PART (A) : PHYSICS

ANSWER KEY

1.	(A)	2.	(B)	3.	(B)	4.	(A)	5.	(C)
6.	(D)	7.	(D)	8.	(A)	9.	(D)	10.	(C)
11.	(AB)	12.	(CD)	13.	(ABC)	14.	(AC)	15.	(3)
16.	(4)	17.	(7)	18.	(6)				

SOLUTIONS

1. (A)

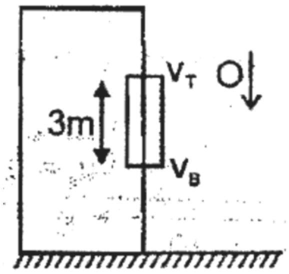
$$s=\frac{(u+v)}{2}t$$

$$3=\frac{(v_T+v_B)}{2}\times 0.5$$

$$v_T+v_B=12\text{ m/s}$$

Also, $v_B=v_T+(9.8)(0.5)$ .....(2)

$$v_B-v_T=4.9\text{ m/s}$$



2. (B)
Using dimensional analysis

3. (B)
Let a be the retardation produced by resistive force, t_a and t_d be the time of ascent and descent respectively.

$$\therefore\quad \frac{t_a}{t_d}=\sqrt{\frac{g-a}{g+a}}=\sqrt{\frac{10-2}{10+2}}=\sqrt{\frac{2}{3}}$$

4. (A)
$V=a+bx$ (V increases as x increases)

$$\frac{dV}{dt}=b\frac{dx}{dt}=bV$$

Hence, acceleration increases as V increase with x.

5. (C)

For A, $\dfrac{ds}{dt}=V_A=\dfrac{1}{\sqrt{3}}$

For B, $\dfrac{ds}{dt}=V_B=\sqrt{3}$

$$\frac{V_A}{V_B}=\frac{1}{3}.$$

6. (D)

Unit vector along AB

$$= \frac{(4-1)\hat{i} + 4\hat{j} + 12\hat{k}}{\sqrt{3^2 + 4^2 + 12^2}}$$

$$= \frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13}$$

Speed = 65 m/s,

Time = 2 s

Distance = 130 m along AB

$$\therefore \quad \overrightarrow{Displacement} = \frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13} \times 130$$

$$= 30\hat{i} + 40\hat{j} + 120\hat{k}$$

$\therefore$ triangle position $k = (30+1)\hat{i} + 1 + 0\hat{j} + 120\hat{k}$

7. (D)

$$\tan 90^\circ = \frac{v\sin\theta}{u + v\cos\theta}$$

$$\therefore \quad u + v\cos\theta = 0$$

$$R = \sqrt{u^2 + v^2 + 2uv\cos\theta} = \frac{1}{2}v$$

$$\Rightarrow \quad (-v\cos\theta)^2 + v^2 + 2(-v\cos\theta)v\cos\theta = \frac{v^2}{4}$$

$$\Rightarrow \quad v^2\cos^2\theta + v^2 - 2v^2\cos^2\theta = \frac{v^2}{4}$$

$$\Rightarrow \quad 1 - \cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \quad \sin^2\theta = \frac{1}{2} \qquad \sin\theta = \pm\frac{1}{2}$$

$$\theta = 30^\circ, 150^\circ$$

8. (A)

Using Integration by substitution

9. (D)

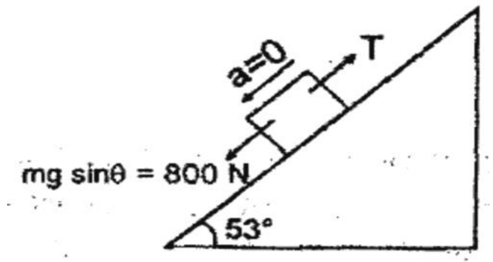
Ranges for complementary angles are same

$$\therefore \quad \text{Required angle} = \frac{\pi}{2} - \frac{5\pi}{36} = \frac{13\pi}{36}$$

10. (C)

For clock to be stationary, T = 800 N

If man moves up by acceleration ‘a’



$T - mg = ma \quad \Rightarrow \quad 800 - 500 = 50a \quad \Rightarrow \quad a = 6\ \text{m/s}^2$

11. (AB)

$S_1 - S_2 = 125\text{m}$ if $S_1 > S_2$ then

$50t - \frac{1}{2}\times 10t^2 = 125$

$10t - t^2 = 25$

$t^2 - 10t + 25 = 0$

$t = 5\text{sec}.$

$S_2 - S_1 = 125\text{m}$ if $S_2 > S_1$ then,

$\frac{1}{2}\times 10t^2 - 50t = 125$

$t^2 - 10t - 25 = 0$

$t = \dfrac{10 + \sqrt{100+100}}{2}$

$t = 5(1+\sqrt{2})\text{sec}.$

12. (CD)

Displacement = 0 ($\because$ initial position = final position)

Average velocity = 0 ($\because$ total displacement = 0)

13. (ABC)

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(1-1) - \hat{j}(2-1) + \hat{k}(2-1) = -\hat{j} + \hat{k}$$

Unit vector $\perp$ to $\vec{A}$ and $\vec{B}$ is $\left(\dfrac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)$

14. (AC)

For painter;

$R + T - mg = ma$

$R + T = m(g+a)$ ...(1)

For the system;

$2T - (m+M)g = (m+M)a$

$2T = (m+M)(g+a)$ ...(2)

Where; m = 100 kg

$M = 50$ kg
$a = 5$ m/sec^2
$\therefore \quad T = \dfrac{150\times 15}{2} = 1125\,\text{N}$ and $R = 375\,\text{N}$

15. (3)
Taking motion in vertical direction

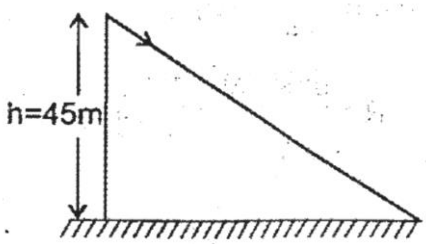
$u = 0,\ g = 10\ \text{m/s}^2,\ h = 45\,\text{m}$

$h = ut + \dfrac{1}{2}gt^2$

$\Rightarrow \quad h = 0 + \dfrac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 45}{10}}$

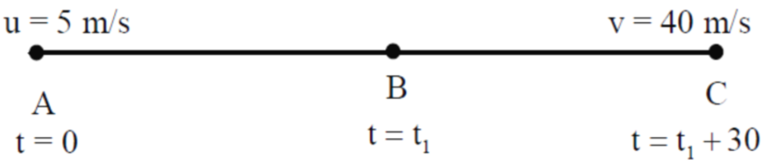
$\Rightarrow \quad t = 3\,\text{sec}.$



16. (4)
$v\sin 30 = 10;\ v =$ 20 km/hr

17. (7)
Using $S_y = -70,\ U_y = 25,\ A_y = -10$

18. (6)



Consider AC, $u = 5$ m/s
$v = 40$ m/s
$a = 0.5$ m/s^2

$t = \dfrac{v-u}{a} = \dfrac{40-5}{0.5} = 70\,\text{s}$

For $AB,\ u = 5\ \text{m/s},\ a = 0.5\ \text{m/s}^2$

$t = 70 - 30 = 40\,\text{s}$

$AB = s = ut + \dfrac{1}{2}at^2 = 5\times 40 + \dfrac{1}{2}\times 0.5\times 40^2$

= 600 m

PART (B) : CHEMISTRY

ANSWER KEY

19.	(A)	20.	(B)	21.	(A)	22.	(B)	23.	(D)
24.	(D)	25.	(D)	26.	(C)	27.	(D)	28.	(C)
29.	(ABC)	30.	(ABD)	31.	(BD)	32.	(ABD)	33.	(4)
34.	(6)	35.	(8)	36.	(7)				

SOLUTIONS

19. (A)

$$D = \frac{M}{V} \Rightarrow 1.42 = \frac{100}{V}$$

$$\Rightarrow V = \frac{100}{1.42}$$

Molarity of HNO_3 = $\frac{70\times1.42\times1000}{63\times100} = 15.8$

20. (B)

$$AgNO_3 + Cl^- \rightarrow \underset{1.4\ g}{AgCl\downarrow} + NO_3^-$$

$$= \frac{1.4}{143.5}\text{mol}$$

$$= \frac{1.4}{143.25}\times35.5\ \text{gm Cl} = 0.35\ \text{gm}$$

21. (A)

$2Na_3PO_4 + 3BaCl_2 \rightarrow Ba_3(PO_4)_2 + 6NaCl$

22. (B)

Rb has least ionization energy or work function.

23. (D)

Number of radial nodes of an orbital = n – l – 1

Number of angular nodes of an orbital = l

24. (D)

Conceptual

25. (D)

4f electron does imperfect shielding on outer electrons therefore effective nuclear charge increase.

26. (C)

27. (D)

28. (C)
Down the group size increases

29. (ABC)

30. (ABD)

31. (BD)
(A) λ can be calculated as : $\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}}{1\times100} = 6.626\times10^{-36}$ m. (very small).
(B) de-Broglie wavelength associated with macroscopic particles is extremely small and so, difficult to observer.
(C) de-Broglie wavelength associated with electron can be calculated by using $\lambda = \frac{h}{mv}$.
(D) $KE_f = 5+20 = 25\,eV$. $\therefore\ \lambda = \sqrt{\frac{150}{KE_f}} = \sqrt{\frac{150}{25}} = \sqrt{6}\,Å$.

32. (ABD)

33. (4)
Balanced chemical equation is
$$4XeF_6 + -(-CH_2-CH_2-)_n- \rightarrow -(-CF_2-CF_2-)_n- + 4nHF + 4nXeF_4$$
$$n_{teflon} = \frac{100}{100n} = \frac{1}{n}$$
$\therefore$ n_{XeF_6} required $= \frac{1}{n}\times 4n = 4$ moles

34. (6)
$$A + \frac{1}{2}B_2 \longrightarrow AB,\quad 100\ Kcal$$
x $\quad x/2 \quad$ x
$$A + 2B_2 \longrightarrow AB_4,\quad 200\ Kcal$$
$(1-x)$ $\quad 2(1-x) \quad$ $(1-x)$
$100x + 200(1-x) = 140$
$200 - 100x = 140$
$x = \frac{60}{100} = 0.6$
Ans. $= 0.6 \times 10 = 6$

35. (8)

In H–atom, 4 lines are observed in Balmer series. So, electron is in n = 6(6 $\rightarrow$ 2, 5 $\rightarrow$2, 4 $\rightarrow$ 2, 3 $\rightarrow$ 2).
In He^+ ion, one line is observed in Paschen series. So electron is in n = 4(4 $\rightarrow$ 3).
$(H)_{6 \rightarrow 2} = (He^+)_{12 \rightarrow 4}$
$\therefore$ electron in He^+ will jump from n = 4 to n = 12.

36. (7)

$$\sqrt{\ell+(\ell+1)}\,\frac{h}{2\pi}=\sqrt{3}\,\frac{h}{\pi}$$
$\Rightarrow \ell=3$
No. of orientation $=2\ell+1=7$

PART (C) : MATHEMATICS

ANSWER KEY

37.	(D)	38.	(C)	39.	(A)	40.	(C)	41.	(D)
42.	(D)	43.	(B)	44.	(B)	45.	(B)	46.	(D)
47.	(ABC)	48.	(ABD)	49.	(ACD)	50.	(AB)	51.	(2)
52.	(8)	53.	(1)	54.	(6)				

SOLUTIONS

37. (D)

$\alpha+\beta=3 \qquad \alpha\beta=7$

Now, $\dfrac{\alpha^{2022}+\beta^{2022}}{\dfrac{1}{\alpha^{2022}}+\dfrac{1}{\beta^{2022}}}=(\alpha\beta)^{2022}=7^{2022}$

38. (C)

Substitute $x\to\dfrac{1}{x}$ $\qquad \therefore \left(\dfrac{1}{x}\right)^3-3\left(\dfrac{1}{x}\right)+1=0$

$\Rightarrow\ 1-3x^2+x^3=0$

$\therefore\ x^3-3x^2+1=0$

39. (A)

$y=ax^2+bx+c$ has no real roots and $c<0$

Hence, $y<0\ \forall\ x\in R$ and graph is downwards opening parabola. ($a<0$).

40. (C)

$\cos\theta\cdot\cos(60^\circ-\theta)\cdot\cos(60^\circ+\theta)=\dfrac{\cos 3\theta}{4}$

$\therefore\ \cos 12^\circ\cdot\cos 48^\circ\cdot\cos 72^\circ\cdot\cos 60^\circ=\dfrac{\cos 36^\circ}{4}\cdot\cos 60^\circ=\dfrac{\sqrt{5}+1}{16}\times\dfrac{1}{2}=\dfrac{\sqrt{5}+1}{32}$

41. (D)

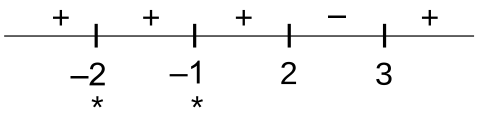
$\dfrac{\sin^2 A}{\cos A(\sin A-\cos A)}+\dfrac{\cos^2 A}{\sin(\sin A-\cos A)}-\dfrac{1}{\sin A\cos A}$

$\Rightarrow\ \dfrac{\sin^3 A-\cos^3 A}{\sin A\cdot\cos A(\sin A-\cos A)}-\sec A\operatorname{cosec} A$

$$= \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A} - \sec A \operatorname{cosec} A$$

$$= \frac{1}{\sin A \cos A} + 1 - \sec A \operatorname{cosec} A = 1$$

42. (D)



$\therefore \; x \in [2, 3) \cup \{-1\}$

43. (B)

This is possible only iff $\cos x = 1$ and $\cos(\pi x) = 1$ both at the same time.

Now, $\cos x = 1 \Rightarrow x \Rightarrow 2n\pi \Rightarrow x = -2\pi, 0, 2\pi, 4\pi, 6\pi, ...$

$\cos(\pi x) = 1 \Rightarrow \pi x = 2k\pi \Rightarrow x \Rightarrow -2, 0, 2, 4, 6, ...$

$\therefore$ only one $(x = 0)$ satisfy the equation.

44. (B)

$\log_2(3x-2) = \log_{1/2} x$

$\Rightarrow \log_2(3x-2) = \log_2\left(\frac{1}{x}\right)$

$\Rightarrow 3x - 2 = \frac{1}{x} \Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (x-1)(3x+1) = 0 \;\; x = 1, -\frac{1}{3}$

But $x = -\frac{1}{3}$ is rejected (Argument of log can't be negative)

$\therefore$ $x = 1$ is the only solution.

45. (B)

$|x^2 - 4x - 21| = -(x^2 - 4x - 21)$ if

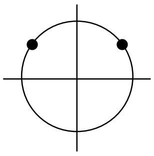
$x^2 - 4x - 21 \le 0$

$\therefore$ $(x+3)(x-7) \le 0$

$\therefore$ $x \in [-3, 7]$

$\therefore$ Sum = – 3 – 2 – 1 + 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 = 22

46. (D)



$$2\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2}$$

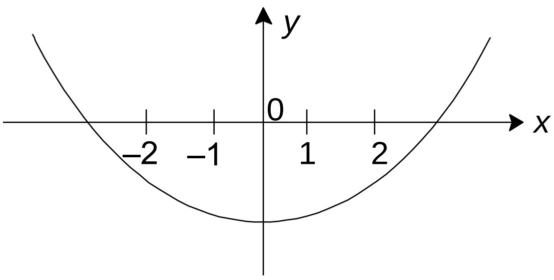
$\therefore$ Four values of x.

47. (ABC)

Since, the equation has two roots.

$\therefore\ b^2 - 4ac > 0$ ...**(A)**

$f(x) = ax^2 + bx + c \quad (a > 0)$



$\therefore\ f(0) = c < 0$ ...**(B)**

$f(1) = a + b + c < 0,\ f(-1) < 0$

$\therefore\ a + |b| + c < 0$ ...**(C)**

$f(2) < 0$ and $f(-2) < 0$

$\therefore\ 4a + 2|b| + c < 0$

48. (ABD)

$$f_n(\theta) = 2^{n+1} \cdot \cos\theta \cdot \cos 2\theta \cdot \cos 4\theta \cdot ...... \cdot \cos 2^n\theta \cdot \sin\theta$$

$$= 2^{n+1} \cdot \frac{\sin\left(2^{n+1} \cdot \theta\right)}{2^{n+1}\sin\theta} \cdot \sin\theta = \sin\left(2^{n+1}\theta\right)$$

$$f_n(\theta) = \sin\left(2^{n+1}\theta\right)$$

$$\therefore\ f_4\left(\frac{\pi}{64}\right) = \sin\left(2^5 \cdot \frac{\pi}{64}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$f_5\left(\frac{\pi}{64}\right) = \sin\left(2^6 \cdot \frac{\pi}{64}\right) = \sin(\pi) = 0$$

$$f_9\left(\frac{\pi}{4096}\right) = \sin\left(2^{10} \cdot \frac{\pi}{4096}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

49. (ACD)

$$4\sin\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{6}\right)=a^2+\sqrt{3}\sin 2x-\cos 2x$$

$$\Rightarrow \quad 2\left[\sin\left(2x+\frac{\pi}{3}-\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)\right]=a^2+\sqrt{3}\sin 2x-\cos 2x$$

$$\Rightarrow \quad 2\left[\sin\left(2x+\frac{\pi}{6}\right)+\sin\frac{\pi}{2}\right]=a^2+\sqrt{3}\sin 2x-\cos 2x$$

$$\Rightarrow \quad 2\left[\sin 2x\cos\left(\frac{\pi}{6}\right)+\cos 2x\sin\frac{\pi}{6}+1\right]=a^2+\sqrt{3}\sin 2x-\cos 2x$$

$$\Rightarrow \quad \sqrt{3}\sin 2x+\cos 2x+2=a^2+\sqrt{3}\sin 2x-\cos 2x$$

$$\Rightarrow \quad 2\cos 2x=a^2-2$$

$$\therefore \quad -2\le a^2-2\le 2$$

$$0\le a^2\le 4$$

50. (AB)

$$x^{\left[(\log_5 x)^2-\frac{9}{2}\log_5 x+5\right]}=(5)^{\frac{3}{2}}$$

Taking log with base 5 both sides.

$$\left[(\log_5 x)^2-\frac{9}{2}(\log_5 x)+5\right]\log_5 x=\frac{3}{2}$$

Take $\log_5 x=y$

$$\therefore \quad y^3-\frac{9}{2}y^2+5y-\frac{3}{2}=0$$

$$2y^3-9y^2+10y-3=0$$

$$(y-1)\left(2y^2-7y+3\right)=0$$

$$\therefore \quad y=1,3,\frac{1}{2}$$

$$\therefore \quad \log_5 x=1;\ \log_5 x=3;\ \log_5 x=\frac{1}{2}$$

$$x=5; \qquad x=125; \qquad x=\sqrt{5}$$

51. (2)

Let $x^4=y$

$$\therefore \quad y+\sqrt{y+20}=22$$

$$\left(\sqrt{y+20}\right)^2=(22-y)^2$$

$$\therefore \quad y+20=484+y^2-44y$$

$\Rightarrow \quad y^2 - 45y + 464 = 0$

$y = 16, 29$.

$\therefore \quad x^4 = 16 \Rightarrow x^4 = 2^4 \Rightarrow x = \pm 2$ (two integral roots).

52. (8)

$$\frac{5}{4}\cos^2 2x + 1 - 2\sin^2 x \cos^2 x + 1 - 3\sin^2 x \cdot \cos^2 x = 2$$

$$\Rightarrow \quad \frac{5}{4}\cos^2 2x - \frac{\sin^2 2x}{2} - \frac{3}{4}\sin^2 2x = 0$$

$$\Rightarrow \quad \frac{5}{4}\cos^2 2x - \frac{5}{4}\sin^2 2x = 0$$

$$\Rightarrow \quad \cos^2 2x - \sin^2 2x = 0$$

$$\Rightarrow \quad \cos 4x = 0$$

$$\therefore \quad 4x = (2n+1)\frac{\pi}{2} \quad n \in I$$

$$\therefore \quad x = (2n+1)\frac{\pi}{8}$$

$$\therefore \quad x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

53. (1)

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{2\cos^2\alpha}} = \frac{1}{7} \Rightarrow \tan\alpha = \frac{1}{7}$$

$$\sqrt{\frac{2\sin^2\beta}{2}} = \frac{1}{\sqrt{10}} \Rightarrow \sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3}$$

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\times\frac{1}{3}}{1-\frac{1}{9}} = \frac{2/3}{8/9} = \frac{2\times 9}{3\times 8} = \frac{3}{4}$$

$$\tan(\alpha+2\beta) = \frac{\tan\alpha + \tan 2\beta}{1-\tan\alpha\cdot\tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{3}{28}} = \frac{4+21}{25} = 1$$

54. (6)

$$(\log_5 x)^2 + \frac{\log_5 5 - \log_5 x}{\log_5 5 + \log_5 x} = 1$$

Take $\log_5 x = t$

$\therefore \quad t^2 + \frac{1-t}{1+t} = 1$

$t^2 + t^3 + 1 - t = 1 + t$

$\Rightarrow \quad t^3 + t^2 - 2t = 0$

$\Rightarrow \quad t\left(t^2 + t - 2\right) = 0$

$\Rightarrow \quad t(t-1)(t+2) = 0$

$\Rightarrow \quad t = 0, 1, -2$

$\therefore \quad \log_5 x = 0 \Rightarrow x = 1$.

$\log_5 x = 1 \Rightarrow x = 5$

$\log_5 x = -2 \Rightarrow x = 5^{-2} = \frac{1}{25}$

$\therefore$ Sum = 1 + 5 = 6