Time: 75 Minutes

Number of Questions: 12

Max Marks: 40

INSTRUCTIONS

1. Use of mobile phones, smartphones, ipads, calculators, programmable wrist watches is **STRICTLY PROHIBITED.** Only ordinary pens and pencils are allowed inside the examination hall.

2. The correction is done by machines through scanning. On the OMR Sheet, darken bubbles completely with a **black or blue ball pen**. Please **DO NOT use a pencil or a gel pen**. Darken the bubbles completely, only after you are sure of your answer; else, erasing may lead to the OMR sheet getting damaged and the machine may not be able to read the answer. .

3. The name, email address, and date of birth entered on the OMR sheet will be your login credentials for accessing your score.

4. Incompletely, incorrectly or carelessly filled information may disqualify your candidature.

5. Each question has a one or two digit number as answer. The first diagram below shows improper and proper way of darkening the bubbles with detailed instructions. The second diagram shows how to mark a 2-digit number and a 1-digit number.

6. The answer you write on OMR sheet is irrelevant. The darkened bubble will be considered as your final answer.

7. Questions 1 and 2 carry 2 marks each; questions 3 to 9 carry 3 marks each; questions 10 to 12 carry 5 marks each.

8. All questions are compulsory.

9. There are no negative marks.

10. Do all rough work in the space provided below for it. You also have blank pages at the end of the question paper to continue with rough work.

11. After the exam, you may take away the Candidate's copy of the OMR sheet.

12. Preserve your copy of OMR sheet till the end of current olympiad season. You will need it later for verification purposes.

13. You may take away the question paper after the examination.

1. Three parallel lines L_1, L_2, L_3 are drawn in the plane such that the perpendicular distance between L_1 and L_2 is 3 and the perpendicular distance between L_2 and L_3 is also 3. A square $ABCD$ is constructed such that A lies on L_1, B lies on L_3 and C lies on L_2. Find the area of the square.

25

2. Ria writes down the numbers $1, 2, \ldots, 101$ in red and blue pens. The largest blue number is equal to the number of numbers written in blue and the smallest red number is equal to half the number of numbers written in red. How many numbers did Ria write with red pen?

3. Consider the set T of all triangles whose sides are distinct prime numbers which are also

5. In parallelogram $ABCD$ the longer side is twice the shorter side. Let $XYZW$ be the quadrilateral formed by the internal bisectors of the angles of $ABCD$. If the area of $XYZW$ is 10, find the area of $ABCD$.

6. Let x, y, z be positive real numbers such that $x^2 + y^2 = 49$, $y^2 + yz + z^2 = 36$ and $x^2 + \sqrt{3}xz + z^2 = 25$. If the value of $2xy + \sqrt{3}yz + zx$ can be written as $p\sqrt{q}$ where p, q are integers and q is not divisible by square of any prime number, find $p + q$.

7. Find the number of maps $f : \{1, 2, 3\} \longrightarrow \{1, 2, 3, 4, 5\}$ such that $f(i) \le f(j)$ whenever $i < j$.

8. For any real number t, let $\lfloor t \rfloor$ denote the largest integer $\le t$. Suppose that N is the greatest integer such that

$$\left\lfloor \sqrt{\left\lfloor \sqrt{\left\lfloor \sqrt{N} \right\rfloor} \right\rfloor} \right\rfloor = 4$$

Find the sum of digits of N.

9. Let $P_0 = (3, 1)$ and define $P_{n+1} = (x_n, y_n)$ for $n \geq 0$ by

$$x_{n+1} = -\frac{3x_n - y_n}{2}, \qquad y_{n+1} = -\frac{x_n + y_n}{2}$$

Find the area of the quadrilateral formed by the points $P_{96}, P_{97}, P_{98}, P_{99}$.

10. Suppose that P is the polynomial of least degree with integer coefficients such that $P\left(\sqrt{7} + \sqrt{5}\right) = 2\left(\sqrt{7} - \sqrt{5}\right)$. Find $P(2)$.

11. In how many ways can four married couples sit in a merry-go-round with identical seats such that men and women occupy alternate seats and no husband seats next to his wife?

12. A 12×12 board is divided into 144 unit squares by drawing lines parallel to the sides. Two rooks placed on two unit squares are said to be non attacking if they are not in the same column or same row. Find the least number N such that if N rooks are placed on the unit squares, one rook per square, we can always find 7 rooks such that no two are attacking each other.