PART (C) : MATHEMATICS

Answer Key & Solution

61. (A)

$3\tan\theta = \cot\theta \Rightarrow \tan^2\theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$ according to options $\pm 30^\circ$

62. (C)

$\sin\theta + \cos\theta = 1$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\therefore$ P.S. of $\theta + \frac{\pi}{4} = \frac{\pi}{4}$

63. (A)

$1 - 2\sin^2\theta = 2\sin^2\theta$

$4\sin^2\theta = 1$

$\sin\theta = \pm\frac{1}{2},\ \theta = \pm 30^\circ$

64. (C)

$\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$\pi\cos\theta + \pi\sin\theta = \frac{\pi}{2} \Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}} \Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

65. (B)

$\sin A = \sin^2 B, 2\cos^2 A = 3\cos^2 B$

$\Rightarrow 2\cos^2 A = 3(1 - \sin A)$

$\Rightarrow 2 - 2\sin^2 A - 3 + 3\sin A = 0$

$\Rightarrow 2\sin^2 A - 3\sin A + 1 = 0$

$\Rightarrow 2\sin^2 A - 2\sin A - \sin A + 1 = 0$

$\Rightarrow (2\sin A - 1)(\sin A - 1) = 0$

$\Rightarrow \sin A = \frac{1}{2},$

$\therefore\ A = \frac{\pi}{6}$ $\qquad (\because A \text{ is acute})$

66. (A)

$\tan(5x - 4x) = 1$

$\Rightarrow \tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}, n \in Z$

67. (A)

$\cot^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta + 1 = 0$

$\Rightarrow \cot^2\theta + \sqrt{3}\cot\theta + \frac{1}{\sqrt{3}}\cot\theta + 1 = 0$

$\Rightarrow \cot\theta\left(\cot\theta + \sqrt{3}\right) + \frac{1}{\sqrt{3}}\left(\cot\theta + \sqrt{3}\right) = 0$

$\Rightarrow \left(\cot\theta + \sqrt{3}\right)\left(\cot\theta + \frac{1}{\sqrt{3}}\right) = 0$

$\Rightarrow \cot\theta = -\sqrt{3}; \cot\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}};\ \tan\theta = -\sqrt{3}$

$\theta = n\pi - \frac{\pi}{6}; n\pi - \frac{\pi}{3}$

68. (A)

$f\left(\frac{1}{2}\right) = 0$

$4\left(\frac{1}{2}\right)^4 - (a-1)\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 a - 6\left(\frac{1}{2}\right) + 1 = 0$

$\frac{1}{4} - \frac{(a-1)}{8} + \frac{a}{4} - 2 = 0$

$\frac{1}{4} + \frac{1}{8} - \frac{a}{8} - 2 = 0$

$\frac{a}{8} = \frac{3}{8} - 2 \Rightarrow a = -13$

69. (B)

$|x-3|^{3x^2-10x+3} = |x-3|^0$

$3x^2 - 10x + 3 = 0$

$\Rightarrow \ x = 3 \ \text{or} \ \frac{1}{3}, \ x \neq 3$

So, $x = \frac{1}{3}$

Also, $x = 4$ (i.e. Base = 1 satisfied)

& $x = 2$

70. (A)

$7\log_{10}\left(\frac{16}{15}\right) + 5\log_{10}\frac{25}{24} + 3\log_{10}\frac{81}{80}$

$= 7\log_{10}\left(2^4\right) - 7\log_{10}3 - 7\log_{10}5 + 5\log_{10}5^2 - 5\log_{10}2^3 - 5\log_{10}3 + 3\log_{10}3^4 - 3\log_{10}5 - 3\log_{10}2^4$

$= 28\log_{10}2 - 7\log_{10}3 - 7\log_{10}5 + 10\log_{10}3 - 3\log_{10}5 - 12\log_{10}2$

$= \log_{10}2 + 0\log_{10}3 + 0\log_{10}5$

$= \log_{10}2$

71. (B)

$\log_{10}(7x-9)^2 + \log_{10}(3x-4)^2 = 2$

$\log_{10}(7x-9)^2(3x-4)^2 = 2$

$2\log_{10}\left|(7x-9)(3x-4)\right| = 2$

$\left|(7x-9)(3x-4)\right| = 10$

$21x^2 - 28x - 27x + 36 = 10, -10$

$21x^2 - 55x + 46 = 0 \ \text{or} \ 21x^2 - 55x + 26 = 0$

$\Delta_1 = (55)^2 - 4(21)(46) < 0$

$\Delta_2 = (55)^2 - 4(21)(26) > 0$

2 solution.

72. (A)

$\log_{abc} bc^2 + \log_{abc} ca^2 + \log_{abc} ab^2$

$= \log_{abc}(abc)^3 = 3$

73. (C)

$\log_{10}\left(3x^2 + 12x + 19\right) - \log_{10}(3x+4) = 1$

$\log_{10}\left(\frac{3x^2 + 12x + 19}{3x+4}\right) = 1$

$\frac{3x^2 + 12x + 19}{3x+4} = 10^1$

$3x^2+12x+19=30x+40$

$x^2-6x-7=0$

$x=7$ or $x=-1$

74. (B)

$2-\log_2\left(x^2+3x\right)\geq 0$

$2\geq \log_2\left(x^2+3x\right)$

$4\geq x^2+3x>0$ (domain)

$x^2+3x-4\leq 0$

$(x+4)(x-1)\leq 0$

$x\in[-4,1]$

$x^2+3x>0$

$x(x+3)>0$

$x\in(-\infty,-3)\cup(0,\infty)$

Ans. $x\in[-4,-3)\cup(0,1]$

75. (B)

$\log_e 2\cdot\log_b 625=\log_{10}16\cdot\log_e 10$

So, $\dfrac{\log_e 2}{\log_e 10}\log_b\left(5^4\right)=\log_{10}\left(2^4\right)$

$\log_{10}2\log_b\left(5^4\right)=\log_{10}16$

$\log_b\left(5^4\right)=\dfrac{\log_{10}(16)}{\log_{10}(2)}$

$4\log_b 5=4$

$\Rightarrow\ b=5$

76. (A)

$ax^2+bx+c=0$

$\alpha=\dfrac{p}{1-p}$

$a\alpha^2+b\alpha+c=0$

$a\left(\dfrac{p}{1-p}\right)^2+b\left(\dfrac{p}{1-p}\right)+c=0$

$ap^2+bp(1-p)+c(1-p)^2=0$

$ap^2+bp-bp^2+cp^2+c-2cp=0$

$p^2(a-b+c)+p(b-2c)+c=0$

Equation is

$$x^2(a-b+c)+x(b-2c)+c=0$$

77. (D)

$$y=\frac{4}{4x^2+4x+9}$$

$$y=\frac{4}{(2x+1)^2+8}\le\frac{4}{8}$$

78. (A)

$$(\alpha-\beta)^2=1$$

$$(\alpha+\beta)^2-4\alpha\beta=1$$

$$b^2-4c=1$$

79. (B)

$$\frac{a}{x-a}+\frac{b}{x-b}=1$$

$$\frac{ax+bx-ab-ab}{x^2-ax-bx+ab}=1$$

$$ax+bx-2ab=x^2-ax-bx+ab$$

$$0=x^2-2x(a+b)-3ab=0$$

Sum of roots = 0

$$a+b=0$$

80. (B)

Let α be common root

$$\alpha^2+bx+a=0$$

$$(-)\quad \alpha^2+a\alpha+b=0$$

$$(b-a)\alpha+(a-b)=0$$

$$\alpha=1$$

So, $1+b+a=0$

$$a+b=-1$$

81. (1)

$$2x-1=|x-7|=\begin{cases} x+7, \text{ if } x\ge -7 \\ -(x+7), \text{ if } x<-7 \end{cases}$$

$\therefore$ If $x\ge -7,\ 2x-1=x+7 \Rightarrow x=8$

If $x<-7,\ 2x-1=-(x+7)$

$\Rightarrow 3x=-6$

$\Rightarrow x=-2$, which is not possible.

Therefore, $x=8$ is only solution.

82. (4)

Let x and $x+2$ be two odd natural number.

We have, $x > 10$ ...(i)

and $x+(x+2) < 40$ ...(ii)

On solving (i) and (ii), we get

$10 < x < 19$

So, required pairs are (11, 13), (13, 15), (15,17) and (17, 19).

83. (1)

$x+\sqrt{3-x} \geq \sqrt{3-x}+3$

$\Rightarrow\ x \geq 3$

But $3-x \geq 0$

$\Rightarrow\ x \leq 3$

Hence, $x = 3$ is the only integral solution.

84. (1)

$2^{x/2}+3^{x/2}=\left(\sqrt{13}\right)^{x/2}$

$\Rightarrow \left(\frac{2}{\sqrt{13}}\right)^{x/2}+\left(\frac{3}{\sqrt{13}}\right)^{x/2}=1$

$\Rightarrow \frac{x}{2}=2 \Rightarrow\ x=4$. Only one value of x.

85. (2)

$x=2+2^{1/3}+2^{2/3}$

$(x-2)^3=2+4+3\left(2^{1/3}\right)\left(2^{2/3}\right)(x-2)$

$x^3-6x^2+12x-8=6+6x-12$

$x^3-6x^2+6x=8+6-12$

$=2$

86. (5)

$4^{1/2}+9^2=10^{\log_x 83}$

$83=83^{\log_x 10}$

So, $\log_x 10=1$

$10=x^1$

$p=5$

87. (9)

$x+1=2\log_2\left(2^x+3\right)-\log_2\left(1980-2^{-x}\right)$

So, $2^{x+1}=\dfrac{\left(2^x+3\right)^2}{\left(1980-2^{-x}\right)}$

Let $2^x=t$

$$2t=\frac{(t+3)^2}{\left(1980-\frac{1}{t}\right)}$$

$$3960t-2=t^2+6t+9$$

$$0=t^2-3954t+11$$

Roots are $2^\alpha, 2^\beta$

$$2^{\alpha+\beta}=11$$

$$\alpha+\beta=\log_2 11$$

$$b-a=9$$

88. (2)

Let $\left(5+2\sqrt{6}\right)^{x^2-3}=t$

$$\left(5-2\sqrt{6}\right)^{x^2-3}=\frac{1}{t}$$

So, $t+\dfrac{1}{t}=10$

$$t^2-10t+1=0$$

$$t=5\pm2\sqrt{6}$$

$$t=5+2\sqrt{6},\ 5-2\sqrt{6}=\frac{1}{5+2\sqrt{6}}$$

$$\left(5+2\sqrt{6}\right)^{x^2-3}=\left(5+2\sqrt{6}\right)^1 \text{ or } \left(5+2\sqrt{6}\right)^{-1}$$

$$x^2-3=1,-1$$

$$x=\pm2 \text{ or } \pm\sqrt{2}$$

$$A=2$$

89. (0)

So, let α be common root

$$a\alpha^2+c\alpha+b=0$$

$$(-)\quad a\alpha^2+2b\alpha+c=0$$

$$2(c-b)\alpha=c-b$$

$$\alpha=\frac{1}{2}$$

$$\therefore\quad a\left(\frac{1}{2}\right)^2+2c\left(\frac{1}{2}\right)+b=0$$

$$a+4c+4b=0$$

90. (6)

$924 = 3\times 308$

$= 3\times 11\times 28 = 3\times 11\times 7\times 4$

N is divisible by 3 and 11 by divisibility N must be divisible by 4.

$\Rightarrow\ \alpha = 4$ or 8.

N can be 444444 or 888888 check for divisibility with '7'.

$444444 \div 7 = 63492$

$\alpha = 4$ or 8

So, λ can be $= 11x - x^2$

$x = 4$ or 8

$\lambda = 44 - 16$ or $88 - 64$

$\lambda = 28$ or 24

$M(112) = 28\times 24$

$M = 6$