PART (C) : MATHEMATICS

Answer Key & Solution

41. (D)

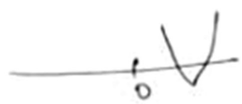
Case-I: ex-1 +ve root

$f(0) < 0$

$\Rightarrow K < -5$

Case-II: ex-2 +ve root

(i) $f(0) > 0$



$\Rightarrow k > -5$

(ii) $\dfrac{-B}{2A} > 0 \Rightarrow \dfrac{-2(k-1)}{2} > 0$

$\Rightarrow k < 1$

(iii) $D \geq 0$

$\Rightarrow \cancel{4}(k-1)^2 - \cancel{4}(k+5) \geq 0$

$\Rightarrow k^2 - 3k - 4 \geq 0$

$(k-4)(k+1) \geq 0$

$\Rightarrow k \leq -1$ or $k \geq 4$

$k \in (-5, -1]$ For k = -5, one root is 0 & other is positive.

$\therefore k \in (-\infty, -1]$

42. (A)

$$\frac{\sin\left(9 \cdot \frac{\pi}{19}\right)}{\sin\left(\frac{\pi}{19}\right)} \cdot \cos\left[\frac{\pi}{19} + \frac{8\pi}{19}\right]$$

$$= \frac{\sin \frac{18\pi}{19}}{2\sin \frac{\pi}{19}} = \frac{1}{2}$$

43. (C)

$y = \sin^4\theta + \cos^2\theta$

$= S^2 - S^2 + 1$; where $S = \sin^2\theta$

$= \left(S^2 - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\left(S^2 - \frac{1}{2}\right)^2 \in \left[0, \frac{1}{4}\right]$

$y \in \left[\frac{3}{4}, 1\right]$

44. (B)

$$\frac{x^2(3x-4)^3 \cdot (x-2)^4}{(x-5)^5 \cdot (2x-7)^6} \le 0$$

$x \ne 5, \frac{7}{2}$

$x = 2, 0$

$$\Rightarrow \frac{(3x-4)^3}{(x-5)^5} \le 0$$

$x \in \left[\frac{4}{3}, 5\right) \cup \{0\} - \left\{\frac{7}{2}\right\}$

Integers $\in \{2, 3, 4, 0\}$

45. (B)

$f(1) = 1 + a + b + c$

$f(0) = c$

$f(0) \cdot f(1) = c[1 + a + b + c] > 0$

46. (C)

$\cos x = \frac{3}{\sqrt{10}}$

$\sin x = \frac{1}{\sqrt{10}}$

$\log_{10}(s.c.\tan x)$

$= \log_{10}\left(\frac{1}{\sqrt{10}} \cdot \frac{3}{\sqrt{10}} \cdot \frac{1}{3}\right)$

$= \log_{10}\left(\frac{1}{10}\right) = -1$

47. (B)

$\tan\frac{\pi}{16} + 2\tan\frac{\pi}{8} + 4$

$\cot\theta - \tan\theta = 2\cot 2\theta$

$\therefore \cot\frac{\pi}{16} - 2\cot\frac{\pi}{8} + 2\tan\frac{\pi}{8} + 4$

$= \cot\frac{\pi}{16} - \left(4\cot\frac{\pi}{4}\right) + 4$

$$= \cot\frac{\pi}{16}$$

48. (A)

$$x^3 - Ax^2 + Bx - C = 0\begin{cases}\alpha - 1\\ \beta - 1\\ \gamma - 1\end{cases} \quad y$$

$$x^3 + Px^2 + Qx - 19 = 0\begin{cases}\alpha\\ \beta\\ \gamma\end{cases} \quad x$$

$y = x - 1$ or $x = y + 1$

$$(y+1)^3 + P(y+1)^2 + Q(y+1) - 19 = 0$$

$$x^3 - Ax^2 + Bx - C = 0$$

$$1 = \frac{-A}{3+1} = \frac{B}{3+2P+Q} = \frac{-C}{P+Q-18}$$

$\therefore\ A + B + C$

$= -3 - P + 3 + 2P + Q - P - Q + 18$

$= 18$

49. (D)

$$\frac{\cos 40^\circ + \cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{\cos 40^\circ + 2\sin 30 \cdot \sin\left(-10^\circ\right)}{\sin 20^\circ}$$

$$= \frac{\cos 40^\circ - \cos 80^\circ}{\sin 20^\circ}$$

$$= \frac{2\sin 60^\circ \cdot \sin 20^\circ}{\sin 20^\circ}$$

$$= \sqrt{3}$$

50. (C)

$$x^3 + qx + q = 0$$

$$\sum\alpha = 0; \sum\alpha\beta = +q; \alpha\beta\gamma = -q$$

$$\sum\frac{1}{\alpha+\beta} = \sum\frac{1}{-\gamma}$$

$$= -\left(\frac{\sum\alpha\beta}{\alpha\beta\gamma}\right)$$

$$= \frac{-q}{-q} = 1$$

51. (A, B, C, D)

(A) $a < 0; c < 0$

$\frac{-b}{2a} < 0 \Rightarrow b < 0$

$\therefore$ $abc < 0$

(B) $a < 0; c > 0$

$\frac{-b}{2a} > 0 \Rightarrow b > 0$

$\therefore$ $abc < 0$

(C) $a > 0; c > 0$

$\frac{-b}{2a} > 0 \Rightarrow b < 0$

$\therefore$ $abc < 0$

(D) $a < 0, c < 0$

$\frac{-b}{2a} < 0 \Rightarrow b < 0$

$abc < 0$

52. (A, B, C)

$2\cos 4x \cdot \cos 8x - 2\cos 5x \cdot \cos 9x = 0$

$\Rightarrow$ $\cos 12x + \cos 4x - \cos 14x - \cos 4x = 0$

$\Rightarrow$ $\cos 12x - \cos 14x = 0$

$\Rightarrow$ $2\sin 13x \cdot \sin x = 0$

53. (A, B, C, D)

Let x be common root

$4\alpha^2 - 11\alpha + 2k = 0$

$4\alpha^2 - 12\alpha - 4k = 0$

$-\quad +\quad +$

$\alpha = -6K$

$\therefore$ $x^2 - 3x - k = 0$ satisfies 'α'

$\Rightarrow 36k^2 + 18k - k = 0$

$36k^2 + 17k = 0$

$k = 0, \frac{-17}{36}$

$\alpha = 0, \frac{17}{6}$

54. (A, C)

$|x^2 - x - 6| = x + 2$

$|x - 3| \cdot |x + 2| = x + 2$

Case-1: $x \geq 3$ or $x \leq -2$

$(x-3)\cdot(x+2)-(x+2)=0$

$(x+2)(x-4)=0$

$x=-2, 4$

Case-2: $x \in (-2, 3]$

$-(x-3)\cdot(x+2)=x+2$

$\Rightarrow (x+2)(1+x-3)=0$

$(x+2)(x-2)=0$

$x=-2, 2$

$x=2$

$x \in \{2, -2, 4\}$

55. (B, C)

$x+2\sqrt{x-1}=\left(\sqrt{x-1}+1\right)^2$

$\dfrac{1}{\left|\sqrt{x-1}+1\right|}+\dfrac{1}{\left|\sqrt{x-1}-1\right|}$ $\quad x-1 \geq 1$

$=\dfrac{1}{\sqrt{x-1}+1}+\dfrac{1}{\sqrt{x-1}-1}; x>2$

$=\dfrac{2\sqrt{x-1}}{(x-2)}; x>2$

$\dfrac{1}{\sqrt{x-1}+1}-\dfrac{1}{\left(\sqrt{x-1}-1\right)};\ x \in [1, 2)$

$\dfrac{-2}{x-2}$' $x \in [1, 2)$

56. (2)

$f_k(x)=\dfrac{1}{k}\left(\sin^k x+\cos^k x\right)$

$f_4(x)-f_6(x)$

$=\dfrac{1}{4}\left(s^4+c^4\right)-\dfrac{1}{6}\left(s^6+c^6\right)$

$=\dfrac{1}{4}\left(1-2c^2s^2\right)-\dfrac{1}{6}\left(1-3c^2s^2\right)$

$=\dfrac{1}{4}-\dfrac{1}{6}=\dfrac{1}{12}=\lambda$

$\therefore\ 24\lambda=2$

57. (2)

$\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cdot\cos\beta$

$\Rightarrow \left(\sin^2\alpha - 2\cos^2\beta\right)^2 + \left(2\sin\alpha\cos\beta - \sqrt{2}\right)^2 = 0$

$\therefore\ \sin^2\alpha = 2\cos^2\beta\ \ \&\ \ \sin\alpha\cos\beta = \dfrac{1}{\sqrt{2}}$

$\Rightarrow\ \dfrac{1}{2\cos^2\beta} = 2\cos^2\beta$

$\Rightarrow\ \cos^4\beta = \dfrac{1}{2^2}\ \Rightarrow\ \cos\beta = \pm\dfrac{1}{\sqrt{2}}$

$\sin\alpha = \pm 1\ \Rightarrow\ \sin\alpha = +1\ \ \&\ \cos\beta = +\dfrac{1}{\sqrt{2}}$ only

$\therefore\ \cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\cdot\sin\beta = -2\cdot 1\cdot\dfrac{1}{\sqrt{2}} = -\sqrt{2}$

58. (2)

$1 - 2s^2c^2 = sc$

$\Rightarrow\ 2t^2 + t - 1 = 0;$ where $t = s\cdot c$

$(2t-1)(t+1) = 0$

$t = \dfrac{1}{2}$ or $t = -1$

$\sin x\cdot\cos x = \dfrac{1}{2}$ or $\sin 2x = -2(\text{rej})$

$\sin 2x = 1$

$[0, 2\pi] \to 2x = \dfrac{\pi}{2}\ \&\ \dfrac{5\pi}{2}$

$x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

$\therefore$ 2 solution

59. (1)

$f(n,\theta) = \left(\sec\dfrac{\theta}{2}\cdot\sec\theta\cdot\ldots.\sec\left(2^{n-1}\theta\right)\right)\cdot(1+\cos\theta)\cdot(1+\cos 2\theta)\cdot(1+\cos 4\theta)\ldots\ldots\ldots(n+1)$ terms

$= \left(\sec\dfrac{\theta}{2}\cdot\sec\theta\cdot\ldots\ldots\cdot\sec 2^{n-1}\theta\right)\times 2\cos^2\dfrac{\theta}{2}\cdot 2\cos^2\theta\ldots\ldots.(n+1)$ term

$= 2^{n+1}\cos\dfrac{\theta}{2}\cdot\cos\theta\ldots\ldots\cos 2^{n-1}\theta$

$= \cancel{2^{n+1}} \times \dfrac{\sin\left[2^{n+1}\cdot\dfrac{\theta}{2}\right]}{\cancel{2^{n+1}}\sin\dfrac{\theta}{2}}$

$$=\frac{\sin\left(2^n\theta\right)}{\sin\frac{\theta}{2}}$$

$$f\left(3,\frac{2\pi}{17}\right)=\frac{\sin\left(8\cdot\frac{2\pi}{17}\right)}{\sin\frac{\pi}{17}}=1$$

60. (0)

$-x^2+x-1=\sin^4 x$

Range of quadratic

$$\in\left(-\infty,-\frac{(1-4)}{-4}\right]$$

$$\in\left(-\infty,-\frac{3}{4}\right]$$

But $\sin^4 x>0$

$\therefore$ 0 Solution