PART (C) : MATHEMATICS

Answer Key & Solution

41. (B)

$2x^2+3x+4=0$ has imaginary roots, conjugate powers.

$\therefore$ both roots common

$$\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$$

42. (C)

$$\frac{\log^2(ab)\cdot\log(c)+\log^2(bc)\cdot\log(a)+\log^2(ca)\cdot\log(b)}{\log(a)^{\log(c)\cdot\log(b)}}$$

$$=\frac{\log^2(ab)\cdot\log(c)+\log^2(bc)\cdot\log(a)+\log^2(ca)\cdot\log(b)}{\log(c)\cdot\log(b)\cdot\log(a)}$$

(Split)

$$=\sum\frac{\log^2(ab)}{\log(a)\cdot\log(b)}$$

$$=\sum\frac{\log^2(ab)}{\log(a)\cdot\log(b)}$$

$$=\sum\frac{(\log a+\log b)^2}{\log a\cdot\log b}\quad[\because \log(abc)=0\Rightarrow \log a+\log b+\log c=0]$$

$$=\sum\frac{(-\log c)^2}{\log a\cdot\log b}$$

$$=\sum\frac{\log^2 c}{\log a\cdot\log b}$$

$$=\frac{\log^3 c+\log^3 b+\log^3 a}{\log a\cdot\log b\cdot\log c}$$

$$=3$$

43. (D)

$x\in[0,2\pi)$

$\cos x+\cos 2x+\cos 3x+\cos 4x=0$

$$\therefore\ 2\cos\frac{5x}{2}\cos\frac{3x}{2}+2\cos\frac{5x}{2}\cos\frac{x}{2}=0$$

$$2\cos\frac{5x}{2}\left(\cos\frac{3x}{2}+\cos\frac{x}{2}\right)=0$$

$$2\cos\left(\frac{5x}{2}\right)\cdot 2\cos x\cdot\cos\frac{x}{2}=0$$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2} \Big/ \frac{3\pi}{2}$

$\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$

$\cos\frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = \frac{\pi}{2} \Big/ \frac{3\pi}{2}; \frac{7\pi}{2} \Big/ \frac{9\pi}{2}$

7 values.

44. (B)

$K > 0$

$\alpha + \beta = 4\sqrt{2}\, K$

$\alpha \cdot \beta = 2e^{4\ln k} - 1$

$= 2k^4 - 1$

$\alpha^2 + \beta^2 = 66$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 66$

$\Rightarrow 32k^2 - 4k^4 + 2 = 66$

$\Rightarrow k^2 = 4 \Rightarrow k = 2$

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (8\sqrt{2})^3 - 3.31.8\sqrt{2}$

$= 8\sqrt{2}[128 - 93]$

$= 280\sqrt{2}$

45. (B)

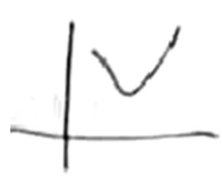
$f(x) = ax^2 + 2bx - 5c$

$f(2) = 4a + 4b - 5c$

Given, $4a + 4b + 4c > 9c$

$\Rightarrow 4a + 4b - 5c > 0$

$\therefore f(2) > 0 \;\&\; D > 0$



$f(x) > 0 \;\forall\; x \in R$

$f(0) > 0$

$\Rightarrow -5c > 0$

$\Rightarrow c < 0 \;\&\; a > 0$

46. (B)

$$f(0) = \frac{1-\cos 2\theta}{2} + \frac{1-\cos[2\theta + 4\pi/3]}{2} + \frac{1-\cos[2\theta + 8\pi/3]}{2}$$

$$=\frac{1}{2}\left[3-\left\{\cos 2\theta+\cos\left(2\theta+\frac{4\pi}{3}\right)+\cos\left(2\theta+\frac{8\pi}{3}\right)\right\}\right]$$

$$=\frac{3}{2}$$

47. (A)

$$\sec^2(\alpha)\left(2-\sec^2\alpha\right)-\operatorname{cosec}^2\alpha\left(2-\operatorname{cosec}^2\alpha\right)=\frac{15}{4}$$

$$\Rightarrow\ \left(1+\tan^2\alpha\right)\left(1-\tan^2\alpha\right)-\left(1+\cot^2\alpha\right)\left(1-\cot^2\alpha\right)=\frac{15}{4}$$

$$\Rightarrow\ 1-\tan^4\alpha-\left(1-\cot^4\alpha\right)=\frac{15}{4}$$

$$\Rightarrow\ \cot^4\alpha-\tan^4\alpha=\frac{15}{4}$$

$$\frac{1}{x}-x=\frac{15}{4}$$

$$\Rightarrow\ 4-4x^2=15x$$

$$4x^2+16x-x-4=0$$

$$x=-4\ \text{ or }\ x=\frac{1}{4}$$

$$\therefore\ \tan^4\alpha=\frac{1}{4}$$

$$\Rightarrow\ \tan\alpha=\pm\frac{1}{\sqrt{2}}$$

48. (A)

Case-1: $x\ge 3$	Case-2: $x\in(-2,3)$	Case-3: $x\le -2$
$x-3-x-2\ge 5$	$-x+3-x-2\ge 5$	$-x+3+x+2\ge 5$
$\Rightarrow -5\ge 5$	$-2x+1\ge 5$	$\Rightarrow\ 5\ge 5$
(rej)	$-2x\ge 4$	true
	$x\le -2$	$x\le -2$
	(rej)	

49. (B)

$$\sqrt{2\cot\alpha+\frac{1}{\sin^2\alpha}}$$

$$=\sqrt{\frac{2c}{s}+\frac{1}{s^2}}$$

$$=\sqrt{\frac{1+\sin 2\alpha}{s^2}}$$

$$=\sqrt{\left(\frac{c+s}{s}\right)^2}$$

$= \sqrt{(1+\cot\alpha)^2}$

$= |1+\cot\alpha|$

$= -(1+\cot\alpha)$

50. (A)

Let $\theta = \frac{\pi}{2^{10}}$

$2\theta = \frac{\pi}{2^9}$

$(\cos\theta\cdot\cos 2\theta ....9 \text{ terms})\times\sin\theta$

$= \frac{\sin(2^9\cdot\theta)}{2^9\sin\theta}\cdot\sin\theta$

$= \frac{\sin\left[2^9\cdot\frac{\pi}{2^{10}}\right]}{2^9}$

$= \frac{1}{2^9}$

51. (A)

$8\cos x\left[\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right]=1$

$8\cos x\left[\frac{1}{4}-\sin^2 x\right]=1$

$\cos 3x = \frac{1}{2}$

$x\in[0,\pi]$

$3x\in[0,3\pi]$

$\therefore$ 3 Solutions.

52. (C)

$\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$

Let $\sqrt{x}=t$

$|t-2|+t(t-4)+2=0$

C-1: $t\ge 2$

$t-2+t^2-4t+2=0$

$\Rightarrow t^2-3t=0$

$t=0, t=3$

$\Rightarrow \sqrt{x}=3 \Rightarrow x=9$

C-2: $t < 2$

$\Rightarrow -t+2+t^2-4t+2=0$

$\Rightarrow t^2-5t+4=0$

$\Rightarrow t=1$ or $t=4$

$\sqrt{x}=1$

$x=1$

$\therefore$ Sum of solution $=1+9=10$

53. (A)

$$x^2+5=2x-4\cos(a+bx)$$

$$\Rightarrow \underbrace{(x-1)^2+4}_{[4,\infty)}=\underbrace{-4\cos(a+bx)}_{[-4,4]}$$

$$\therefore (x-1)^2+4=4 \ \& \ -4\cos(a+bx)=4$$

$$\Rightarrow x=1$$

$$-4\cos(a+b)=4$$

$$\Rightarrow \cos(a+b)=-1$$

$$\Rightarrow a+b=2n\pi+\pi$$

$a,b\in(0,5)$ $\qquad (a+b)\big|_{\max}=3\pi$

$a+b\in(0,10)$

54. (C)

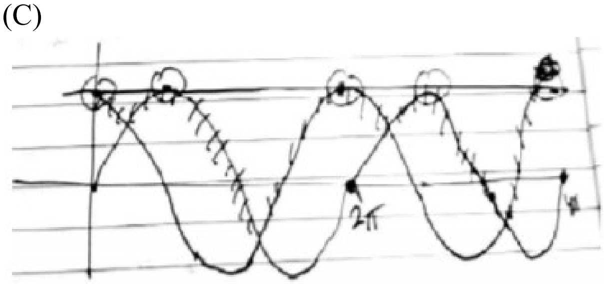
$$B=\frac{1}{\tan 66^\circ\cdot\tan 78^\circ}$$

$$=\frac{\tan 6^\circ\cdot\tan 54^\circ}{\tan 6^\circ\cdot\tan 66^\circ\cdot\tan 54^\circ\cdot\tan 78^\circ}$$

$$=\frac{\tan 6^\circ\cdot\tan 54^\circ\cdot\tan 42^\circ}{\tan 18^\circ\cdot\tan 78^\circ\cdot\tan 42^\circ}$$

$$=\frac{\tan 6^\circ\cdot\tan 54^\circ\cdot\tan 42^\circ}{\tan 54^\circ}=A$$

55. (C)



56. (B, C)

$y = \dfrac{3x}{1+x^2}$

$yx^2 - 3x + y = 0$

$\because\ x \in R$

$D \geq 0$

$\Rightarrow\ 9 - 4y^2 \geq 0$

$\Rightarrow\ y^2 \leq \dfrac{9}{4}$

$y \in \left[-\dfrac{3}{2}, -\dfrac{3}{2}\right]$

57. (B, C, D)

$4^{\sqrt{\log_{16} 2}} = 4^{\sqrt{\log_{2^4} 2}}$

$= 4^{\sqrt{\frac{1}{4}}}$

$= 4^{\frac{1}{2}} = 2$

(A) $2^{\sqrt{\frac{1}{4}}} = 2^{\frac{1}{2}} = \sqrt{2} < 2$

(B) $16^{\sqrt{\frac{1}{4}}} = 16^{\frac{1}{2}} = 4 > 2$

(C) $16^{\sqrt{2}} > 2$

(D) $2^{\sqrt{\frac{1}{2}\times 4}} = 2^{\sqrt{2}} > 2$

58. (B, C)

$\alpha + \beta = \dfrac{\pi}{3}$

$\cos\alpha + \cos\beta = 1$

$\Rightarrow\ 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cdot\cos\left(\dfrac{\alpha-\beta}{2}\right) = 1$

$\Rightarrow\ \cos\left(\dfrac{\alpha-\beta}{2}\right) = \dfrac{1}{\sqrt{3}}$

So, $\cos(\alpha-\beta) = 2\times\dfrac{1}{3} - 1 = \dfrac{-1}{3}$

Now, $|\cos\alpha - \cos\beta|^2$

$= (\cos\alpha + \cos\beta)^2 - 4\cos\alpha\cdot\cos\beta$

$= 1^2 - 2\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

$= 1 - 2\left[\dfrac{1}{2} - \dfrac{1}{3}\right]$

$$=1-\frac{2}{6}=\frac{2}{3}$$

59. (B, C, D)

$\because\ a=1>0$

$\therefore\ \frac{-D}{4a}=\frac{13+36K-9K^2}{4}$

$\Rightarrow\ D=9K^2-36K-13$

$=(3K-13)(3K+1)$

(A) $D>0\ \ K\in\left(-\infty,-\frac{1}{3}\right)\cup\left(\frac{13}{3},\infty\right)$

(B) $D<0$

$\Rightarrow\ \ K\in\left(-\frac{1}{3},\frac{13}{3}\right)$

(D) $D=0$

$\Rightarrow\ K=-\frac{1}{3},\frac{13}{3}$

60. (A, B)

$\sin x+\sin 3x-3\sin 2x=\cos x+\cos 3x-3\cos 2x$

$\Rightarrow\ \sin 2x(2\cos x-3)=\cos 2x(2\cos x-3)$

$\Rightarrow\ 2\cos x-3=0$ or $\sin 2x=\cos 2x$

(rej) $\tan 2x=1$

$2x=n\pi+\frac{\pi}{4}$

$x=\frac{n\pi}{2}+\frac{\pi}{8}$