PART (C) : MATHEMATICS

SECTION – I : SINGLE CORRECT ANSWER TYPE
(Maximum Marks : 45)

This section contains 15 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Marking Scheme : +3 for correct answer, 0 if not attempted and -1 in all other cases.

41. If equations $ax^2+bx+c=0,\left(a,b,c\in R,a\neq 0\right)$ and $2x^2+3x+4=0$ have a common root then $a:b:c$ equals
(A) 1 : 2 : 3
(B) 2 : 3 : 4
(C) 4 : 3 : 2
(D) 3 : 2 : 1

42. If log $\left(abc\right)=0$, then the value of $\dfrac{\log^2\left(ab\right).\log c+\log^2\left(bc\right)\log a+\log^2\left(ca\right)\log b}{\log\left(a\right)^{\log\left(b\right)^{\log\left(c\right)}}}$ (where a, b, c are not unity) is equal to –
(A) 1
(B) 2
(C) 3
(D) 6

43. If $0\leq x<2\pi$, then the number of real values of x, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$, is
(A) 9
(B) 3
(C) 5
(D) 7

44. If α and β are roots of the equation, $x^2-4\sqrt{2}kx+2e^{4\ln k}-1=0$ for some k, and $\alpha^2+\beta^2=66$, then $\alpha^3+\beta^3$ is equal to
(A) $248\sqrt{2}$
(B) $280\sqrt{2}$
(C) $-32\sqrt{2}$
(D) $-280\sqrt{2}$

45. If $a+b+c>\frac{9c}{4}$ and equation $ax^2+2bx-5c=0$ has non-real complex roots, then

(A) $a>0, c>0$
(B) $a>0, c<0$
(C) $a<0, c<0$
(D) $a<0, c>0$

46. If $f(\theta)=\sin^2\theta+\sin^2\left(\theta+\frac{2\pi}{3}\right)+\sin^2\left(\theta+\frac{4\pi}{3}\right)$, then $f\left(\frac{\pi}{15}\right)$ is equal to

(A) $\frac{2}{3}$
(B) $\frac{3}{2}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$

47. If $2\sec^2\alpha-\sec^4\alpha-2\operatorname{cosec}^2\alpha+\operatorname{cosec}^4\alpha=\frac{15}{4}$, then $\tan\alpha$ is equal to [Given : $\alpha\in(0,\pi/2)$]

(A) $\frac{1}{\sqrt{2}}$
(B) $\frac{1}{2}$
(C) $\frac{1}{2\sqrt{2}}$
(D) $\frac{1}{4}$

48. If $|x-3|-|x+2|\geq 5$, then x

(A) $(-\infty-2]$
(B) $[2,\infty)$
(C) $(-\infty,-5]$
(D) $(-\infty,-5)$

49. If $\frac{3\pi}{4}<\alpha<\pi$, then $\sqrt{2\cot\alpha+\frac{1}{\sin^2\alpha}}$ is equal to

(A) $1+\cot\alpha$
(B) $-1-\cot\alpha$
(C) $1-\cot\alpha$
(D) $-1+\cot\alpha$

50. The value of $\cos\left(\frac{\pi}{2^2}\right)\cdot\cos\left(\frac{\pi}{2^3}\right)........\cos\left(\frac{\pi}{2^{10}}\right)\cdot\sin\left(\frac{\pi}{2^{10}}\right)$ is

(A) $\frac{1}{512}$

(B) $\frac{1}{1024}$

(C) $\frac{1}{256}$

(C) $\frac{1}{2}$

51. If number of solutions of the equations $8\cos x\left[\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right]=1$ in $[0, \pi]$ is k, then k is equal to

(A) 3
(B) 2
(C) 4
(D) 1

52. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0, (x>0)$ is equal to

(A) 7
(B) 8
(C) 10
(D) 12

53. If $x^2+5=2x-4\cos(a+bx)$, where $a, b\in(0,5)$ is satisfied for atleast one real x, then the maximum value of $a+b$ is equal to

(A) 3π
(B) 2π
(C) π
(D) None of these

54. If $A=\tan 6^\circ\cdot\tan 42^\circ$ and $B=\cot 66^\circ\cdot\cot 78^\circ$, then

(A) $A=2B$

(B) $A=\frac{1}{3}$

(C) $A=B$

(D) $3A=2B$

55. If $\max(\sin x, \cos x) = 1$ has exactly 5 solutions in interval $\left[0, \frac{k\pi}{2}\right]$, then value of k is equal to

(A) 6
(B) 7
(C) 8
(D) 9

SECTION – II : MULTIPLE CORRECT ANSWER TYPE
(Maximum Marks : 15)

This section contains 5 multiple choice questions. Each question has 4 options (A), (B), (C) and (D) for its answer, out of which ONE OR MORE than ONE option can be correct.
Marking Scheme : +3 for correct answer, 0 if not attempted and 0 in all other cases.

56. What can be said about the range of the rational expression $\frac{3x}{1+x^2}$?

(A) The maximum value of this expression is 2
(B) The minimum value of this expression is $-3/2$
(C) The largest integer that lies in the range of this rational function is 1.
(D) None of these

57. Which of the following is/are greater than $4^{\sqrt{\log_{16} 2}}$?

(A) $2^{\sqrt{\log_{16} 2}}$
(B) $16^{\sqrt{\log_{16} 2}}$
(C) $16^{\sqrt{\log_{2} 4}}$
(D) $2^{\sqrt{\log_{4} 16}}$

58. If $\alpha + \beta = \frac{\pi}{3}$ and $\cos\alpha + \cos\beta = 1$, then –

(A) $\cos(\alpha - \beta) = \frac{1}{3}$
(B) $|\cos\alpha - \cos\beta| = \sqrt{\frac{2}{3}}$
(C) $\cos(\alpha - \beta) = -\frac{1}{3}$
(D) $|\cos\alpha - \cos\beta| = \frac{1}{2\sqrt{3}}$

59. If the range of a quadratic polynomial $P(x)$ with leading coefficient one is $\left[\frac{13+36k-9k^2}{4}, \infty\right) \forall x \in R$, then interval of k for which $P(x)=0$ has –

(A) real & unequal roots is $\left[-\frac{1}{3}, \frac{13}{3}\right]$

(B) imaginary roots is $\left(-\frac{1}{3}, \frac{13}{3}\right)$

(C) real & unequal roots is $\left(-\infty, -\frac{1}{3}\right) \cup \left(\frac{13}{3}, \infty\right)$

(D) equal roots is $\left\{-\frac{1}{3}, \frac{13}{3}\right\}$

60. For the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

(A) Number of principal solutions are 4

(B) General solution is $\frac{n\pi}{2} + \frac{\pi}{8}, n \in I$

(C) General solution is $n\pi + \frac{\pi}{8}, n \in I$

(D) Number of principal solutions are exactly 2.