PART (A) : PHYSICS

Answer Key & Solution

1. (A)

$$\frac{dx}{d\theta}=a(1+\cos\theta);\ \frac{dy}{d\theta}=a(0+\sin\theta)$$

$$\frac{dy}{dx}=\frac{dy}{d\theta}\Big/\frac{d\theta}{dx}=\frac{a\sin\theta}{a(1+\cos\theta)}$$

2. (C)

$$\left[\frac{\log(2+3x)}{3}\right]_2^5$$

$$\frac{1}{3}(\log 17-\log 8)=\frac{1}{3}\ln\frac{17}{8}$$

3. (B)

$$\frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta I}{I}=\frac{5}{100}+\frac{0.2}{10}$$

$$\frac{\Delta R}{R}\times 100=7\%$$

4. (A)

$$a=V\frac{dv}{dx}$$

5. (A)

For A

$$S_5=\frac{a_1}{2}(2\times 5-1)$$

For B

$$S_3=\frac{a_2}{2}(2\times 3-1)$$

Given that $\frac{a_1}{2}(2\times 5-1)=\frac{a_2}{2}(2\times 3-1)$

$$9a_1=5a_2$$

6. (B)

Initial velocity,

$$u=17\ \text{m/s}$$

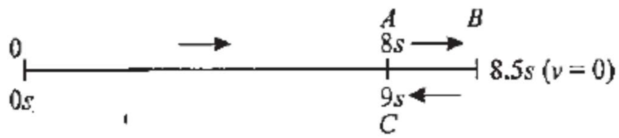
Acceleration, $a=-2\ \text{m/s}^2$

Since, the particle continuously experiencing retardations, let the time at which its velocity becomes zero is 't'.

Using $v = u + at$

$0 = 17 - 2t$

$t = 8.5\,s$



Distance covered by particle in 9[th] second

$= AB + BC$

Velocity of particle of A,

$v_A = 17 - 2(8)$

$v_A = 1\text{ m/s}$

Distance AB, $s_{AB} = \dfrac{v_B^2 - v_A^2}{2a} = \dfrac{0-(1)^2}{2(-2)} = \dfrac{1}{4} = 0.25m$

Distance BC, $s_{BC} = \dfrac{v_C^2 - v_B^2}{2a} = \dfrac{(1)^2 - 0}{2(2)} = \dfrac{1}{2} = 0.25m$

Thus, distance covered in 9[th] second of its motion

$= 0.25m + 0.25m$

$= 0.5\,m$

7. (A)

$|\Delta\vec{\mathbf{v}}| = |\vec{\mathbf{g}}.\Delta t| = (10)(0.5) = 5\text{ m/s}$

This change is in vertically downward direction (in the direction of acceleration $\vec{\mathbf{g}}$).

8. (A)

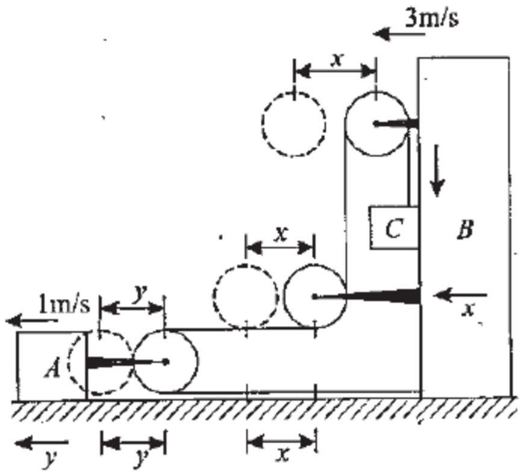
For second particle $v_x = v_y$

$\therefore \quad v_2 = \sqrt{2}v_y$

For first particle, $v_1 = v_y$

$\therefore \quad \dfrac{K_A}{K_B} = \dfrac{v_1^2}{v_2^2} = \dfrac{1}{2}$

9. (A)



C comes by $(3x-2y)$

$$v_c = 3x - 2y = (3\times 3) - (2\times 2)$$
$$v_c = 9 - 4$$
$$v_c = 5 \text{ m/s}$$

10. (B)

$$(3u)^2 = u^2 + 2gh$$
$$\therefore\ h = \frac{4u^2}{g}$$

11. (C)

$$x_1 = \frac{1}{2}a(10)^2 = 50a$$
$$x_2 = \frac{1}{2}a(20)^2 - \frac{1}{2}(a)(10)^2 = 150a$$
$$x_3 = \frac{1}{3}a(30)^2 - \frac{1}{2}a(20)^2 = 250a$$
$$\therefore\quad x_1 : x_2 : x_3 = 1:3:5$$

12. (D)

Let v be the velocity of projectile at this instant. Horizontal component of velocity remains unchanged. Therefore,

$$v\cos 30^\circ = 10\cos 60^\circ \text{ or } v\frac{\sqrt{3}}{2} = \frac{10}{2}$$
$$\therefore\quad v = \frac{10}{\sqrt{3}} \text{ m/s}$$

13. (A)

Substituting the proper values in equation,

$$S = ut + \frac{1}{2}at^2$$

We have,

$$-h = (10)(11) - \frac{1}{2}(10)(11)^2$$

or $h = 495$ m

14. (D)

$\because \vec{v}_{S,B} = \vec{v}_S - \vec{v}_B$

$\because \vec{v}_S = \vec{v}_{S,B} + \vec{v}_B$

$$= 10\hat{j} + 5\hat{j} = 15\hat{j}$$

Velocity of stone w.r.t. ground = 15 m/s upwards

$\because v = u + at$

$$= 15 - gt = 15 - 20 = -5\,\text{m/s}$$

15. (B)

$P = F/A = MLT^2/L^2 = ML^{-1}T^{-2}$

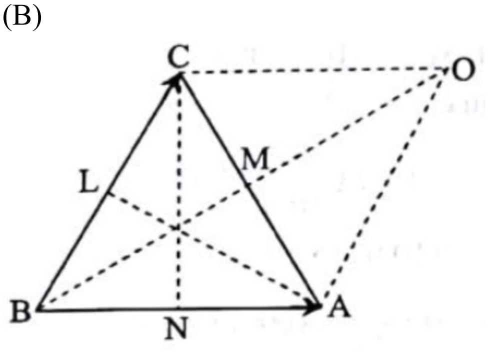
$b = \frac{t^2}{Px} = \frac{T^2}{ML^{-1}T^{-2}L} = M^{-1}T^4$

$a = Pbx = T^2$

$\frac{a}{b} = M^1L^0T^{-2}$

16. (C)

17 (B)

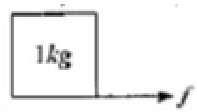


$\because \overrightarrow{BO} = \overrightarrow{BA} + \overrightarrow{BC}$

$$= 2\overrightarrow{BM}$$

18. (A)

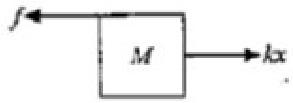
Let initial acceleration of system is 'a'



$f = ma$

$\mu mg = ma$

$a = 2\ \text{m/s}^2$



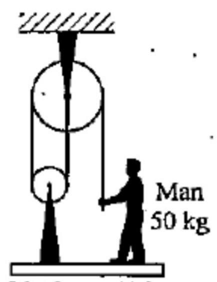
$kx - f = ma$

$1000x - 2 = 4 \times 2$

$1000x = 10$

$x = \dfrac{1}{100}\ \text{m} = 1\ \text{cm}$

19. (C)

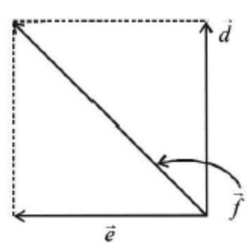


$3T = 900$

$T = 300\ \text{N}$

20. (C)

Using the law of vector addition, $(\vec{d} + \vec{e})$ is as shown in the figure.



$\therefore\ \vec{d} + \vec{e} = \vec{f}$

21. (2)

Total area around fountain

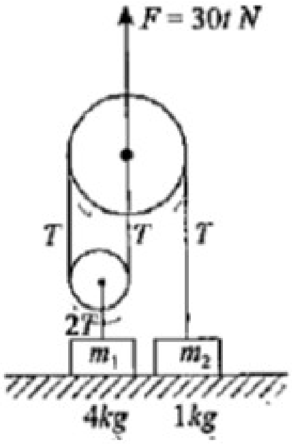
$A = \pi R_{max}^2$

Where, $R_{max} = \dfrac{v^2 \sin 2\theta}{g} = \dfrac{v^2 \sin 90^\circ}{g} = \dfrac{v^2}{g}$

$$\therefore \quad A = \pi \frac{v^4}{g^2}$$

22. (2)

Block B loses contact when normal reaction on it becomes zero.



i.e. $2T = 40$

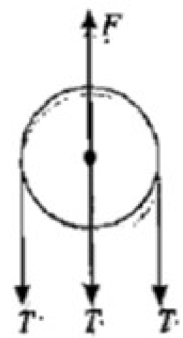
$T = 20\,\text{N}$

$F = 3T$

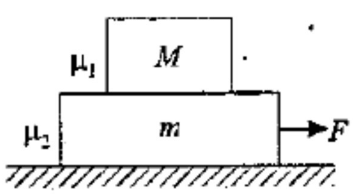
$30t = 3(20)$

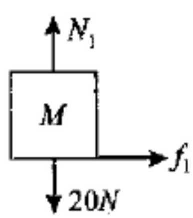
$30t = 60$

$t = 2s$



23. (22.5)



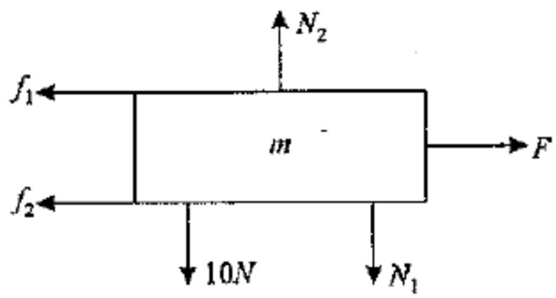


$f_{1_{max}} = \mu_1 N_1 = 0.25 \times 20 = 5\,\text{N}$

$f_{2_{max}} = \mu_2 N_2$

$f_2 = 0.5(10+20)$

$f_2 = 0.5 \times 30 = 15\,\text{N}$



When both board and block start sliding at $F > 15$ N their acceleration will be

$$a = \frac{F-15}{3}$$

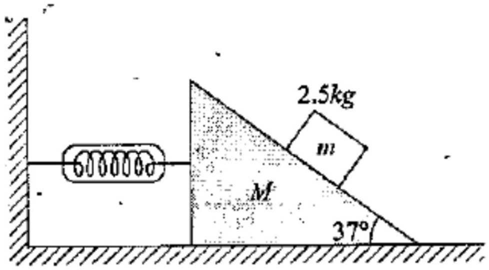
to start sliding between board and block we use

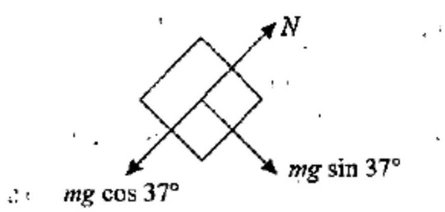
$$f_{1_{\max}} = Ma$$

$$5 = 2\left(\frac{F-15}{3}\right)$$

$\Rightarrow$ $F = 22.5\,\text{N}$

24. (12)

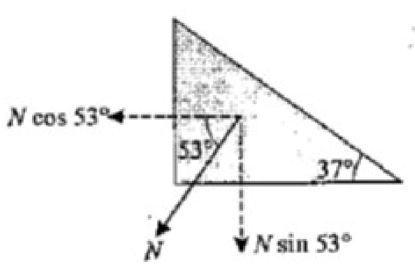




$$N = mg\cos 37^\circ\ \text{N}$$

$$N = 2.5 \times 10 \times \frac{4}{5}$$

$$N = 20\,\text{N}$$



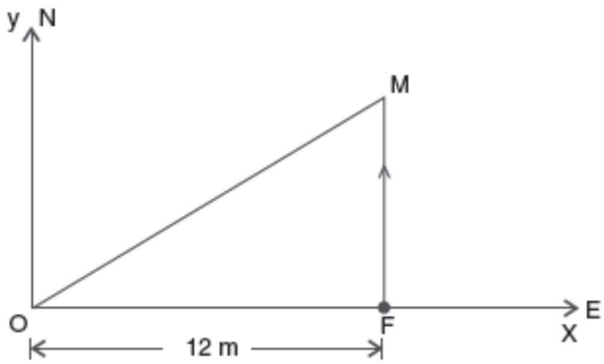
Reading of spring balance,

$$R = N\cos 53^\circ$$

$R = 20 \times \frac{3}{5}$

$R = 12\,\text{N}$

25. (4)



The ball was hit at 0 and caught by the fielder at point M.
Fielder runs for 2.4 s

$\therefore\ FM = 2.4 \times 5 = 12m$

OM = $12\sqrt{2}$ m = 3v → v = $4\sqrt{2}$ m/s

26. (150)

$$\frac{B}{2} = \sqrt{A^2 + B^2 + 2AB\cos\theta} \qquad ...(\text{i})$$

$$\therefore \quad \tan 90^\circ = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$$

$$\therefore \quad \cos\theta = -\frac{A}{B}$$

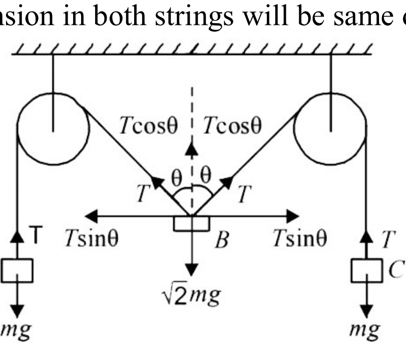
Hence, from (i) $\frac{B^2}{A} = A^2 + B^2 - 2A^2 \;\Rightarrow\; A = \sqrt{3}\frac{B}{2}$

$$\Rightarrow \quad \cos\theta = -\frac{A}{B} = -\frac{\sqrt{3}}{2}$$

$$\therefore \quad \theta = 150^\circ$$

27. (45)

The tension in both strings will be same due to symmetry.



For equilibrium in vertical direction for body B we have

$$\sqrt{2}mg = 2T\cos\theta$$

$\therefore \quad \sqrt{2}mg = 2(mg)\cos\theta \qquad [\because T = mg, (\text{at equilibrium})]$

$\therefore \quad \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$

28. (625)

Here, $m = 0.5$ kg, $u = -10$ m/s;

$t = 1/50$ s, $v = +15$ ms^{-1}

Force $= m(v-u)/t = 0.5(10+15)\times 50 = 625\,\text{N}$

29. (6)

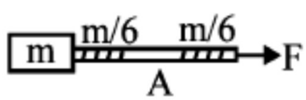
$v = u - at \Rightarrow t = \frac{u}{a}$[As $v = 0$]

$t = \frac{u\times m}{F} = \frac{30\times 1000}{5000} = 6\,\text{sec}$

30. (7)

The acceleration of the system is

$a = \frac{F}{m+\frac{m}{3}} = \frac{3F}{4m}$



The tension in the middle of the rope (i.e., at point A) is

$T = \left(m\times\frac{m}{6}\right)a = \frac{7m}{6}\times\frac{3F}{4m} \Rightarrow T = \frac{7F}{8}$