PART (A) : PHYSICS

ANSWER KEY

1.	(C)	2.	(D)	3.	(C)	4.	(C)	5.	(A)
6.	(A)	7.	(ABCD)	8.	(AD)	9.	(BD)	10.	(AB)
11.	(BC)	12.	(BD)	13.	(C)	14.	(B)	15.	(A)
16.	(C)	17.	(B)	18.	(A)				

SOLUTIONS

1. (C)

The retardation is given by $\frac{dv}{dt} = -av^2$

Integrating between proper limits $\Rightarrow -\int_u^v \frac{dv}{v^2} = \int_0^t a\,dt$ or $\frac{1}{v} = at + \frac{1}{u}$

$\Rightarrow \frac{dt}{dx} = at + \frac{1}{u} \qquad \Rightarrow dx = \frac{u\,dt}{1+aut}$

Integrating between proper limits $\Rightarrow \int_0^s dx = \int_0^t \frac{u\,dt}{1+aut} \Rightarrow S = \frac{1}{a}\ln(1+aut)$

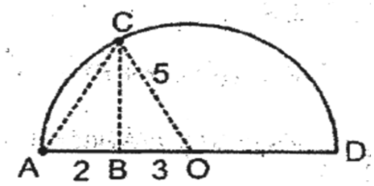
2. (D)

From triangle $BCO \Rightarrow BC = 4$

From triangle $BCA \Rightarrow AC = \sqrt{2^2+4^2} = 2\sqrt{5}$

$AC = u_1 t\,;\ BC = u_2 t$

$\therefore \frac{u_1}{u_2} = \frac{AC}{BC} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{\sqrt{4}}$



3. (C)

Angle is dimensionless but has unit (radian or degree)

4. (C)

Position vector of point $A = 2\hat{i} + \hat{j}$

Position vector of point $B = 9\hat{i} + 25\hat{j}$

$\therefore \overrightarrow{AB} = (9\hat{i} + 25\hat{j}) - (2\hat{i} + \hat{j}) = 7\hat{i} + 24\hat{j}$

Unit vector in the direction of $\overrightarrow{AB}$

$AB = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{7\hat{i}+24\hat{j}}{25}$

$\therefore \vec{v} = 50AB = 14\hat{i} + 48\hat{j}$

5. (A)

$$V = \frac{ds}{dt} = 3t^2 - 12t + 3$$

$$a = \frac{dv}{dt} = 6t - 12 = 0$$

$t = 2\,\text{s}$

At. $t = 2\,\text{s}$,

$V = 3\times 2^2 - 12\times 2 + 3 = 12 - 24 + 3 = -9$ m/s

6. (A)

$[g] = LT^{-2}$ and numerical value $\propto \frac{1}{\text{unit}}$

7. (ABCD)

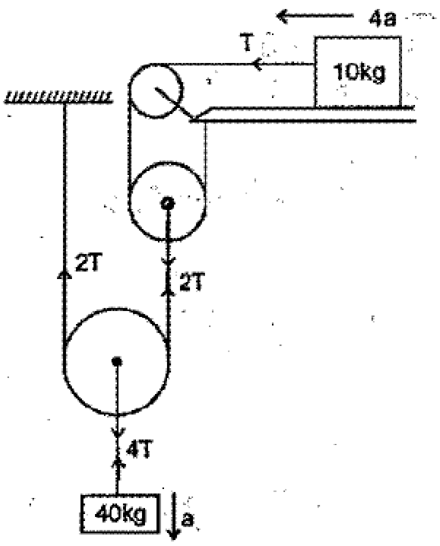
60 = 15t ; t = 4 sec, therefore U_y = 20 m/s

8. (AD)

Slope of given v-t graph (i.e. acceleration) is constant.
From 0 to 10 seconds, velocity is in positive and then negative. That means the particle turns around at t = 10 sec.
The positive and negative areas are not equal. So displacement is not zero.
Area of v-t graph from t = 0 to t = 10 sec is same as that from t = 10 to 20 sec.
Hence average speed is same.

9. (BD)

Applying NLM on 40 kg block
400 – 4T = 40a
For 10 kg block T = 10.4 a
Solving a = 2 m/s^2
T = 80 N

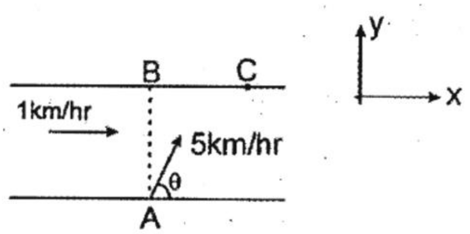


10. (AB)

$AB = BC = 400\,\text{m} = 0.4\,\text{km}$

$v_x = 5\cos\theta + 1$

$v_y = 5\sin\theta$



tame taken $(t) = \dfrac{AB}{v_y} = \dfrac{BC}{v_x}$

$\Rightarrow \; v_y = v_x \quad \Rightarrow \; 5\sin\theta = 5\cos\theta + 1$

$\Rightarrow \; \theta = 53^\circ$ and $t = \dfrac{0.4}{5\sin(53^\circ)} = 0.1\,\text{hr} = 6\,\text{min}$.

11. (BC)

For equilibrium $N_A \cos 60^\circ + N_B \cos 30^\circ = Mg$

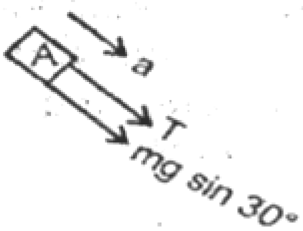
and $N_a \sin 60^\circ = N_B \sin 30^\circ$

On solving $N_B = \sqrt{3} N_A$

$N_A = \dfrac{Mg}{2}$

12. (BD)

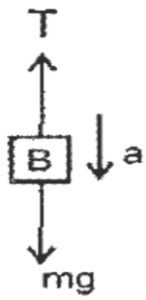
Let a be acceleration of system and T be tension in, the string.



F.B.D. of block A

$mg\sin 30^\circ + T = ma$

$\dfrac{mg}{2} + T = ma \qquad \ldots(\text{i})$



F.B.D. of B

$mg - T = ma \qquad \ldots(\text{ii})$

Adding equation (i) & (ii); we get

$2ma = \dfrac{3mg}{2} \;\Rightarrow\; a = \dfrac{3}{4}g$

From equation (i); $T = \dfrac{mg}{4}$

13. (C)

14. (B)

15. (A)

16. (C)

$x = t^2 - 4$

$y = t - 4$

$x = (y+4)^2 - 4$

$x = y^2 + 8y + 12$

17. (B)

$t^2 - 4 = 0 \Rightarrow t = 2\,\text{sec}$

18. (A)

$t = 2\,\text{sec}$

$V_x = \frac{dx}{dt} = 2t = 4 \text{ m/s}$

$V_y = \frac{dy}{dx} = 1$

$V = \sqrt{4^2 + 1^2} = \sqrt{17}$

PART (B) : CHEMISTRY

ANSWER KEY

19.	(B)	20.	(D)	21.	(A)	22.	(A)	23.	(D)
24.	(C)	25.	(ABCD)	26.	(AD)	27.	(ABC)	28.	(AC)
29.	(ABD)	30.	(C)	31.	(A)	32.	(A)	33.	(B)
34.	(C)	35.	(B)	36.	(B)				

SOLUTIONS

19. (B)

$P_4 + O_2 \rightarrow P_4O_6 + P_4O_{10}$

31 gram 32 gram

According to question weight of P is conserved to

Let Mole of P_4O_6 = a

Mole of P_4O_{10} = b

Initial weight of P = Final weight of P.

$31 = [a \times 4] \times 31 + [b \times 4] \times 31$

$4a + 4b = 1]$ $(1) \times 3$

Initial weigh of oxygen = Final weight of oxygen

$32 = [a \times 6] \times 16 + [a \times 10] \times 16$

$3a + 5b = 1]$ $(2) \times 4$

$12a + 20b = 4$

$12a + 12b = 3$ So, $b = \dfrac{1}{8}$

$8b = 1$

Similarly $a = \dfrac{1}{8}$

So weight of $P_4O_6 = \dfrac{1}{8} \times 220 = 27.5$ $P_4O_{10} = \dfrac{284}{8} = 35.5$

20. (D)

21. (A)

22. (A)

23. (D)

24. (C)

25. (ABCD)

	Silica	H_2O	Impurities
% in original clay $\Rightarrow$	40	19	100 – (40 + 19) = 41
% after partial drying $\Rightarrow$	a	10	100 – (a + 10) = 90 – a

On heating, only water evaporates from clay, whereas silica and impurities are left as it is. Therefore, % ratio of silica and impurities remains unchanged, i.e.

$$\frac{40}{a} = \frac{41}{90-a}, \quad \therefore\ a = 44.4\%$$

% of impurities after partial drying = (90 – a) = (90 – 44.4) = 45.6%

26. (AD)

(A) Weight of $CaCO_3$ = (0.22 g CO_2)

$$\left(\frac{1\,\text{mol } CO_2}{44\text{ g } CO_2}\right)\left(\frac{1\,\text{mol } CaCO_2}{\text{mol } CO_2}\right)\left(\frac{100\text{ g } CaCO_3}{\text{mol } CaCO_3}\right)\frac{0.22\times100}{44} = 0.5\text{ g } CaCO_3$$

(B) Moles of $CaCO_3$ = moles of Ca = $\left(\frac{0.22}{44}\right) = 0.005$ mol

Weight Ca = 0.005 × 40 = 0.2 g Ca

(D) % of Ca $= \frac{0.2}{1.0}\times100 = 20\%$ Ca

Hence (C) is wrong.

27. (ABC)

(A) For n = 5, $\ell_{\min} = 0$. $\therefore$ Orbital angular momentum $= \sqrt{\ell(\ell+1)}\,\hbar = 0$. (False(

(B) Outermost electronic configuration = $3s^1$ or $3s^2$. $\therefore$ possible atomic number = 11 or 12 (False).

(C) Mn_{25} = [Ar] $3d^5\ 4s^2$. $\therefore$ 5 unpaired electrons. $\therefore$ Total spin $= \pm\frac{5}{2}$ (False).

(D) Inert gases have no unpaired electrons. $\therefore$ spin magnetic moment = 0 (True).

28. (AC)

29. (ABD)

30. (C)

31. (A)

n = 1	n = 2	n = 3	n = 4
$\ell = 0 \rightarrow 1s$	$\ell = 0 \rightarrow 2s$	$\ell = 0 \rightarrow 3s$	$\ell = 0 \rightarrow 4s$
$\ell = 1 \rightarrow 1p$	$\ell = 1 \rightarrow 2p$	$\ell = 1 \rightarrow 3p$	$\ell = 1 \rightarrow 4p$
$\ell = 2 \rightarrow 1\,d$	$\ell = 2 \rightarrow 2\,d$	$\ell = 2 \rightarrow 3d$	$\ell = 2 \rightarrow 4d$
	$\ell = 3 \rightarrow 2f$	$\ell = 3 \rightarrow 3f$	$\ell = 3 \rightarrow 4f$
		$\ell = 4 \rightarrow 3g$	$\ell = 4 \rightarrow 4g$
			$\ell = 5 \rightarrow 4\,h$

Electronic configuration $\rightarrow$ 1s, 1p, 2s, 1d, 2p, 3s, 3d, 3p, 4s, 3d
For third period $3s^3$, $3d^{15}$, $3p^9$
number of element = 27

32. (A)
$1s^3$, $1p^9$, $2s^3$, $1d^{14}$

33. (B)
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34. (C)

35. (B)

36. (B)

PART (C) : MATHEMATICS

ANSWER KEY

37.	(A)	38.	(B)	39.	(C)	40.	(D)	41.	(B)
42.	(D)	43.	(ABCD)	44.	(BCD)	45.	(ABC)	46.	(ABD)
47.	(A)	48.	(BCD)	49.	(D)	50.	(C)	51.	(A)
52.	(C)	53.	(B)	54.	(A)				

SOLUTIONS

37. (A)

$|3x-9|+2\geq 2$ OR $|3x-9|+2\leq -2$

$|3x-9|\geq 0$ $\qquad |3x-9|\leq -4$

$x\in R$

$\therefore \quad x\in(-\infty,\infty)$

38. (B)

$a^2-1=0$ and $a^2-4a+3=0$ and $a^2-100a+99=0$

$a=1,-1$ and $a=1,3$ and $a=1,99$

$\therefore\ a=1$ (only one value possible).

39. (C)

$$\sin(\theta+\phi-\phi)=5\sin(\theta+\phi)$$

$$\Rightarrow \sin(\theta+\phi)\cdot\cos\phi-\cos(\theta+\phi)\sin\phi=5\sin(\theta+\phi)$$

$$\Rightarrow \tan(\theta+\phi)\cdot\cos\phi-\sin\phi=5\cdot\tan(\theta+\phi)$$

$$\Rightarrow \tan(\theta+\phi)=\frac{\sin(\phi)}{\cos\phi-5}$$

40. (D)

$$N=\frac{4\times4^5\times6\times6^5}{3\times3^5\times2\times2^5}=\frac{4^6\times6^6}{3^6\times2^6}=\left(\frac{4\times6}{3\times2}\right)^6=4^6$$

$\therefore\ \log_2 4^6=12$

41. (B)

$$8\cos x\cdot\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$$

$$8\cos x\left(\frac{3}{4}-\sin^2 x-\frac{1}{2}\right)=1$$

$$8\cos x\left(\frac{1}{4}-\sin^2 x\right)=1$$

$$2\cos x-8\cos x\left(1-\cos^2 x\right)=1$$

$$2\cos x-8\cos x+8\cos^3 x=1$$

$$8\cos^3 x-6\cos x-1=0$$

$$2\left(4\cos^3 x-3\cos x\right)=1$$

$$2\cos 3x=1$$

$$\cos 3x=\frac{1}{2}$$

$$\therefore\ 3x=2n\pi\pm\frac{\pi}{3}\quad n\in I$$

$$x=\frac{2n\pi}{3}\pm\frac{\pi}{9}$$

$$n=0,\ x=\frac{\pi}{9},\ n=1,\ x=\frac{2\pi}{3}-\frac{\pi}{9}=\frac{5\pi}{9},\ \frac{2\pi}{3}+\frac{\pi}{9}=\frac{7\pi}{9}$$

$$\therefore\ \frac{\pi}{9}+\frac{5\pi}{9}+\frac{7\pi}{9}=\frac{13\pi}{9}$$

42. (D)

$$x^2-3kx+2\cdot k^2-1=0$$

$$2k^2-1=7\ \Rightarrow\ 2k^2=8\ \Rightarrow\ k=\pm 2.$$

$\therefore$ Sum = $3k$ = 6. (k = –2 rejected).

43. (ABCD)

Case 1: $x>0$

$$\frac{x^2-7x+10}{x^2-6x+9}<0$$

$$\frac{(x-2)(x-5)}{(x-3)^2}<0$$

+ | – | – | +
2 3 5

$\therefore\ x\in(2,3)\cup(3,5)$

Case 2: $x<0$

$$\frac{x^2+7x+10}{x^2-6x+9}<0$$

$$\frac{(x+2)(x+5)}{(x-3)^2}<0$$

+ | – | + | +
–5 –2 3

$\therefore\ x\in(-5,-2)$

44. (BCD)

$$\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}=\frac{5}{2} \text{ and } \tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}=\frac{3}{2}$$

$$\therefore\ \tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)=\frac{\frac{5}{2}}{1-\frac{3}{2}}=\frac{\frac{5}{2}}{\frac{-1}{2}}=-5$$

$$\sin(\alpha+\beta)=\frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)}=\frac{-10}{1+25}=\frac{-10}{26}=\frac{-5}{13}$$

$$\tan(\alpha+\beta)=\frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)}=\frac{-10}{1-25}=\frac{-10}{-24}=\frac{5}{12}$$

$$=\frac{\frac{-32}{11}}{1-\frac{256}{121}}=\frac{\frac{-32}{11}}{\frac{-135}{121}}=\frac{352}{135}$$

$$\therefore\quad \cos(2\alpha+2\beta)=\frac{1-\tan^2(\alpha+\beta)}{1+\tan^2(\alpha+\beta)}=\frac{1-\frac{25}{144}}{1+\frac{25}{144}}=\frac{119}{169}$$

45. (ABC)

$$2\sin\left(\frac{x}{2}\right)\left(\cos^2 x-\sin^2 x\right)=\cos^2 x-\sin^2 x$$

$$2\sin\frac{x}{2}\cdot\cos 2x-\cos 2x=0$$

$$(\cos 2x)\left(2\sin\frac{x}{2}-1\right)=0$$

$$\therefore\ \cos 2x=0 \text{ or } \sin\frac{x}{2}=\frac{1}{2}$$

$$\therefore\ 2x=2n\pi\pm\frac{\pi}{2} \text{ or } \therefore\ \frac{x}{2}=n\pi+(-1)^n\cdot\frac{\pi}{6}$$

$$x=n\pi\pm\frac{\pi}{4} \qquad x=2n\pi+(-1)^n\cdot\frac{\pi}{3} \qquad n\in I$$

46. (ABD)

$$5^{x+1}=7^x$$

$$(x+1)\log 5=x\cdot\log 7$$

$$x+1=x\cdot\log_5 7$$

$$x(1-\log_5 7)=-1$$

$$\therefore\ x=\frac{1}{\log_5 7-1}=\frac{1}{\frac{\log_3 7}{\log_3 5}-1}=\frac{\log_3 5}{\log_3 7-\log_3 5}$$

$$\therefore\ x=\frac{\log_3 5}{\log_3(7/5)}=\log_{7/5}5=\log_{1.4}5$$

47. (A)

$$D>0\ \Rightarrow\ 1-4a^2\geq 0$$

$$\Rightarrow\ 4a^2-1\leq 0$$

$$\Rightarrow\ a\in\left(-\frac{1}{2},\frac{1}{2}\right)-\{0\}$$

Now, $|x_1-x_2|<1$

$$\left|\frac{\sqrt{D}}{a}\right|<1\ \Rightarrow\ \left|\frac{\sqrt{1-4a^2}}{a}\right|<1\ \Rightarrow\ a\in\left(-\infty,-\frac{1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\infty\right)$$

Now taking intersection

$$\therefore\ a\in\left(-\frac{1}{2},-\frac{1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\frac{1}{2}\right)$$

48. (BCD)

$$\log x=k(b-c),\ \log y=k(c-a),\ \log z=k(a-b)$$

$$\therefore\ x=10^{k(b-c)},\ y=10^{k(c-a)},\ z=10^{k(a-b)}$$

Then $xyz=10^{o}=1$.

$$x^a\cdot y^b\cdot z^c=10^{o}=1$$

$$x^{b+c}\cdot y^{c+a}\cdot z^{a+b}=10^{k\left(b^2-c^2+c^2-a^2+a^2-b^2\right)}=1$$

$$x^{b^2+bc+c^2}\cdot y^{c^2+ac+a^2}\cdot z^{a^2+ab+b^2}=10^{k\left(b^3-c^3+c^3-a^3+a^3-b^3\right)}$$

$$=10^{o}=1.$$

Solution for Passage 1 Que. No. 49, 50 & 51

49. (D)

50. (C)

51. (A)

$$y=ax^2+bx+c$$

Vertex $\equiv\left(\frac{-b}{2a},\frac{-D}{4a}\right)\equiv(3,-2)$

$\therefore\ \frac{-b}{2a}=3$ and $\frac{-D}{4a}=-2$ and $c=10$.

$\therefore\ b=-6a$ and $4ac-b^2=-8a$ and $c=10$.

$b=-6a$ and $40a-b^2=-8a$

Now, $a\neq 0$

$\therefore\ 40a-36a^2=-8a$

$\therefore\ 40-36a=-8 \Rightarrow 36a=48$

$\therefore\ b=-6\times\frac{4}{3}=-8$ (**Ans. of Q. 49**)

$\therefore$ Quadratic equation is $\frac{4}{3}x^2-8x+10=0$

or $4x^2-24x+30=0\begin{cases}\alpha\\ \beta\end{cases}$

$\therefore\ \alpha+\beta=\frac{24}{4}=6;\ \alpha\beta=\frac{30}{4}=\frac{15}{2}$

$\therefore\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=36-15=21$

$\therefore\ S=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{21\times 2}{15}=\frac{14}{5}$

$p=1\ \therefore$ equation $\Rightarrow x^2-\frac{14}{5}x+1=0 \Rightarrow 5x^2-14x+5=0$ (**Ans. of Q.50**)

$y\geq\frac{-2}{3}$.

$\frac{4}{3}x^2-8x+10\geq\frac{-2}{3}$

$4x^2-24x+30\geq -2$

$4x^2-24x+32\geq 0$

$x^2-6x+8\geq 0$

$(x-2)(x-4)\geq 0$

$x\in(-\infty,2]\cup[4,\infty)$ (**Ans. of Q.51**)

Solution for Passage 2 Que. No. 52, 53 & 54

52. (C)

53. (B)

54. (A)

$x^2+y^2=1 \Rightarrow x=\cos\theta,\ y=\sin\theta$

$\therefore\ P=(3\cos\theta-4\cos^3\theta)^2+(3\sin\theta-4\sin^3\theta)^2$

$P=\cos^2 3\theta+\sin^2 3\theta=1$

$Q=\cos^6\theta+\sin^6\theta=1-3\cos^2\theta\sin^2\theta$

$$=1-\frac{3}{4}\sin^2 2\theta$$

$$Q_{\min}=\frac{1}{4};\quad Q_{\max.}=1$$

$$R=3\left(\cos^4\theta+\sin^4\theta\right)-2\left(\cos^6\theta+\sin^6\theta\right)-1$$

$$=3\left(1-2\cos^2\theta\sin^2\theta\right)-2\left(1-3\cos^2\theta\sin^2\theta\right)-1$$

$$=3-6\sin^2\theta\cos^2\theta-2+6\sin^2\theta\cos^2\theta-1$$

$$R=0$$