{j}-5t^2\hat{j}$

$\therefore\ x = 30\text{ t},\ 0=50t-5t^2, \therefore\ t = 10s$ $x = 300\text{ m}$

Illustration 16. **Particle A is projected with speed $10\sqrt{2}$ m/s at angle 45^0 with horizontal from the top of a tower of height 10 m. Simultaneously, particle B is projected, horizontally with speed 10m/s from the top of another tower of height 20m. The 2 particles collide mid air. Find the horizontal distance between the towers.**

Solution:



$\overrightarrow{u_A} = 10\hat{i} + 10\hat{j}$ $\quad \overrightarrow{u_B} = -10\hat{i}$

$\overrightarrow{u_{AB}} = 20\hat{i} + 10\hat{j}$

$\overrightarrow{a_{AB}} = 0$

$\overrightarrow{s_{AB}} = R\hat{i} + 10\hat{j}$

$$\overrightarrow{s_{AB}} = \overrightarrow{u_{AB}}\, t + \frac{1}{2}\overrightarrow{a_{AB}}\, t^2$$

$$R\hat{i} + 10\hat{j} = \left(20\hat{i} + 10\hat{j}\right) \times t$$

$R = 20\,t$ $\qquad 10 = 10\,t$

$\therefore t = 1s$ $\qquad R = 20$ m

EXERCISE - 1

Single Option Correct:

1. A lift ascends with a constant acceleration of $4\,m/s^2$, then with a constant velocity v and then moves with a constant retardation of $4m/s^2$ to finally come to rest If the total height ascended be 20 m and the total time taken be 6 s, then the time during which the lift was moving with constant velocity v is
   (a) 2 s (b) 3 s (c) 4 s (d) 5 s

2. A particle moves along a straight line such that its displacement x changes with time t as $x = \sqrt{at^2 + 2bt + c}$ where a, b and c are constants, then the acceleration varies as
   (a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{1}{x^3}$ (d) $\frac{1}{x^4}$

3. A particle moves along a straight line such that the relation between time t and displacement s is $s^2 = t$, then
   (a) acceleration is positive and directly proportional to v^2
   (b) acceleration is positive and directly proportional to v^3
   (c) acceleration is negative and directly proportional to v^2
   (d) acceleration is negative and directly proportional to v^3

4. A particle starts from rest, with an acceleration $a = \frac{\lambda}{x^2}$, where $\lambda > 0$ and x is the distance of the particle from a fixed point O. The particle is at a distance μ from O, when it is at rest. Its velocity when at a distance 2μ from O is

(a) $\sqrt{\frac{\lambda}{\mu}}$ (b) $\sqrt{\frac{\lambda}{2\mu}}$ (c) $\sqrt{\frac{2\lambda}{\mu}}$ (d) none of these

5. A particle starts from a point x = 0 along the positive X – axis with a velocity v varying with x as $v = \mu\sqrt{x}$, the average velocity of the particle over the first s meters of its path is

(a) $\mu\sqrt{s}$ (b) $\mu\sqrt{\frac{s}{2}}$ (c) $2\mu\sqrt{s}$ (d) $\frac{\mu}{2}\sqrt{s}$

6. The displacement x and time t for a particle moving in one dimension is given by $t = ax^2 + bx$, where a and b are constants. The deceleration of the particle is

(a) bv^2 (b) $2av^3$ (c) $2b^2v^2$ (d) $2abv^3$

7. The acceleration 'a' for a particle depends on displacement s as a = 5 + s. At t = 0, s = 0 and velocity v = 5. Then the velocity v, corresponding to displacement s is given by

(a) v = 5 + s (b) $v = \sqrt{5+s}$ (c) $v = \sqrt{s^2 + 10s}$ (d) v = s - 5

8. Position of a particle moving along x – axis is given by $x = 2 + 8t - 4t^2$, where t is time in sec. The distance traveled by the particle in the first two seconds is:

(a) 2 units (b) 8 units (c) 10 units (d) 16 units

9. For a velocity versus time graph, which of the following statement is true

(a) The curve can be a circle

(b) The area under the curve and above the x- axis, between any two instants gives the average acceleration.

(c) The slope at any instant yields the rate of change of acceleration at the instant

(d) None of these

10. The displacement time graph for a particle is a straight line parallel to time axis. It necessarily implies that:

(a) The object is stationary with velocity zero

(b) the acceleration of the object is zero

(c) Both (a) and (b)

(d) none of the above

11. For motion with uniform velocity :

(a) The velocity time graph is a straight line parallel to the time axis.

(b) The position time graph is a parabola with its axis as the time axis

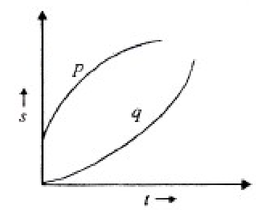
(c) The acceleration time graph is a straight line inclined with the time axis

(d) None of the above

12. A train takes t sec to perform a journey. It travels for $\left(\frac{t}{n}\right)$ sec with uniform acceleration, then for $(n-3)\frac{t}{n}$ sec with uniform speed v and finally it comes to rest with uniform retardation. The average speed of the train is

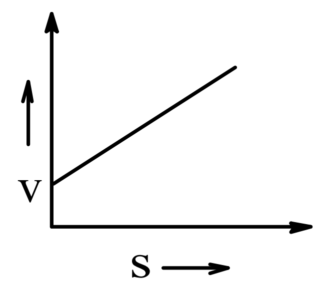
(a) $(3n-2)\frac{v}{2n}$ (b) $(2n-3)\frac{v}{2n}$ (c) $(3n-2)\frac{v}{3n}$ (d) $(2n-3)\frac{v}{3n}$

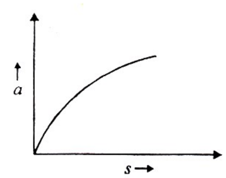
13. The time dependence of the position of two bodies moving along a straight line is given by curves p and q respectively. Then

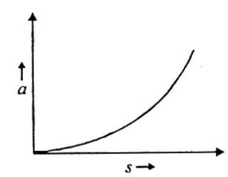


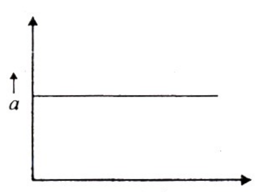
(a) curve q corresponds to decelerated motion
(b) curve p corresponds to accelerated motion
(c) velocity at the end of the motion is more for the body corresponding to curve q
(d) none of these

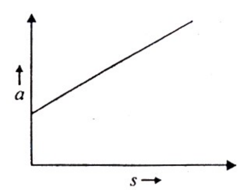
14. The graph given to the right is the velocity (v) versus displacement (s) graph of a particle moving in straight line. Which of the following graphs best represents the acceleration versus displacement graph?



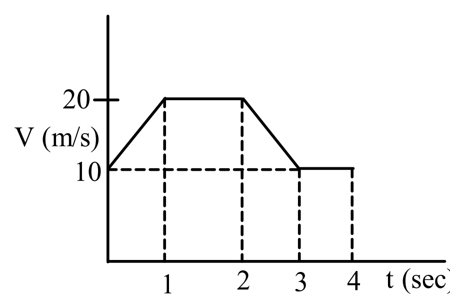
(a)


(b)


(c)


(d)


15. Figure given below shows the variation of velocity with time for a particle moving along a straight line, the average velocity, during the entire motion is

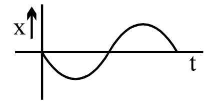


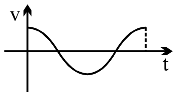
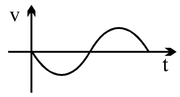
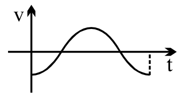
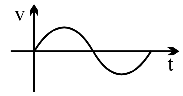
(a) 15 m/s (b) 7.5 m/s
(c) 6.25 m/s (d) 13.75 m/s

16. A train moving with a velocity of 30km/hr had to slow down to15 km/hr due to repairs along the way and then after some time regains its original speed. If the distance covered during retardation be one km and that during acceleration be half km, the time lost in the journey is

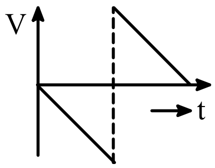
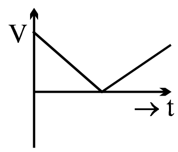
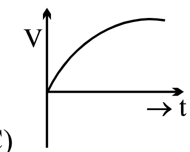
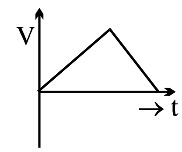
(A) 3 min. (B) 4 min. (C) 2 min. (D) 1 min.

17. If position time graph of a particle is sine curve as shown, what will be its velocity-time graph?

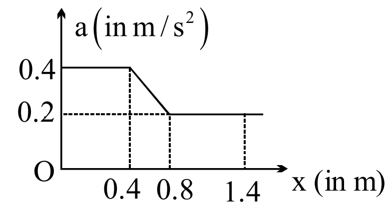


(A)  (B)  (C)  (D) 

18. The velocity- time graph of a body falling from rest under gravity and rebounding the same height from a solid surface is represented by which of the following graphs?

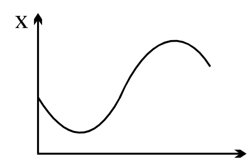
(A)  (B)  (C)  (D) 

19. The acceleration of a particle which moves along the positive x-axis varies with its position as shown. If the velocity of the particle is 0.8 m/s at x = 0, the velocity of the particle at x = 1.4 is(in m/s)



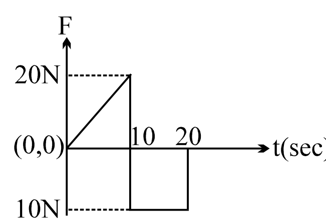
(A) 1.6 (B) 1.2
(C) 1.4 (D) none of these

20. The graph of position *x* versus time *t* represents the motion of a particle. If *b* and *c* are both positive constants and b > c, which of the following expressions best describes the acceleration a of the particle?



(A) a = b – ct (B) a = + b (C) a = – c (D) a = b + ct

21. A particle of mass 1 kg is acted upon by a force 'F' which varies with time as shown in the figure. If initial velocity of the particle is $10 ms^{-1}$, the maximum velocity attained by the particle during the period is

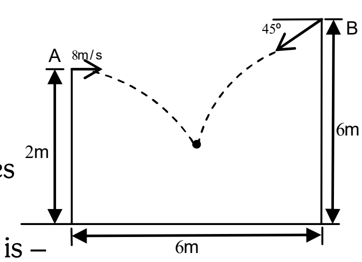


(A) 210 ms^{-1} (B) 110 ms^{-1}
(C) 100 ms^{-1} (D) 90 ms^{-1}

22. A particle moves in the xy plane with velocity $V = k_1\hat{i} + k_2 x\hat{j}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors along the x and y axes, and k_1 and k_2 are constants. At the initial moment of time the particle was located at the point x = y = 0, then the equation of the particle's trajectory y (x) is -

(A) $y = \frac{k_1}{2k_2}x^2$ (B) $y = \frac{k_2}{2k_1}x^2$ (C) $y = \frac{2k_1}{k_2}x^2$ (D) $y = \frac{2k_2}{k_1}x^2$

23. From points A and B at the respective heights of 2m and 6m, two bodies are thrown simultaneously towards each other; one is thrown horizontally with a velocity of 8m/s and the other, downward at an angle of 45° to the horizontal at an initial velocity such that the bodies collide in flight. The horizontal distance between points A and B equals 6m. The initial velocity v_0 of the body thrown at an angle 45° is –



(A) $4\sqrt{2}\, m/s$ (B) $8\sqrt{2}\, m/s$ (C) $16\sqrt{2}\, m/s$ (D) $32\sqrt{2}\, m/s$

24. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a, b are constants, and x and y are respectively the horizontal and vertical distances of the projectile form the point of projection. The maximum height attained is .... and the angle of projection with the horizontal is ....

(A) $\frac{a^2}{2b}, \tan^{-1}(a)$ (B) $\frac{a^2}{2b}, \tan^{-1}(b)$ (C) $\frac{a^2}{4b}, \tan^{-1}(a)$ (D) $\frac{a^2}{4b}, \tan^{-1}(b)$

EXERCISE - 2

More than one option is Correct:

1. The displacement x of a particle varies with time t as $x = \alpha t^2 - \beta t^3$

(A) particle will return to its starting point after time $\frac{\alpha}{\beta}$.

(B) the particle will come to rest after time $\frac{2\alpha}{3\beta}$

(C) the initial velocity of the particle was zero but its initial acceleration was not zero.

(D) no net force act on the particle at time $\frac{\alpha}{3\beta}$

2. A particle moves along x-axis and displacement varies with time t as $x = (t^3 - 3t^2 - 9t + 5)$. Then

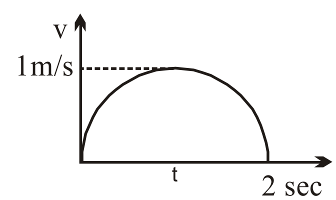
(A) in the interval $3 < t < 5$, the particle is moving in $+x$ direction

(B) the particle reverses its direction of motion twice in entire motion if it starts at $t = 0$

(C) the average acceleration from $1 \le t \le 2$ seconds is $6\,m/s^2$.

(D) in the interval $5 \le t \le 6$ seconds, the distance travelled is equal to the displacement.

3. Velocity-time graph for a car is semicircle as shown here. Which of the following is correct ?



(A) Car must move in circular path.

(B) Acceleration of car is never zero.

(C) Mean speed of the particle is $\pi/4$ m/s.

(D) The car makes a turn once during its motion.

4. 2 bodies start moving in straight line simultaneously from point O. The first body moves with constant velocity of $40\,m/s$ & and the second body starts from rest with a constant acceleration of $4\,m/s^2$

(a) Time that elapses before the second body catches up with the first body is $20s$.

(b) Greatest distance between the two bodies prior to their meeting is $200m$

(c) Time elapsed when the distance between them is maximum is $10s$

(d) All above statements are false.

5. Velocity V of a moving point, on x – axis varies with its x – coordinate as $V=\beta x^{\frac{1}{3}}$ when β is a positive constant. Assuming the particle to start from origin.
(a) Acceleration of particle is variable.
(b) Mean velocity of point averaged over time T from starting is $\left(\frac{2\beta}{3}\right)^{\frac{3}{2}}\sqrt{T}$
(c) Mean velocity of point averaged over the time taken by the point to move from the origin to the point where its x – coordinate becomes x_0 is $\frac{2\beta x_0^{\frac{1}{3}}}{3}$
(d) All above statements are false.

6. A particle starts rectilinearly from station A with acceleration varying according to $a=\alpha-\beta s$ & comes to halt at station B, α & β are positive constants & s is its distance from station A,
(a) Distance between A & B is $\frac{2\alpha}{\beta}$
(b) Maximum velocity of particle is $\frac{\alpha}{\sqrt{\beta}}$
(c) Distance yet to be covered, when particle attains its maximum velocity is $\frac{\alpha}{\beta}$
(d) Maximum velocity is 2α

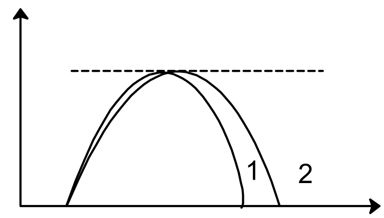
7. A platform is moving upwards with constant acceleration of $2\,m/s^2$. At time $t=0$, a boy standing on platform throws a ball upwards with speed of $8\,m/s$ (with respect to himself). At this instant, platform was at a height of $4\,m$ from ground & was moving with $2\,m/s$ speed. Take $g=10\,m/s^2$
(a) Ball strikes platform at height of $\frac{76}{9}$ m
(b) Ball strikes platform in time $\frac{4}{3}$ s
(c) Maximum height attained by ball from ground is $9\,m$.
(d) Maximum distance of ball from platform is $\frac{8}{3}$ m

8. A man is standing on a truck moving with a constant velocity of $15\,m/s$ on a horizontal road. The man throws a ball in such a way that it returns to the truck after the truck has moved $60\,m$
(Take $g=10\,m/s^2$)
(a) Speed of the ball as seen from the truck is $20\,m/s$
(b) Angle of projection as seen from the truck is 90^o with horizontal.
(c) Speed of ball as seen from the ground is $25\,m/s$
(d) Angle of projection as seen from the ground is 53^0

9. A car is moving rectilinearly on a horizontal path with acceleration a_0. A person sitting inside the car observes that an insect S is crawling up the screen with an acceleration a. If θ is the inclination of the screen with the horizontal, the acceleration of the insect;
(a) parallel to screen is $(a+a_0\cos\theta)$
(b) along the horizontal is $(a_0-a\cos\theta)$
(c) perpendicular to screen is $a_0\sin\theta$
(d) perpendicular to screen is $a_0\tan\theta$

10. A particle is projected from a point P with a velocity v at an angle θ with horizontal. At a certain point Q it moves at right angle to its initial direction. Then:
(a) velocity of particle at Q is v sin θ
(b) velocity of particle at Q is v cot θ
(c) time of flight from P to Q is (v/g) cosec θ
(d) time of flight from P to Q is (v/g) sec θ

11. Trajectories of two projectiles are shown in figure. Let T_1 and T_2 be the time periods and u_1 and u_2 be the speeds of projection of the two projectiles respectively.



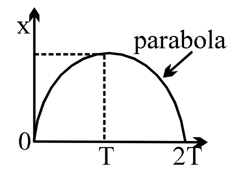
Then :
(a) $T_2 > T_1$ (b) $T_2 = T_1$
(c) $u_1 > u_2$ (d) $u_1 < u_2$

12. Choose the correct alternative(s).
(a) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone is $2h$
(b) The angle of projection with the horizontal for a projectile whose range R is n times the maximum height H is $\tan^{-1}$ (4/n)
(c) The time of flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R \tan\theta$, where θ is the angle of projection
(d) A ball is thrown vertically up. Another ball is thrown at an angle θ with the vertical. Both of them are thrown from horizontal ground and both remain in air for the same period of time. Then the ratio of maximum heights attained by the two balls is 1:1

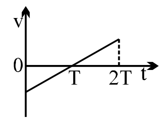
EXERCISE - 3

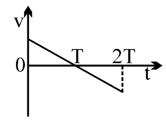
COMPREHENSION – 1

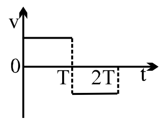
The x-t graph of a particle moving along a straight line is shown in figure

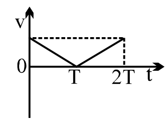


1. The v-t graph of the particle is correctly shown by

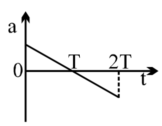
(A)


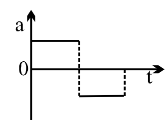
(B)


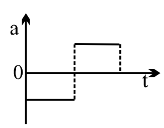
(C)


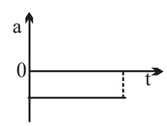
(D)


2. The a-t graph of the particle is correctly shown by

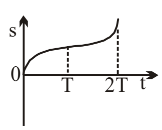
(A)


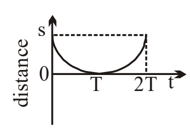
(B)


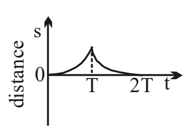
(C)


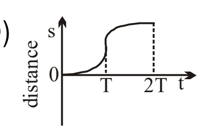
(D)


3. The distance-time graph of the particle is correctly shown by

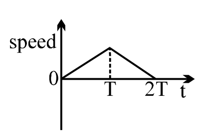
(A)


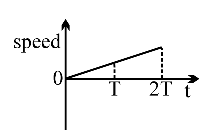
(B)


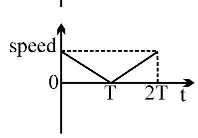
(C)


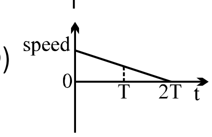
(D)


4. The speed-time graph of the particle is correctly shown by

(A)


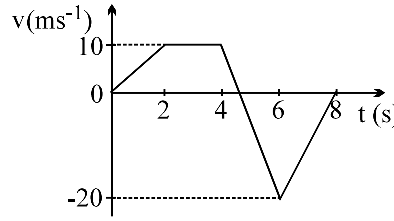
(B)


(C)


(D)


COMPREHENSION – 2

The figure shows a velocity-time graph of a particle moving along a straight line



5. The particle is not at rest at
(A) t = 0 s (B) t = 5 s (C) t = 8 s (D) all of the above

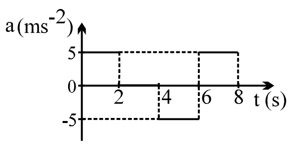
6. Identify the region in which the rate of change of velocity $\left|\frac{\Delta \vec{v}}{\Delta t}\right|$ of the particle is maximum
(A) 0 to 2s (B) 2 to 4s (C) 4 to 6 s (D) 6 to 8 s

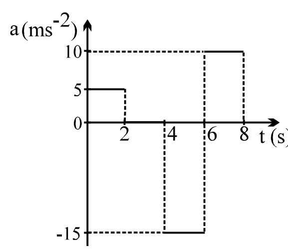
7. If the particle starts from the position $x_0 = -15$ m, then its position at t = 2s will be x=
(A) –5 m (B) 5 m (C) 10 m (D) 15 m

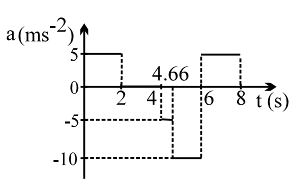
8. The maximum displacement of the particle is
(A) 33.3 m (B) 23.3 m (C) 18.3 m (D) zero

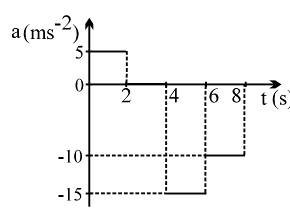
9. The total distance travelled by the particle is
(A) 66.6 m (B) 51.6 m (C) zero (D) 36.6 m

10. The correct acceleration-time graph of the particle is shown as

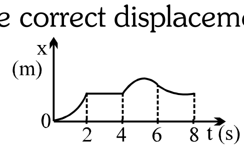
(A)


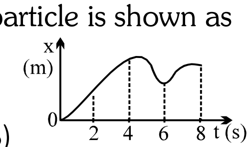
(B)


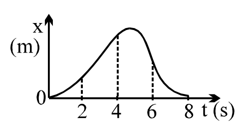
(C)


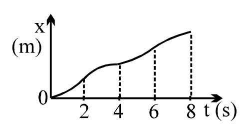
(D)


11. The correct displacement-time graph of the particle is shown as

(A)


(B)


(C)


(D)


COMPREHENSION – 3

The distance from A to B is l. A plane files at constant speed v relative to air along straight line from A to B and then back from B to A. Calculate the total time taken for this round trip, if a wind is blowing in three directions with velocity u. The direction of wind for the three cases is given below.

12. Along the line from A to B

(A) $\dfrac{2l}{v\left(1+\dfrac{2u^2}{v^2}\right)}$ (B) $\dfrac{2l}{v\left(1-\dfrac{u^2}{v^2}\right)}$ (C) $\dfrac{2l}{v\sqrt{1-\dfrac{u^2}{v^2}}}$ (D) $\dfrac{2l}{v\sqrt{1+\dfrac{u^2}{v^2}}}$

13. Perpendicular to line AB,

(A) $\dfrac{2l}{v\left(1+\dfrac{u^2}{v^2}\right)}$ (B) $\dfrac{2l}{v\left(1-\dfrac{u^2}{v^2}\right)}$ (C) $\dfrac{2l}{v\sqrt{1-\dfrac{u^2}{v^2}}}$ (D) $\dfrac{2l}{v\sqrt{1+\dfrac{u^2}{v^2}}}$

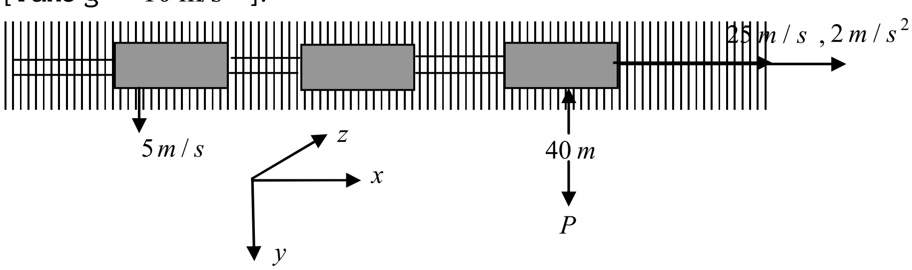
14. At an angle θ to line AB

(A) $\dfrac{2l\sqrt{1-\left(\dfrac{u^2}{v^2}\right)\sin^2\theta}}{v\times\left(1-\dfrac{u^2}{v^2}\right)}$ (B) $\dfrac{2l}{v}\times\dfrac{\sqrt{1-\dfrac{u^2}{v^2}\cos^2\theta}}{1-\dfrac{u^2}{v^2}}$

(C) $\dfrac{2l}{v}\times\dfrac{\sqrt{\dfrac{u}{v}\cos^2\theta}}{1-\dfrac{u^2}{v^2}}$ (D) $\dfrac{2l}{v}\times\dfrac{\sqrt{1-\dfrac{u^2}{v^2}}}{1-\left(\dfrac{u^2}{v^2}\right)\sin^2\theta}$

COMPREHENSION – 4

A train is travelling on a straight horizontal track with a constant acceleration of $2\ m/s^2$ across a bridge over a river. When the velocity of the train is 25 m/s, a man inside one of the cars throws a stone horizontally out of a window in a direction perpendicular to direction of motion of the train with a speed of 5 m/s relative to himself. In the absence of air resistance the stone hits the water at point P, 40 m in horizontal direction from track, as shown in the figure.
Consider point of projection as origin and Cartesian co – ordinate system as shown in figure.
[Take $g = 10\ m/s^2$].



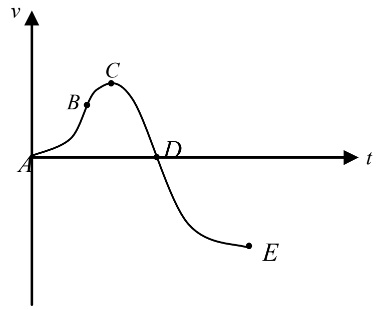
15. The coordinates of point P are :
(A) (200 m , 0, 320 m) (B) (200 m , 40m , 320 m)
(C) (40 m , 200m , 640 m) (D) (264 m , 0 , 0)

16. At the instant when the stone hits the water, the coordinates of the car window from which stone is projected is :
(A) (264 m , 0 , 0) (B) (40 m , 0 , 0) (C) (200 m , 0 , 0) (D) (200 m , 0 , 320 m)

17. If air resistance is not negligible and it exerts a constant force on stone, due to which the acceleration imparted to stone is $\vec{a} = \left(-5\hat{i} - \hat{j}\right)$, then the coordinates of the point where the stone hits the water in this case are :
(A) (40 m , 200 m , 640 m) (B) (40 m , 40 m , 320 m)
(C) (40 m , 40 m , 640 m) (D) (40 m , 8m , 320 m)

MATCH LIST – 1

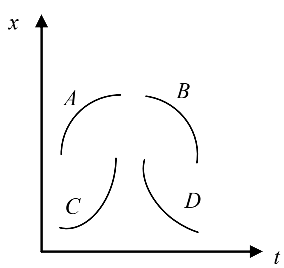
The velocity – time graph of a particle moving along X – axis is shown in the figure. Match the entries of Column I with entries of Column – II .



Column – I	Column – II
(A) For AB , particle is	(P) moving in +ve x – direction with an increasing speed
(B) For BC , particle is	(Q) moving in +ve x – direction with a decreasing speed
(C) For CD, particle is	(R) moving in –ve x – direction with an increasing speed
(D) For DE, particle is	(S) moving in – ve x – direction ith a decreasing speed

MATCH LIST – 2

Column (I) shows the position - time graph of the particle moving along x- axis .

Column – I	Column – II
	(P) Accelerating (Q) Decelerating (R) Speeding up (S) Slowing down

EXERCISE - 4

1. A train starts from rest and moves with a constant acceleration for the first 1 km. For the next 3 km, it has a constant velocity and for the last 2 km, it moves with constant retardation to come to rest after a total time of motion of 10 min. Find the maximum velocity and the three time intervals in the three types of motion.

2. Two cars are moving in the same direction with the same speed of 30 km/hr. They are separated by 5 km. What is the speed of a car moving in the opposite direction if it meets these two cars at an interval of 4 minutes?

3. A thief on a bike moving with uniform velocity passes a stationary policeman who is hiding behind a bill board with a motorcycle. After a 2.0 sec. delay (reaction time) the policeman accelerates to his maximum speed of 150 km/hr in 12sec. The policeman moves with uniform speed of 150 km/hr from this instant onwards and catches up with the thief 1.5 km beyond the billboard. Find the speed of the thief in km/hr.

4. A truck travelling along a straight road with a constant speed of 72 km/hr passes a car at time t = 0 moving much slower. At the instant the truck passes the car, the car starts accelerating with constant acceleration $1 m/s^2$ and overtakes the truck 0.6km further down the road. The car moves with uniform velocity from this instant onwards. Find the distance between them at time t = 50 sec.

5. A particle moves along a straight line such that its position x changes with time as :
$12x = 4t^3 - 15t^2 + 12t + 6$, where x is in m and t in sec. Find the distance covered by it at the end of t = 2 s.

6. The instantaneous velocity 'v' of a particle at time t is given by $v = 4t^3 - 2t + 1$ where t is in sec and v in cm/s. Find the average velocity of the particle during the 3rd second.

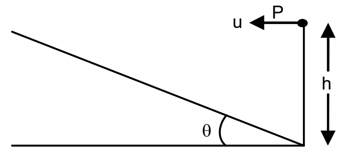
7. Two swimmers leave point A on one bank of the river to reach point B which is directly opposite A on the other bank. One of them crosses the river along the straight line AB while the other swims at right angles to the stream and then walks the distance that he has been carried away by the stream to get to point B. Find the velocity (u) with which the man should walk if both swimmers reached the destination simultaneously? The stream velocity is 2 km/hr and the velocity of each swimmer with respect to water equals 2.5 km/hr.

8. A shell is fired from a point O at an angle of 60° with the horizontal with a speed of 40m/s. It strikes a horizontal plane through O, at a point A. The gun is fired a second time with the same angle of elevation as before but a different speed v. The target starts to rise vertically upward from A with a constant speed $9\sqrt{3}\, m/s$ at the same instant as the shell is fired. If the shell strikes the target, find v(take $g = 10 m/s^2$)

9. A particle is projected vertically upwards from the top of a tower and strikes the ground in 5 s. The average velocity during its motion is 5 m/s. Find the average speed of the particle. (Take $g = 10 m/s^2$)

10. A particle is projected with a velocity of 29.4m/s at an angle of 60° to the horizontal from the bottom of an inclined plane of angle 30^0 to horizontal. Find the range of the particle up the incline. (Take g = 9.8 m/s^2)

11. Determine the velocity with which a stone must be projected horizontally from a point P, so that it may hit the inclined plane perpendicularly. The inclination of the plane with the horizontal is θ and P is h meter vertically above the foot of the incline as shown in the figure.



12. A projectile is projected from the base of a hill whose shape is that of right circular cone, whose axis is vertical. The projectile grazes the vertex and strikes the hill again at a point on the base. If θ be the semi – vertical angle of the cone, h its height, u the initial speed of projection and α the angle of projection, show that

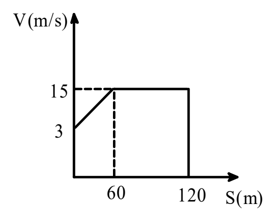
(i) $\tan\theta = 2\cot\alpha$ (ii) $u^2 = \dfrac{gh\left(4+\tan^2\theta\right)}{2}$

13. A particle is projected from O at an angle α with the horizontal. After t seconds it has an elevation β as seen from the point of projection. Prove that its initial velocity is $\dfrac{gt\cos\beta}{2\sin(\alpha-\beta)}$

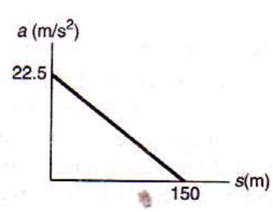
14. At the initial moment two particles are located at one point at an elevated position and move with velocities 3m/s and 4m/s horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular. $\left(g=10\,m/s^2\right)$

15. A body falling freely from a given height 'H' hits an inclined plane in its path at a height 'h'. As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h/H) will the body take maximum time to reach the ground?

16. The v-s graph describing the motion of a motorcycle is shown in figure. Construct the a-s graph of the motion and determine the time needed for the motorcycle to reach the position s=120m. Given ln5 =1.6.



17. A jet plane starts from rest at s=0 and is subjected to acceleration as shown in figure. Determine the speed of the plane when it has travelled 60m.



Previous IIT - JEE Problems

1. A car accelerates from rest at a constant rate α for sometime after which it decelerates at a constant rate β to come to rest. If the total time lapse is t seconds, evaluate (a) the maximum velocity reached and (b) the total distance travelled. **[1978]**

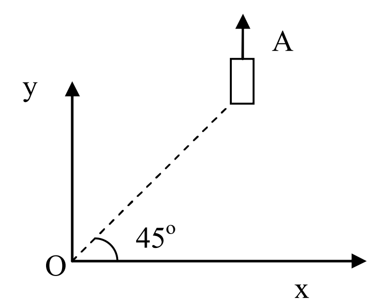
2. The displacement (x) of a particle moving in one dimension, under the action of a constant force is related to the time t by the equation $t = \sqrt{x} + 3$, where x is in metres and t is in seconds. Find the displacement of the particle when its velocity is zero. **[1979]**

3. A man standing on the edge of cliff throws a stone straight up with speed u and then throws another stone straight down with the same intial speed u from the same position. Find the ratio of speeds, the stone would have attained when they hit the ground at the base of the cliff?

4. A particle is moving east - wards with a velocity of 5m/sec. In 10 seconds the velocity changes to 5 m/sec northward. The average acceleration in this time is
   (a) zero
   (b) $\frac{1}{\sqrt{2}} m/sec^2$ towards north-west
   (c) $\frac{1}{\sqrt{2}} m/s^2$ towards north-east
   (d) $\frac{1}{\sqrt{2}} m/sec^2$ towards north

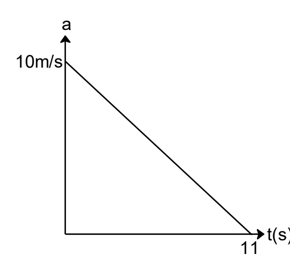
5. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed (neglect air resistance). [T/F] **[1983]**

6. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1) m/s$. At a particular instant, when line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and hits the trolley.

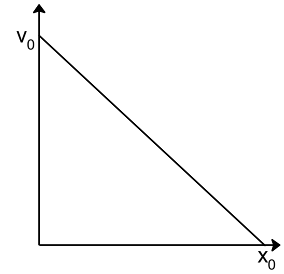


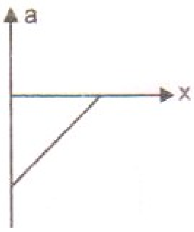
   (a) The motion of the ball is observed from the frame of the trolley. Calculate the angle made by the velocity vector of the ball with the x - axis in this frame.
   (b) Find the speed of the ball with respect to the surface if $\phi = \frac{4\theta}{3}$. **[2002]**

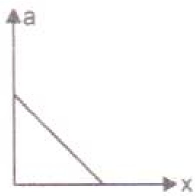
7. A body starts from rest at time t = 0, the acceleration time graph is shown in the figure. The maximum velocity attained by the body will be: **[2004]**
   (a) 110 m/s (b) 55 m/s (c) 650 m/s (d) 550 m/s



8. The velocity displacement graph of a particle moving along a straight line is shown. The most suitable acceleration-displacement graph will be: **[2005]**



(a)


(b)


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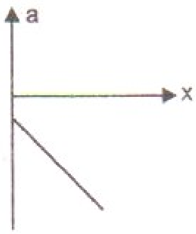
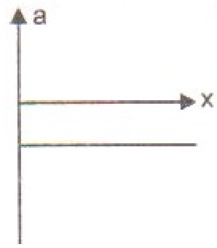
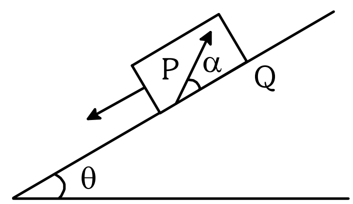


(d)


9. Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}\ \text{ms}^{-1}$ at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find (a) the time-interval between the firings, and (b) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane.

[IIT – 95]

10. A large, heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u, and the direction of projection makes an angle α with the bottom, as seen by an observer inside the box, as shown in figure.

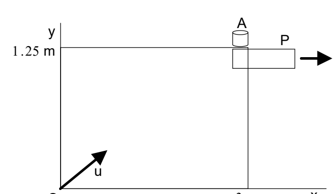


(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance)

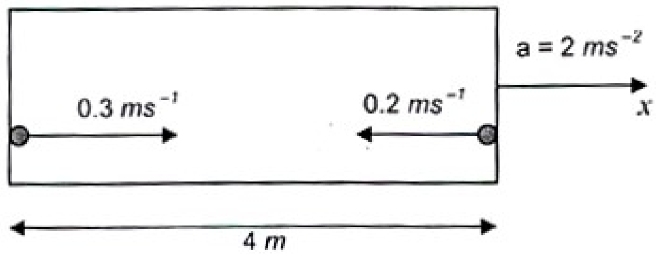
(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when particle was projected.

[IIT - 98]

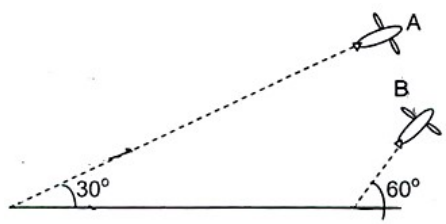
11. An object A is kept fixed at the point x = 3 m and y = 1.25 m on a plank P raised above the ground. At time t = 0 the plank starts moving along the + x direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downwards motion at an angle of 45° to the horizontal. All the motions are in x-y plane. Find u and the time after which the stone hits the object. Take g = 10 m/s^2. **[IIT – JEE 2000]**



12. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hits each other is **(2014)**



13. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30^0 and 60^0 with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}ms^{-1}$. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is **(2014)**



ANSWER KEY

Exercise – 1

1. (C)	2. (C)	3. (D)	4. (A)	5. (D)	6. (B)	7. (A)
8. (B)	9. (D)	10. (C)	11. (A)	12. (B)	13. (C)	14. (D)
15. (A)	16. (D)	17. (C)	18. (A)	19. (B)	20. (A)	21. (B)
22. (B)	23. (C)	24. (C)				

Exercise – 2

1. (A, B, C, D) 2. (A, D) 3. (C) 4. (A, B, C) 5. (A, B, C)
6. (A, B, C) 7. (A, B, C, D) 8. (A, B, C, D) 9. (B, C)
10. (B, C) 11. (B, D) 12. (A, B, C, D)

Exercise – 3

Comprehension Type

1. (B)	2. (D)	3. (A)	4. (C)	5. (B)	6. (C)	7. (A)
8. (A)	9. (A)	10. (B)	11. (C)	12. (B)	13. (C)	14. (A)
15. (B)	16. (A)	17. (D)				

Match List – 1

A – p: B – p: C – q: D – R

Match List – 2

A – q, s: B – p, r: C – p, r: D – q, s

Exercise – 4

1. (15 m/s, 133.33 s, 200 s, 266.67 s) 2. (45 km / hr,) 3. (122.7 km/hr)
4. (300 m) 5. (0.793 m) 6. (61 cm/s) 7. (3 km/hr)
8. (50 m/s) 9. (13 m/ s) 10. (58.8 m) 11. $\left(\sqrt{\dfrac{2gh}{2+\cot^2\theta}}\right)$
14. $\dfrac{7\sqrt{3}}{5}$m 15. $\left(\dfrac{1}{2}\right)$ 16. 12.0s 17. 46.47 ms^{-1}

Previous IIT – JEE Questions

1. (a) $\left(\dfrac{\alpha\beta t}{\alpha+\beta}\right)$ (b) $\left(\dfrac{\alpha\beta t^2}{2(\alpha+\beta)}\right)$ 2. displacement = 0 3. 1:1 4. B

5. T 6. (a) $\theta = 45^0$ (b) 2 m/s 7. B 8. A

9. (i) 1 s . ; (ii) $\left(5\sqrt{3}, 5\right)$ 10. (a) $\dfrac{u^2\sin 2\alpha}{g\cos\theta}$, (b) $\dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

11. u = 7.29 m/s, t = 1 s. 12. (2 OR 8) 13. (5)

PREFACE

Isaac Newton, born in England in the year of Galileo's death, is the principal architect of classical mechanics. He carried to full fruition the ideas of Galileo and others who preceded him. His three laws of motion were first presented (in 1686) in his Principia Mathematica Philosophiae Naturalis. Before Galileo's time most philosophers thought that some influence or "force" was needed to keep a body moving. They thought that a body was in its "natural state" when it was at rest. For a body to move in a straight line at constant speed, for example, they believed that some external agent had to continually propel it ; otherwise it would "naturally" stop moving. In this chapter we try to find ways of calculating the forces that act on particles from the properties of the particles and its environment ; that is, we look for force laws.

This booklet consists of summarized text coupled with sufficient number of solved examples of varying difficulties, which enables the students to develop problem solving ability along with emphasis on physical concept.

The end–of–chapter problems are categorized into five section, namely Exercise – I (objectives where only one of the option is correct), Exercise – II (objectives where more than one option may be correct), Exercise – III (matrix matches and paragraph type questions), Exercise – IV (subjective questions) and Exercise- V (old JEE questions) to help the student assess his understanding of the concept and further improvise on his problem solving skills. Solutions to all the questions in the booklet are available and will be provided to the students (at the discretion of the professor). Every possible attempt has been made to make the booklet flawless. Any suggestions for the improvement of the booklet would be gratefully accepted and acknowledged.

(Dept. of Physics)

IIT –ian'sPACE

IIT–JEE SYLLABUS

Newton's laws of motion; Inertial and uniformly accelerated frames of references; Static and dynamic friction.

CONTENTS

Laws of motion & Friction : Tentative Lecture Flow (Board Syllabus & Booklet Discussion Included)	
Lecture no.1	Newtons laws, Tension, Normal force, Concept of momentum, Free body diagram
Lecture no.2	Connected bodies, constraint relation, principle of virtual work.
Lecture no.3	Problem based on connected bodies, springs
Lecture no.4	Concept of friction, Static, Kinetic, Solved examples
Lecture no.5	Pseudo force, wedge problem
Lecture no.6	Miscellaneous examples

LAWS OF MOTION

In this chapter we shall observe the motion of bodies along with the cause of motion under the assumptions of the mass being constant and all parts of the body having the same state of motion so that we may treat bodies as point masses called particles.

NEWTON'S FIRST LAW

Every body in the universe continues its state of motion (rest or uniform velocity) until some external force acts upon it to change its state of motion. Action of the external source is a physical quantity known as force.

1. This law is also called law of inertia. Inertia is a property by virtue of which a body opposes the change in state of rest or motion
2. Force is such a factor, which is essential for change in translatory motion of a body.
3. The first law of motion defines the force.

Example

1. A book kept on the table remains there, until we lift it.
2. Removal of dust particles from a cloth by shaking it.
3. Banking of the passengers (towards the motion of bus), sitting in a bus on applying sudden brakes

Momentum

It is the product of mass *m* and velocity $\vec{v}$ of the body usually denoted as $(\vec{p})$ since velocity is a vector quantity and mass is a scalar thus momentum is a vector quantity $\vec{p} = m\vec{v}$.

Illustration -1

A 2kg ball is moving on a floor (taken as X – Y plane) with velocity given as $\vec{v} = (5\hat{i} + 5\hat{j})$ m/s at an instant. Find the magnitude of momentum at that instant.

Solution: $\vec{p} = m\vec{v} = 2(5\hat{i} + 5\hat{j})$ kg m/s $= 10\hat{i} + 10\hat{j}$

$$|\vec{p}| = |10\hat{i} + 10\hat{j}| = 10\sqrt{2} \text{ kg m/s}$$

NEWTON'S SECOND LAW

The rate of change of momentum of a body is directly proportional to the applied force.

$\vec{F} \,\alpha\, \frac{d\vec{p}}{dt}$ (Rate of change of a vector quantity is also a vector thus force is a vector)

or $\vec{F} = K\frac{d\vec{p}}{dt}$ $(\because K = 1)$ [K is a proportionality constant that equals one in S.I. units]

$\vec{F} = \frac{d\vec{p}}{dt}$ $(\because K = 1) \Rightarrow$ [when mass is constant] $= m\vec{a} = m\left(\frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}\right)$

Component Form of Newton's second law

$$F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = m\frac{dv_x}{dt}\hat{i} + m\frac{dv_y}{dt}\hat{j} + m\frac{dv_z}{dt}\hat{k}$$

$$\Rightarrow \quad F_x = m\frac{dv_x}{dt} \quad ; \quad F_y = m\frac{dv_y}{dt} \quad ; \quad F_z = m\frac{dv_z}{dt}$$

$$a_x = \frac{F_x}{m} \quad ; \quad a_y = \frac{F_y}{m} \quad ; \quad a_z = \frac{F_z}{m}$$

[For motion in a plane only two components, say x and y are needed]

Important Points

1. *Force is a vector quantity, whose unit is Newton (N) or kg ms^{-2} (in MKS) & dyne or gmcms^{-2}(in CGS)*
2. *The dimension of force is [MLT^{-2}]*
3. *The second law of motion gives the magnitude and unit of force.*
4. *If m is not constant* $\vec{F}=\frac{d(m\vec{v})}{dt}=m\frac{d\vec{v}}{dt}+\vec{v}_{rel}\frac{dm}{dt}$. *As in case of rocket propulsion, the mass of the fuel varies with respect to time.*
5. *The second law is obviously consistent with the first law as **F = 0** implies **a = 0**. (for m = constant)*

Illusration- 2

A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 m/s . If the mass of the ball is 0.15 kg and duration of interaction between the ball and the bat is 0.5 s. Determine the average force imparted to the ball.

Solution: - Let us consider initial motion directed along x-axis.

$\vec{p}_i = m\vec{u} = 0.15\times(12\hat{i}) = 1.8\hat{i}$ Ns $\qquad$ $\vec{p}_f = m\vec{v} = 0.15\times(-12\hat{i}) = -1.8\hat{i}$ Ns

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -3.6\hat{i}$ Ns $\qquad$ Average force $=\frac{\Delta\vec{p}}{\Delta t}=\frac{-3.6\hat{i}}{0.5}=(-7.2\hat{i})$ N

Illustration -3

Momentum of a block as function of time is given as $\vec{p}=(10t^2\,\hat{i}+5t\,\hat{j})$ **kgm/s. Find the force acting on block at** $t=2$ **sec.**

Solution: $\vec{F}=\frac{d\vec{p}}{dt}=\frac{d}{dt}(10t^2\,\hat{i}+5t\,\hat{j})\text{ N}=\frac{d(10t^2\hat{i})}{dt}+\frac{d(5t\,\hat{j})}{dt}\text{ N}$

At t = 2 s $\Rightarrow$ $\vec{F}=20t\,\hat{i}+5\hat{j}\text{ N}=(40\hat{i}+5\hat{j})\text{ N}$

Illustration- 4

A block of mass 2 kg is initially moving on a floor (taken as X – Y plane) with velocity $10\hat{i}$ **. At** $t=0$ **a constant force of magnitude 10N parallel to y-axis begins acting on block for 3 second and then ceases. (a) Find the velocity of block at t = 2 s**

(b) Find the velocity of block at t = 4 s

Solution : (a) $0<t\le 3$ s $\qquad$ $\vec{F}=10\hat{j}$

Velocity at $t=0$, $\qquad$ $\vec{u}=10\hat{i}$

For $0\le t\le 3$ $\qquad$ $\vec{a}=\frac{\vec{F}}{m}=5\;\hat{j}$ m/s^2

As acceleration is constant $\vec{v}=\vec{u}+\vec{a}t$

$\Rightarrow$ $\vec{v}=10\hat{i}+5\hat{j}t=10\hat{i}+10\hat{j}\ ms^{-1}$ $\qquad$ (At t = 2s)

(b) At $t=3$, let velocity of block be $\vec{v}_1$

$\Rightarrow$ $\vec{v}_1=10\hat{i}+15\hat{j}$ $\qquad$ for $t>3$ $\vec{F}=0\Rightarrow$ $\vec{a}=0$

It means velocity of block remains constant $=\vec{v}_1$ $\qquad$ or $\qquad$ $\vec{v}_1=(10\hat{i}+15\hat{j})$ m/s .

Hence velocity at 4 s is $\vec{v}_1=10\hat{i}+15\hat{j}$ m/s.

TRANSLATIONAL EQUILIBRIUM

If the net force on body is zero then the body is said to be in translational equilibrium.

$\vec{F}_{net} = \sum \vec{F}_i = (\sum F_{xi})\hat{i} + (\sum F_{yi})\hat{j} + (\sum F_{zi})\hat{k} = 0$ [F_i represents i^{th} force]

$\Rightarrow \quad F_x = \Sigma F_{xi} = 0,\ F_y = \Sigma F_{yi} = 0,\ F_z = \Sigma F_{zi} = 0$

$\Rightarrow \quad a_x = 0,\ a_y = 0,\ a_z = 0$

* If body is in equilibrium at rest then it is called *static equilibrium*

* If body is in equilibrium moving with uniform velocity then it is called *dynamic equilibrium.*

NEWTON'S THIRD LAW

For every action (Force) there is an equal and opposite reaction (Force). Forces always occur in pairs. Force is the mutual interaction between two bodies. Mutual forces between two bodies are always equal and opposite.

Let body B exert force $\vec{F}_{AB}$ on body A, body A exert a force $\vec{F}_{BA}$ on body B. These forces are related by $\vec{F}_{AB} = -\vec{F}_{BA}$. The negative sign represents opposite direction.

Illusration -5 :

An object having mass of 5 kg, far away from the surface of the earth has acceleration of 6 m/s^2 towards center of earth.Find the force exerted by the object on the earth.

Solution : Force on object by the earth is $\vec{F}_{OE} = m_{ob}.\vec{a}_{ob} = 5 \times 6(N) = 30\,N$

Force exerted by earth on object is 30 N.

Force between earth and object makes an action reaction pair thus force exerted by object on earth is also 30N, directed away from the center of earth.

Important Points :

1. *Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.*
2. *There is no cause-effect relation implied in the Third Law. The force on A by B and force on B by A act at the same instant . Any one of them may be called action and the other reaction.*
3. *Action and reaction forces act on different bodies , not on the same body. Thus if we are considering the motion of any one body (A or B) , only one of the two forces is relevent. It is an error to add up the two forces and claim that the net force is zero.*

   However, if you are considering the system of two bodies as a whole, $\mathbf{F}_{AB}$ and $\mathbf{F}_{BA}$ are internal forces of the system (A+B). They add up to give a null force. Internal forces in a body or a system of particles thus get cancelled in pairs.

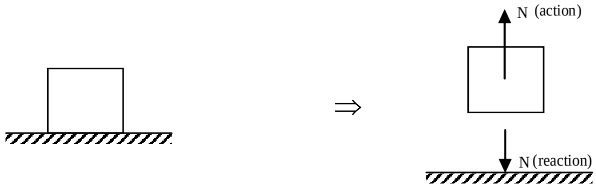
Steps To Apply Laws Of Motion

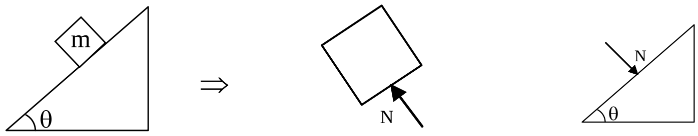
(a) If there are many bodies in the system and they are placed or connected in such a way that all bodies in the system have same acceleration, then apply Newton's second law on the system. After calculating acceleration (a), draw FBD for each body in the system. Then use Newton's second law for each of them to get unknowns.

(b) If system has number of bodies with different accelerations, then first find constraint relations (relationship between their accelerations). Draw FBD for each of the body in the system. Then use Newton's second law for each of them to get the unknowns.

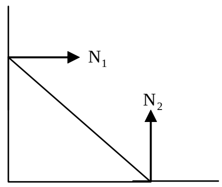
FREE BODY DIAGRAMS (Normal Reaction)

If two bodies are in contact, each body experiences a Normal Force which is Perpendicular to contact surface area.





If contact surface area is not defined and only point of contact is defined then the normal force is taken perpendicular to the surface at the point of contact.



Tension

When a rope (string) is connected to a body and pulled taut, the rope is said to be under tension. It pulls the body with a force T, whose direction is away from the body and along the length of the rope. A rope is usually regarded to be massless and unstretchable. The rope exists only as a connections between two bodies. It pulls on the body at each end with the same magnitude of force T.

(i) If a string is massless, the tension in it is same everywhere. However, if a string has a mass, tension at different points may be different.

(ii) If there is friction between string and pulley, tension may be different on two sides of the pulley, but if there is no friction between pulley and string, tension will be same on both sides of the pulley.

(iii) If two persons are pulling a rope opposite to each other, each applying a force equal to 100N, then tension in the rope will be 100 N.

Illusration- 6

A block of mass 4 kg lies over a horizontal surface

(a) Find the normal reaction between the block and the surface.

(b) Now another block of mass 2 kg is placed over the first block. Find the normal reaction between the first block and the ground surface in the new case.($g = 10\ m/s^2$)

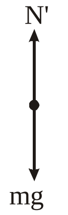
Solution

(a) $N - Mg = 0$

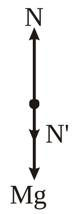
$\Rightarrow \quad N = Mg = 40N$

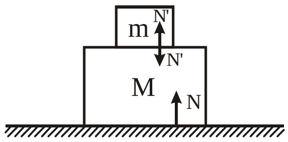
(b) Let normal reaction between the first and second block be N'

F.B.D. of m



F.B.D. of M





$\Rightarrow \quad N' = mg$

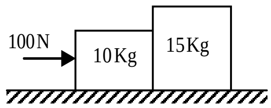
$N - N' = Mg$

$\Rightarrow \quad N = Mg + N' = N(M+m)g = 60\ N$

Illustration- 7 : **Find**

(a) common acceleration of the system

(b) normal contact force (pushing force or the reaction force)



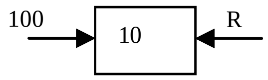
smooth surface

Solution R is Reaction (normal contact force) between the two blocks,

N_1 and N_2 are the normal reactions (contact force) between the blocks and the ground.

Considering only the horizontal forces

FBD of 10 Kg



Equation of motion

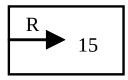
$100 - R = 10\ a$ ..... (I)

$R = 15\ a$ ..... (II)

$100 = 25\ a$

$\Rightarrow \quad a = 4\ m/s^2$

FBD of 15 Kg



Equation of Motion

$R = 15\ a$ ..... (II)

$\Rightarrow\ R = 15 \times 4 = 60N$

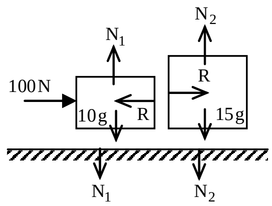
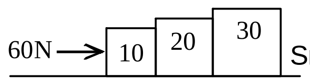


Illustration - 8 :In the diagram given

(a) find the acceleration of the system.



Smooth surface

(b) Find the reaction forces between 10, 20 and 20, 30 kg blocks.

Solution

From Newton's second law, we can write

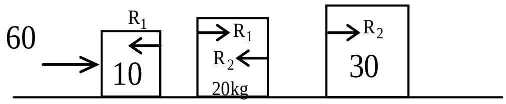
$60 - R_1 = 10a$ (i)

$R_1 - R_2 = 20a$ (i)

$R_2 = 30a$ (iii)

$\Rightarrow \quad 60 = 60a$

$\Rightarrow \quad a = 1\ m/s^2$ $\quad\Rightarrow\quad R_1 = 50\ N;\quad R_2 = 30\ N$

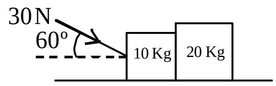


FBD's

Illustration- 9 **In the diagram given find**

(a) acceleration of the system

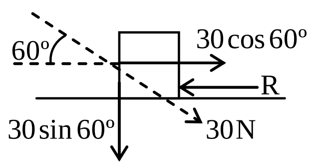
(b) normal contact force between the blocks.



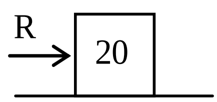
Smooth surface

Solution

We need to resolve the applied force



FBD of 10 kg



FBD of 20 Kg

Here for resolving the 30N force we first extend it and then resolve it into two mutual perpendicular components

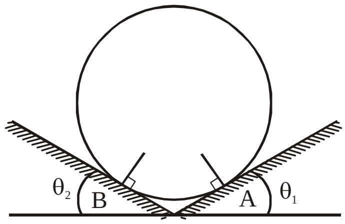
Eq^{ns}. of motion would be

$30\cos 60^0 - R = 10a$ (i)

R = 20 a (ii)

Solving (i) and (ii)

$\Rightarrow$ a = 1/2 m/s^2 $\Rightarrow$ $R = 20\times\frac{1}{2} = 10N$



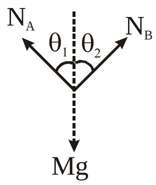
Illusration - 10

A sphere of mass M lies between two smooth incline as shown. Find the normal reaction at contact points A and B.

Solution

Let normal reaction at contact points A and B be N_A and N_B respectively

F.B.D. of sphere is

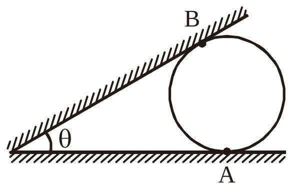


Considering 2 forces in vertical direction $N_A \cos\theta_1 + N_B \cos\theta_2 - Mg = 0$ ... (ii)

Considering forces in horizontal direction $N_B \sin\theta_2 - N_A \sin\theta_1 = 0$ ...(ii)

solving equation (i) and (ii) we get

$$N_A = \frac{Mg\sin\theta_2}{\sin(\theta_1+\theta_2)} \qquad N_B = \frac{Mg\sin\theta_1}{\sin(\theta_1+\theta_2)}$$



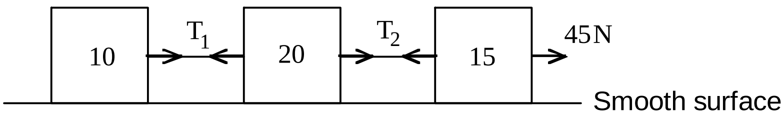
Exercise ***A sphere of mass M lies between a smooth horizontal surface and smooth, light inclined surface as shown. Find the normal reactions at points A and B.***

Answers $N_A = Mg$; $N_B = 0$

Illusration - 11

In the diagram shown

Find (i) acceleration of each block (ii) Tension in the each string



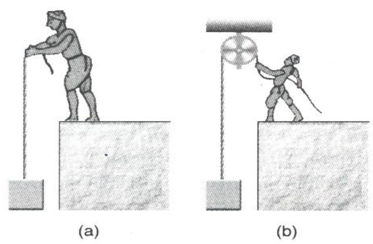
Solution

$T_1 = 10a$ (i) $T_2 - T_1 = 20a$ (ii) $45 - T_2 = 15a$ (iii)

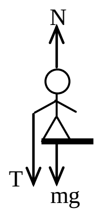
a = 1 m/s^2 $\Rightarrow$ $T_1 = 10N$, $T_2 = 30$ N

Illusration - 12

Normal force exerted on man, when he pulls a block as shown in the figure (a) is greater as compared to the normal force exerted on a man pulling block as shown in figure (b).



Solution

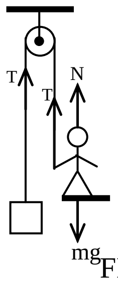


$N = T + mg$

$T = Mg$

(where M is the mass of block)

FBD for figure(a)



$N + T = mg$

$N = mg - T$

$T = Mg$ (where M=mass of block)

FBD for Figure (b)

STRING CONSTRAINS

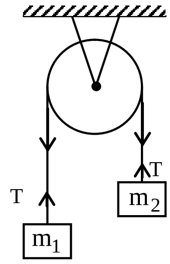
Length of the string always remains constant.

Consider an example.

If point 1 of the string moves by a distance x_1 and point 2 moves by a distance of x_2 in the same time interval then total elongation in the string is $x_1 - x_2$ which should be equal to zero, as string is inextensible.

$\Rightarrow \quad x_1 - x_2 = 0 \qquad x_1 = x_2 \qquad \frac{d^2x_1}{dt^2} = \frac{d^2x_2}{dt^2} \quad \Rightarrow \quad a_1 = a_2$

SIMPLE PULLEY SYSTEM (Also refered as Atwood machine)



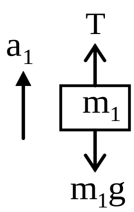
$\Rightarrow$ Draw the free body diagram of m_1 & m_2

$\Rightarrow$ Assume m_1 to be accelerating in upward dir. with an acceleration of a_1

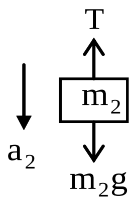
$\Rightarrow$ Assume m_2 to be accelerating in downward direction with an acceleration of a_2

$\Rightarrow$ Tension is the same all throughout the string as the pulley is smooth and string is massless.

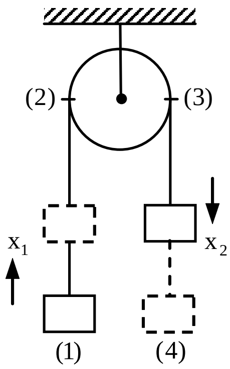
$\Rightarrow$ Tension acts away from m_1 with respect to m_1 and away from m_2 with respect to m_2



$T - m_1g = m_1a_1$ .... (1)



$m_2g - T = m_2a_2$ .... (2)



By constraint point (1) moves towards the centre of the segment by a distance of x_1 while point (2) is at rest.

Hence elongation is this segment is ($-x_1$). Point (4) is moving away (2) from the centre of this segment by a distance of x_2 and hence elongation in this segment is (x_2). Total elongation is the string is $x_2 - x_1$ which should be equal to zero.

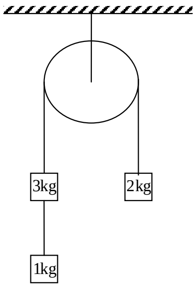
$x_2 - x_1 = 0 \quad \Rightarrow \quad x_2 = x_1 \qquad \Rightarrow a_1 = a_2 = a$ (say) $\quad$ (3)

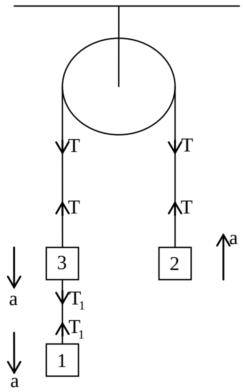
Using (1), (2) & (3)

$a = \frac{m_2 - m_1}{m_2 + m_1} g \quad , \qquad T = \frac{2m_1m_2g}{m_1 + m_2}$

Illusration - 13

Find the acceleration of all the masses and tension in both the strings

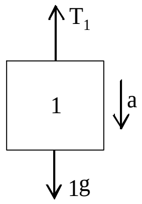




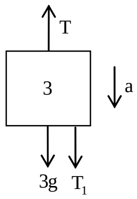
Solution

By constraint , all masses have the same acceleration(in magnitude).

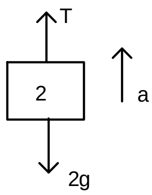
Free body diagram of 1kg block is shown below:



$1g - T_1 = 1a \longrightarrow (1)$



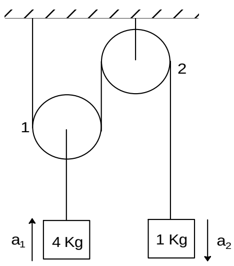
$3g + T_1 - T = 3a \longrightarrow (2)$

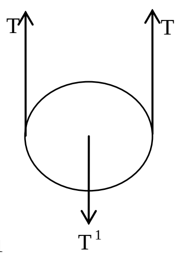


$T - 2g = 2a \longrightarrow (3)$

Solving eq. (1), (2) & (3) $a = g/3\ m/s^{2,}$, $T_1 = 2g / 3$ and $T_2 = \frac{8g}{3}$.

Illusration - 14 *Find the acceleration of each mass .*



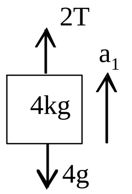


Solution

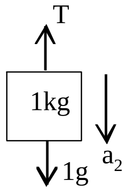
Pulley (1)

As pulley is of negligible mass $2T = T^1$

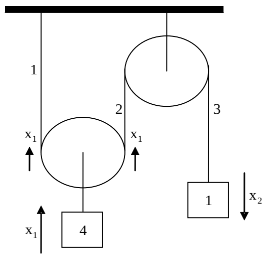
$2T - 4g = 4a_1$ (1)



$1g - T = 1a_2$ (2)



To find the relation between a_1 & a_2
we use constraint relation
Elongation in segment (1) is $(-x_1)$
Elongation in segment (2) is $(-x_1)$
Elongation in segment (3) is $(+x_2)$



Hence total elongation in the string

$-x_1 - x_1 + x_2 = 0 \Rightarrow \quad x_2 = 2x_1$

$\Rightarrow \quad a_2 = 2a_1$

$2T - 4g = 4a_1 \longrightarrow (1) \qquad 1g - T = 1a_2 \longrightarrow (2) \qquad a_2 = 2a_1 \longrightarrow (3)$

Solving eq. (1), (2), & (3)

$a_1 = \frac{-g}{4}$ and $a_2 = \frac{-2g}{4} = -g/2$

$\Rightarrow$ 4kg mass accelerates with an acceleration of g/4 in downward direction.

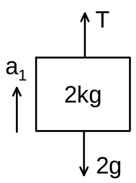
$\Rightarrow$ 1kg mass accelerates in upward direction with an acceleration of g/2.

Illusration - 15

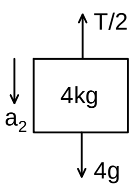
Find the acceleration of each mass

Solution

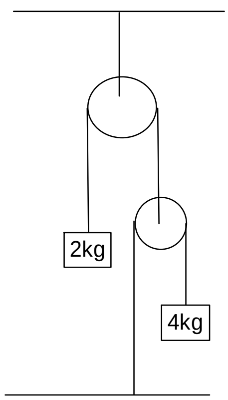
Draw the free body diagrams



$T - 2g = 2a_1 \longrightarrow (1)$



$4g - T/2 = 4a_2 \longrightarrow (2)$



To find the relation between a_1 & a_2 we use constraint relations.

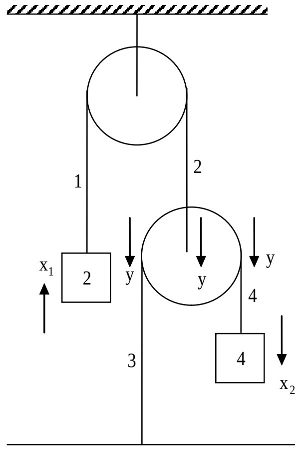
Let us assume 2kg mass moves up by a distance of x_1, 4kg mass moves down by x_2 and pulley moves down by a distance of y.

Elongation in segment (1) = $-x_1$
Elongation in segment (2) = y ;

Total elongation $-x_1 + y = 0$ or $x_1 = y$

Elongation is segment (3) = $(-y)$
Elongation is segment (4) = $(x_2 - y)$

Hence $x_2 - y - y = 0$
$x_2 = 2y$ or $x_2 = 2x_1$
$a_2 = 2a_1 \longrightarrow (3)$
Solving (1), (2) & (3) we get
$a_1 = g/3\ m/s^2$
$a_2 = 2g/3\ m/s^2$



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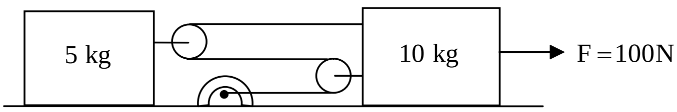
Positive acceleration for a_1 & a_2 implies the directions assumed are correct.

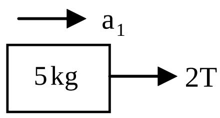
Illusration - 16 Find the acceleration of 5kg & 10kg mass.



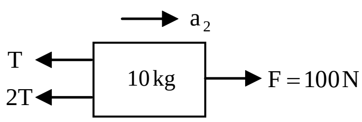
Solution

Draw the free body diagram of 5kg & 10kg mass

$2T = 5a_1 \longrightarrow (1)$

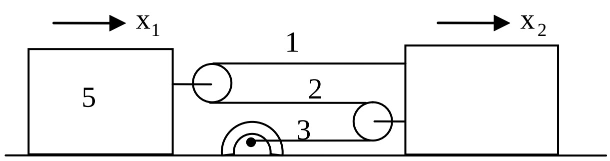


$F - 3T = 10a_2$



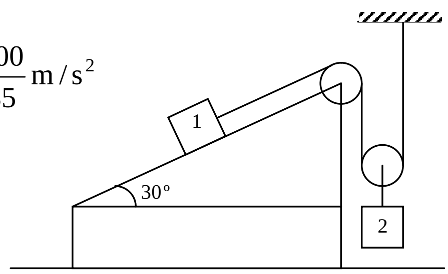
$100 - 3T = 10a_2 \longrightarrow (2)$

To find the relation between a_1 & a_2 we use constraint relation
Elongation in segment (1) $x_2 - x_1$
Elongation in segment (2) $x_2 - x_1$
Elongation in segment (3) $+x_2$
$2(x_2 - x_1) + x_2 = 0$
$2x_2 - 2x_1 + x_2 = 0$
$3x_2 = 2x_1$
$3a_2 = 2a_1 \longrightarrow (3)$



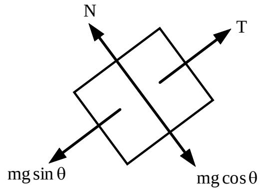
Solving (1), (2), & (3) $\quad a_1 = \dfrac{600}{85} m/s^2 ; a_2 = \dfrac{400}{85} m/s^2$

Illusration - 17 The wedge is fixed and smooth.
Find the acceleration of 2kg & 1kg mass.



Solution -

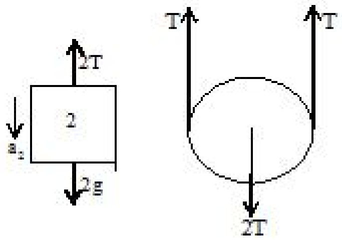
F.B.D (free body diagram) of 1kg mass



Assuming that 1kg mass moves up the plane by an acceleration of a_1.

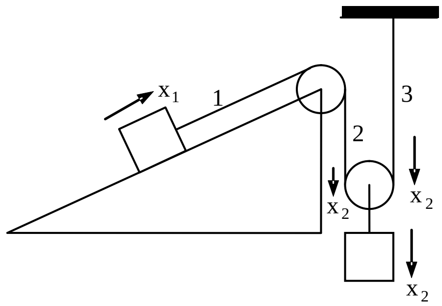
$$T - mg\sin 30^o = ma_1$$
$$T - g\sin 30^o = 1a_1 \longrightarrow (1)$$

Assuming that 2kg mass moves down by an acceleration a_2



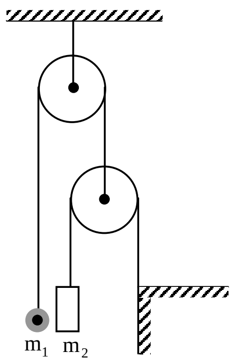
$$2g - 2T = 2a_2 \longrightarrow (2)$$

To find the relation between a_1 & a_2 we use constraint relations as shown.



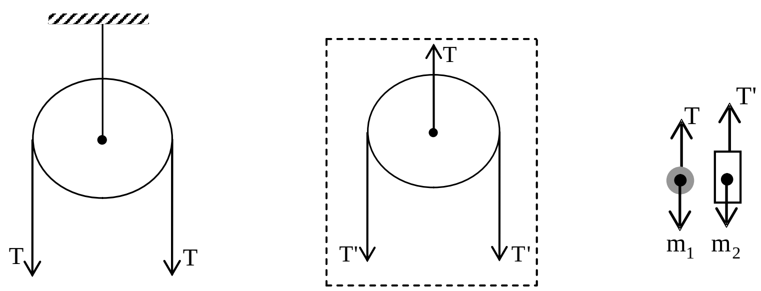
Elongation in segment (1) $(-x_1)$
Elongation in segment (2) (x_2)
Elongation in segment (3) (x_2)
Total elongation in the string
$-x_1 + x_2 + x_2 = 0$
$2x_2 = x_1$
$2a_2 = a_1 \qquad \longrightarrow (3)$
Solving for (1), (2) & (3) we get
$a_2 = g/6\ m/s^2 \quad \& \quad a_1 = g/3\ m/s^2$

Illusration - 18



In the arrangement shown in figure the mass of the ball is $\eta = 1.8$ **times as great as the rod. The length of the rod is l =100cm. The masses of the pulleys and the threads, as well as the friction are negligible. The ball is set on the same level as the lower end of the rod and then released. How soon will the ball be opposite to the other end of the rod.**

Solution



Let the acceleration of the mass m_1 is upward, then the acceleration of m_2, will be 2a downward.
From FBD, we have

$T - m_1g = m_1a \qquad \ldots(i)$

$m_2g - T' = m_2(2a) \qquad \ldots(ii)$

and $\quad T = 2T^1 \qquad \ldots(iii)$

Solving above equations, we get $a = \left[\dfrac{2m_2 - m_1}{m_1 + 4m_2}\right]g$

given, $m_1 = 1.8m_2$ $\therefore$ $a = \dfrac{g}{29}\ m/s^2$

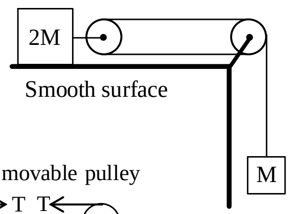
Acceleration of rod relative to ball = 2a – (–a) = 3a

Displacement, $\ell = ut + \dfrac{1}{2}(3a)t^2$ or $1 = 0 + \dfrac{1}{2}3x\dfrac{g}{29}x\,t^2$

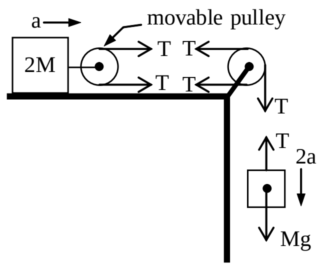
$\therefore$ $t = 1.40s$

Illusration - 19

Draw free body diagram and write equations of force for each block and find the force on the clamp by the pulley.



Solution



For block 2M

$2T = 2M(a)$ … (i)

For block M

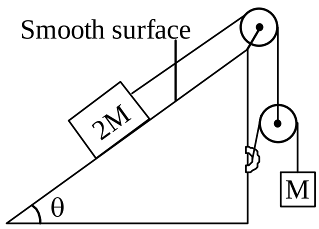
$Mg - T = M(2a)$ …(ii)

Force on clamp which hold the pulley

$F = \sqrt{(2T)^2 + T^2} = \sqrt{5}T$

Illusration - 20

Draw free body diagram and write equations of force for each block in the following devices.



Solution

For block 2M ;

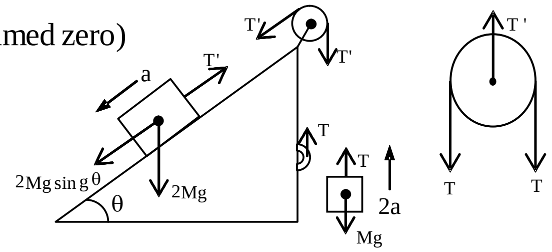
$2Mg\sin\theta - T' = 2M(a)$ … (i)

For movable pulley ; (As mass of pulley is assumed zero)

$T' - 2T = 0$ … (ii)

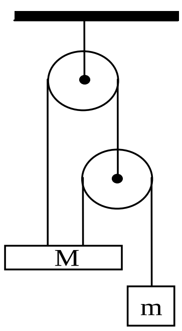
For block M ;

$T - Mg = M(2a)$ …(iii)

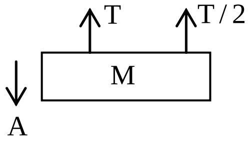


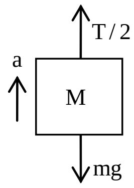
Illusration - 21

Find the acceleration of the both masses m & M.



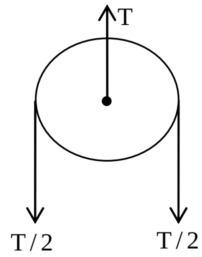
Solution





$$Mg - \frac{3T}{2} = Ma \quad (1)$$

$$\frac{T}{2} - mg = ma \longrightarrow \quad (2)$$



To find the relation but A & a we apply constrains.

Elongation in segment (1) = x_1

Elongation in segment (2) = y

Total elongation of the string is

$x_1 + y = 0$ or $x_1 = -y \ldots$(i)

Elongation in segment (3) = $x_1 - y$

Elongation in segment (4) = $-x_2 - y$

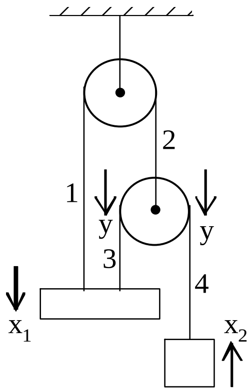
Total elongation= $x_1 - y - x_2 - y = 0$

$x_1 - x_2 - 2y = 0$

$x_1 - x_2 + 2x_1 = 0$

or $x_2 = 3x_1$

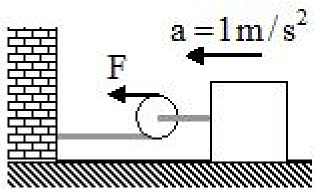
$a = 3A \longrightarrow$ (3)



Solving (1), (2), & (3) $A = \dfrac{M - 3m}{M + 9m} g$ and $a = \dfrac{3M - 9m}{M + 9m} g$

Illusration - 22

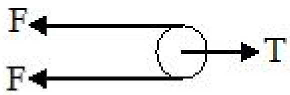
A block of mass 200kg is set into motion on a frictionless horizontal surface with the help of frictionless pulley and a rope system as shown in figure (a). What horizontal force should be applied to produce in the block an acceleration of $1 m/s^2$?

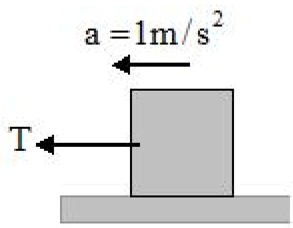


(a)

Solution

As shown in fig (b), when force F is applied at the end of the string, the tension in the lower part of the string is also F. If T is the tension in string connecting the pulley and the block, then,





(b)

$T = 2F$

But $T = ma = (200)(1) = 200$ N

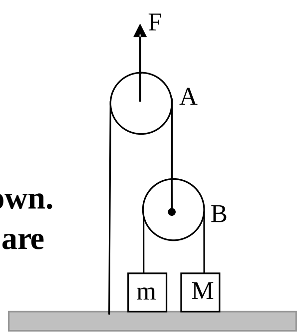
$\therefore$ $2F = 200$ N

or $F = 100$ N

Illusration - 23

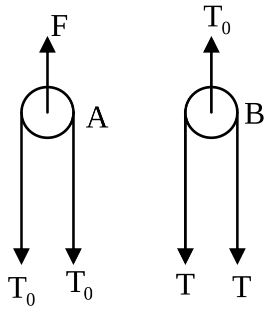
Two blocks of mass m = 5kg and M = 10kg are connected by a string passing over a pulley B as shown. Another string connects the centre of pulley B to the floor and passed over another pulley A as shown. An upward force F is applied at the centre of pulley A.Both the pulley are massless.Find the acceleration of blocks m and M if F is:

(a) 100 N (b) 300 N (c) 500 N (take $g = 10 m/s^2$)



Solution

Let T_0 = tension in the string passing over A
T = tension in the string passing over B



$2T_0 = F$ and $2T = T_0$
$\Rightarrow$ T = F/4

(a) T = F / 4 = 25N.
weights of block are; mg = 50N
Mg = 100N
As T < mg and Mg both, the blocks will remain stationary on the floor.

(b) T = F / 4 = 75N.
As T < Mg and T > mg, M will remain stationary on the floor, whereas m will move.

acceleration of m, $a = \frac{T - mg}{m} = \frac{75 - 50}{5} = 5 m/s^2$

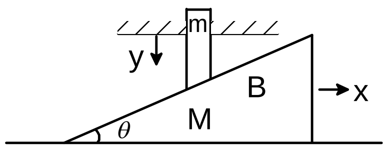
(c) T = F / 4 = 125 N
As T > mg and Mg, both the blocks will accelerates upwards.

acceleration of m, $a_1 = \frac{T - mg}{m} = \frac{125 - 50}{5} = 15 m/s^2$

Acceleration of M, $a_2 = \frac{T - Mg}{M} = \frac{125 - 100}{10} = 2.5 m/s^2$

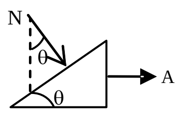
Illusration- 24

Find the acceleration of rod A and wedge B,
If M = mass of wedge ; m = mass of rod.
All surfaces are smooth and the rod is restricted to move in the vertical direction only .



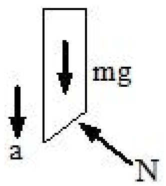
Solution

Let the acceleration of wedge be 'A'



$N \sin\theta = MA.$ .... (1)

Let acceleration of rod be 'a'



$mg - N\cos\theta = ma.$ .... (2)

$\tan\theta = y/x$

$y = x\tan\theta$

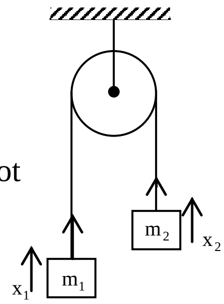
$\frac{d^2y}{dt^2} = \frac{d^2x}{dt^2}\tan\theta$

$a = A\tan\theta$............(3)

Solving (1) (2) & (3), we get

$$A = \frac{mg}{M\cot\theta + m\tan\theta} \qquad \text{and} \qquad a = \frac{mg\tan\theta}{M\cot\theta + m\tan\theta}$$

CONSTRAINTS [The Virtual Work]



Consider the Atwood machine shown. If one holds both the masses such that the string is slack. There will not be any tension in the string. Once released the string will become taut and it will oppose the motions of one block where as it will support the motion of other one.

Conclusion: It is transferring the effort of one block to the other one. So, on its own it will not do any work on the system of two blocks. Work done by tension on block having mass $m_1 = \vec{T}\cdot\vec{x}_1$ where $\vec{x}_1$ is the displacement of block m_1. Similarly work done by tension on block m_2 will be $\vec{T}\cdot\vec{x}_2$. Since sum of these two has to be zero ; we have

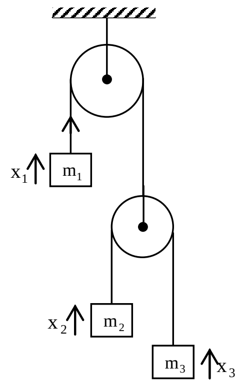
$$\vec{T}\cdot\vec{x}_1 + \vec{T}\cdot\vec{x}_2 = 0$$

$$\Rightarrow \quad \vec{x}_1 + \vec{x}_2 = 0$$

$$\Rightarrow \quad \vec{x}_1 = -\vec{x}_2 \qquad \text{(i)}$$

Differentiating equation (i) we have

$$\vec{V}_1 = -\vec{V}_2 \quad \text{(ii)} \qquad \text{and} \qquad \vec{a}_1 = -\vec{a}_2 \quad \text{(iii)}$$



Consider the figure shown

Tension on block $m_1 = T$

Tension on block $m_2 = T/2$

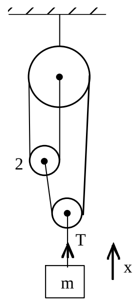
Tension on block $m_3 = T/2$

Assuming displacements of masses as shown in the figure.

Now using sum of work done by tension on the entire system = 0.

$$Tx_1 + \frac{T}{2}x_2 + \frac{T}{2}x_3 = 0$$

$$\Rightarrow \quad 2x_1 + x_2 + x_3 = 0$$



Differentiating it twice we get the relation among the accelerations of three blocks

$$2a_1 + a_2 + a_3 = 0$$

In the next example we will take a little complicated situation.

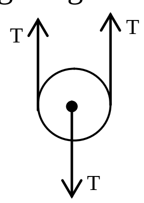
In this system we have only one block on which tension will work on.

$$T\,x = 0$$

$$\Rightarrow \quad x = 0 \quad \text{and} \quad a = 0$$

$$\Rightarrow \quad 2a_1 + a_2 + a_3 = 0$$

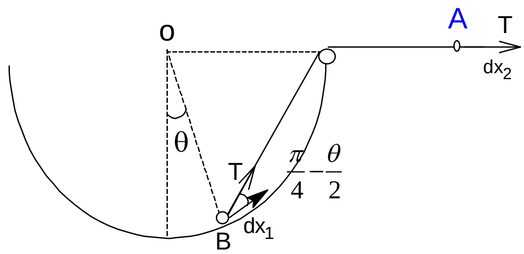
We have made a mistake which gives a wrong result. We cancelled T against zero that was our mis take. We should always make sure the quantity getting cancelled itself is non zero. Looking at the F.B.D. of pulley (2) we realize the mistake



Since the pulley is massless

$$2T - T = 0\,.\,a_{pulley} \quad \Rightarrow \quad T = 0$$

Hence the mass *m* is going to fall freely under gravity with which we have over burdended the system. Massless pulleys and strings are hypothetical entities.



Example:

A mass *m* is being pulled on a circular track with the help of an inextensible string as shown above. We will find the velocity of mass *m* using the same procedure.

Mass *m* will move along the circular track hence the elemental displacement will be along the track, say dx_1 and displacement of point A is dx_2.

using $\vec{T}\cdot d\vec{x}_1 + \vec{T}.d\vec{x}_2 = 0$

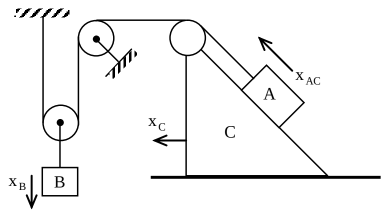
we get $\Rightarrow$ $T\ dx_1\ .\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right) - T\ dx_2 = 0\ V_1\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right) = V_2$

Think why can't we find, the acceleration of mass *m*, if acceleration of point A is known, using the same procedure.

Illusration - 25

Find the relation between accelerations of blocks A, B and wedge C, of the figure shown below.

Solution



Displacment of block A will depend on the displacement of wedge C, we can apply $\vec{T}\cdot\vec{x} = 0$ for relative displacements as well.

For block A $\quad \vec{T}_A \cdot \vec{x}_{AC} = T\ x_{AC}$

For blcok B $\quad \vec{T}_B \cdot \vec{x}_B = -2T\ x_B$

For wedge C $\quad \vec{T}_C \cdot \vec{x}_C = T\ x_C$

$\Rightarrow \quad x_{AC} + x_C - 2x_B = 0 \quad \Rightarrow \quad V_{AC} + V_C - 2V_B = 0 \quad \Rightarrow \quad a_{AC} + a_C - 2a_B = 0$

Try the same problem by using absoute values of tensions and displacement.

ACCELERATING REFERENCE FRAMES

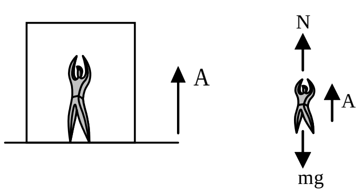
While solving problem in this chapter, it is important to note that Newton's laws are valid only for inertial or non- accelerating reference frames.

If we want to solve a problem from a non inertial frame (accelerating frame) we should introduce an imaginary force called pseudo force in a direction opposite to that of the accelerating frame, along with all the real forces acting on the body. The magnitude of this pseudo force is depends on the mass of body and the magnitude of acceleration of non- inertial frame. Pseudo force on a body can be defined as

$$\vec{F}_p = -m\vec{a}$$

Accelerating Elevators

(a) **<u>Lift is accelerating upwards</u>**:- Imagine a man in an elevator, which is accelerating upwards. Let us first describe the motion of the man with respect to ground. A person on the ground observes the man is accelerating along with the elevator.



Hence $N - mg = mA$

$N = m(g + A)$ ...... (i)

Hence, the man experiences an increase in his weight.

To describe the same motion with respect to an observer in the elevator, introduce a pseudo force opposite to the direction of acceleration of the elevator. To this observer the man is in equilibrium

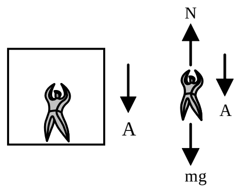
Hence $N = mg + mA$ .... (ii)

Note that above equation (ii) is identical to equation (i)

Remember: Normal force on the man is the weight experienced by him.

(b) **Lift is accelerating downwards** :- Now consider this cars when the lift is accelerating downwards.

Applying Newton's second law with respect to ground, we get



$$mg - N = mA$$

$$N = m(g - A) \qquad (i)$$

Note that in a freely falling lift (i.e. $A = g$) man will experience weightlessness

Applying Newton's second law with respect to elevator, we obtain

N+mA⟶ 

Pseudo force

$N + mA = mg \qquad N = m(g - A) \quad$ (ii)

Note that equation (ii) is in agreement with equation (i).

Illusration - 26

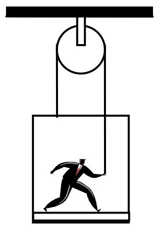
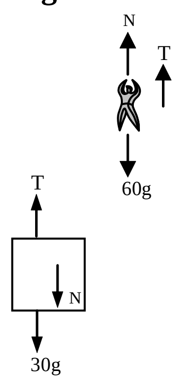


Figure shows a man of mass 60 kg standing on a light weighing machine kept in a box of mass 30 kg. The box is hanging from a pulley fixed to the ceiling through a light rope, the other end of which is held by the man himself. If the man manages to keep the box at rest, what is the weight shown by the machine ? What force should he exert on the rope to get his correct weight on the machine ?

Solution

(a) If the man manages to keep the box at rest.



FBD of Box

$N + T = 60\text{ g}$ .... (1)

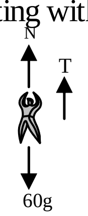
$T = N + 30\text{ g}$ .... (2)

$N + N + 30g = 60g$

$2N = 30g$

$N = 15\text{ g} = 150\text{ N}$

(b) Force to be applied by the man $\Rightarrow$ Tension in the rope to get his correct weight. This will be a condition of inequilibrium and both the man and the lift will be accelerating with same acceleration



$\Rightarrow \qquad N + T - 60g = 60a \qquad (1)$

$N = 60\text{ g}$

$\Rightarrow \qquad T = 60\text{ a}$

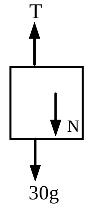
$T - N - 30g = 30a \qquad (2)$

$60a - 60g - 30g = 30a$

$30a = 90g$

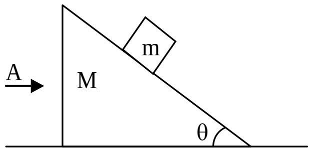
$a = 3g$

$\Rightarrow \quad T = 60 \times (3g) = 1800N$



Illusration - 27

What should be the acceleration given to mass M, so that m may not slide on the wedge ? (All surface are smooth)



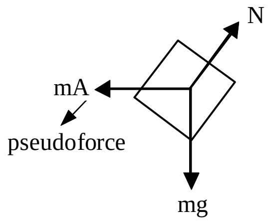
Solution

F.B.D. of *m* with respect to wedge

$\Rightarrow$ m is in equilibrium with respect to wedge

$\Rightarrow \quad \sum F = 0$

$\Rightarrow \quad \sum F_x = 0 \quad \sum F_y = 0$

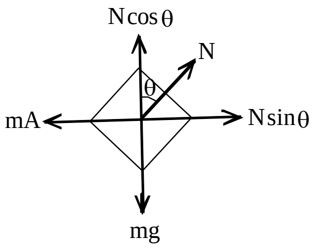


$N\sin\theta = mA$

$N\cos\theta = mg$

$\tan\theta = A/g$

$A = g\tan\theta$



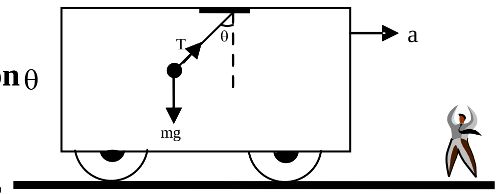
Illusration - 28

Consider a simple pendulum hanging in an accelerated railroad car. We wish to determine the angle of deflection θ **of the pendulum with respect to vertical.**

In inertial frame : To an observer on the ground.

In non-inertial frame: To an observer in the railroad car.

also find the forces on the bob of the pendulum in the both frame.



Solution

In inertial frame : The problem can be visualised in two frames : inertial frame, i.e., with respect to ground and non-inertial frame i.e., with respect to car.

(i) Weight of bob, mg acts vertically downward

(ii) Tension in the string T.

Thus for the motion of bob relative to an observer on ground,

$T\sin\theta = ma$ .... (i)

and $T\cos\theta = mg$ ..... (ii)

Dividing equation (i) by (ii), we get, $\tan\theta = \dfrac{a}{g}$

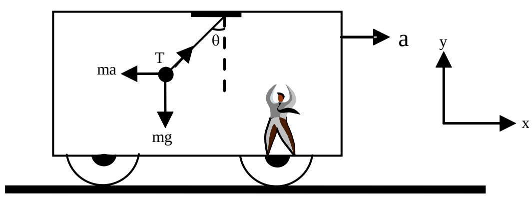
In non-inertial frame : To an observer inside the railroad car, the forces on the bob are ;

(i) Weight of bob, mg acts vertically downward.

(ii) Tension in the string T and

(iii) Pseudo force ma opposite of the acceleration of car

As bob is stationary relative to the observer inside car, so



$\sum F_x = 0$;

or $\quad T\sin\theta - ma = 0$

$\therefore \quad T\sin\theta = ma$ ...... (1)

and $\sum F_y = 0$

or $T\cos\theta - mg = 0$

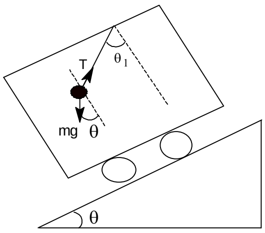
$\therefore$ $T\cos\theta = mg$ ...... (2)

Dividing (1) by (2), we get, $\tan\theta = \dfrac{a}{g}$

Pendulum hanging from the ceiling of a trolley moving down the inclined plane

(i) With constant velocity

Let θ_1 is the angle the string makes with the perpendicular drawn from ceiling of the trolley.



We have

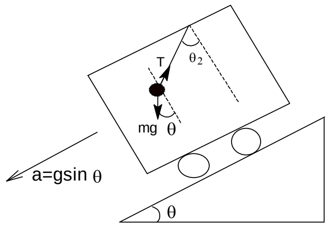
$T\cos\theta_1 = mg\cos\theta$ .... (1)

and $mg\sin\theta - T\sin\theta_1 = m\times 0$ .... (2)

Dividing (2) by (1), we get $\theta_1 = \theta$

(ii) Constant acceleration g sin θ

Let θ_2, is the angle which the string makes with the perpendicular drawn from ceiling of the trolly.



We have

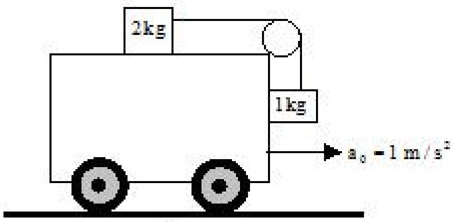
$T\cos\theta_2 = mg\cos\theta$ .... (1)

and $mg\sin\theta - T\sin\theta_2 = m(g\sin\theta)$ .... (2)

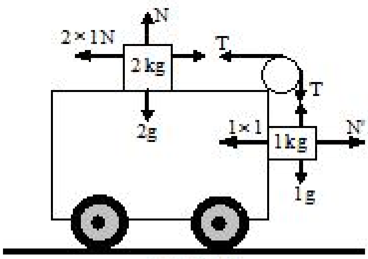
from (2), we get $\theta_2 = 0^0$

Illusration - 29

A cart carries two blockes of masses 2 kg and 1 kg which are connected by a string passing over a pulley, as shown in figure. The cart is moving towards right with an acceleration of 1 m/s^2. Find the acceleration of blocks with espect to ground and tension in the string. (Take g = 10 m/s^2)



(a)



(b) FBD

Solution

Let acceleration of the 2 kg block is 'a' towards right with respect to cart. The forces on the blocks in accelerated frame are shown in figure.

For 2 kg block ; $T - 2\times 1 = 2a$ .... (1)

For 1 kg block ; $1g - T = 1a$ .... (2)

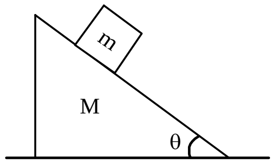
Solving equation (1) and (2), we get $a = \dfrac{8}{3}\,m/s^2$ and $T = \dfrac{22}{3}\,N$

Acceleration of blocks w.r.t. ground ; Acceleration of 1 kg block = $\sqrt{1+(8/3)^2}\ m/s^2$

Acceleration of 2 kg block ; $a = \dfrac{8}{3} + 1 = \dfrac{11}{3}\,m/s^2$.

Illusration - 30

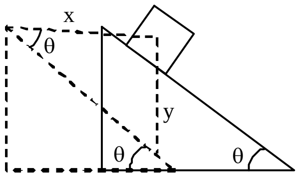
With what acceleration must M be moved so that 'm' appears to fall freely.



Solution $\Rightarrow$ By the time M moved by a distance of x, m must fall by a distance of y

$\tan\theta = y/x$

$y = x\tan\theta$



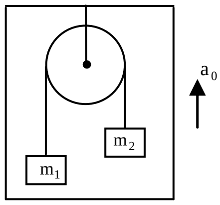
$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta$$

$$\frac{d^2y}{dt^2} = \frac{d^2x}{dt^2}\tan\theta$$

$g = A\tan\theta \qquad \Rightarrow \qquad A = g\cot\theta$

Illusration - 31

A simple pulley in an elevator accelerating in the upward direction. Find acceleration of both the masses.



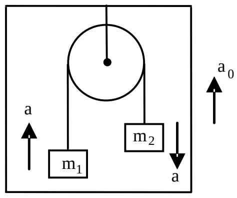
Solution :

Both the masses m_1 & m_2 will have the same acceleration a in opposite directions with respect to the elevator

Hence

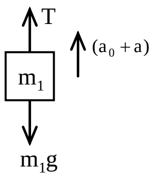
$\Rightarrow$ Acceleration of m_1 with respect to ground is $(a_0 + a)$ in upward direction.

$\Rightarrow$ Acceleration of m_2 with respect to ground is $(a_0 - a)$ in upward direction

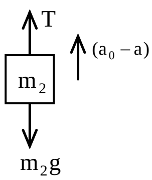


Remember: both masses accelerate in upward direction with respect to ground.

$\Rightarrow \qquad T - m_1g = m_1(a_0 + a) \qquad (1)$



$\Rightarrow \qquad T - m_2g = m_2(a_0 - a) \qquad (2)$



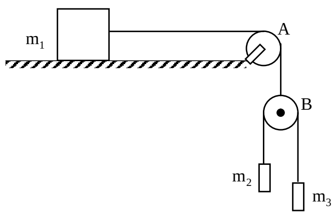
Solving (1) and (2)

$$a = \frac{m_2 - m_1}{m_1 + m_2}g + \frac{m_2 - m_1}{m_1 + m_2}a_0$$ $\Rightarrow$ $\vec{a}$ is the acceleration of m_1 & m_2 with respect to pulley

Exercise : Solve the above problem with the lift accelerating in downward direction.

Illusration- 32

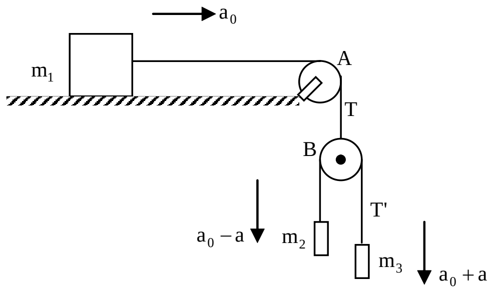
Three blocks of masses m_1, m_2 and m_3 are connected as shown in the figure. All the surfaces are frictionless and the string and the pulleys are light. Find the acceleration of m_1.



Solution

Suppose the acceleration of m_1 is a_0 towards right. That will also be the downward acceleration of the pulley B because the string connecting m_1 and B is constant in length. The upward acceleration of m_2 with respect to B equals the downward acceleration of m_3 with respect to B. Let this acceleration be a. The acceleration of m_2 with respect to the ground $a_0 - a$ (downward) and the acceleration of m_3 ith respect to the ground = $a_0 + a$ (downward).

As the mass of the pulley is negligible,



$$2T' - T = 0$$

giving $\quad T' = T/2 \qquad (1)$

Applying Newton's second law for each block,

$T = m_1 a_0 \qquad (2)$

$m_2 g - T/2 = m_2(a_0 - a) \qquad (3)$

$m_3 g - T/2 = m_3(a_0 + a) \qquad (4)$

Putting T from (2) in (3) and (4)

$$a_0 - a = \frac{m_2 g - m_1 a_0/2}{m_2} = g - \frac{m_1 a_0}{2m_2}$$

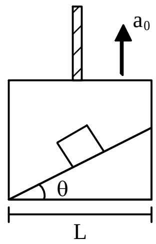
and $a_0 + a = \dfrac{m_3 g - m_1 a_0/2}{m_3} = g - \dfrac{m_1 a_0}{2m_3}$ $\qquad$ Adding, $2a_0 = 2g - \dfrac{m_1 a_0}{2}\left(\dfrac{1}{m_2} + \dfrac{1}{m_3}\right)$

or, $\quad a_0 = g - \dfrac{m_1 a_0}{4}\left(\dfrac{1}{m_2} + \dfrac{1}{m_3}\right)$

$$\Rightarrow a_0\left[1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)\right] = g \quad ; \qquad \text{or,} \qquad a_0 = \frac{g}{1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)}$$

Illusration - 33

A particle slides down a smooth inclined plane of elevation θ, fixed in an elevator going up with an acceleraton a_0. The base of the incline has a length L. Find the time taken by the particle to reach the bottom.



(a)

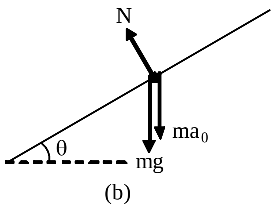
Solution

In the elevator frame. a pseudo force ma_0 in the downward direction is to be applied on the particle of mass m together with the real forces. Thus, the forces on m are (figure b)

(i) N normal force,

(ii) mg downward (by the earth),

(iii) ma_0 downward (pseudo).



(b)

Let a be the acceleration of the particle with respect to the incline. Taking components of the forces parallel to the incline and applying Newton's law,

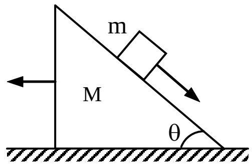
$$mg\sin\theta + ma_0\sin\theta = ma \quad ; \qquad \text{or,} \qquad a = (g + a_0)\sin\theta.$$

This is the acceleration with respect to the elevator. In this frame, the distance travelled by the particle is $L/\cos\theta$.

Hence, $\quad \dfrac{L.}{\cos\theta} = \dfrac{1}{2}(g + a_0)\sin\theta . t^2 \quad ; \qquad$ or, $\quad t = \left[\dfrac{2L}{(g + a_0)\sin\theta\cos\theta}\right]^{1/2}$

Illusration - 34

All the surfaces shown in figure are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block 'm' with respect to the prism.



Solution

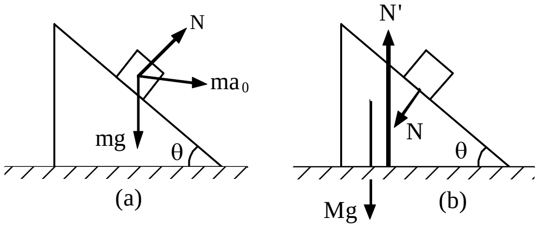
Let the acceleration of the prism be a_0 in the backward direction. Consider the motion of the smaller block 'm' from the frame of the prism.

The force on the block are (figure a)

(i) N normal force,

(ii) mg downward (gravity)

(iii) ma_0 forward (pseudo).



The block slides down the plane. Components of the forces parallel to the incline give

$$ma_0\cos\theta + mg\sin\theta = ma$$

or, $\quad a = a_0\cos\theta + g\sin\theta. \qquad (1)$

Components of the force perpendicular to the incline give

$$N + ma_0\sin\theta = mg\cos\theta \qquad (2)$$

Now consider the motion of the prism from the ground frame. The forces are (figure b) (i) Mg down ward, (ii) N normal to the incline (by the block), (iii) N' upward (by the horizontal surface). Horizontal components give,

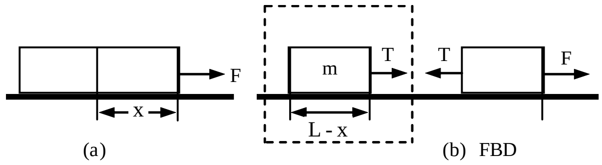
$$N\sin\theta = Ma_0 \quad \text{or,} \quad N = {Ma_0}/{\sin\theta} \qquad (3)$$

Putting in (2) $\quad \dfrac{Ma_0}{\sin\theta} + ma_0\sin\theta = mg\cos\theta \quad$ or, $\quad a_0 = \dfrac{mg\sin\theta\cos\theta}{M + m\sin^2\theta} \quad$ from (1),

$$a = \frac{mg\sin\theta\cos^2\theta}{M + m\sin^2\theta} + g\sin\theta \quad \Rightarrow \quad a = \frac{(M+m)g\sin\theta}{M + m\sin^2\theta}$$

Illusration - 35

A homogeneous rod of lenght L and mass M is placed on smooth horizontal surface. It is acted by a force F at its one end. Find the stretching force in the cross section of a rod at a distance x from the end where the force is applied.



Solution - The acceleration of rod $a = \dfrac{F}{M}$

Now cut the rod in two parts and connect them by hypothetical massless string as shown in figure.

The mass of part of the rod of length $(L-x)$, is $m = \dfrac{M}{L}(L-x)$.

By Newton's second law, for the part inside dotted box

T = ma

$$\Rightarrow \quad T = \frac{M}{L}(L-x) \times \frac{F}{M} \qquad \text{or} \qquad T = F\left(1 - \frac{x}{L}\right) \qquad \textbf{Ans.}$$

At x = 0, T = F and x = L, T = 0.

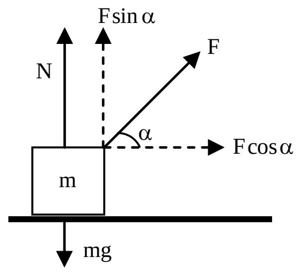
Illusration- 36

At the moment t = 0 the force F = at is applied to a small body of mass m resting on a smooth horizontal plane ('a' is constant). The permanent direction of this force from an angle α with horizontal. Find

(a) The velocity of the body at the moment of its breaking off the plane.

(b) The distance travelled by the body upto this amount.

Solution



(a) The normal reaction of the body

$N = mg - F\sin\alpha$

$= mg - (at)\sin\alpha$

At the instant of breaking off the plane, N = 0

$\therefore \quad 0 = mg - (at)\sin\alpha \quad$ or $\quad t = \left(\dfrac{mg}{a\sin\alpha}\right)$

The acceleration of the block

$$\frac{dv}{dt} = \frac{F\cos\alpha}{m} = \frac{(at)\cos\alpha}{m} \quad \text{or} \quad dv = \frac{(at)\cos\alpha}{m}dt \qquad .....(1)$$

Integrating equation (1), we have

$$\int_0^{v_0} dv = \frac{a\cos\alpha}{m}\int_0^t t\,dt \quad \text{or} \quad |v|_0^{v_0} = \frac{a\cos\alpha}{m}\left|\frac{t^2}{2}\right|_0^t \quad \text{or} \quad v_0 = \frac{a\cos\alpha}{2m}t^2 \qquad ......(2)$$

$$= \frac{a\cos\alpha}{2m}\left(\frac{mg}{a\sin\alpha}\right)^2 \qquad = \frac{mg^2\cos\alpha}{2a\sin^2\alpha}$$ **Ans.**

(b) We also have

$$v\frac{dv}{ds} = \frac{at\cos\alpha}{m} \qquad ..... (3)$$

From equation (2), we have

$$v = \frac{a\cos\alpha t^2}{2m} \qquad \therefore \qquad t = \left[\frac{2mv}{a\cos\alpha}\right]^{1/2}$$

Thus from (3) $v\dfrac{dv}{ds} = \dfrac{a}{m}\left[\dfrac{2mv}{a\cos\alpha}\right]^{1/2}\cos\alpha \quad$ or $\quad ds = \left[\dfrac{m}{2a\cos\alpha}\right]^{1/2} v^{1/2}dv$

After integrating, we get $s = \dfrac{2}{3}\left(\dfrac{m}{2a\cos\alpha}\right)^{1/2} v^{3/2}$

Here, v = 0 for s = 0. At the moment of break off the plane $v_0 = \dfrac{mg^2\cos\alpha}{2a\sin^2\alpha}$

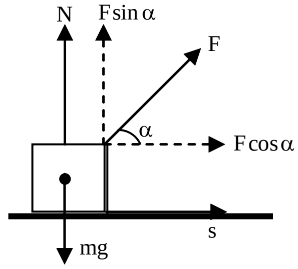
Hence, the distance travelled by the body upto break off

$$s_0 = \frac{2}{3}\left(\frac{m}{2a\cos\alpha}\right)^{1/2}\left[\frac{mg^2\cos\alpha}{2a\sin^2\alpha}\right]^{3/2} = \frac{m^2g^3\cos\alpha}{6a^2\sin^3\alpha}$$ **Ans.**

Illusration - 37

A bar of mass m resting on a smooth horizontal plane starts moving due to the force F = mg/3 of constant magnitude. In the process of its rectilinear motion the angle α between the direction of this force and the horizontal varies as α = as, where a is a constant, and s is the distance traversed by the bar from its initial position. Find the velocity of the bar as a function of the angle α.

Solution Let any distances, the α in the inclination of force, From Newton's second law, we have



$$F\cos\alpha = m\frac{dv}{dt}$$

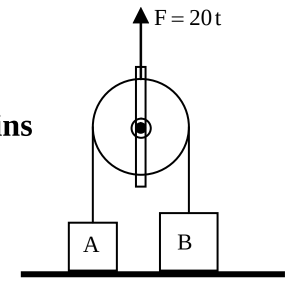
or $\frac{mg}{3}\cos(as) = m\left(v\frac{dv}{ds}\right)$ or $vdv = \frac{g}{3}\cos(as)ds$ (1)

Integrating equation (1), we get

$\int_0^v vdv = \frac{g}{3}\int_0^s \cos(as)ds$ or $\left|\frac{v^2}{2}\right|_0^v = \frac{g}{3a}|\sin(as)|_0^s$

or $\left(\frac{v^2}{2} - 0\right) = \frac{g}{3a}(\sin as - \sin 0)$ $v = \left[\frac{2g}{3a}\sin as\right]^{1/2}$ $\Rightarrow$ $= \left[\frac{2g}{3a}\sin\alpha\right]^{1/2}$

Illusration - 38



Two blocks A and B of mass 1 kg and 2 kg respectively are connected by a string, passing over a light frictionless pulley. Both the blocks are resting on a horizontal floor and the pulley is held such that string remains just taut. At moment t = 0, a force F = 20 t newton starts acting on the pulley along vertically upward direction as shown in figure. Calculate velocity of A when B loses contact with the floor. (Take g = 10 m/s^2)

Solution

Let T be the tension in the string. Then

$2T = 20\,t$ or $T = 10\,t$ newton.

Let the block A loses its contact with the floor at time $t = t_1$. This happens when the tension in string becomes equal to the weight of A. Thus, $T = mg$ or $10t_1 = 1\times 10$ or $t_1 = 1s$ ..... (1)

Similarly, for block B, we have $10t_2 = 2\times 10$ or $t_2 = 2s$ ...... (2)

i.e., the block B loses contact after 2 second. For block A, at time t such that $t \geq t_1$ let a be its acceleration in upward direction. Then

$10t - 1\times 10 = 1\times a = (dv/dt)$ or $dv = 10(t-1)dt$ ...... (3)

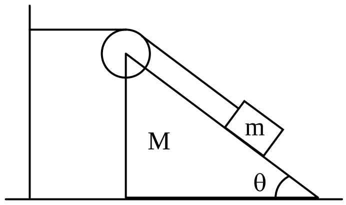
Integrating this expression, we get

$\int_0^v dv = 10\int_1^t (t-1)dt$ or $v = 5t^2 - 10t + 5$ .... (4)

Substituting $t = t_2 = 2$ s $v = 20 - 20 + 5 = 5$ m/s .... (5)

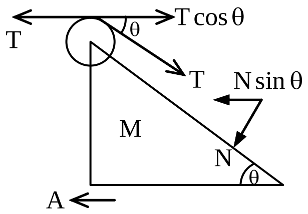
Illustration - 39

Find the acceleration of 'm'. All surfaces are smooth.



Solution

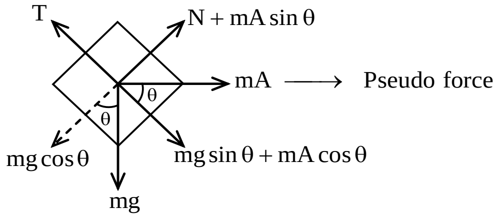
F.B.D of M w.r.t ground,



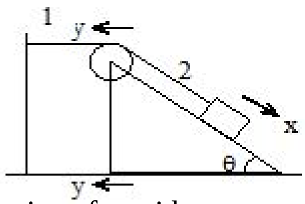
$$T - T\cos\theta + N\sin\theta = MA \qquad (1)$$

F.B.D of m w.r.t wedge,

$$mg\sin\theta + mA\cos\theta - T = ma \qquad (2)$$



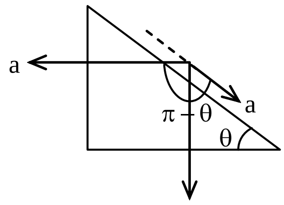
$$mg\cos\theta = N + mA\sin\theta \qquad (3)$$



Elongation in segment 1 is -y
Elongation in segment 2 is x
x - y = 0
x = y
a = A (4)

This is the acceleration of *m* with respect to *M*. , To find the acceleration of m with respect to ground.

$$\vec{a}_{m,G} = \vec{a}_{m,M} + \vec{a}_{M,G}$$



$$= \sqrt{a^2 + a^2 + 2a^2\cos(\pi - \theta)} \qquad = \sqrt{2a^2 - 2a^2\cos\theta}$$

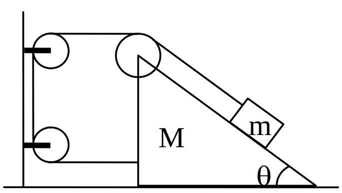
$$= \sqrt{2a^2(1-\cos\theta)} = \sqrt{2a^2(2\sin^2\theta/2)} = 2a\sin\frac{\theta}{2}$$

By eq (1), (2) and (3), we have

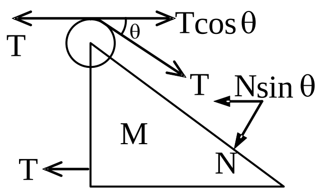
$$|\vec{a}_{m,G}| = 2\left[\frac{mg\sin\theta}{M + 2m(1-\cos\theta)}\right]\sin\frac{\theta}{2}$$

Illusration - 40

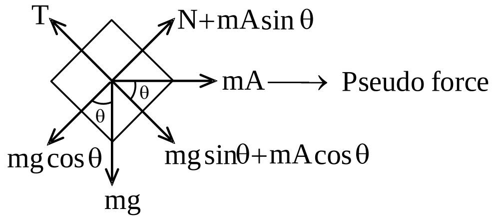
Draw the free body diagram & write all the dynamic equations & constrain relations.



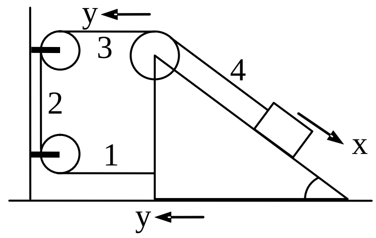
F.B.D of M



$$2T + N\sin\theta - T\cos\theta = MA \qquad (1)$$
$$N + mA\sin\theta = mg\cos\theta \qquad (2)$$
$$mg\sin\theta + mA\cos\theta - T = ma \qquad (3)$$

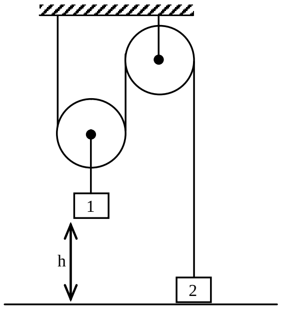


Elongation in segment (1) = - y
Elongation in segment (2) = 0
Elongation in segment (3) = - y
Elongation in segment (4) = x
x - 2y = 0 or x = 2y or a = 2A (4)



Illusration - 41

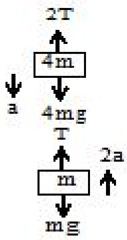
In the arrangement shown in figure. The mass of body 1 is η = 4.0 times as great as that of body 2. The height h = 20 cm. The masses of the pulleys and the threads, as well as the friction, are negligible. At a certain moment body 2 is released and the arrangement is set in motion. What is the maximum height that body 2 will go up to ?



Solution :

When body (1) reaches the ground, body (2) rises to a height of '2h' (by constraints). To find the acceleration of the blocks when the tension in the string is not yet zero.

4 mg - 2T = 4ma (1)
T - mg = m(2a) (2)



Solving for (1) & (2) $\Rightarrow$ a = g/4

Velocity acquired by body 2 in travelling a distance of 2h with an acceleration of g/4

$$v^2 = u^2 + 2\,(2a)\,(2h)$$

$$= 0 + 2\times\frac{2g}{4}\times 2h \qquad \Rightarrow \qquad v = \sqrt{2gh}$$

The body (2) rises further with a retardation g ($\because$ T = 0 when body (1) reaches the ground) till

its velocity becomes zero i..e., height to which body 2 rises $= \dfrac{u^2}{2g} = \dfrac{2gh}{2g} = h$

Total height body 2 will go up to = 2h + h = 3h

SPRING BLOCK SYSTEM

Consider an ideal spring connected with a block as shown in Figure 1. If an external force displaces the block to the right by an amount *x*. The spring will get extension by the same amount as shown in Figure 2. If the external force is removed, the block bounces back to the left and goes beyond the original position (O) of the block and comes to rest on the other side at point B.Refer to figure 2, the force which accelerates the block towards its equilibrium position is known as restoring force. It has been observed experimentally that restoring force is directly proportional to 'x'

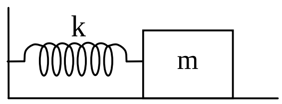


Figure1

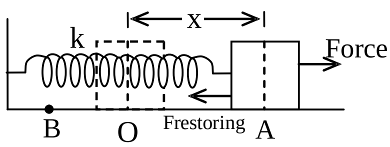


Figure2

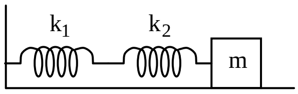
$\Rightarrow$ |restoring force| = k x

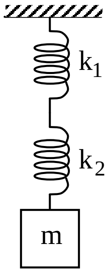
Since the directions of *x*(displacement) and the restoring force (spring force) are opposite, so we put a –ve sign in the expression for restoring force.

$\Rightarrow$ restoring force = – k x

At position A, in figure (2), after removal of external force acceleration of the block is $-\dfrac{kx}{m}$.

Series Combination

(a) 

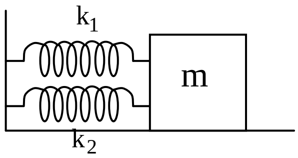
(b) 

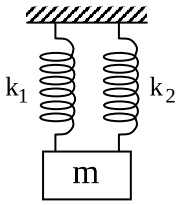
$$x_1 + x_2 = x_{eq}$$

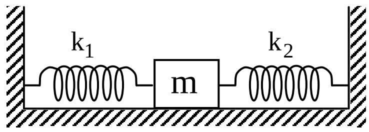
$$\frac{T}{k_1} + \frac{T}{k_2} = \frac{T}{k_{eq}}$$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Parallel Combination

(a) 

(b) 

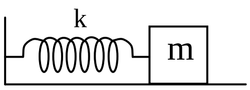
(c) 

$T = k_1x + k_2x$;

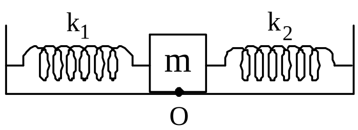
$K_{eq}\, x = (k_1 + k_2)\, x.$; or $K_{eq} = k_1 + k_2$

Illusration - 42

(a) In the given diagram when the spring is extended by x = 2 cm find the acceleration of the block when it is released.(k = 100 Nm^{-1} ; m = 2 kg)



(b) Find the net force on the block and the acc^n. when it has an extension of 1 cm to the left of point 'O' where both the springs are in their relaxed positions.
Take the value of k_1 = 100 N/m , k_2 = 200 N/m , m = 1 kg.



Solution

(a) $F_{restoring} = -kx = -100 \times 0.02 = -2$ N and acc$^n = -\dfrac{2}{2} = -1$ m/s^2

(b) As we know $k_{eq} = k_1 + k_2$ = 300 N/m

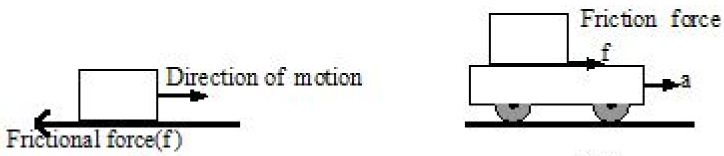
$$F_{restoring} = -k_{eq}x = -300\left(-\frac{1}{100}\right) = 3 \text{ N}$$

(+ve sign shows, net force is towards right)

Also, $\text{acc}^n = \dfrac{\text{Force}}{\text{mass}} = \dfrac{3}{1} = 3 \text{ m/s}^2$.

FRICTION

1. Consider a block placed on a horizontal floor and give it an initial push (figure a). The block will stop after travelling some distance. According to Newton's second law, a retarding force must be acting on the block. This opposing force is called the friction force. The friction force always acts along tangential direction at the point of contact and in a direction opposite to the direction of relative motion between the two surfaces.



(a) (b)

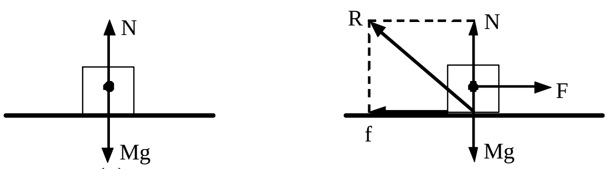
2. Now consider a block placed at rest in an accelerating cart as shown in figure (b). The block in fact accelerating along with the cart. Which force causes the acceleration of the block ? It is clear that the only force in the horizontal direction is the frictional force. If there were no friction, the surface of the cart would slip and the block would remain at its initial position by inertia. Thus we can say that sometimes friction is required to start the motion. Its direction may be backward or forward of the motion of the body. Friction can be defined as a force which opposes the relative motion between the surfaces in contact.

Friction can be defined as a force which opposes the relative motion between the surfaces in contact.

Contact Force : The force at the point of contact of two bodies is called contact force.

1. Consider a block is placed on a rough surface figure (a). The contact force on it is equal to ;

R = N = Mg



(a) (b)

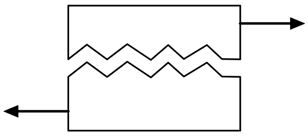
2. Now consider the same block is acted by a horizontal force F. Let f be the frictional force on the block.

Then contact force $R = \sqrt{N^2 + f^2} = \sqrt{(Mg)^2 + f^2}$

The component of the contact force *R* perpendicular to the contact surface is called normal force and its component parallel to the contact surface is called frictional force.

ORIGIN OF FRICTION

The frictional force arises due to molecular interactions between the surfaces at the points of actual contact. When two bodies are placed one over the other, the actual area of contact is much smaller than the total surface area of bodies (see figure). The molecular forces starts operating at the actual points of contacts of the surfaces. Molecular bonds are formed at these contact points. When one body is pulled over the other, these bonds are broken, and the material gets deformed and new bonds are formed. The local deformation sends vibrations into the bodies. These vibrations ultimately damp out the energy of vibrations appears as heat. Hence, to start or to carry-on the motion, there is need of a force.



IMPORTANT Points :

1) When two surfaces in contact have relative motion or have tendency of motion with each other then a force acts at the point of contact of the object and this force is called friction force.
2) The force of friction is always in a direction opposite to tendency of relative slipping.
3) It is parallel to the surface.
4) Frictional force is independent of the area of surface in contact.
5) The force of friction depends on the nature of material of the surfaces in contact.
6) Friction is a non-conservative force i.e. work done against friction is path dependent.
7) It is a misconception that friction opposes the motion of a moving body but it favour the motion of a body. It opposes the relative motion between the two bodies. When a person walks may, he pushes the ground backward. The rough surface of ground exerts a forward force which causes the motion of the person.

STATIC FRICTION

In cases where there is no relative motion between the surfaces in contact, friction is known as static friction. (abbreviated as f_s). Static friction is an adjustable force which may have any value from zero to certain maximum value, under a given case. The maximum possible value of this static friction is known as limiting static friction.

$\Rightarrow 0 \le f_s \le$ Limiting static friction.

(Liming static friction) $\propto$ (Normal reaction between the surfaces in contact)

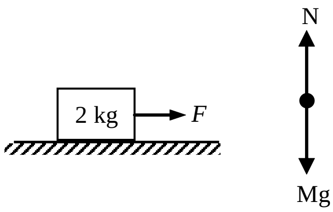
$f_{s\,Lim} = \mu_s N$

where μ_s is a constant, (characteristic of the surfaces in contact) known as coefficient of static friction.Thus

$0 \le f_s \le \mu_s N$

Let us consider a case to understand it. A block of mass 2 kg lies over a rough horizontal surface. The coefficient of static friction between the block and surface is 0.5

$N = Mg$ $\qquad 0 \le f_s, \le N = 10\text{N}$



Now a horizontal force say F is applied on block. F is gradually increased from zero

			Friction
for	F = 0		f = 0
for	F = 5 N		f = 5 N
for	F = 9.99 N		f = 9.99 N
for	F = 10 N	f = 10 N	when F > 10 N

Now when force is increased beyond 10N relative motion starts and friction becomes kinetic

KINETIC FRICTION :

Whenever there is relative motion between the surfaces in contact, friction is known as kinetic friction (f_k) which is a constant force such that

$f_k = \mu_k N$

where μ_k is known as the coefficient of kinetic friction between the surfaces.

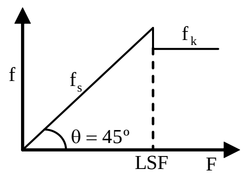
In most of the cases $\mu_k < \mu_s$, but if in a problem only μ is given for a surface, then we take

$\mu_s = \mu_k = \mu$

Let us consider the same case as discussed earlier for $\mu_k = 0.48$

for $F > 10$ N ; $f_k = \mu_k N = 9.6N$

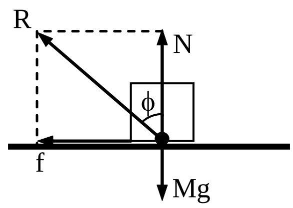
Friction suddenly decreases from 10 N to 9.6 N just as motion starts for $F > f_{LSF}$ (f_{LSF}= limiting static friction)



f_s reduces from f_s to f_k suddenly, plot of friction against applied force is as shown above

ANGLE OF FRICTION (ϕ) : The angle of friction is defined as the angle which the resultant contact force *R* makes with the normal reaction, N. From the figure, the angle of friction (in the case of limiting static friction or kinetic friction).

$\tan\phi = \dfrac{f}{N}$ we have $\mu = \dfrac{f}{N}$ $\therefore\ \mu = \tan\phi$



CONTACT FORCE (R)

$N \le R \le \sqrt{N^2 + (\mu_s N)^2}$

or $N \le R \le N\sqrt{1+\mu_s^2}$

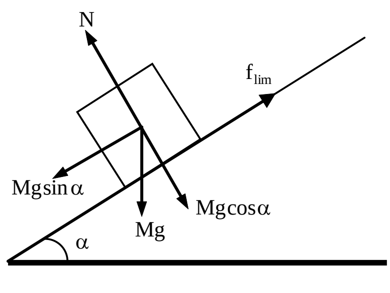
ANGLE OF REPOSE (α) : It is the angle that an inclined plane makes with the horizontal when a body placed on it is in limiting equilibrium. Let us consider a block placed on a rough inclined plane of inclination α . If block is just about to slide, then we have

$F_{lim} = Mg\sin\alpha$ .... (i)

and $N = Mg\cos\alpha$ .... (ii)

Dividing equation (i) by (ii), we get

$\dfrac{f_{lim}}{N} = \tan\alpha$



Also we have $\mu_s = \dfrac{f_{lim}}{N}$ $\therefore \mu_s = \tan\alpha$

As $\mu_s = \tan\phi$; $\therefore \alpha = \phi$

Force (Parallel to Incline) Required to Keep a Block at Rest on an Inclined Plane

In the figure (a), f_{ext} will be the *minimum* force required (parallel to incline) to keep the block at rest such that

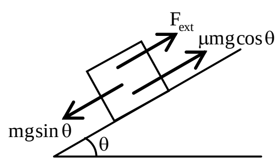


figure -(a)

$F_{min} = mg\sin\theta - \mu mg\cos\theta$

In the figure (b), F_{ext} in this case is *maximum* force required to keep the block at rest such that

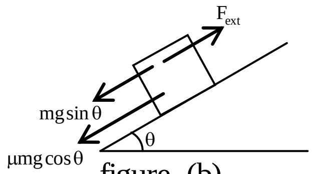


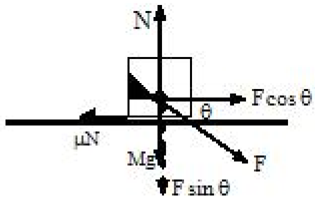
figure -(b)

$$F_{max} = mg\sin\theta + \mu mg\cos\theta$$

Illusration - 43

Prove its easier to pull a body on a rough surface than to push it.

Solution

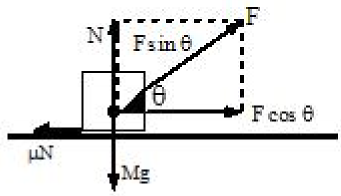


Push : Consider a block of mass *m* placed on a rough horizontal surfac
The coefficient of static friction between the block and surface is
Let a push force F is applied at an angle θ with the horizontal.

As the block is in equilibrium along y-axis. Thus we have $\sum F_y = 0$; or $N = mg + F\sin\theta$

To just move the block along x-axis, we have $F\cos\theta = \mu\ N = \mu(mg + F\sin\theta)$

or $$F = \frac{\mu mg}{\cos\theta - \mu\sin\theta} \qquad \ldots\ldots.\ (i)$$



Pull : Along y-axis we have $\sum F_y = 0$;

$\Rightarrow \quad \therefore \quad N = mg - F\sin\theta$

To just move the block along x-axis, we have

$F\cos\theta = \mu\ N = \mu(mg - F\sin\theta)$

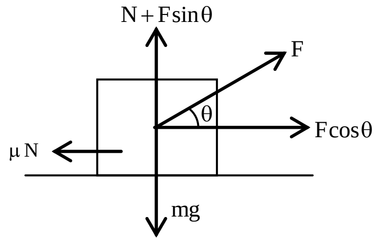
or $$F = \left(\frac{\mu mg}{\cos\theta + \mu\sin\theta}\right) \qquad \ldots\ldots.\ (ii)$$

It is clear from equation (i) and (ii) that pulling force is smaller than pushing force.

Illusration - 44

Find the angle θ for which the force applied is minimum to move the block on the horizontal surface.

Solution



$\Rightarrow \quad F\cos\theta > \mu N$

$F\cos\theta > \mu(mg - F\sin\theta)$

To just move the block,

$F(\cos\theta + \mu\sin\theta) = \mu mg$

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

For F to be minimum, the denominator should be maximum

$$y = \cos\theta + \mu\sin\theta \quad ; \quad \frac{dy}{d\theta} = -\sin\theta + \mu\cos\theta = 0 \qquad \Rightarrow \quad \mu = \tan\theta$$

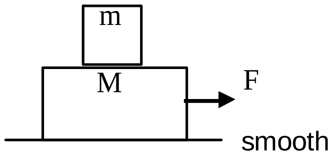
$\theta = \tan^{-1}(\mu)$ is the angle at which the body should be pulled for minimum force condition.

and the minimum force required is

$$F_{min} = \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Sliding Conditions

To decide if m slides over M.

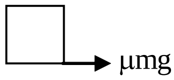


First find the acceleration of combined masses, assuming both to move together.

$$\Rightarrow A = \frac{F}{m+M}$$

Pseudo force on m is mA $= \dfrac{mF}{m+M}$

Now calculate the maximum limiting friction on 'm' which drags m in the direction of F.

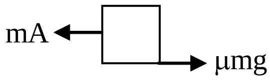


If the Pseudo force is less than mg both the bodies move together.

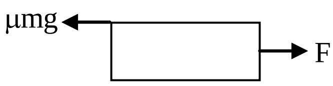
$\Rightarrow$ If Pseudo force is greater than mg, then both the bodies don't move together.

Draw F.B.D of m & M separately and find the accelerations of both the masses.

$$a_r = \frac{mA - \mu mg}{m}$$ (a_r is the acceleration with respect to M)

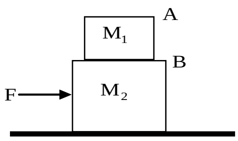


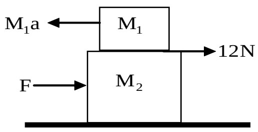
$$A = \frac{F - \mu mg}{M}$$ (A is the acceleration of M)



Illusration - 45

A block of mass 4 kg is placed on another block of mass 5 kg, and the block B rests on a smooth horizontal table. The limiting friction between A and B is 12 N. What maximum horizontal force can be applied on B so that both A and B move together ? Also find out the acceleration produced by this force.





Here $M_1 = 4$ kg and $M_2 = 5$ kg

The limiting friction between the blocks, $f_{lim} = 12$ N

Let F is the force applied on the block B.

The acceleration produced in the system

$$a = \frac{F}{M_1 + M_2} = \frac{F}{4+5}\,\text{m/s}^2 = \frac{F}{9}\,\text{m/s}^2$$

Because of this acceleration the block A experiences a pseudo force of magnitude

$$F_{pseudo} = M_1\, a = 4 \times \frac{F}{9}$$

As block A moves together with B,

$\therefore \quad F_{pseudo} = f_{lim}$

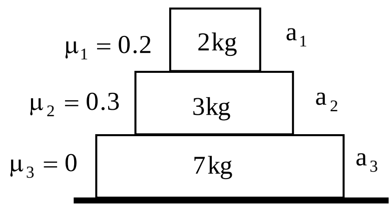
For maximum value of applied force

$$F_{pseudo} = f_{lim} \quad \text{or} \quad \frac{4F}{9} = 12 \quad \text{or} \quad F = 27\text{ N}$$

Thus the acceleration of blocks $= \dfrac{27}{4+5} = 3$ m/s^2

Illusration - 46

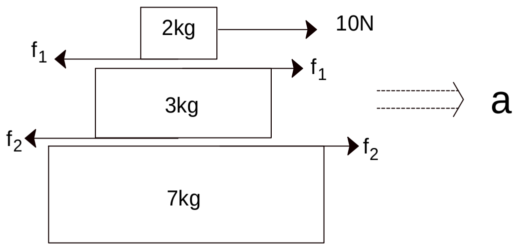
Find the acceleration a_1, a_2, a_3 of the three blocks shown in figure, if a horizontal force of 10N is applied on
(a) 2 kg block (b) 3 kg block
(c) 7 kg block (Take g = 10 m/s^2)



Solution

Limiting friction between 2 kg and 3 kg block = $f_{1\,lim}$ = 4N
Limiting friction between 3 kg and 7 kg block = $f_{2\,lim}$ = 15N

(a) When force of 10N is applied on 2kg block.
Assumption : there is no slipping between any blocks and all of them move with same acceleration



For the whole system: 10= 12a
a =5/6 m/s^2

Now for the 3+7 kg block combined : f_1=10a

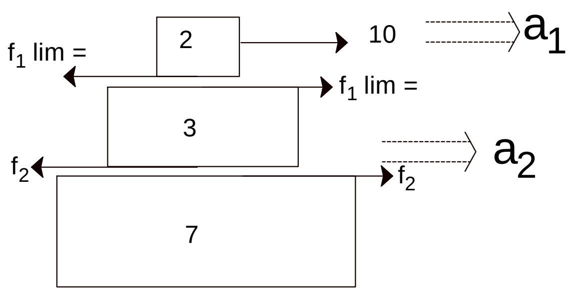
$$f_1 = 10 \times \frac{5}{6} = \frac{20}{3} N$$

Now only for the 7 kg block : f_2=7a

$$f_2 = 7 \times \frac{5}{6} = \frac{35}{6} \text{N}$$

Here we see that f_1 exceeds $f_{1\,lim}$ which is not possible, therefore our assumption must be wrong and there must be slipping between 2 kg and 3 kg block

So now assuming that there is slipping between 2 and 3 kg block but no slipping between 3 and 7 kg block
Acceleration of 2 kg block = a_1
Acceleration of 3 + 7 kg block = a_2
2 kg block : 10- $f_{1\,lim}$ = 2a_1



10-4 = 2 a_1
a_1=3 m/s^2
3+7 kg block: $f_{1\,lim}$= 10a_2

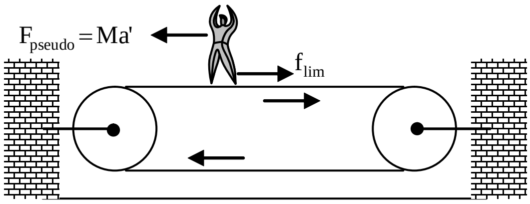
4 = 10a_2
a_2 = 2/5 m/s^2

for the 7 kg block : f_2= 7a_2
f_2=7x2/5
f_2=14/5 m/s^2

Now f_2 is coming within the limits which means this assumption is correct .The other two parts of the question can be done in a similar fashion.

Illusration - 47

Figure shown a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1m/s². What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, upto what acceleration of the belt can the man continue to be stationary relative to the belt ? Mass of the man = 65 kg. [g=9.8 m/s²]



Solution

As the man is stationary relative to belt,
the acceleration of man = acceleration of belt
$= 1\ m/s^2$
Mass of the man, $M = 65$ kg
Net force on the man $= M\,a = 65 \times 1 = 65$ N
Limiting friction force between the man's shoes and the belt

$$f_{lim} = \mu_s N = 0.2 \times Mg$$

Because of the acceleration of the belt, man experiences a pseudo force opposite to motion of belt which is $F_{pseudo} = Ma'$
Where *a'* is the acceleration of the belt.
The man remain stationery relative to belt.

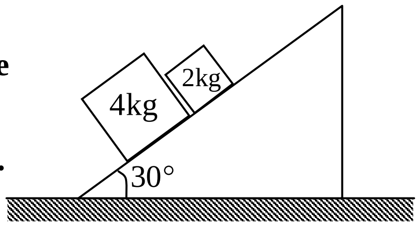
$$F_{pseudo} \le f_{lim}$$

For maximum vlaue of *a'*

$$Ma' = 0.2 \times Mg \qquad \therefore \qquad a' = 0.2 \times 9.8 = 1.96\ m/s^2$$

Illusration - 48

Figure shows two blocks in contact sliding down an inclined surface of inclination 30°. The friction coefficient between the block of mass 2.0 kg and the incline is $\mu_1 = 0.20$ and that between the block of mass 4.0 kg and the incline is $\mu_2 = 0.30$. Find the acceleration of 2.0 kg block. (g = 10 m/s)².



Solution

Since, $\mu_1 < \mu_2$, acceleration of 2 kg block down the plane will be more than the acceleration of 4 kg block if allowed to move separately. But the 2.0 kg block is behind the 4.0 kg block so both of them will move with same acceleration say 'a'. Taking both the blocks as a system.

Force down the plane on the system $= (4 + 2)g \sin 30^\circ$

$$= (6)(10)\left(\frac{1}{2}\right) = 30\ \text{N}$$

Force up the plane on the system $= \mu_1 (2)(g) \cos 30^\circ + \mu_2 (4)(g) \cos 30^\circ$

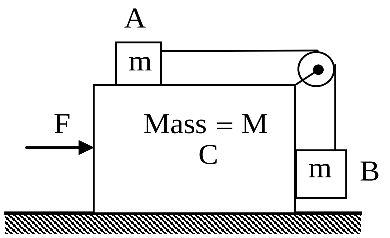
$$= (2\mu_1 + 4\mu_2)\, g \cos 30^\circ$$
$$= (2 \times 0.2 + 4 \times 0.3)(10)(0.86)$$
$$\approx 13.76\ \text{N}$$

$\therefore$ Net force down the plane is $F = 30 - 13.76 = 16.24$ N

$\therefore$ Acceleration of both the blocks down the plane will be $a = \dfrac{F}{4+2} = \dfrac{16.24}{6} = 2.7\ m/s^2$

Illusration- 49

Consider the situation shown in figure. The horizontal surface below the bigger block is smooth. The coefficient of friction between the blocks is μ. Find the minimum and the maximum force F that can be applied in order to keep the smaller blocks at rest with respect to the bigger block.



Solution

Suppose the minimum force needed to prevent slipping between the blocks is F.

Considering A + B + C as the system, the acceleration of the system is

$$a = \frac{F}{M + 2m} \qquad \text{......... (1)}$$

Now, consider the FBD of A. The forces on A shown in figure are :

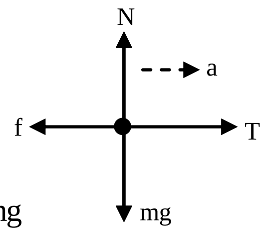
(i) tension T by the string towards right,

(ii) friction f by the block C towards left,

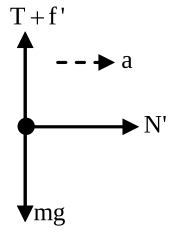
(iii) weight mg downward and

(iv) normal force N upwards.

For vertical equilibrium $N = mg$



FBD of A



FBD of B

As the minimum force needed to prevent slipping is applied, the friction is limiting. Thus,

$f = \mu N = \mu mg$

As the block moves towards right with an acceleration

$T - f = ma$

or, $T - \mu mg = ma$ .... (2)

Now, consider the FBD of B. The forces on B shown in figure are :

(i) tension T upwards, (ii) weight mg downward,

(iii) normal force N' towards right, and (iv) friction f' upwards.

As the block moves towards right with an acceleration a,

$N' = ma$

As the friction is limiting, $f' = \mu N' = ma$ .... (3)

For vertical equilibrium $T + f' = mg$

Solving these equations, we get $a_{min} = \frac{1-\mu}{1+\mu}g$.

When a large force is applied the block A tends to slip on C towards left and the block B tends to slip on C in the upward direction. The friction on A is towards right and that on B is downwards. Solving as above, the acceleration in this case is

$$a_{max} = \frac{1+\mu}{1-\mu}g$$

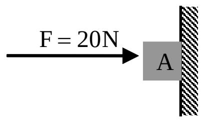
Thus, a lies between $\frac{1-\mu}{1+\mu}g$ and $\frac{1+\mu}{1-\mu}g$. From Eq. (1) the force F should lie between

$$\frac{1-\mu}{1+\mu}g\,(M + 2m) \text{ and } \frac{1+\mu}{1-\mu}g\,(M + 2m)$$

Illusration - 50

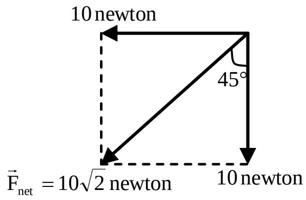
A block of mass 1 kg is pushed against a rough vertical wall with a force of 20N, coefficient of static friction being 1/4. Another horizontal force of 10N is applied on the block in a direction parallel to the wall. Will the block move ? If yes, in which direction ? If no, find the frictional force exerted by the wall on the block. (g = 10 m/s^2)

Solution



Normal reaction on the block from the wall will be :

$$N = F = 20\text{ N}$$



Therefore, limiting friction $f_L = \mu N$

$$f_L = \left(\frac{1}{4}\right) \times (20) = 5\text{N}$$

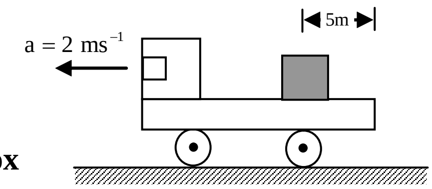
Weight of the block is $W = mg = (1)(10) = 10\text{N}$

A horizontal force of 10N is applied to the block. The resultant of these two forces will be $10\sqrt{2}$ N in the direction shown in figure. Since this resultant is greater than the limiting friction. The block will move along wall in the direction of $\vec{F}_{net}$ with acceleration.

$$a = \frac{F_{net} - f_L}{m} = \frac{10\sqrt{2} - 5}{1} = 9.14\, m/s^2$$

Illusration - 51

The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in figure. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms^{-2}, find the time when box falls off the truck. (g = 9.8 m/s^2)



Solution

Mass of box,	$m = 40$ kg
Acceleration of truck,	$a = 2\text{ ms}^{-2}$
Distance of the box from the rear end,	$s = 5$ m
Coefficient of friction,	$\mu = 0.15$

As the truck is an accelerated frame, so box experiences a backward force when it is seen from truck.

$$F = ma$$

Motion of the box is opposed by the frictional force

$$f = \mu mg$$

$\therefore$ Net force on the box in the backward direction is $F_{net} = F - f = ma - \mu mg$

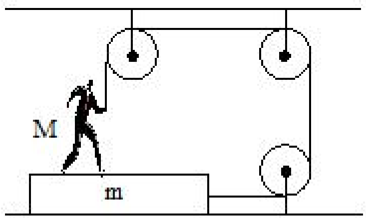
$$= 40(2 - 0.15 \times 9.8) = 21.2\text{ N}$$

Acceleration of the box relative to truck in the backward direction is

$$a_r = \frac{F_{net}}{m} = \frac{21.2}{40} = 0.53\text{ms}^{-2} \quad ; \quad t = \sqrt{\frac{2s}{a_r}} \quad (s = 5\text{ m}) \quad = \sqrt{\frac{2 \times 5}{0.53}} = 4.34\text{s}$$

Illusration- 52

The friction coefficient between the board and the floor shown in the figure is μ. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor.



(a)

Solution

Let T be the force exerted by the man on the rope.

In the vertical direction. $\sum f_v = 0$;

or $N + T = (M + m)g$

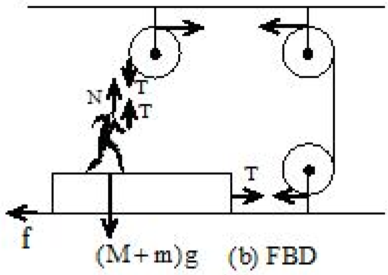
or $N = (M + m)g - T$

The board will not slip over the floor, if $T \le f_{lim}$.

For maximum value of T for which board does not slip on the floor, we have

$T = f_{lim} = \mu N = \mu[(M + m)g - T] = \mu(M + m)g - \mu T$

or $$T = \left[\frac{\mu(M+m)g}{1+\mu}\right]$$



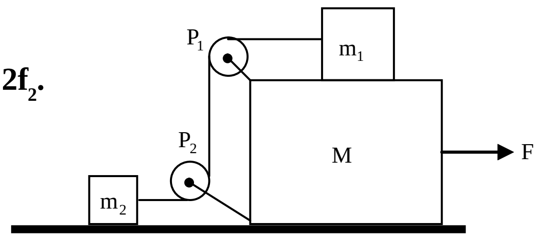
(b) FBD

Illusration - 53

In the figure m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the string are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applied to mass M. Take $g = 10\ m/s^2$

(a) Draw free-body diagram for mass M, clearly showing all the forces.

(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2f_2$. Find f_1 and f_2. Write down equation of motion of all the masses. Find F, tension in the string and the acceleration of the masses.



(a)

Solution

(a) FBD is shown in the figure (b)

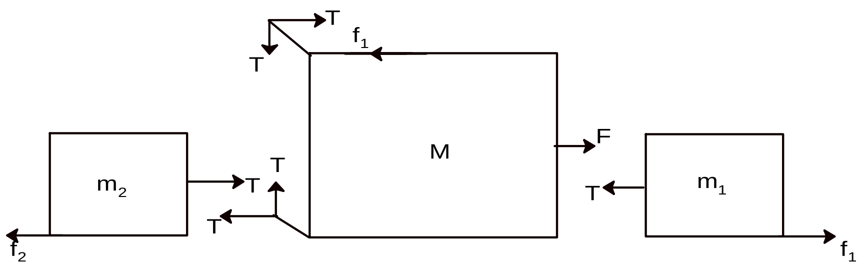


figure (b)

(b) Supposing all the blocks are in motion with relative slipping.

(i) $f_{1_{max}} = \mu_1 N_1 = \mu_1 m_1 g$

$= 0.3 \times 20 \times 10 = 60N$ and $f_{2_{max}} = \mu_2 N_2 = \mu_2 m_2 g = 0.3 \times 5 \times 10 = 15N$

Here $f_1 = 4f_2$ so this cant be the answer we require.

(ii) Let all the blocks are at rest

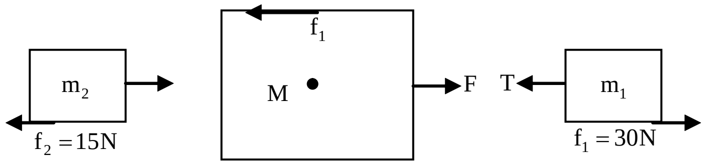
$$F - f_1 = 0 \quad \text{and} \quad T - f_1 = 0 \qquad \text{and} \qquad T - f_2 = 0$$

which gives $f_1 = f_2$, hence does not satisfy the given condition.

(iii) Since m_1 can not move over the block M, therefore all the blocks move together and $f_2 = 15N$ so $f_1 = 30N$

So the only possibility is that m_1 is stationary with respect to M and m_2 is moving on the ground .i.e. the whole system is moving towards right with the same acceleration

$$30 - T = 20a \qquad \text{(i)}$$
$$F - 30 = 50a \qquad \text{(ii)}$$
and
$$T - 15 = 5a \qquad \text{(iii)}$$



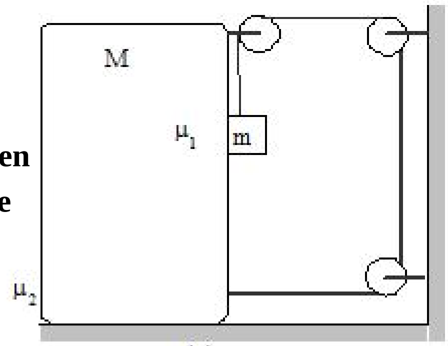
figure(c)

After solving these equations, we get

$$a = \frac{3}{5}\,m/s^2,\ T = 18N,\ F = 60N$$

Illusration - 54

Find the acceleration of the block of mass M in the situation of figure shown. The coefficient of friction between two blocks is μ_1 and that between the bigger block and the ground is μ_2.



(a)

Solution

Suppose block *M* moves towards right with an acceleration *a*. The acceleration of block *m* in downward direction will be 2a in addition to its acceleration a towards right together with block *M*. see the FBD.

For the motion of block *m*

Along horizontal direction $N_1 = ma$

Along vertical direction $mg - (\mu_1 N_1 + T) = m(2a)$ (i)

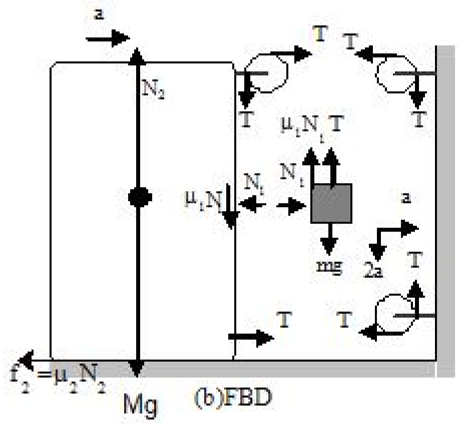
After substituting value of N_1 in equation (i), we have

$mg - (\mu_1 ma + T) = m(2a)$ (ii)

For the block *M* ;Along vertical direction $\sum F_V = 0$;

or $N_2 = T + \mu_1 N_1 + Mg$ (iii)

Along horizontal direction $2T - (N_1 + \mu_2 N_2) = Ma$ (iv)



(b)FBD

From equation (i) and (iii), we have

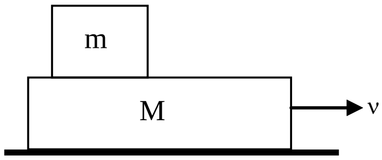
$2T - [N_1 + \mu_2 (T + \mu_1 N_1 + Mg)] = Ma$

or $2T - [ma + \mu_2 T + (ma) + Mg] = Ma$ (v)

Now solving equations (ii) and (v), we get $a = \dfrac{(2m - \mu_2(M + m))g}{M + m[5 + 2(\mu_1 - \mu_2)]}$

Illusration- 55

Figure shows a small block of mass m kept at the left end of a larger block of mass M length l. The system can slide on a horizontal road. The system is started towards right with an initial velocity v. The friction coefficient between the road and the bigger block is μ and that between the blocks is $\mu/2$. Find the time elapsed before the smaller block separates from the bigger block.



(a)

Solution :

Suppose the acceleration of lower block is a_0 towards left ($-a_0$ toward right). The block m experiences a pseudo force of magnitude ma_0 relative to block M. Let acceleration of block m is ‘a’ relative to M. The situation is figure.

For block *m* $\quad N_1 = mg \qquad$ (i)

and $\quad ma_0 - \dfrac{\mu mg}{2} = ma \qquad$ (ii)

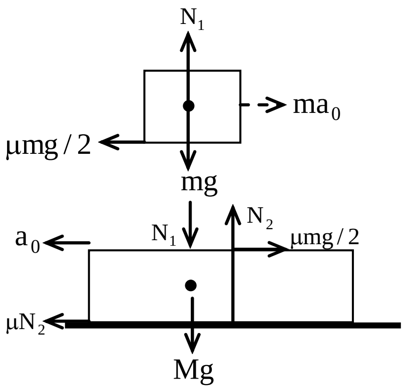
For block *M* $\quad N_2 = Mg + N_1$

$= Mg + mg = (M + m)\,g$

and $\quad \mu N_2 - \dfrac{\mu mg}{2} = Ma_0 \qquad$ (iii)

or $\quad \mu(M+m)g - \dfrac{\mu mg}{2} = Ma_0$

$\Rightarrow \quad a_0 = \dfrac{\mu(2M+m)g}{2M}$



(b) FBD

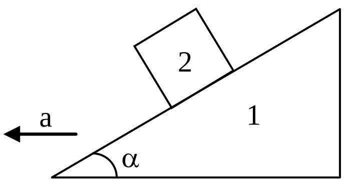
Substituting the value of a_0 in equation (ii), we get $\quad a = \dfrac{\mu(M+m)g}{2M}$

Now using second equation of motion for the motion of m w.r.t. M, we have

$$\ell = 0 + \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\frac{2\ell}{a}} = \sqrt{\frac{2\ell}{\left(\dfrac{\mu(M+m)g}{2M}\right)}} = \sqrt{\frac{4M\ell}{(M+m)\mu g}}$$

Illusration - 56

The prism 1 and bar 2 of mass m placed on it, gets a horizontal acceleration ‘a’ directed towards the left. At what maximum value of this acceleration will the bar be still stationary relative to the prism, if the coefficient of friction between them is μ ? (where $\mu < \cot\alpha$)



(a)

Solution

For maximum value of acceleration of prism, the tendency of the bar is sliding up the inclined plane, and therefore, frictional force on bar acts down the incline plane.

For the equilibrium of bar along y-axis, we have

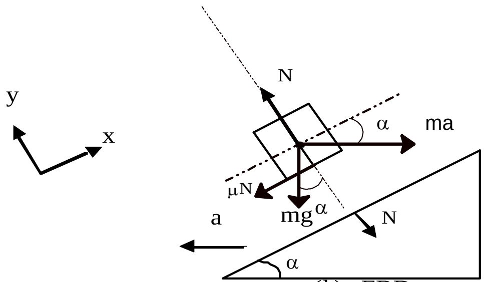
$\sum F_y = 0$ or, $N = mg\cos\alpha + ma\sin\alpha$ (i)

For the equilibrium of the bar relative to prism along x-axis, we have

$\sum F_x = 0$

or $mg\sin\alpha + \mu N = ma\cos\alpha$ (ii)

Solving equations (i) and (ii), we get $a = \left[\dfrac{g(1+\mu\cot\alpha)}{\cot\alpha - \mu}\right]$



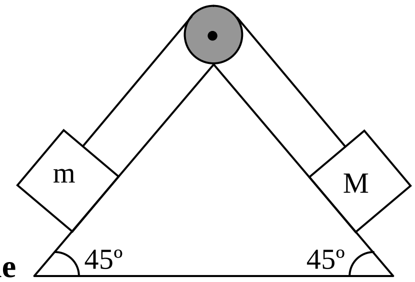
(b) FBD

Illusration - 57

Figure shows two blocks connected by a light string placed on the two inclined surfaces of a fixed triangular structure. The coefficient of static and kinetic friction are 0.28 and 0.25 respectively at each of the surfaces.

(a) Find the minimum and the maximum values of m for which the system remain at rest.

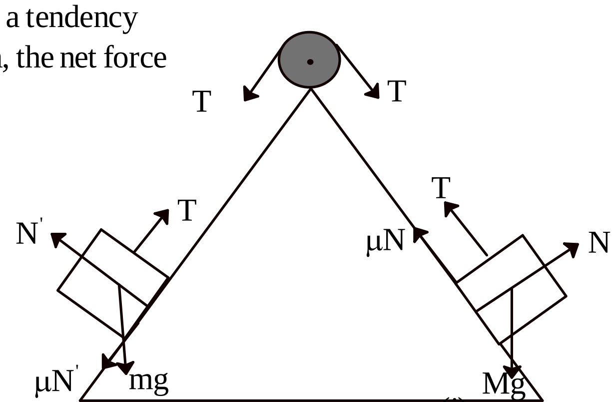
(b) Find the acceleration of the either block if m is given the minimum value calculated in the first part and is gently pushed up the incline for a short while. (Given M= 2 Kg)



Solution

Here we have, $M = 2$ kg, $\mu_s = 0.28$ and $\mu_k = 0.25$

(a) When *m* has its minimum value, 2 kg block has a tendency to move down. As the blocks are in equilibrium, the net force on them must be zero.



for M ; $N = Mg\cos 45^\circ = \dfrac{Mg}{\sqrt{2}}$

and $T + \mu_s N = Mg\sin 45^\circ = \dfrac{Mg}{\sqrt{2}}$

From above equations, we have

$T = \dfrac{Mg(1-\mu_s)}{\sqrt{2}}$ ....(i)

for m ; $N' = mg\cos 45^\circ = \dfrac{mg}{\sqrt{2}}$ and $T = \mu_s N' + mg\sin 45^\circ$ $= \mu_s N' + \dfrac{mg}{\sqrt{2}}$

From the above equations, we have $T = \dfrac{mg(1+\mu_s)}{\sqrt{2}}$ ....(ii)

Now from equations (i) and (ii), we have $\dfrac{Mg(1-\mu_s)}{\sqrt{2}} = \dfrac{mg(1+\mu_s)}{\sqrt{2}}$

Here $M = 2$ kg, $\mu_s = 0.28$; After solving, we get $m = \dfrac{9}{8}$ kg

When m has its maximum value its tendency to slip down along the plane. The direction of friction are reversed. Thus we have

$$m = \frac{(1+\mu_s)M}{(1-\mu_s)} \quad \text{or} \quad m = \frac{32}{9}\text{kg}$$

(b) If $m = \frac{9}{8}$kg and the system is gently pushed, kinetic friction starts acting. Thus,

For block M ; $Mg \sin 45^\circ - T - \mu_k N = Ma$ ....(iii)

For block m ; $T - mg \sin 45^\circ - \mu_k N' = ma$ ....(iv)

Solving equation (iii) and (iv), we get $a = 0.31$ m/s^2

Illusration - 58

A small bar starts sliding down an inclined plane forming an angle α with the horizontal. The friction coefficient depends on the distance x covered as $\mu = kx$, where k is a consent. Find the distance covered by the bar till it stops and its maximum velocity over this distance.

Solution

Applying Newton's second Law along the incline.

$mg \sin\alpha - kx\, mg \cos\alpha = ma$

or $a = g \sin\alpha - kxg \cos\alpha$ …(i)

where *a* is the acceleration of bar.

We can write $a = v\, dv/dx$,

or $\int_0^v v\, dv = \int_0^x (g \sin\alpha - kxg\cos\alpha)dx$

$$v = \sqrt{(2x\sin\alpha - kx^2\cos\alpha)g} \quad \text{…(ii)}$$

It can be seen that the velocity again becomes zero after covering a distance $x = 2\tan\alpha/k$

Therefore, the distance covered by the bar till it stops is $\frac{2\tan\alpha}{k}$ **Ans.**

Further, the maximum velocity of the bar will be at $a = 0 \Rightarrow x = \frac{\tan\alpha}{k}$ (from Eq. (i))

Substituting this in Eq. (ii), maximum velocity $v_m = \sqrt{\left(\frac{2\sin\alpha\tan\alpha}{k} - \frac{\tan^2\alpha\cos\alpha}{k}\right)g}$

or $v_m = \sqrt{\frac{g}{k}\tan\alpha\sin\alpha}$

Illusration - 59

A plank of mass m_1 with a bar of mass m_2 placed on it lies on a smooth horizontal plane. A horizontal force growing with time t as F = kt (k is a constant) is applied to the bar. Find how the acceleration of the plank and of the bar depend on t, if the coefficient of fiction between the plank and the bar is equal to μ.

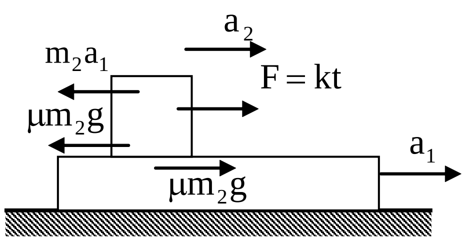
Solution

As the force is proportional to time, initially this force will be very less hence the bar and the plank will move together.. Thus initially combined acceleration of the system will be

$$A = \frac{F}{m_1 + m_2} = \frac{kt}{m_1 + m_2}$$

After some time since the force keeps on growing the static frictional bond will be broken and the two blocks will move separately with different acceleration. Let us take the acceleration of m_1 is a_1 on ground and that of m_2 on it is a_2 with respect to m_1.

Consider the forces acting on the two bodies shown in figure. When m_2 starts sliding over the plank, friction on it will oppose its motion with respect to plank, so $\mu m_2 g$ will act in backward direction and its reaction on the plank is in forward direction which will move the plank with acceleration a_1 as it is the only force acting on plank, which can be given as



$a_1 = \frac{\mu m_2 g}{m_1}$ ....(1)

As plank is accelerated, it becomes a non-inertial frame for m_2, with respect to it a pseudo force m_2a_1 is applied on the bar. As it is sliding with acceleration a_2 with respect to plank, we have its motion equation as

$$F - \mu m_2 g - m_2 a_1 = m_2 a_2$$

or $\quad a_2 = \frac{kt}{m_2} - \mu g - a_1$

This acceleration a_2 of the bar is with respect to plank, hence net acceleration of bar is $a_2 + a_1$. Thus the net acceleration of bar is

$$a_{bar} = a_2 + a_1 = \frac{kt}{m_2} - \mu g$$

or it could have been calculated directly from the frame of the ground which would have been simpler

i.e. $kt - \mu m_2 g = m_2 a_{bar}$

Now we need to calculate the instant when the bar the starts sliding on plank. It is the instant when the net force on the bar from the frame of the plank just becomes equal to limiting static friction i.e. when the external force on bar just balances the friction plus pseudo force on it. Let us take this instant be t_1, and it is given by

$$kt_1 = m_2 a_1 + \mu m_2 g$$

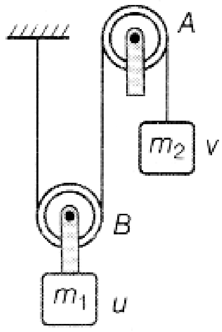
Now by equation (1) we have,

$$\Rightarrow \quad t_1 = \frac{\mu m_2 g(m_1 + m_2)}{m_1 k}$$

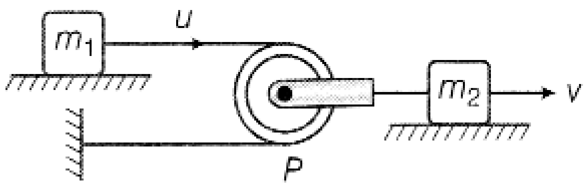
PROBLEMS ON CONSTRAINTS

1. Find the relation between u and v in following cases.

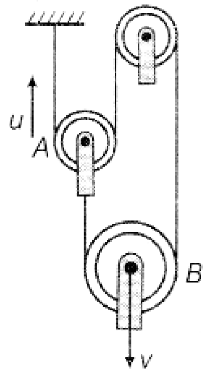
i. The pulley A is smooth and fixed. The pulley B is free to move. The velocity of block m_1 is u in downward direction and velocity of m_2 is v.



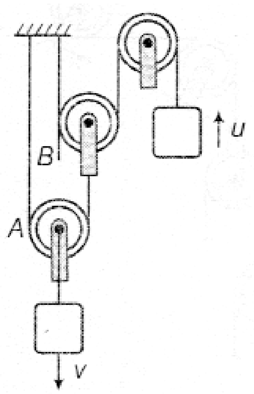
ii. Movable pulley P is smooth and light.



iii. Pulley A and B are movable. The velocity of pulley B is v and velocity of A is u.

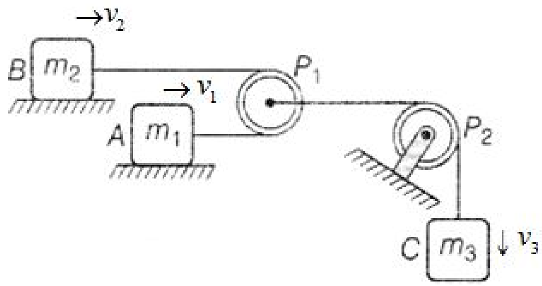


iv. Pulley A and B are light movable.

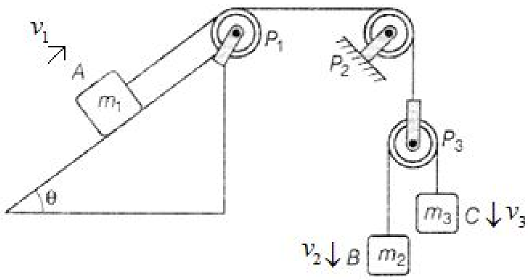


2. Find the relation among v_1, v_2 and v_3 in the following cases.

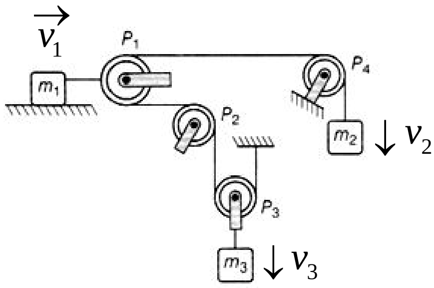
i. Blocks A and B are placed on the smooth horizontal table. A smooth light pulley P_1 is movable but smooth pulley P_2 is fixed. The velocities of blocks A, B and C are v_1, v_2 and v_3 respectively.



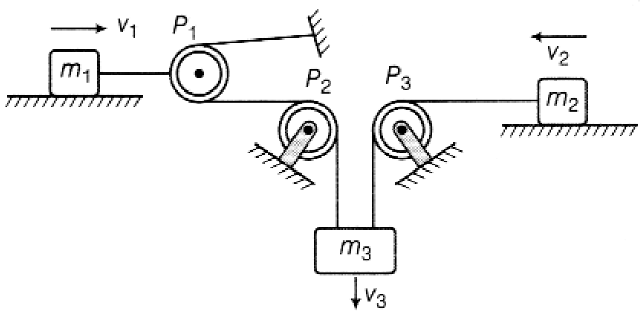
ii. All ideal pulleys are fixed excepts pulley P_3. The velocities of A, B and C are v_1, v_2 and v_3, respectively.



iii. All ideal pulleys are fixed except pulleys P_1 and P_3. The velocities of masses m_1, m_2 and m_3 are v_1, v_2 and v_3, respectively.

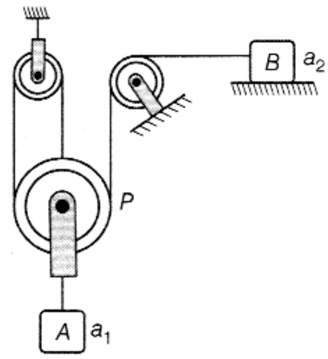


iv. For the situation shown in figure, blocks are connected through light strings. Light pulley P_1 is movable while other smooth pulleys are fixed.

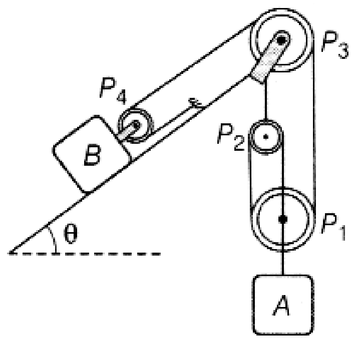


3. Find the relation between a_1 and a_2 in the following cases.

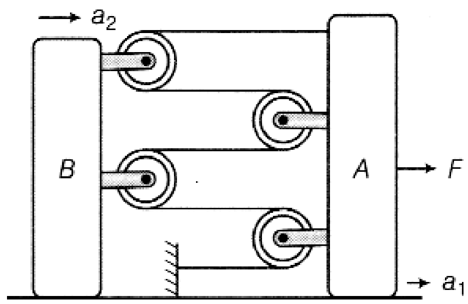
i. The smooth and light pulley P is movable. The acceleration of blocks A and B are a_1 and a_2, respectively.



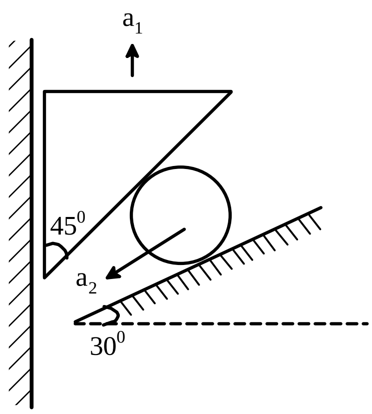
ii. All pulleys are smooth and light. The pulleys P_2 and P_3 are fixed, but P_1 and P_4 are movable.



iii. Block A and B are connected through light string passing through light smooth pulleys as shown in figure.



iv. Find relation between acceleration of triangular block and sphere.



ANSWER KEY

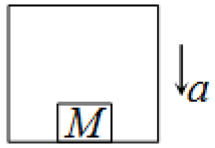
1. (***i***) $v = 2u$, (***ii***) $u = 2v$, (***iii***) $u = 2v$, (***iv***) $u = 4v$

2. (***i***) $v_1 + v_2 = 2v_3$, (***ii***) $v_2 + v_3 = 2v_1$, (***iii***) $2v_1 = v_2 + 2v_3$, (***iv***) $v_2 = v_3 = 2v_1$

3. (***i***) $a_2 = 3a_1$, (***ii***) $3a_1 = 2a_2$, (***iii***) $5a_1 = 4a_2$, (***iv***) $a_1 \sin 45^0 = a_2 \sin 15^0$

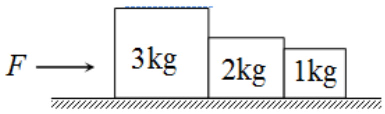
EXERCISE # I

1. With what acceleration 'a' should the box in the figure descend so that the block of mass M exerts a force $Mg/4$ on the floor of the box?



(A) $g/4$ (B) $g/2$ (C) $3g/4$ (D) $4g$

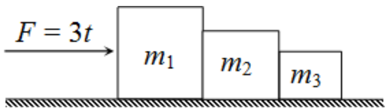
2. Consider the following statement about the blocks shown in the diagram that are being pushed by a constant force on a frictionless table.
A. All blocks move with the same acceleration.
B. The net force on each block is the same
Which of these statement are/is correct?



(A) A only (B) B only (C) both A and B (D) neither A nor B

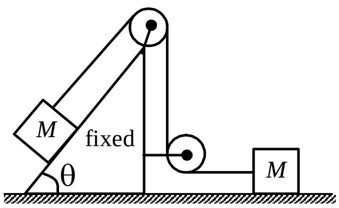
3. A body of mass 2 kg moves vertically downwards with an acceleration a = 19.6 m/s^2. The force acting on the body simultaneously with the force of gravity is (g = 9.8 m/s^2, neglect air resistance)
(A) 19.6 N (B) 19.2 N (C) 59.2 N (D) 58.8 N

4. A time dependent force $F = 3t$ (F in Newton and t in second) acts on three blocks m_1, m_2 and m_3 kept in contact on a rough ground as shown. Co-efficient of friction between blocks and ground is 0.4. If m_1, m_2 and m_3 are 3 kg, 2 kg and 1 kg respectively, the time after which the blocks start to move is (g = 10 ms^{-2})



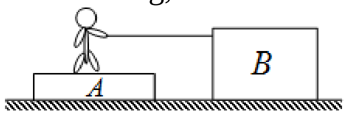
(A) 4 sec (B) 8 sec (C) $\frac{8}{3}$ sec (D) $\frac{4}{3}$ sec

5. Two blocks, each having a mass M, rest on frictionless surface as shown in the figure. If the pulleys are light and frictionless, and M on the incline is allowed to move down, then the tension in the string will be



(A) $\frac{2}{3}Mg\sin\theta$ (B) $\frac{3}{2}Mg\sin\theta$ (C) $\frac{Mg\sin\theta}{2}$ (D) $2\,Mg\sin\theta$

6. In adjacent figure, a boy, on a horizontal platform *A*, kept on a smooth horizontal surface, holds a rope attached to a box *B*. Boy pulls the rope with a constant force of 50 N. The co-efficient of friction between boy and platform is 0.5.
(Mass of boy = 80 kg, mass of platform 120kg, mass of box = 100 kg)

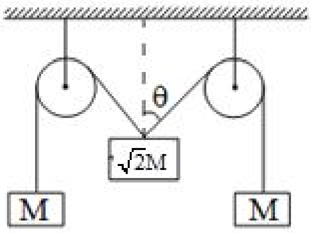


(A) velocity of platform relative to box after 4 s is 3 m/s.
(B) velocity of boy relative to platform after 4 s is 2 m/s.
(C) friction force between boy and platform is 40 N.
(D) friction force between boy and platform is 50 N.

7. A body of mass m is kept stationary on a rough inclined plane of inclination θ. The magnitude of force acting on the body by the inclined plane is

(A) mg (B) $mg \sin\theta$ (C) $mg \cos\theta$ (D) $mg\sqrt{1+\cos^2\theta}$

8. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be :

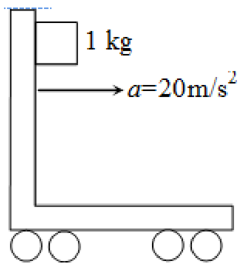


(A) 0° (B) 30° (C) 45° (D) 60°

9. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet of water at a rate of 1kgs^{-1} at a speed of 5 ms^{-1}. The initial acceleration of the block is

(A) $\frac{2}{5}$ ms^{-2} (B) $\frac{5}{2}$ ms^{-2} (C) 5 ms^{-2} (D) $\frac{1}{5}$ ms^{-2}

10. A block of mass 1 kg just remains in equilibrium with the vertical wall of a cart accelerating uniformly with 20 m/s^2 as shown. The co-efficient of friction between block and wall is (g = 10 m/s^2)

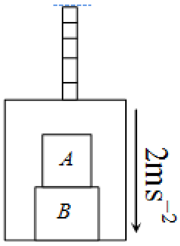


(A) 0.1 (B) 0.2 (C) 0.5 (D) 1

11. A block of mass m is attached to a massless spring of spring constant K. This system is accelerated upward with acceleration a. The elongation in spring will be

(A) $\frac{mg}{K}$ (B) $\frac{m(g-a)}{K}$ (C) $\frac{m(g+a)}{K}$ (D) $\frac{ma}{K}$

12. The elevator shown in figure is descending with an acceleration of 2 m s^{-2}. The mass of the block A = 0.5 kg. The force exerted by the block A on the block B is (g = 10 ms^{-2})



(A) 2N (B) 4N (C) 6N (D) 8N

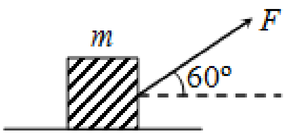
13. A man slides down a light rope whose breaking strength is η times his weight $(\eta < 1)$. The minimum acceleration of the man so that the rope doesn't break is

(A) $g(1-\eta)$ (B) $g(1+\eta)$ (C) $g\eta$ (D) $\frac{g}{\eta}$

14. A body of mass 1.5 kg is thrown vertically upwards with an initial velocity of 40 m/s reaches its highest point after 3 s. The air resistance acting on the body during the ascent is (assuming air resistance to be uniform, g = 10 m/s^2)

(A) 35 N (B) 25 N (C) 15 N (D) 5 N

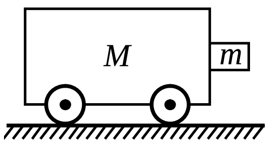
15. A mass m rests on a horizontal surface. The coefficient of friction between the mass and the surface is μ. If the mass is pulled by a force F as shown in figure, the limiting friction between the mass and the surface will be



(A) μmg (B) $\mu[mg-(\sqrt{3}/2)F]$

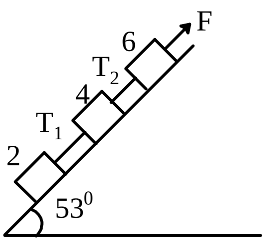
(C) $\mu[mg-(F/2)]$ (D) $\mu[mg+(F/2)]$

16. A wagon of mass M has a block of mass m attached to it as shown in the figure. The coefficient of friction between the block and wagon is μ. The minimum acceleration of the wagon so that the block m does not fall is



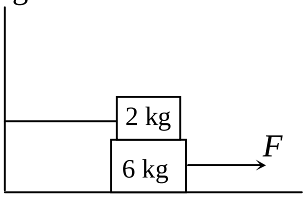
(A) $\frac{g}{\mu}$ (B) $\frac{\mu}{g}$ (C) μg (D) $\frac{M\mu g}{m}$

17. Three blocks of masses 2kg, 4kg and 6kg are connected by string and resting on a frictionless incline of 53° as shown. A force of 120N is applied upward along the incline to the 6 kg block. If the strings are ideal, the ratio T_1/T_2 will be (g = 10 ms^{-2})



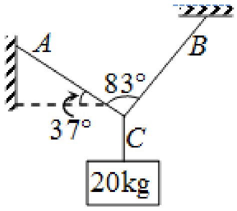
(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 4

18. A block of mass 2 kg is resting over another block of mass 6 kg. 2 kg block is connected to one end of a string fixed to a vertical wall as shown. If the coefficient of friction between the blocks is 0.4, the force required to pull out the 6 kg block with an acceleration of 1.5 m/s^2 will be (g = 10 ms^{-2})



(A) 17 N (B) 9 N (C) 8 N (D) 1 N

19. A block of mass 20 kg is balanced by three strings A, B&C as shown in figure. Ratio of tensions in string A and B (T_A/T_B) is

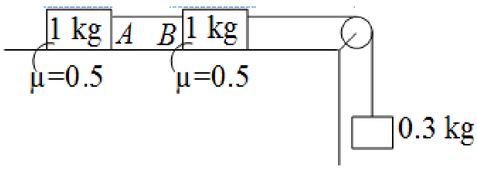


(A) $\frac{5}{8}$ (B) $\frac{5\sqrt{3}}{8}$ (C) $\frac{5}{6}$ (D) $\frac{8}{5}$

20. A block of mass 0.1 kg is held against a wall by applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is

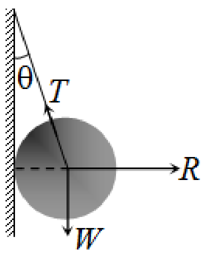
(A) 2.5 N (B) 0.98 N (C) 4.9 N (D) 0.49 N

21. Consider the situation shown in figure, find the tension in string AB consider pulley and string as frictionless and massless:



(A) 3 N (B) 2 N (C) 8 N (D) Zero

22. A metal sphere is hung by a string fixed to a wall. The force acting on the sphere is shown in figure. Which of the following statement is incorrect?



(A) $\vec{R}+\vec{T}+\vec{W}=0$ (B) $T^2=R^2+W^2$ (C) $T=R+W$ (D) $R=W\tan\theta$

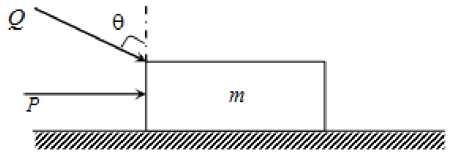
23. A string of length L and mass M is lying on a horizontal table. A force F is applied at one of its ends. Tension in the string at a distance x from the ends at which force is applied is

(A) Zero (B) F (C) $F(L-x)/L$ (D) $F(L-x)/M$

24. A box of mass 8 kg is placed on a rough inclined plane of inclination θ. Its downward motion can be prevented by applying an upward pull F and it can be made to slide upwards by applying a force $2F$. The coefficient of friction between the box and the inclined plane is

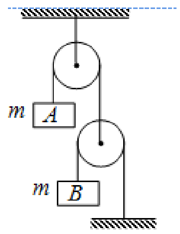
(A) $\frac{1}{3}\tan\theta$ (B) $3\tan\theta$ (C) $\frac{1}{2}\tan\theta$ (D) $2\tan\theta$

25. A block of mass m, lying on a rough horizontal plane, is acted upon by a horizontal force P and another force Q inclined at an angle θ to the vertical as shown. The block will remain in equilibrium, if the minimum coefficient of friction between it and the surface is



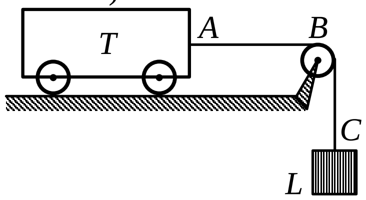
(A) $(P + Q\sin\theta) / (mg + Q\cos\theta)$ (B) $(P\cos\theta + Q) / (mg - Q\sin\theta)$
(C) $(P + Q\cos\theta) / (mg + Q\sin\theta)$ (D) $(P\sin\theta - Q) / (mg - Q\cos\theta)$

26. Two blocks A and B of equal masses m are suspended with ideal pulley and string arrangement as shown. The acceleration of mass B is



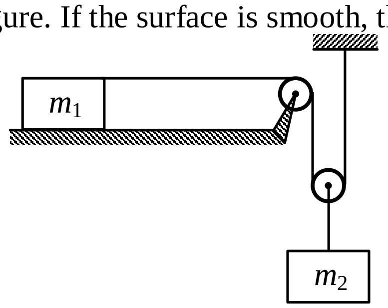
(A) $\frac{g}{3}$ (B) $\frac{5g}{3}$ (C) $\frac{2g}{3}$ (D) $\frac{2g}{5}$

27. A trolley T (mass 5 kg) on a horizontal smooth surface is pulled by a load L (2 kg) through a uniform rope ABC of length 2 m and mass 1 kg. As the load falls from $BC = 0$ to $BC = 2$m, its acceleration (inms^{-2}) changes from (Take $g = 10$ ms^{-2}).



(A) 20/6 to 20/5 (B) 20/8 to 30/8 (C) 20/5 to 30/6 (D) none of these

28. In the arrangement shown in figure. If the surface is smooth, the acceleration of the block m_2 will be

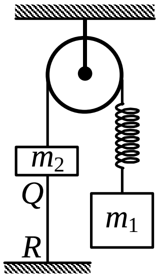


(A) $\frac{m_2 g}{4m_1 + m_2}$ (B) $\frac{2m_2 g}{4m_1 + m_2}$ (C) $\frac{2m_2 g}{m_1 + 4m_2}$ (D) $\frac{2m_1 g}{m_1 + m_2}$

29. A car starts from rest to cover a distance x. The coefficient of friction between the road and tyresis μ. The minimum time in which the car can cover distance x is proportional to

(A) μ (B) $\frac{1}{\sqrt{\mu}}$ (C) $\sqrt{\mu}$ (D) $\frac{1}{\mu}$

30. In the shown system, $m_1 > m_2$. Thread QR is holding the system. If this thread is cut, then just after cutting.

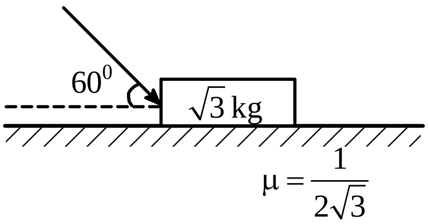


(A) acceleration of mass m_1 is zero and that of m_2 is directed upward
(B) acceleration of mass m_2 is zero and that of m_1 is directed downward
(C) acceleration of both the blocks will be zero
(D) acceleration of both the blocks will be non-zero& same in magnitude

31. A lift is moving downwards with an acceleration equal to acceleration due to gravity. A body of mass M kept on the floor of the lift is pulled horizontally. If the co-efficient of friction is μ, then the frictional resistance offered by the body is

(A) Mg (B) μMg (C) $2\mu Mg$ (D) zero

32. What is the maximum value of the force F such that the block shown in the arrangement, does not move?

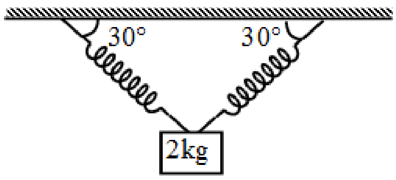


(A) 20 N (B) 10 N (C) 12 N (D) 15 N

33. A particle moves on a rough horizontal ground with initial velocity v_0. If half of its velocity is decreased due to friction in time t_0, then coefficient of friction between the particle and the ground is

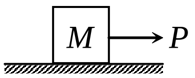
(A) $\frac{v_0}{2gt_0}$ (B) $\frac{v_0}{4gt_0}$ (C) $\frac{3v_0}{4gt_0}$ (D) $\frac{v_0}{gt_0}$

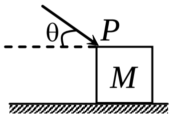
34. A block of mass 2 kg is hanging with two identical massless springs as shown in figure. The acceleration of the block just at the moment, the right spring breaks is (g = 10 m/s^2)

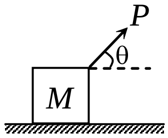


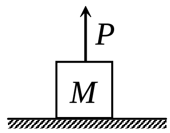
(A) 10 m/s^2 (B) 5 m/s^2 (C) 25 m/s^2 (D) 4 m/s^2

35. Figure shows the four different ways in which a constant force P may be applied to a block of mass M, kept on a rough surface. The situation in which the block can be moved with least effort is

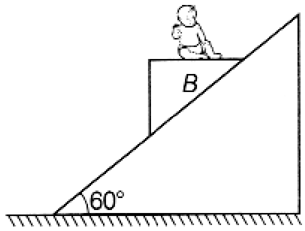
(A) 

(B) 

(C) 

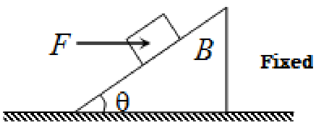
(D) 

36. A baby is sitting on the horizontal surface of triangular block B, which is sliding on a frictionless fixed inclined plane. Find minimum coefficient of friction between baby and the triangular block B, so that baby remains stationary with respect to the triangular block B.



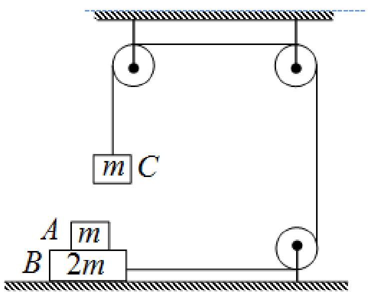
(A) Zero (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) $\sqrt{3}$

37. A horizontal force F is applied to a block of mass m kept on a smooth inclined plane of inclination θ as shown in figure. The resultant force on the block (up the plane) is



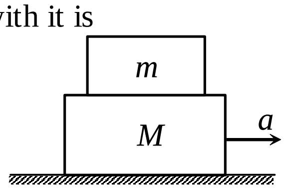
(A) $F + mg \tan\theta$ (B) $F\cos\theta - mg\sin\theta$
(C) $F\sin\theta - mg\cos\theta$ (D) $F\sin\theta + mg\cos\theta$

38. In the arrangement shown in the figure, there is a friction force between the blocks of masses m and $2m$. Block of mass $2m$ is kept on a smooth horizontal plane. The mass of the suspended block is m. If block A is stationary with respect to block of mass $2m$. The minimum value of coefficient of friction between m and $2m$ is



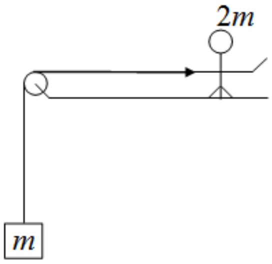
(A) 1/2 (B) $1/\sqrt{2}$ (C) 1/4 (D) 1/3

39. A block of mass m is placed on the top of another block of mass M as shown in the figure. The coefficient of friction between them is μ. The maximum acceleration with which the block M may move so that m also moves along with it is



(A) μg (B) g/μ (C) μ^2/g (D) g/μ^2

40. A man of mass $2m$ is pulling up a block of mass m with constant velocity. The acceleration of man is (neglect any friction).

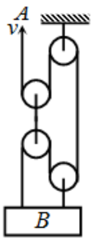


(A) g (B) $2g$ (C) $3g$ (D) $\frac{g}{2}$

41. Two blocks of masses m and M are placed on a horizontal frictionless table connected by a spring as shown in figure. Mass M is pulled to the right with a force F. If the acceleration of mass m is a, the acceleration of M will be

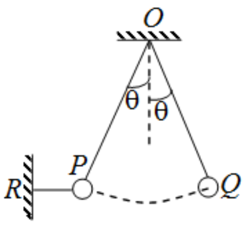
(A) $\frac{F-ma}{M}$ (B) $\frac{F+ma}{M}$ (C) $\frac{F}{M}$ (D) $\frac{am}{M}$

42. In the arrangement shown, end A of light inextensible string is pulled up with constant velocity v. The velocity of block B is



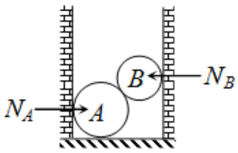
(A) $v/2$ (B) v (C) $v/3$ (D) $3v$

43. A ball of mass 1 kg is at rest in position P by means of two light strings OP and RP. The string RP is now cut and the ball swings to position Q. If $\theta = 45°$. Find the ratio of tensions in the strings in positions OP (when RP was not cut) and OQ (when RP was cut). Take $g = 10$ m/s^2.



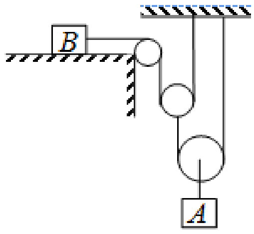
(A) 1 (B) 2 (C) 3 (D) 1.5

44. Two spheres A and B are placed between two vertical walls as shown in figure. Friction is absent everywhere. The ratio of N_A to N_B is



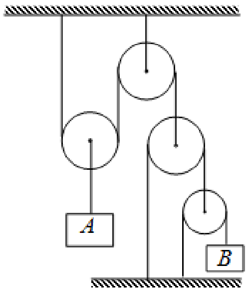
(A) 1 (B) 2 (C) 4 (D) cannot be determined

45. The block B has a mass of 10 kg. The coefficient of friction between block B and the surface is $\mu = 0.5$. Determine the acceleration of the block A of mass 16 kg. Neglect the mass of the pulleys and cords. (Take $g = 10$ m/s^2).



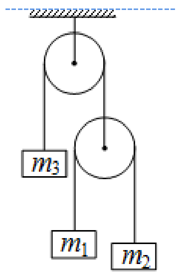
(A) zero (B) 2 m/s^2 (C) 1 m/s^2 (D) none of these

46. Block A moves upward with acceleration $\frac{1}{2}$ m/s^2. The acceleration of block B in downward direction will be



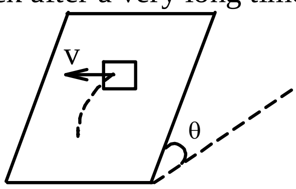
(A) 2 m/s^2 (B) 3 m/s^2 (C) 4 m/s^2 (D) 6 m/s^2

47. In the figure, pulleys are smooth and strings are massless, $m_1 = 1$ kg and $m_2 = \frac{1}{3}$ kg. To keep m_3 at rest, mass m_3 should be



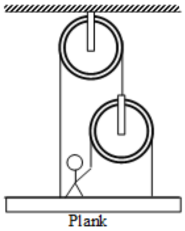
(A) 1 kg (B) $\frac{2}{3}$ kg (C) $\frac{1}{4}$ kg (D) 2 kg

48. A block is placed on a incline plane at angleθ. The coefficient of friction between the block and the plane is$\mu = \tan\theta$. The block is given a kick so that it gets initial velocity v horizontally along the plane. What is the speed of the block after a very long time?



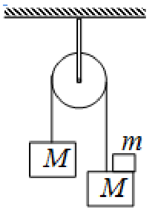
(A) v (B) $\frac{v}{2}$ (C) $2v$ (D) $\frac{v}{4}$

49. In the figure, the force with which the man should pull the rope to hold the plank in position is F. If weight of the man is 60 kg, the plank and pulleys have negligible masses, then (g = 10 m/s^2)



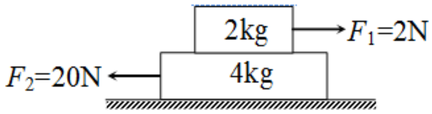
(A) F = 150 N (B) F = 300 N (C) F = 600 N (D) F = 1200 N

50. Two identical blocks of mass M are linked by a thread wrapped around a pulley-block with a fixed axis. A small block of mass m is placed on one of the blocks as shown. If pulley and strings are ideal, the force exerted by small block m on block M is,



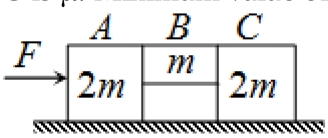
(A) $\dfrac{mMg}{2M+m}$ (B) $\dfrac{2mMg}{2M+m}$ (C) $\dfrac{2mMg}{M+m}$ (D) $\dfrac{2m^2g}{2M+m}$

51. In the arrangement shown in figure, coefficient of friction between the two blocks is μ = ½. The force of friction acting between the two blocks is [Horizontal surface of ground is smooth]



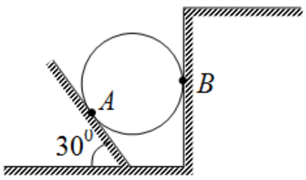
(A) 8 N (B) 10 N (C) 6 N (D) 4 N

52. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from slipping is



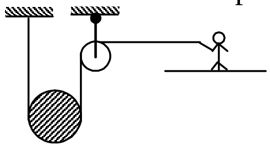
(A) $\left(\dfrac{3}{2\mu}\right)mg$ (B) $\left(\dfrac{5}{2\mu}\right)mg$ (C) $\left(\dfrac{5}{2}\right)\mu mg$ (D) $\left(\dfrac{3}{2}\right)\mu mg$

53. The 50 kg homogeneous smooth sphere rests on the 30^0 incline A and bears against the smooth vertical wall B. The contact force at B is (g = 10 m/s^2)



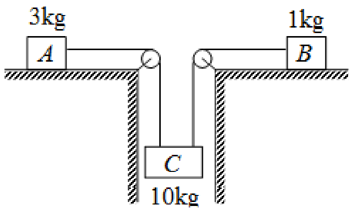
(A) 250 N (B) zero (C) $\dfrac{500}{\sqrt{3}}$ N (D) 500 N

54. In the diagram shown, the shaded pulley has mass $2M$. A man of mass M pulls the pulley upwards with constant velocity. If friction is not present between the man and the ground then what will be the acceleration of the man? (Neglect friction between pulley and string)



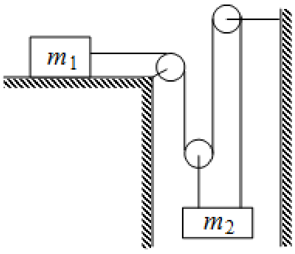
(A) $2g$ (B) $3g/2$ (C) $2g/3$ (D) g

55. All the surfaces shown in figure are smooth. If T_A and T_B are the tension in strings connected to block A and B respectively, then T_A/T_B is (Pulley and strings are ideal)



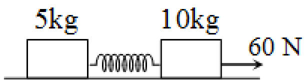
(A) 3 : 1 (B) 1 : 1 (C) 2 : 3 (D) 3 : 2

56. Two blocks m_1 and m_2 of equal masses as shown in figure. Assume ideal pulleys and strings and neglect friction at all the surfaces. The acceleration of the two blocks will be



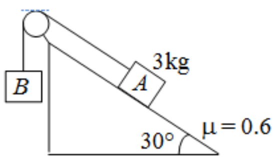
(A) $\frac{4g}{13}, \frac{g}{13}$ (B) $\frac{2g}{7}, \frac{g}{7}$ (C) $\frac{3g}{10}, \frac{g}{10}$ (D) $\frac{g}{4}, \frac{g}{4}$

57. Two masses of 5 kg and 10 kg respectively are connected by a mass less spring as shown in figure. A force of 60 N acts on the 10 kg mass. At the instant shown the 10 kg mass has acceleration 5 ms^{-2}. The acceleration of 5 kg mass is



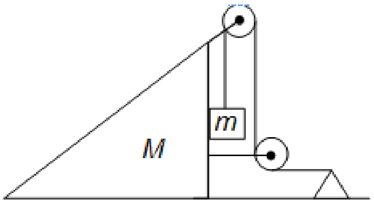
(A) 2 ms^{-2} (B) 4 ms^{-2} (C) 5 ms^{-2} (D) 12 ms^{-2}

58. Two blocks A and B of equal mass 3 kg each are connected over a massless pulley as shown in figure. The block A is placed on a rough inclined plane of angle 30°. The coefficient of friction between block A and inclined plane is 0.6. The friction force acting on the block A is



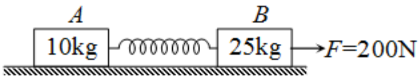
(A) zero (B) 15.6 N (C) 18 N (D) 15 N

59. In the given arrangement the mass of wedge is M = 3kg and of the body is m = 1kg. The masses of the pulley and thread are negligible. Find the acceleration of m relative to wedge (Neglect friction)



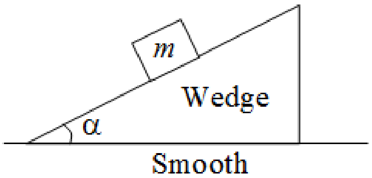
(A) 10 m/s^2 (B) 6 m/s^2 (C) 2 m/s^2 (D) 1.6 m/s^2

60. Two blocks A and B, attached to each other by a massless spring, are kept on a smooth horizontal surface and pulled by a force F = 200 N as shown. If at some instant, the 10kg mass has acceleration of 10 m/s^2, the acceleration of 25 kg mass at that instant will be



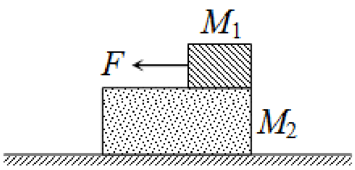
(A) 2.5 m/s^2 (B) 4.0 m/s^2 (C) 3.6 m/s^2 (D) 1.2 m/s^2

61. A block of mass m slides down a smooth wedge of inclination α placed on a horizontal smooth surface, the horizontal force required to keep the wedge stationary is



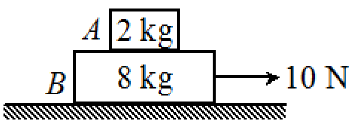
(A) $mg\cos\alpha$ (B) $mg\sin\alpha$ (C) $mg\tan\alpha$ (D) $mg\cos\alpha\sin\alpha$

62. A block of mass M_1 = 10 kg is placed on a slab of mass M_2 = 30 kg. The slab lies on a frictionless horizontal surface as shown in figure. The coefficient of static friction between the block and slab is μ_s = 0.25 and that of dynamic friction is μ_k = 0.12. A force F = 40 N acts on block M_1. The acceleration of the slab will be (g = 10 m/s^2)



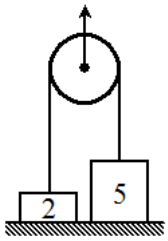
(A) 0.5 m/s^2 (B) 0.4 m/s^2 (C) 1 m/s^2 (D) $\frac{5}{6}$ ms^{-2}

63. Block A of mass 2 kg is placed over a block B of mass 8 kg. The combination is placed on a rough horizontal surface. If g = 10 ms^{-2}, coefficient of friction between B and floor = 0.5, coefficient of friction between A and B = 0.4 and a horizontal force of 10 N is applied on 8 kg block, then the force of friction between A and B is



(A) 10 N (B) 5 N (C) 4 N (D) None of these

64. Two blocks of masses 2 kg and 5 kg are at rest on ground. The masses are connected by a string passing over a frictionless pulley which is under the influence of a constant upward force F = 50 N. The accelerations of 5 kg and 2 kg masses are

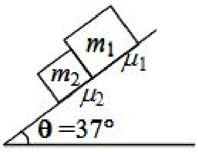


(A) 0, 2.5 ms^{-2}
(B) 0, 0
(C) 2.5 ms^{-2}, 2.5 ms^{-2}
(D) 1 ms^{-2}, 2.5 ms^{-2}

65. A body of mass 10 kg is lying on a rough plane inclined at an angle of 30° to the horizontal and the co-efficient of friction is 0.5. The minimum force required to pull the body up the plane is (g = 9.8 m/s^2)

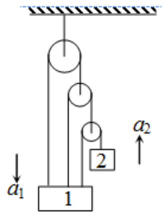
(A) 914 N (B) 91.4 N (C) 9.14 N (D) 0.914 N

66. Two blocks m_1 and m_2 are resting on a rough inclined plane of inclination 37° as shown in figure. The contact force between the blocks is, (m_1 = 4 kg, m_2 = 2kg, μ_1 = 0.8, μ_2 = 0.5, g = 10 m/s^2, sin 37° = 3/5)



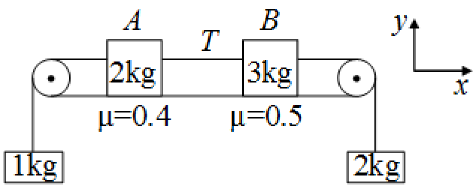
(A) 3.2 N (B) 3.6 N (C) 7.2 N (D) Zero

67. Using constraint equations relation between a_1 and a_2 will be



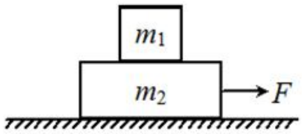
(A) $a_1 = 3a_2$ (B) $a_2 = 3a_1$ (C) $a_2 = 6a_1$ (D) $a_2 = 7a_1$

68. A block system is shown in figure. Frictional force on 2kg is f_1 and on 3kg is f_2 and T is tension in the string connecting block A and B. Which of the following is correct?



(A) $f_1 = 8\hat{i},\ f_2 = -15\hat{i}, T = 2N$
(B) $f_1 = -8\hat{i},\ f_2 = -15\hat{i}, T = 2N$
(C) $f_1 = 5\hat{i},\ f_2 = -15\hat{i}, T = 5N$
(D) $f_1 = -5\hat{i},\ f_2 = -15\hat{i}, T = 5N$

69. A bar of mass m_1 is placed on a plank of mass m_2, which rests on a smooth horizontal plane. The coefficient of friction between the surfaces of bar and plank is k. The plank is subjected to a horizontal force F depending on time t as $F = at$, where a is a constant. The moment of time t_0 at which the plank starts sliding relative to bar is

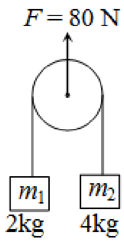


(A) $\dfrac{akg}{m_1+m_2}$ (B) $\dfrac{(m_1+m_2)kg}{a}$ (C) $\dfrac{(m_1+m_2)g}{ka}$ (D) $\dfrac{ka}{(m_1+m_2)g}$

70. A block of mass 0.5 kg. rests against a wall exerting a horizontal force of 15 N on the wall. If the coefficient of friction between the wall and the block is 0.5 then the frictional force acting on the block will be

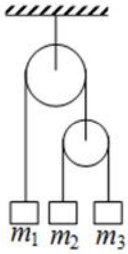
(A) 0.49 Newton (B) 4.90 Newton (C) 9.8 Newton (D) 49.9 Newton

71. The acceleration of blocks m_1 and m_2 with respect to pulley in the condition shown in figure will be (Pulley and string are ideal. g = 10 m/s^2)



(A) 10 m/s^2 (B) 5 m/s^2 (C) $\dfrac{10}{3}$ m/s^2 (D) zero

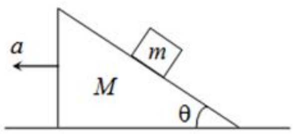
72. In the arrangement, shown in figure, pulleys are massless and frictionless and threads are inextensible. Blocks of mass m_1 will remain at rest if



(A) $\dfrac{1}{m_1} = \dfrac{1}{m_2} + \dfrac{1}{m_3}$ (B) $\dfrac{4}{m_1} = \dfrac{1}{m_2} + \dfrac{1}{m_3}$

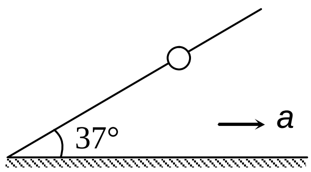
(C) $m_1 = m_2 + m_3$ (D) $\dfrac{1}{m_3} = \dfrac{2}{m_2} + \dfrac{3}{m_1}$

73. A block of mass m is resting on a wedge of mass M and inclination θ. Wedge is accelerated with $a = g$ tanθ as shown in figure.
The acceleration of block with respect to wedge is
(assume all surfaces to be smooth)



(A) zero (B) $2g$ sin θ
(C) g sin θ (1 – tan θ) (D) g sin θ (1 + tan θ)

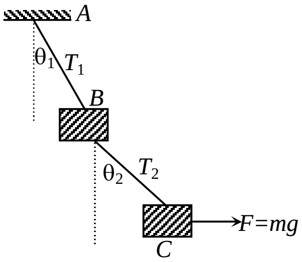
74. A bead is fitted onto a rod and can slide over it. The co-efficient of friction between bead and rod is 0.9. At the initial moment the bead is in the middle of the rod. The rod moves translationally in a horizontal plane with an acceleration $a = \dfrac{60}{67}$ m/s^2 in a direction making angle of 37° with the horizontal. The acceleration of bead relative to rod is



(A) $\dfrac{10}{3}$ m/s^2 (B) $\dfrac{20}{3}$ m/s^2 (C) 0 m/s^2 (D) $\dfrac{5}{3}$ m/s^2

EXERCISE # II

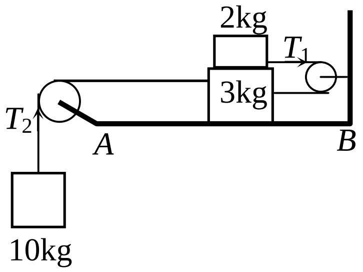
1. The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant force mg acting on C.



(A) $\tan\theta_1 = 1/2$ (B) $\tan\theta_2 = 1/2$ (C) $T_1 = \sqrt{5}mg$ (D) $T_2 = \sqrt{2}mg$

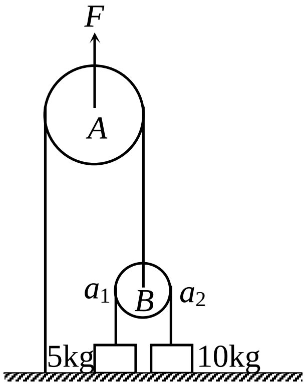
2. Four forces act on a point object. The object will be in equilibrium if
(A) all of them are in the same plane
(B) they are opposite to each other in pair
(C) the sum of x, y and z components of force is zero separately
(D) they from a closed polygon of 4 sides

3. Coefficient of friction between the two blocks is 0.3 where as the surface AB is smooth. (g =10 ms^{-2})



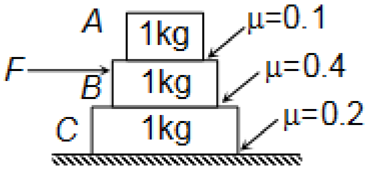
(A) Acceleration of the system of masses is 5.86 ms^{-2}
(B) Tension T_1 in the string is 17.7 N
(C) Tension T_2 in the string is about 41.4 N
(D) Acceleration of 10 kg mass is 7.55 ms^{-2}

4. In the given figure pulleys are massless and frictionless and strings are light and inextensible. A force is applied on pulley A vertically upward. At any time acceleration of 5 kg is a_1 (upward) and 10 kg is a_2 (upward) then (g = 10 m/s^2)



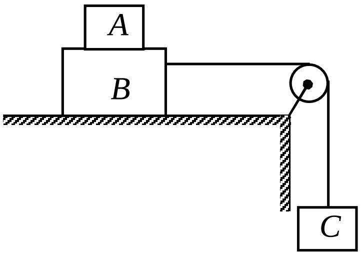
(A) $a_1 = 0$, $a_2 = 0$ if F = 100 N
(B) $a_1 = 5$ m/s^2 and $a_2 = 0$ if $F = 300$ N
(C) $a_1 = 15$ m/s^2 $a_2 = 2.5$ m/s^2 if $F = 500$ N
(D) acceleration of the masses is independent of F

5. In the given figure, the force is applied on block B. The co-efficient of friction for different contact surfaces is as shown in figure. Choose the correct option (s)



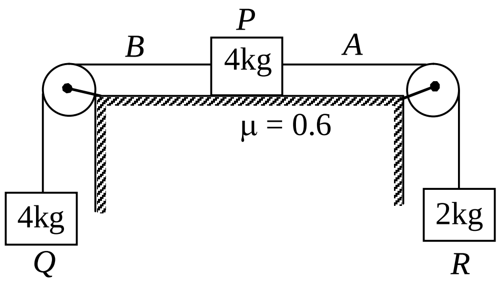
(A) if $F = 6$ N, there is no motion at any part of the system.
(B) if $F > 12$ N, there is motion between A and B.
(C) if $F > 11$ N, there is motion between B and C.
(D) if $F < 6$ N, there is motion between A and B.

6. Two block A and B each of mass m is placed as shown, coefficient of friction between A and B is 0.5 and surface is smooth. Block B is connected to a block C of mass M with the help of massless string. Then



(A) If $M = 2m$, acceleration of block A and B is $g/2$.
(B) If $M = 2m$, friction force between A and B is $\frac{1}{2}mg$.
(C) Relative motion start between blocks A and B if $M>2m$
(D) For any value of M acceleration of block A and B are equal.

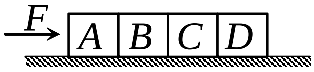
7. A block P of mass 4 kg is placed on horizontal rough surface with coefficient of friction $\mu = 0.6$. And two blocks R and Q of masses 2 kg and 4 kg connected with the help of massless strings A and B respectively passing over frictionless pulleys as shown, then ($g = 10\text{m/s}^2$)



(A) acceleration of block P is zero.
(B) tension in string A is 20 N.
(C) tension in string B is 40 N.
(D) contact force on block P by table is $20\sqrt{5}$ N.

8. Which of the following statements is/are incorrect?
(A) when a person walks on a rough surface the frictional force exerted by the surface on the person is opposite to the direction of his motion.
(B) we can predict the direction of motion of a body from the direction of the force acting on it.
(C) a body is said to be in translational equilibrium if no net force acts on the body
(D) in case of non-uniform circular motion angle between velocity and acceleration may be constant.

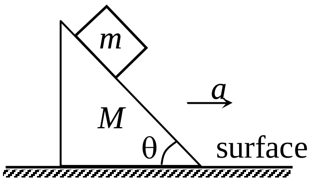
9. Four identical blocks each of mass m are kept on a horizontal frictionless plane in contact with adjacent blocks as shown in figure. A force F is applied on the system



(A) acceleration of each block is $\frac{F}{4m}$ (B) net force on the block C is $\frac{F}{4}$

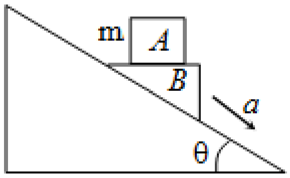
(C) net force on the block A is $\frac{3F}{4}$ (D) force by the block C on the D is $\frac{F}{8}$

10. A block of mass m is placed on inclined smooth surface of wedge of mass M as shown. Wedge is accelerating horizontally with an acceleration a such that block is relatively at rest on the inclined surface.



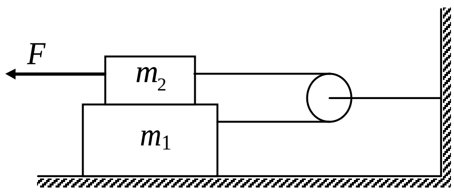
(A) the value of a is g cot θ.
(B) the value of a is g tan θ.
(C) normal force on the wedge due to surface is $(m + M)g$.
(D) normal force on the surface due to wedge is Mg.

11. In the given figure a block A *of mass* m rests on a smooth triangular block B and the block B is given an acceleration of $a = 2\ \text{m/s}^2$ along the plane



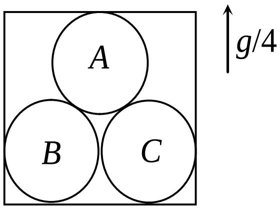
(A) normal force on block A due to block B is $m(g - a)$
(B) acceleration of block A relative to block B is a cos θ.
(C) If friction is present between block and A and B, the coefficient of friction should be greater than $\frac{a}{g}\cos\theta$, for no relative motion between A and B
(D) If friction is present between block and A and B, the coefficient of friction should be greater than $\frac{a\cos\theta}{g - a\sin\theta}$, for no relative motion between A and B.

12. Two blocks each of mass 1 kg are placed as shown. They are connected by a string which passes over a smooth (massless) pulley. There is no friction between m_1 and the ground and the coefficient of friction between m_1 and m_2 is 0.2. A force F is applied to m_2. Which of the following statements is/are correct?



(A) The system will be in equilibrium if $F < 4\ \text{N}$.
(B) If $F > 4\ \text{N}$ the tension in the string will be $4\ \text{N}$.
(C) If $F > 4\ \text{N}$ the frictional force between the block will be $2\ \text{N}$.
(D) If $F = 6\ \text{N}$ the tension in the string will be $3\ \text{N}$.

13. Three identical spheres, each of mass m, are kept in contact inside a box as shown in figure. If box is moving vertically upward with an acceleration $g/4$, then (neglect friction) (Sphere A is not touching the box)



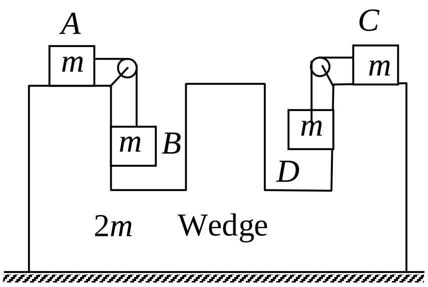
(A) Normal force applied by the spheres on the bottom of the box is $\frac{9}{4}mg$.

(B) Normal force applied by the spheres on the bottom of the box is $\frac{15}{4}mg$.

(C) Normal force between spheres A and B is $2\sqrt{3}\,mg$.

(D) Normal force between spheres A and B is $\frac{5mg}{4\sqrt{3}}$.

14 In the given figure, all surfaces are frictionless and strings pulleys are massless, then

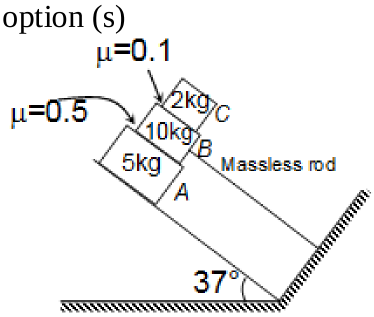


(A) acceleration of block B is $g/2$.
(B) acceleration of block B is zero.
(C) acceleration of block wedge is zero.
(D) acceleration of block wedge is $g/2$.

15. Two blocks A and B are placed rough horizontal surface and are connected by a string. If two unequal force F_1 & F_2 are applied ($F_1 > F_2$) on block A and B in opposite direction. Choose the correct alternatives.

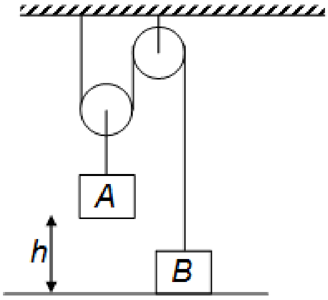
(A) friction on both the blocks act leftward
(B) friction on B always act leftward
(C) friction on A may have any direction i.e. left or right
(D) tension on the string may be zero

16. In the given figure, the inclined plane is frictionless and co-efficient of friction between the blocks is shown. Then choose the correct option (s)



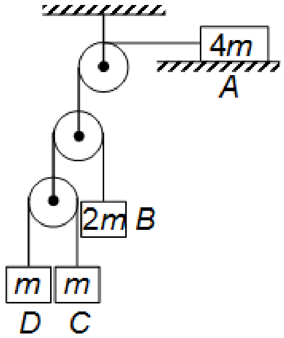
(A) friction force on block A is 30 N
(B) friction force on block C is 1.6 N
(C) tension in the rod is 91.6 N
(D) block A will accelerate down the inclined.

17. In the given arrangement the mass of particle A is $\eta = 4$ times as great as that of body B. The height $h = 20$ cm. The masses of the pulleys and the threads, as well as the friction, are negligible. At a certain moment body A is released and the arrangement set in motion. What is the maximum height that body B will go upto?



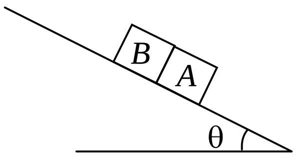
(A) 60 cm (B) 50 cm (C) 40 cm (D) 30 cm

18. In the given arrangement, strings and pulleys are massless. Find the speed of block A at $t = 2s$. Assuming at t = 0 system is released from rest (neglect friction every where)



(A) 8 m/s (B) 9 m/s (C) 10 m/s (D) none of these

19. Two blocks having same mass are placed on rough incline plane and the coefficient of friction between A and incline is $\mu_1 = 1.0$ and between block B and incline is $\mu_2 = \frac{1}{\sqrt{3}}$. As the inclination of the plane 'θ' with respect to horizontal increases. Choose the correct answers(s).



(A) there is no contact force between block A and B if $\theta \le 30^\circ$

(B) there is no contact force between blocks A and B if $\theta \le 45^\circ$

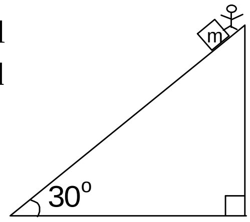
(C) acceleration of blocks is independent of θ.

(D) they starts moving at an angle $\theta = \tan^{-1}\left(\frac{\mu_1 + \mu_2}{2}\right)$

EXERCISE # III

COMPREHENSION TYPE

COMPREHENSION – I



When we apply a horizontal force on a body kept on horizontal floor, a horizontal force starts opposing the net applied force. This force is called frictional force and is always tangential to the contact surface. Frictional force on a moving body is called kinetic friction which is directly proportional to the normal force [$f_k = \mu_k N$, where μ_k is a constant depending upon the surfaces and is called the coefficient of kinetic friction]. Frictional force on a static body is called static friction. Maximum value of static friction force is $f_s = \mu_s N$, where μ_s is coefficient of static friction. Now a man wants to slide down a block of mass m which is kept on a fixed inclined plane of inclination 30° as shown in the figure. Initially, the block is not sliding. To just start sliding, the man pushes the block down the incline with a force F. Now, the block starts accelerating. To move it downwards with constant speed, the man starts pulling the block with same force. Surfaces are such that ratio of maximum static friction to kinetic friction is 2.Now, answer the following questions.

1. What is the value of F?

(A) $\frac{mg}{4}$ (B) $\frac{mg}{6}$ (C) $\frac{mg\sqrt{3}}{4}$ (D) $\frac{mg}{2\sqrt{3}}$

2. What is the value of μ_s

(A) $\frac{4}{3\sqrt{3}}$ (B) $\frac{2}{3\sqrt{3}}$ (C) $\frac{3}{3\sqrt{3}}$ (D) $\frac{1}{2\sqrt{3}}$

3. If the man continues pushing the block by force F, its acceleration would be

(A) g/6 (B) g/4 (C) g/2 (D) g/3

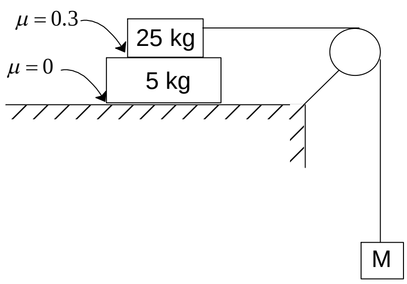
4. If the man wants to move the block up the incline, what minimum force is required to just start the motion?

(A) $\frac{2}{3}$mg (B) $\frac{mg}{2}$ (C) $\frac{7mg}{6}$ (D) $\frac{5mg}{6}$

5. What minimum force is required to move it up the inclined with constant speed?

(A) $\frac{2}{3}$mg (B) $\frac{mg}{2}$ (C) $\frac{7mg}{6}$ (D) $\frac{5mg}{6}$

COMPREHENSION – II



Two blocks of masses 25 kg and 5 kg are placed on a horizontal table as shown in the figure. A massless string passes over a frictionless and massless pulley whose one end is connected to 25 kg block and other end is connected to block M.

The coefficient of friction between two blocks is $\mu = 0.3$ and between the 5 kg block and ground is zero.

The system is released from rest.

(Take g = 10 m/s^2)

6. If M = 45 kg, the force of friction between two blocks will be

(A) 80 N (B) 60 N (C) 30 N (D) 75 N

7. Find the value of M for which there will be slipping between the blocks.
(A) M > 90 kg (B) M > 0 (C) M < 90 kg (D) None of these

8. If M = 45 kg, the acceleration of the system will be
(A) 6 m/s^2 (B) 5 m/s^2 (C) 8m/s^2 (D) 10 m/s^2

COMPREHENSION – III

If a body moves through a liquid or a gas then the fluid applies a force on the body which is called drag force. Direction of the drag force is always opposite to the motion of the body relative to the fluid. At low speeds of the body. drag force (F_D) is directly proportional to the speed.

$F_D = kv$

When K is a proportionally constant and it depends upon the dimension of the body moving in air at relatively high speeds, the drag force applied by air on the body is proportional to v^2 instead of v.

$F_D = k_1 v^2$

Where this proportionality constant k_1 can be given by

$k_1 = \rho CA$

Where ρ is the density of air

C is another constant giving the drag property of air A is area of cross – section of the body

Consider a case that an object of mass m is released from a height h and it falls under gravity. As it's speed increases the drag force starts increasing on the object. Due to this at some instant, the object attains equilibrium. The speed attained by the body at this instant is called " terminal speed" of the body. Assume that the drag force applied by air on the body follows the relation F_D = kv, neglect the force of buoyancy applied by air on the body then answer the following questions.

9. What is the pattern of acceleration change of the body?
(A) It first increases, then decreases to zero (B) It decreases uniformly from starting to zero
(C) It decreases non – uniformly from starting to zero (D) It first increases then becomes constant

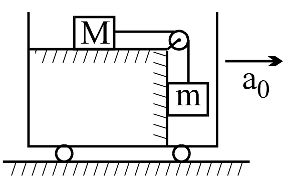
10. What is the terminal speed of the object?

(A) $\frac{mg}{k}$ (B) mgk (C) $\frac{k}{mg}$ (D) $\sqrt{\frac{mg}{k}}$

11. What is the nature of change in speed of the body?
(A) It first decreases and then increases to a constant value
(B) It increases uniformly to a constant value
(C) It increases non – uniformly to a constant value
(D) It continuously decreases to zero.

COMPREHENSION – IV

Imagine the situation in which the given arrangement is placed inside a trolley that can move only in the horizontal direction, as shown in figure. Friction is present only between M and trolley. If the trolley is accelerated horizontally along the positive x-axis with a_0, then



12. Choose the correct statement(s).
(A) There exists a value of a_0 = b at which friction force on block M becomes zero
(B) There exists two values of a_0 = (b + a) and (b – a) at which the magnitudes of friction acting on block M are equal
(C) The maximum value of static friction force acts on the block M at two accelerations a_1 and a_2 such that $a_1 + a_2 = 2b$
(D) The maximum value of friction is independent of the acceleration a_0.

13. If a_{min} and a_{max} are the minimum and maximum values of a_0 for which the blocks remain stationary with respect to the surface, then identify the correct statements

(A) If $a_0 < a_{min}$, the block m accelerates downward

(B) If $a_0 > a_{max}$, the block m accelerates upward

(C) The block m does not accelerate up or down when $a_{\min} \le a_0 \le a_{\max}$

(D) The friction force on the block M becomes zero when $a_0 = \dfrac{a_{\min} + a_{\max}}{2}$

14. Identify the correct statement(s) related to the tension T in the string

(A) No value of a_0 exists at which T is equal to zero

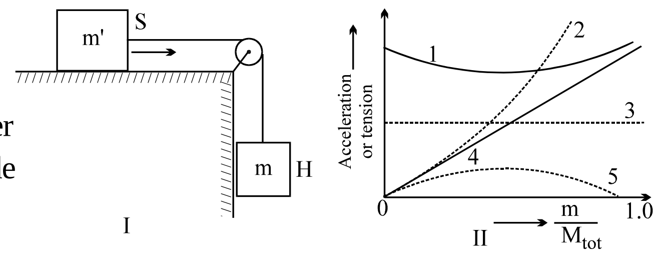
(B) There exists a value of a_0 at which T = mg

(C) If T < mg, then it must be more than μMg

(D) If T > mg, then it must be less than μMg

COMPREHENSION – V

Two containers of sand S and H are arranged like the blocks figure I. The containers alone have negligible mass; the sand in them has a total mass M_{tot}; the sand in the hanging container H has mass m. You are to measure the magnitude a of the acceleration of the system in a series of experiments where m varies from experiment to experiment but M_{tot} does not; that is, you will shift sand between the containers before each trial.



$\dfrac{m}{M_{tot}}$ is taken on the horizontal axis for all the plots.

15. The plot in figure II which gives the acceleration magnitude of the containers (taken on y-axis) as a function of $\dfrac{m}{M_{tot}}$ ratio is:

(A) 1 (B) 3 (C) 4 (D) 5

16. The curve which gives tension in the connecting string (taken on y-axis) as a function of $\dfrac{m}{M_{tot}}$ ratio is:

(A) 1 (B) 2 (C) 4 (D) 5

17. The curve which gives the magnitude of net force, on container H (taken on y-axis) as a function of $\dfrac{m}{M_{tot}}$ ratio is:

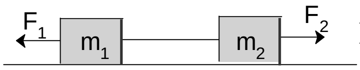
(A) 1 (B) 2 (C) 4 (D) 5

MATCH LIST

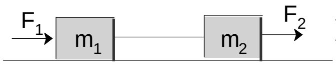
Matrix Match – 1

Column – I gives four different situations involving two blocks of mass m_1 and m_2 placed in different ways on a smooth horizontal surface as shown. In each of the situations horizontal forces F_1 and F_2 are applied on blocks of mass m_1 and m_2 respectively and also $m_2F_1 < m_1F_2$. Match the statements in **column I** with corresponding results in **column – II.**

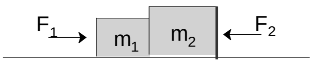
Column I

(A) 

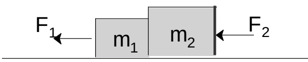
Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is

(B) 

Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is

C) 

The magnitude of normal reaction between the blocks is

(D) 

The magnitude of normal reaction between the blocks is

Column II

(p) $\dfrac{m_1m_2}{m_1+m_2}\left(\dfrac{F_1}{m_1}-\dfrac{F_2}{m_2}\right)$

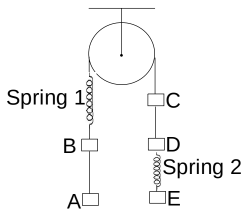
(q) $\dfrac{m_1m_2}{m_1+m_2}\left(\dfrac{F_1}{m_1}+\dfrac{F_2}{m_2}\right)$

(r) $\dfrac{m_1m_2}{m_1+m_2}\left(\dfrac{F_2}{m_2}-\dfrac{F_1}{m_1}\right)$

(s) $m_1m_2\left(\dfrac{F_1+F_2}{m_1+m_2}\right)$

Matrix Match – 2

The system shown below is initially in equilibrium. Masses of the blocks A, B, C, D and E are respectively 3m, 3m, 2m, 2m and 2m. Match the condition in column – I with the immediate effects in column – II



Column – I

(A) After spring 2 is cut, tension in string AB
(B) After spring 2 is cut, tension in string CD
(C) After string between C and pulley is cut, tension in string AB
(D) After string between C and pulley is cut, tension in string CD

Column - II

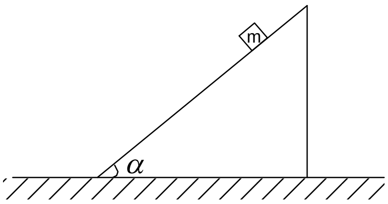
(p) increases
(q) decreases
(r) does not change
(s) zero

Matrix Match – 3

Column - I		Column - II
(A)	Force of friction	(p) Opposes motion
(B)	Normal reaction on a block of mass m kept on horizontal ground	(q) Opposes relative motion
		(r) is always mg
		(s) may be equal to mg

Matrix Match – 4

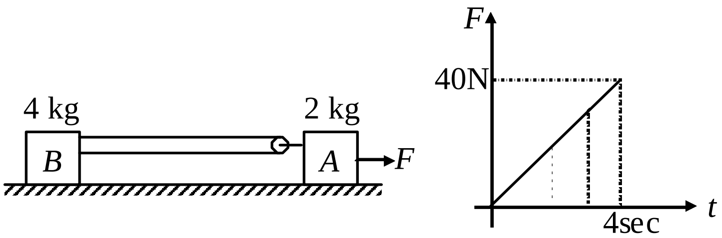
Consider μ as coefficient of friction between block and the fixed inclined plane at an angle α. From figure answer the given questions.



Column - I	Column – II
(A) The force by wedge on block if block m is at rest	(p) mg
(B) The friction force on block if block is at rest	(q) mg sin α
(C) The friction force when the block is sliding with constant velocity	(r) μ mg cos α
(D) The frictional force when the block is accelerating downward	(s) mg cos α
	(t) zero

Matrix Match – 5

Two block system is placed on rough surface having coefficient of friction μ= 0.5. Block *A* is pulled by applied force *F* which varies with time as shown.

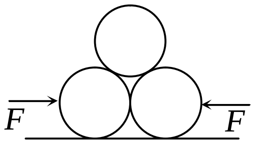


Column I	Column II
(A) Acceleration of block *A* and *B* is zero at	**p.** at $t = 1\text{sec}$
(B) Tension in string is less than 10 N	**q.** at $t = 2\,\text{sec}$
(C) Friction force on block *A* is 10 N	**r.** at $t = 3\text{sec}$
(D) Friction force on block *B* is less than 20 N	**s.** at $t = 4\text{sec}$
	t. at *t* = 2.5 sec

EXERCISE # IV

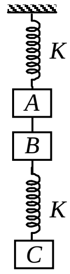
SUBJECTIVE TYPE

1. Two smooth cylindrical bars weighing W N each lie next to each other in contact. A similar third bar is placed over the two bars as shown in figure. Neglecting friction, find the minimum horizontal force (in N) on each lower bar necessary to keep them in contact.(Take $W = 20\sqrt{3}$ N)

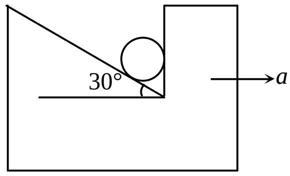


2. A flexible chain of weight 10 N hangs between two fixed points A and B which are at the same horizontal level. The inclination of the chain with the horizontal at both the points of support is 45°. Find the tension (in N) of the chain at the mid point.

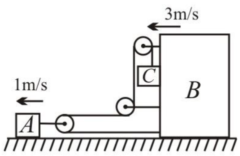
3. The system shown is in equilibrium. Find the ratio of acceleration of the blocks A and B at the instant when the spring between ceiling and A is cut. All the blocks have equal mass and the springs are ideal.



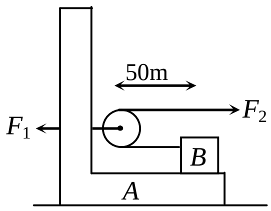
4. A heavy spherical ball is constrained in a frame as shown in figure. The inclined surface is smooth. The maximum acceleration with which the frame can move without causing the ball to leave the frame is $\frac{n}{\sqrt{3}}$. Find value of n.



5. The velocities of A and B are shown in the figure. Find the speed (in m/s) of block C. (Assume that the pulleys and string are ideal)

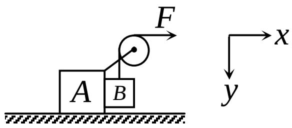


6. A 1 kg block B rests on a bracket A of same mass as shown in figure. Constant forces F_1 = 20N and F_2 = 8N start acting at time t = 0. The distance of block B from pulley is 50 m at t = 0. Determine the time (in s) when block B reaches the pulley.

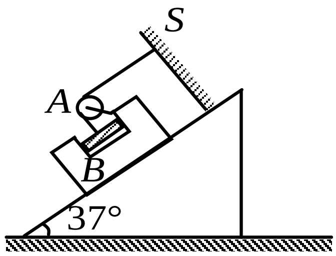


7. Two men A and B of equal mass are holding the free ends of a massless rope which passes over a frictionless light pulley. Man A climbs up the rope with acceleration 10 m/s^2 relative to the rope while man B hangs on without climbing. Find the acceleration (in m/s^2) of the man B with respect to ground.

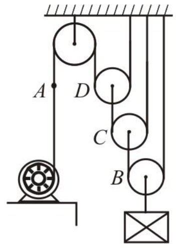
8. In the situation given, all surfaces are frictionless. If $F = Mg/2$, the ratio of accelerations of blocks A and B is $\frac{n}{\sqrt{5}}$. Find value of n. (Both the blocks have mass M)



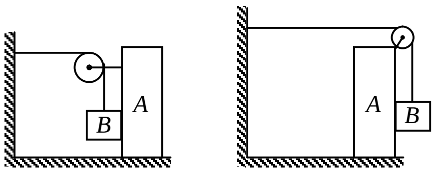
9. A block B is kept on an inclined plane. Another block A is inserted in a slot in the block B through a light string. One end of the string is fixed to a support and other end of the string is attached to A. All the surfaces are smooth. Masses of A and B are same. The acceleration of block B is found to be $4/n$. Find value of n. ($\sin 37° = 3/5$)



10. Starting from rest, the cable is wound onto the drum of the motor at a rate of $v_A = (24t^2)$m/s, where t in seconds. Determine the time (in s) needed to lift the load by a distance of 27m.



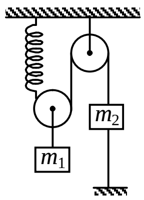
11. A 20 kg block B is suspended from an ideal string attached to a 40 kg block A. The ratio of the acceleration of the block B in cases (i) and (ii) shown in figure immediately after the system is released from rest is $\frac{3n}{2\sqrt{2}}$. Find value of n. (Neglect friction)



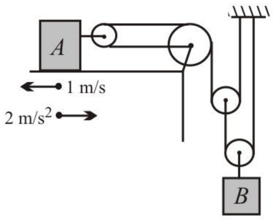
Case (i)   Case (ii)

12. In figure shown, pulleys are ideal. Initially the system is in equilibrium and the string connecting m_2 to rigid support is cut. Find the initial acceleration (in m/s^2) of m_2.

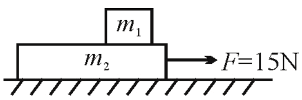
(Take m_1 = 15 kg, m_2 = 5 kg)



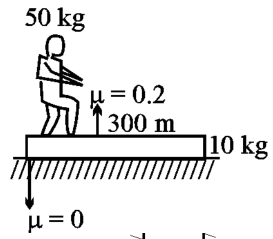
13. In the given figure, find the velocity (in cm/s) and acceleration (in m/s^2) of B, if velocity and acceleration of A are as shown.



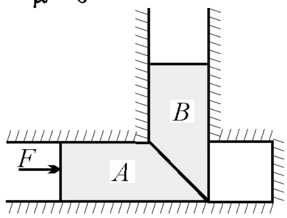
14. A slab of mass 10 kg is resting on a frictionless floor with a block of mass 1 kg on its top. The coefficient of friction between them is 0.1. A horizontal force of 15 N is applied to the slab at t = 0. Calculate the distance (in cm) moved by slab when the block has moved a distance of 0.5 m on the slab.



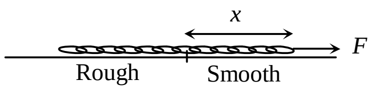
15. A man of mass 50 kg is standing on one end of a stationary wooden plank resting on a frictionless surface. The mass of the plank is 10 kg, its length is 300 m and the coefficient of friction between the man and the plank is 0.2. Find the shortest time (in s) in which the man can reach the other end of the plank starting from rest and stopping at the other end.



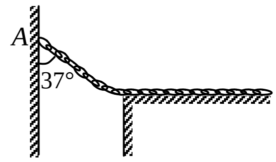
16. A side view of a simplified form of vertical latch B is as shown. The lower member A can be pushed forward in its horizontal channel. The sides of the channels are smooth, but at the interfaces of A and B, which are at 45° with the horizontal, there exists a static coefficient of friction $\mu = 0.4$. What is the minimum force F (in N) that must be applied horizontally to A to start motion of the latch B if it has a mass $m = 0.6$ kg ?



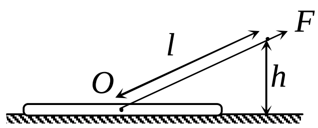
17. A chain of mass per unit length $\lambda = 2$ kg/m is pulled up by a constant force F. Initially the chain is lying on a rough surface and passes onto the smooth surface. The co-efficient of friction between chain and rough surface is $\mu = 0.1$. The length of the chain is L.Find the velocity of the chain when $x = L$.



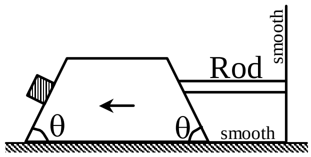
18. A uniform chain of length L has one of its end attached to the wall at point A, while $\frac{3L}{4}$ of the length of the chain is lying on table as shown in figure. Find the minimum co-efficient of friction between table and chain so that chain remains in equilibrium is



19. A log of mass m is pulled at a constant velocity and with a force F by means of a rope of length l. The distance between the end of the rope and the ground is h as shown. Find the co-efficient of friction between the log and the ground.



20. In the diagram shown, no relative motion takes place between the wedge and the block placed on it. The rod slides downwards over the wedge and pushes the wedge to move in horizontal direction. The mass of wedge is same as that of the block and is equal to M. If $\tan\theta = \frac{1}{\sqrt{3}}$, find the mass of rod.



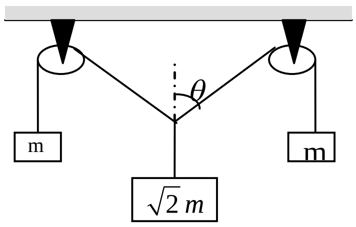
LAST YEAR IIT-JEE QUESTIONS

OBJECTIVE TYPE

1. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. The long piece will have the force constant of:

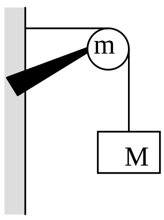
(A) (2/3) k (B) (3/2) k (3) 3 k (D) 6 k **[JEE 1999]**

2. The pulleys and the strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be: **[JEE 2001]**



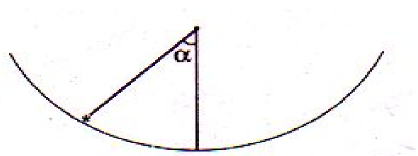
(A) 0^o (B) 30^o (C) 45^o (D) 60^o

3. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the plank is given by: **[JEE 2001]**



(A) $\sqrt{2}$ Mg (B) $\sqrt{2}$ mg (C) $(\sqrt{(M+m)^2+m^2})g$ (D) $(\sqrt{(M+m)^2+M^2})g$

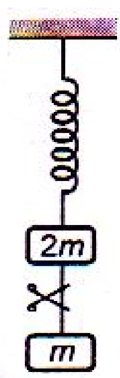
4. An insect crawls up hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface of the insect makes an angle α with the vertical, the maximum possible value of α is given **[2001]**



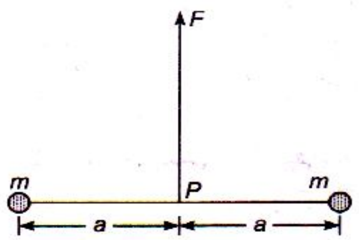
(a) $\cot\alpha = 3$ **(b)** $\tan\alpha = 3$ **(c)** $\sec\alpha = 3$ **(d)** $\text{cosec}\,\alpha = 3$

5. System shown in figure is in equilibrium and at rest. The spring and string are massless, now the string is cut. The acceleration of mass $2m$ and m just after the string is cut will be **[2006]**

(a) g/2 upwards, g downwards
(b) g upwards, g/2 downwards
(c) g upwards, 2g downwards
(d) 2g upwards, g downwards



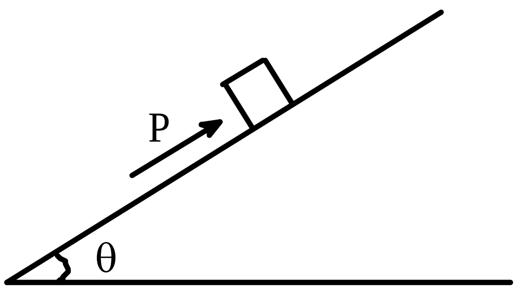
6. Two particles of mass *m* each are tied at the ends of a light string of length 2a. The system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance *a* from the centre *P* (as shown in the figure).

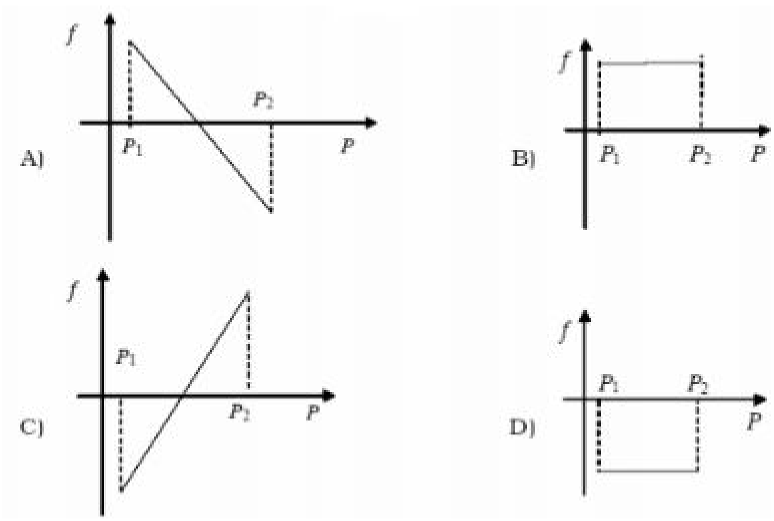


Now, the mid-point of the string is pulled vertically upwards with a small but constant force *F*. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is **[2007]**

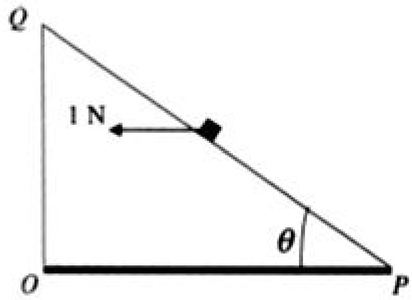
(a) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$ (b) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$ (c) $\frac{F}{2m}\frac{x}{a}$ (d) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$

7. A block of mass *m* is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force *P* parallel to the plane. The direction of force pointing up the plane is taken to be positive. As *P* is varied from $P_1 = mg\,(\sin\theta - \mu\cos\theta)$ to $P_2 = mg\,(\sin\theta + \mu\cos\theta)$, the frictional force f versus *P* graph will look like **[2010]**





8. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2) **[2012]**



(a) $\theta = 45^0$

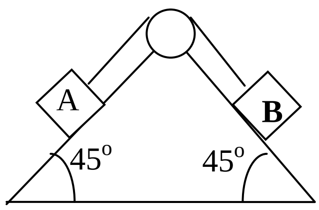
(b) $\theta > 45^0$ and a friction force acts on the block towards P

(c) $\theta > 45^0$ and a friction force acts on the block towards Q

(d) $\theta < 45^0$ and a friction force acts on the block towards Q

SUBJECTIVE TYPE

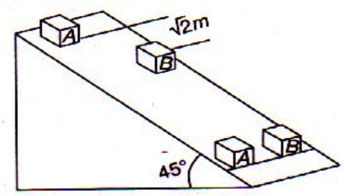
9. A block A of mass m and a block B of mass 2m are placed on a fixed triangular wedge by means of a massless inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45^o to the horizontal on both sides. The coefficient of friction between block A and the wedge is 2/3 and that between block B and wedge is 1/3. If the system A and B is released from rest, find :



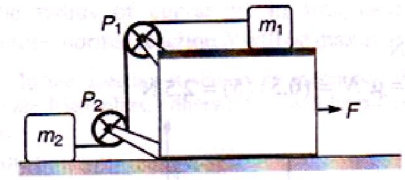
(i) the acceleration of A. (ii) tension in the string.

(iii) the magnitude and direction of friction acting on A. **[JEE 1997]**

10. Two blocks A and B of equal masses are released from an inclined plane of inclination 45^0 at t = 0. Both the blocks are initially at rest. The coefficient of kinetic friction between the block A and the inclined plane is 0.2 while it is 0.3 for block B. Initially the block A is $\sqrt{2}$ m behind the block B. When and where their front faces will come in a line.



11. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively.



The coefficient of friction between M and ground is zero, between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the strings are massless. The string is perfectly horizontal between P_1 and m_1 and P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force is applied to the mass M.

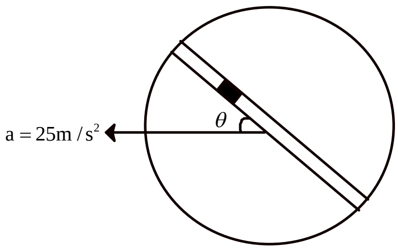
Take $g = 10 m/s^2$ **[JEE 2000]**

(a) Draw a free body diagram for mass M, clearly showing all the forces.

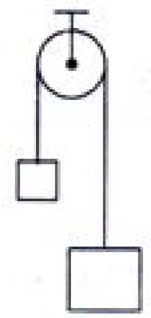
(b) Let the magnitude of force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2f_2$. Find f_1 and f_2. Write down the equations of motion of all the masses. Find F, the tension in the string and the accelerations of the masses.

12. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg placed as shown. The coefficient of friction between the block and all surfaces of groove in contact is $\mu = 2/5$. The disc has an acceleration of $25\,m/s^2$. Find the acceleration of the block with respect to disc.

$\left(\theta = 37^\circ\right)$ **[2006]**



13. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10 m/s^2$ find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **[2009]**

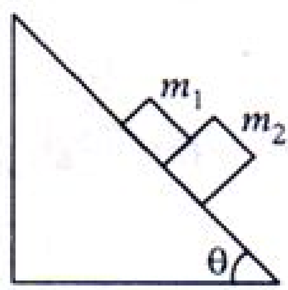


14. A block is moving on an inclined plane making an angle 45^0 with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then N is **[2011]**

MATRIX MATCH

15. A block of mass $m_1 = 1$ kg, another mass $m_2 = 2$ kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List II with an angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.

[Useful information : $\tan(5.5^0) \approx 0.1; \tan(11.5^0) \approx 0.2; \tan(16.5^0) \approx 0.3$]



	List I		List II
P.	$\theta = 5^0$	1.	$m_2 g \sin\theta$
Q.	$\theta = 10^0$	2.	$(m_1 + m_2) g \sin\theta$
R.	$\theta = 15^0$	3.	$\mu m_2 g \cos\theta$
S.	$\theta = 20^0$	4.	$\mu(m_1 + m_2) g \cos\theta$

Code :

(a) P – 1, Q – 1, R – 1, S – 3
(b) P – 2, Q – 2, R – 2, S – 3
(c) P – 2, Q – 2, R – 2, S – 4
(d) P – 2, Q – 2, R – 3, S – 3

ANSWER KEY

EXERCISE – I

1.	C	2.	A	3.	A	4.	B
5.	C	6	A	7.	A	8.	C
9.	B	10.	C	11.	C	12.	B
13.	A	14.	D	15.	B	16.	A
17.	C	18.	A	19	A	20.	B
21.	D	22.	C	23.	C	24.	A
25.	A	26.	D	27.	B	28.	A
29.	B	30.	A	31.	D	32.	A
33.	A	34.	A	35.	C	36.	D
37.	B	38.	C	39.	A	40.	D
41.	A	42.	C	43.	B	44.	A
45.	A	46.	C	47.	A	48.	B
49.	A	50.	B	51.	A	52.	B
53.	C	54.	D	55.	A	56.	C
57.	A	58.	D	59.	C	60.	B
61.	D	62.	B	63.	D	64.	A
65.	B	66.	D	67.	D	68.	C
69.	B	70.	B	71	B	72.	B
73.	B	74.	C				

EXERCISE – II

1.	ACD	2.	CD	3.	ABC	4.	ABC
5.	ABC	6	ABC	7.	ABCD	8.	AB
9.	AB	10.	BC	11.	BD	12.	ACD
13.	BD	14.	AC	15.	BCD	16.	ABC
17.	A	18.	C	19	AD		

EXERCISE – III

1.	B	2.	A	3.	D	4.	C
5.	D	6	C	7.	D	8.	A
9.	C	10.	A	11.	C	12.	ABCD
13.	ABCD	14.	ABC	15.	C	16.	D
17.	B						

MATCH LIST

1. A - Q; B - R; C - Q; D - R

2. A - R; B - Q; C –Q, S; D - Q

3. A - Q,S;B - S

4. A - P; B - Q; C - Q,R; D - R

5. (A) - P,Q,R,T; (B) - P, Q, T; (C) - P,Q,R.S,T.; (D) -P, Q, T.

EXERCISE – IV

1.	**10N**	**2.**	**5N**
3.	**1**	**4.**	**10**
5.	**5 m/s**	**6.**	**5s**
7.	**5 m/s^2**	**8.**	**1**
9.	**3**	**10.**	**3S**
11.	**1**	**12.**	**5 m/s^2**
13.	**50 cm/s; 1 m/s^2**	**14.**	**175 cm**
15	**10s**	**16.**	**14N**
17.	$\sqrt{F-L}\ m/s$	**18.**	$\frac{1}{4}$
19.	$\mu=\dfrac{F\sqrt{l^2-h^2}}{mgl-Fh}$	**20.**	**3M**

Last Year IIT-JEE Questions

OBJECTIVE TYPE

1.	**B**	**2.**	**C**	**3.**	**D**	**4.**	**A**	**5.**	**A**
6.	**B**	**7.**	**A**	**8.**	**B,C**				

SUBJECTIVE TYPE

9. (i) zero (ii) $\dfrac{2\sqrt{2}mg}{3}$ (iii) $\dfrac{mg}{3\sqrt{2}}$ 10. $S_A = 8\sqrt{2}m,\ 2s$

11. (b) $a=\dfrac{3}{5}m/s^2, T=18N, F=60N, f_2=15N\ f_1=30\ N\ F=60N$

12. 10 m/s^2 13. 8 14. 5 15. (d)