Directional Monitoring of Multiple Moving Targets by Multiple Unmanned Aerial Vehicles

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***Abstract*—Unmanned Aerial Vehicles (UAVs) have wide applications in many fields, e.g. multiple Unmanned Aerial Vehicles (UAVs) cooperatively tracking multiple targets. This paper studies the multiple UAVs cooperatively tracking multiple targets by vision surveillance system, where the images/videos of targets have direction requirements. One target is covered by a UAV if and only if its position is within the Field Of View (UAV) as well as the UAV is within a requested angle of the target's face direction. The objective is to maximize the total covered targets number by the UAVs, which can not be solved by existing models. A simple effective distributed, online cooperation algorithm for this problem is designed in this paper. The theoretical analysis shows our algorithm achieves constant factor to the optimal.**

I. Introduction

Last decades witness flourishing applications of Unmanned Aerial Vehicles (UAVs) in many fields, such as social and sport events [1] , entertainment and commercial activity [2], and military surveillance [3]. These applications bring great potential benefits to achieve better security, less labor and lower cost.

Image or video based surveillance is the most popular application of UAVs where directional cameras are often used. The problem of autonomous UAVs equipped with face-down cameras maximizing the total number of targets covered at fixed altitude (as well as fixed coverage range), which is called Cooperative Multi-robot Observation of Multiple Moving Targets (CMOMMT) problem is studied in[4] [5] . UAVs equipped with Face-down camera capture images from a bird's view and its coverage range is proportional to UAV's altitude: The higher altitude, the larger coverage range, but lower resolution of image, and vice visa [6] . [7] studies to balance the covered number of targets and image resolution by adjusting the altitude/distance from UAVs to targets. Except for the bird's view, onboard Pan Tilt Zoom (PTZ) camera [8] provides a more flexible angle to capture images of targets, which was usually achieved by ground robots[9]. UAVs equipped with PTZ cameras are used to cooperatively track moving targets [10] [11].

Inspired by the great demand of Virtual Reality (VR) [12], some state-of-art commercial UAVs are equipped with omni-directional cameras [13]. The omni-directional UAV outperforms UAVs equipped with face-down or PTZ cameras with limited sensing range or Angle Of View (AOV) omni-directional camera provides continuous omni-directional image/video around it. In conventional CMOMMT (where directional cameras are used), to achieve better coverage performance, extra equipments (like device free RF-based signal [14]) are required to detect the motion states of targets, which are out of UAVs' Field Of View (FOV). Omni-directional camera's low resolution images of far areas could be used to get coarse state information of targets [15]. More knowledge on detecting the motion of targets by images is reviewed in [16]. Moreover, conventional CMOMMT assumes targets are non-directional: one target is covered once it is within the UAV's FOV. This leads the conventional coverage model does not fit some face recognition applications [17] or some VR applications where interaction is needed. One UAV should be in a certain angle within the face direction of targets to capture its face image. To the best of our knowledge, no earlier work simultaneously consider the three surveillance factors: 1) distances between UAVs and targets, 2) directions of targets, and 3) mobility of targets. So existing models can not be applied to this scenario.

In this paper, we extend the conventional CMOMMT problem in which the UAV cameras are directional or from a bird's view, with limited sensing range and the moving targets are un-directional. We study the scenario UAVs are equipped with omni-directional camera and capture effective image of a target i.f.f the target is within the sensing range of the UAV as well as the UAV is within a certain facing angle of the target. We design a simple online algorithm called Most Uncovered First (MUTF) to maximize the number of covered targets and provide theoretical analysis of our algorithm. Our theoretical analysis shows MUTF algorithm achieves a constant factor to the optimal. The contributions of this paper is three-fold: 1) We are the first to simultaneous consider three surveillance factors: distance between UAVs and targets, directions of targets and mobility of targets; 2) We propose a simple distributed online algorithm for the UAV cooperation; 3) Theoretical analysis of the algorithm shows it can achieve constant factor to the optimal.

The rest of the paper is organized as follows: Problem formulation is given in Section II and a simple but effective online algorithm called Most Uncovered Targets First (MUTF) algorithm is proposed in Section III. Section IV illustrates a

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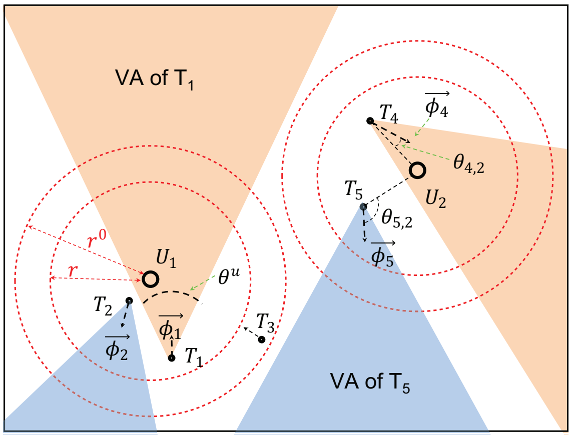

Fig. 1. Monitoring model of Omni-directional camera to directional targets. Targets T_2 and T_4 are not covered although their distances to UAVs is short enough, because they 'show' their backs to nearby UAVs and no UAVs are within their VAs. Target T_1 and T_4 are covered by U_1 and U_2 respectively because U_1 and U_2 are within the targets' VAs as well as distances are shorter than r. T_3 is not covered because it's resolution does not meet the application's requirement, but U_1 can detect T_3's motion state by the low resolution images.

approximate closed expression of the covered targets number distribution in uniform distributed targets moving model, and based on this, we prove our algorithm achieves constant factor to the optimal. In Section V, extensive experiments are conducted to further verify the performance of our algorithm. Section VI gives a short review of cooperative monitoring/tracking of multiple targets by UAV swarms.

II. Problem Formulation

We study using a team of UAVs $\mathbb{M}$ ($|\mathbb{M}| = M \in \mathbb{Z}$) to monitor a group of targets $\mathbb{N}$ ($|\mathbb{N}| = N > M$ and $N \in \mathbb{Z}$) moving in a Surveillance Field $SF \in \mathbb{R}^2$. SF is a in compact convex 2D field with no obstacles. The area of SF is $S \in \mathbb{R}$, where $|\bullet|$ is the set cardinality. Without ambiguity, we denote UAV m and its location at time t as notation $U_m(t) = (x_m(t), y_m(t)) \in SF$ ($m = 1, 2, 3, ..., M$). v^u is the max velocity of UAVs. We assume target's face direction is the same to its motion direction and use a 3-tuple: $T_n(t)$=$<P_n(t)$,$\overrightarrow{\phi_n(t)}$,$v_n(t)>$ $(n = 1, 2, 3, ..., N)$ to denote the motion state of target n at t, where $P_n(t)$, $\overrightarrow{\phi_n(t)}$ and $v_n(t)$ are its location, face direction and velocity. $v_n(t) \in [v^{min}, v^{max}]$ and is uniformly distributed in the range. $\theta^t \in (0, 2\pi]$ is the Visible Angle (VA) of targets: a target is covered by a UAV i.f.f the target in the UAV's Field Of View (FOV) and also the UAV is within the target's angle. Specifically, if $\theta^t = 2\pi$, the target is non-directional as conventional CMOMMT problem. All UAVs have a communication range of R. Two UAVs can communicate with each other if their distance is smaller than R. Two UAVs are neighbors if they can communicate with each other. All UAVs are equipped with a homogeneous omni-directional camera. Denote r as cameras' maximum distance of covering a target, which is determined by application, and denote r^0 as cameras' maximum radius of detecting the motion state of targets, which is determined by the feature of camera as well as the images processing algorithm. Fig. 1 shows the coverage model of omni-directional cameras. Since this paper addresses the application of capturing high resolution of targets' frontal image and target's motion state can be detected by low resolution image of whole target [15], so we assume $r^0 \gg r$. The area of camera's FOV is: $S^u = \pi r^2$. Vectors $\overrightarrow{mn}$ and $\overrightarrow{nm}$ are the vectors from UAV m to target n and n to m respectively. $d_{m,n}$=$|\overrightarrow{mn}|$ is the distance between m and n. $\theta_{n,m}(t)$ is the angle between $\phi_n(t)$ and $\overrightarrow{nm}$ at t ($\theta_{4,2}$ and $\theta_{5,2}$ in Fig.1). The continuous time is discretized into short slots of duration τ: $time = 0\tau, 1\tau, 2\tau, ..., t\tau, ..., T\tau$. At the start of time-slot t, the state $T_n(t)$ ($n \in \mathbb{N}$) of target n can be detect by UAV m if $d_{n,m} \le r^0$ and $T_n(t)$ remains during the slot.

Formally, we define a $m \times n$ matrix $C_{m,n}(t)$:

$$C_{m,n}(t) = \begin{cases} 1, \text{ if } (d_{m,n}(t) \le r) \&\& (\theta_{n,m}(t) \le \frac{\theta^t}{2}) \\ 0, \ others \end{cases} . \quad (1)$$

where $C_{m,n}(t)$=1 indicates target n is covered by UAV m during the t^{th} time slot. Then we define another metric:

$$Cover_n(t) = \begin{cases} 1, \text{ if } \exists m \in \mathbb{M} : C_{m,n}(t) = 1 \\ 0, \ others \end{cases} . \quad (2)$$

where $Cover_n(t) = 1$ indicates target n is covered by one UAV at least and $Cover_n(t) = 0$ means the target is not covered by any UAV. Our problem is to schedule trajectories of the UAV swarm to maximize the expected number of targets during a give time period $[0, T]$:

$$E^c(n) = \frac{\sum_{t=0}^{T} \sum_{n=1}^{N} Cover_n(t)}{T}. \quad (3)$$

To make the problem more general, we make the following more assumptions: 1) $r \ll L$, 2) $MS^u < S$,3) $v^u\tau \ll L$ and 4) $v^u\tau + r \le r^0$ and $v^u\tau + r \le R$. The first assumption is straightforward because if not, only a very small number of cameras at fixed locations could cover all the SF. And similar reason is the second assumption. Targets like human beings change their state on the order of seconds and other targets like vehicles or animals move faster. And the typical velocity of UAVs are on the order of tens m/s (e.g. $20m/s$) [18], so the third and fourth assumptions are reasonable.

III. Most Uncovered Targets First Algorithm

In this section, we introduce a simple distributed online algorithm to maximize the expected covered targets number in Eq.(3). Our best effort solution contains 3 steps:

Step.1: UAV detects the motion states T_n of targets n with $d_{m,n} \le r^0$, the states set is denoted as $\mathbb{N}^*$. Meanwhile, the UAV hears messages from it's neighbor UAVs. A message informs the receiving UAV the location its sender will go during current time-slot, the locations set of the UAV's neighbors is denoted as $\mathbb{M}^*$.

Step.2: according to the information in set $\mathbb{M}^*$ and $\mathbb{N}^*$, the UAV will find out all targets in $\mathbb{N}^*$ covered by its neighbor set $\mathbb{M}^*$. Then for the uncovered targets in $\mathbb{N}^*$, the UAV will searches all the discrete locations it can arrive to find out the location it can cover the most uncovered targets. The MUTF algorithm is detailed presented in **Algorithm 1**.

Step.3: After choosing the destination, the UAV will broadcasting a message containing its decision, meanwhile, it will fly to the destination.

One can refer to [16] for motion state detection by images and to [14], [19]-[20] for motion state detection by RF-based devices in indoor and ourdoor environments in **Step.1**. Moreover, inspired by recent progress in UAV realtime communication system [21][22], we assume the time for receiving the messages from neighbors $\ll \tau$ and is negligible. Before presenting the MTUF algorithm, we first discretize the continuous SF into small cell δ^2 as:

$$\Psi\{b\} = \{(i\delta, j\delta)\} \subset SF \; i, j \in [0, \frac{L}{\delta}], \tag{4}$$

for $b = (i\delta, j\delta) \in \Psi\{b\}$, $m \in \mathbb{M}$ and $n \in \mathbb{N}$, $d_{b,m} = \sqrt{(i\delta - x_m)^2 + (j\delta - y_m)^2}$ is the distance between b and UAV m, and $\theta_{n,b}$ is the angle between vector $\overrightarrow{nb}$ and ϕ_n. Notation $K = \lfloor \frac{2r^0}{\delta} \rfloor$, where $\lfloor x \rfloor \in \mathbb{Z}$ is the max integer satisfying $\lfloor x \rfloor \leq x$. Similarly, $\lceil x \rceil \in \mathbb{Z}$ is the minimum integer satisfying $\lceil x \rceil \geq x$.

Algorithm 1 Most Uncovered Targets First Algorithm (MUTF)

Require: $\mathbb{N}^*$, $\mathbb{M}^*$, $\Psi\{b\}$, $P_m = (i_m\delta, j_m\delta)$, v^u, r, P_n, ϕ_n, θ^t, τ
1: **for** $n^* \in \mathbb{N}^*$ **do**
2:   **for** $m^* \in \mathbb{M}^*$ **do**
3:     **if** $(d_{m^*,n^*} \leq r)\&\&(\theta_{n^*,m^*} \leq \frac{\theta^t}{2})$ **then**
4:       delete n^* from $\mathbb{N}^*$
5:     **end if**
6:   **end for**
7: **end for**
8: **if** $\mathbb{N}^* == NULL$ **then**
9:   Exit
10: **else**
11:   Initialize a $K \times K$ matrix $A[1:K][1:K] = 0$
12:   **for** $n^* \in \mathbb{N}^*$ **do**
13:     **for** $(b = (i\delta, j\delta) \in \Psi\{b\})\&\&(d_{b,m} \leq v^u\tau)\}$ **do**
14:       **if** $\theta_{n*,b} \leq \frac{\theta^t}{2}$ **then**
15:         $A[i - (i_m - \lfloor \frac{v^u\tau}{\delta} \rfloor)][j - (j_m - \lfloor \frac{v^u\tau}{\delta} \rfloor)]$
16:         $= A[i - (i_m - \lfloor \frac{v^u\tau}{\delta} \rfloor)][j - (j_m - \lfloor \frac{v^u\tau}{\delta} \rfloor)] + 1$
17:       **end if**
18:     **end for**
19:   **end for**
20:   $P_m = arg\ max\ A$
21: **end if**
22: **return** P_m

IV. Performance Analysis

In this section, we will theoretically analyze the performance of our MUTF algorithm. We firstly analyze its time and space complexity. Next we get the Probability Density Function (pdf) of the number covered by single UAV and $\theta^t = 2\pi$ if the targets are uniformly distributed in SF and further verify it in Section V. Then we prove that the performance of MUTF in case of $\theta^t < 2\pi$ is not worse than that in the case of $\theta^t < 2\pi$. Based on this, we extend our analysis to non-uniformly scenario and prove MUTF achieves a constant factor to optimal in cases of single as well as multiple UAVs.

A. Complexity of MUTF

Theorem 1: *The mean and worst time complexity of MUTF are:* $\mathcal{O}\left(N\frac{(v^u\tau+r)^2}{S}(M\frac{(v^u\tau)^2}{S} + \frac{(v^u\tau)^2}{\delta^2})\right)$, and $\mathcal{O}\left(N(M + \frac{(v^u\tau)^2}{\delta^2})\right)$, respectively.

Proof: $\mathbb{M}^*$ is the set of targets that may be covered by this UAV and $\mathbb{N}^*$ is the set of UAVs may overlap with this UAV. We denote $|\mathbb{M}^*| = M^*$ and $|\mathbb{N}^*| = N^*$. Clearly $M^* \leq M$ and $N^* \leq N$.Line.1 to Line. 7 is to delete the targets covered by other UAVs $m^* \in \mathbb{M}^*$. And the time is $\mathcal{O}(N^*M^*)$. If all the targets are covered by other UAVs, this UAV has no targets to cover (Line 8 to Line 9). If $\mathbb{N}^* \neq NULL$, this UAV will calculate the number of targets at every discretized location it can go before next time-slot (Line 11 to Line 18), the time is $< \mathcal{O}(N^*\frac{(v^u\tau)^2}{\delta^2})$. Line 19 searches the discretized location set A and its time complexity is $\mathcal{O}(K^2) = \mathcal{O}(\frac{(r^0)^2}{\delta^2})$. So the total time complexity is:

$$\mathcal{O}\left(N^*(M^* + \frac{(v^u\tau)^2)}{\delta^2}\right). \tag{5}$$

If the targets are uniformly distributed in SF, the mean value of N^* and M^* are $N^* = \frac{N(v^u\tau+r)^2}{S}$ and M^*=$M\frac{(v_u\tau)^2}{L^2}$, respectively. By combining them with Exp.(5), we can get the average time complexity of MUTF algorithm. Similarly, in the worst case, the time complexity is achieved when $N^* = N$ and $M^* = M$. So ***Theorem 1*** is proved.

Theorem 2: *The space complexity of MUTF is upper bounded by* $\mathcal{O}(M + N + \frac{(v_u\tau+r)^2}{\delta^2})$.

Proof: The proof can be easily got from **Algorithm. 1**, similar to the proof of ***Theorem 1*** and omitted due to space limitation.

Theorem 1 and ***Theorem 2*** show our algorithm has low time and space complexity, which is very important for the resource limited UAVs to conduct realtime surveillance task in high dynamic environment.

B. Approximate closed-form expression of Probability Distribution Function (PDF) of covered targets number by one UAV

We first get a approximate closed-form expression of the pdf targets number covered by one UAV ($M = 1$), if targets are uniformly distributed and $\theta^t = 2\pi$. Then we extend the result to the cases where targets are non-uniformed distributed and $\theta^t < 2\pi$. In this case, MUTF is: The UAV always choose the location covering the most targets. We assume the UAV m is fixed at the center of SF. The targets number n^1 covered by the fixed m follows the binomial distribution [23]:

$$p(n^{fi}) = C_N^{(n^{fi})} p^{(n^{fi})} \left(1-p\right)^{N-(n^{fi})}, \tag{6}$$

where $p = \frac{S^*}{S}$ is the probability one target is covered by m fixed at the center of SF.

During one time-slot τ, the max distance m can move is $v^u\tau$, and the max distance of target m can cover is $v^u\tau + r$. We denote $S^\tau = \pi(v^u\tau + r)^2$. We divide S^τ into K^τ sub-areas whose areas are all S^*: $K^\tau = \frac{S^\tau}{S^*} \in \mathbb{R}$. So MUTF is to choose the one sub-area with the most targets, which can be approximate by the so-called *largest order statistic*[24] , i.e. finding the largest one of $f(K^\tau)$ i.i.d values. $f(K^\tau) \geq K^\tau$ is a function of K^τ, and the pdf of n^τ is:

$$p(n^{mo}) = f(K^\tau)\{(P^{fi}(n^{mo}))^{f(K^\tau)} - [P^{fi}(n^{mo} - 1)]^{f(K^\tau)}\}, \tag{7}$$

where $n^{mo} = 1, 2, ...N$ and $P^{fi}(n) = \sum_{i=0}^{n} p^{fi}(i)$ is the CDF of $p^{fi}(n)$. By further verification in V, we find that $f(K^\tau) = [K^\tau]$ fits well. Which means the pdf of the targets number covered by the UAV departing from the SF's center can be approximate as:

$$p(n^{mo}) \approx [K^\tau](P(n^{fi})^{[K^\tau]} - P(n^{fi}-1)^{[K^\tau]}), \quad (8)$$

and its expected value is:

$$E(n^{mo}) = \sum_{n^{mo}=0}^{N} n^{mo} p(n^{mo}). \quad (9)$$

Eq.(9) is got when the minimum distance from the choosing location to the edge is $\geq r$. The probability the minimum distance from the choosing location to the edge is $\leq r$ is $< \frac{4rL}{L^2} = \frac{4r}{L}$. So if $M = 1$, $\theta^t = 2\pi$ and targets are uniformly distributed in SF, the expected of number of targets ($E^1(n)$) covered by the UAV is lower bounded by:

$$\frac{E(n^1)}{E(n^{mo})} > (1 - \frac{4r}{L}). \quad (10)$$

We can see the that if $\theta_t < 2\pi$, Eq. (10) still holds, which means the performance of MTUF in the case of $\theta_t < 2\pi$ is not worse than the case of $\theta_t = 2\pi$.

C. Non-uniformly distributed targets and multiple UAVs cases

Based on the result in Subsec.(IV-B) with the assumption that the targets are uniformly distributed, in this part, we will prove that our MUTF algorithm achieves a constant factor to the optimal solution. We first study the case where $M = 1$ and extend the result to $M > 1$. Observing targets' random motion in scenarios like within a town square, we adopt the targets moving model in [25] to derive the location distribution of targets. In Random WayPoint (RWP) moving model in [26], moving targets choose destinations and velocity randomly in SF, and the location distribution of a target is:

$$p^{(X,Y)} = \frac{36(x^2 - Lx)(y^2 - Ly)}{L^6}, \quad (11)$$

where $p^{(X,Y)}$ is independent of targets' velocity [25]. Mean while, since $r \ll L$, when UAV locates are $loc^0 \in SF$, for any $loc^1 \in SF$, where $d_{loc^0,loc^1} \leq r$, we approximate the $p^{(loc^1)} \approx p^{(loc^0)}$. Specifically, $p^{(\frac{L}{2},\frac{L}{2})} = \frac{9}{16L^2}$ is the highest probability, which means the center is the targets densest area of SF.

If $M = 1$, we propose an upper bound of any covering algorithm as: at the start of every time start, the UAV always departs from the center $(\frac{L}{2}, \frac{L}{2})$ of SF. So the expected covered targets number ($E(n^{opt})$) of any covering algorithm is upper bounded as:

$$E(n_1^{opt}) < E(n_1^{(\frac{L}{2},\frac{L}{2})}), \quad (12)$$

where $E(n_1^{(\frac{L}{2},\frac{L}{2})})$ is $E(n_1^{mo})$ in Eq. (9) by replacing p by $p^{(\frac{L}{2},\frac{L}{2})}$ in Eq.(6). We use superscript $(\frac{L}{2}, \frac{L}{2})$ instead of mo to emphasize its dependency on target distribution $p^{(\frac{L}{2},\frac{L}{2})}$. Subscript 1 is used to emphasize $M = 1$.

Before propose the lower bound of our MUTF, we prove ***Theorem 3*** first.

Theorem 3: *If the targets are non-uniformly distributed in SF, and the probability one target locates at any two locations $loc^* \in SF$ and $loc^{**} \in SF$ are p^* and p^{**}, respectively. If the UAV adopts the MTUF algorithm, and the probability the UAV at loc^* and loc^{**} are p_u^* and p_u^{**} , then we have:*

$$\frac{p_u^*}{p_u^{**}} \geq \frac{p^*}{p^{**}}. \quad (13)$$

Proof: Mathematical induction is used to prove ***Theorem 3***. 1) If there is only one UAV in SF ($|\mathbb{N}| = 1$), it is clearly Eq. (13) holds because $\frac{p_u^*}{p_u^{**}} = \frac{\pi r^2 p^*}{\pi r^2 p^{**}} = \frac{p^*}{p^{**}}$. 2) If $|\mathbb{N}| = N > 1$ we denote the probability the UAV chooses loc^* as $p_{u,N}^*$, and targets number UAV covers as N^1($0 < N^1 <= |\mathbb{N}|$). Similarly, when the UAV choose loc^{**} and covers N^1 targets, the probability is denoted as $p_{u,N}^{**}$, we assume $\frac{p_{u,N}^*}{p_{u,N}^{**}} \geq \frac{p^*}{p^{**}}$ holds. 3) When $|\mathbb{N}| = N + 1$, compared with $|\mathbb{N}| = N$, we call the one more target as the $(N+1)^{th}$ target. For any cases the UAV covers the N^1 targets if $|\mathbb{N}| = N$, if $|\mathbb{N}| = N+1$, the probability the the UAV covers the N^1 and the $(N+1)^{th}$ target is $p_{u,N+1}^* = \pi r^2 p^* p_{u,N}^*$. Similarly, $p_{u,N+1}^{**} = \pi r^2 p^{**} p_{u,N}^{**}$. Then we have:

$$\frac{p_{u,N+1}^*}{p_{u,N+1}^{**}} = \frac{p^* p_{u,N}^*}{p^{**} p_{u,N}^{**}} \geq \frac{p^*}{p^{**}}. \quad (14)$$

Theorem 3 shows the UAVs conducting MUTF algorithm has high probability to the dense area of target: not lower than the distribution of targets at least.

We observe that targets are more likely to face the direction of the the densest area (the center of SF in RWP model [25]). If the UAV chooses $(x, y) \in SF$ covering the most targets, we denote the expected covered targets number as $E^{uni}(n)$ and $E^{dir}(n)$ where the targets direction are uniformly distributed and more likely to face a specific direction at $(x, y) \in SF$, respectively. So we can see $E^{uni}(n) \leq E^{dir}(n)$ because the targets are more like to face the common direction. By combining ***Theorem 3***, we can get the performance of our MUTF algorithm is lower bounded by:

$$E_1^c(n) \geq \iint_{SF} E(n^{(x,y)})\, ds, \quad (15)$$

where $E(n^{(x,y)})$ is $E(n^{mo})$ in Eq. (9) by replacing p with $p^{(x,y)}$ in Eq.(6). Subscript 1 is used to emphasize $M = 1$.

Theorem 4: *If travel destinations of targets are randomly distributed in a square deployment field, we have:*

$$\begin{aligned} \frac{E(n_1^c)}{E(n_1^{opt})} &\overset{①}{>} \frac{\iint_{SF} E(n^{(x,y)})\, ds}{E(n^{(\frac{L}{2},\frac{L}{2})})} \\ &\overset{②}{\geq} \frac{\iint_{SF} \sum_{n=0}^{N} n * p^{(x,y)}\, ds}{\sum_{n=0}^{N} n p^{(\frac{L}{2},\frac{L}{2})}} \\ &\overset{③}{=} 0.64. \end{aligned} \quad (16)$$

where $\sum_{n=0}^{N} n * p^{(x,y)}$ and $\sum_{n=0}^{N} n p^{(\frac{L}{2},\frac{L}{2})}$ in the third expression are the expected covered targets number if the UAV is fixed at $(x, y) \in SF$ and $(\frac{L}{2}, \frac{L}{2})$ respectively.

Proof: ① in Eq.(16) is gotten by combining Eq.(12) and Eq.(15). ③ is gotten by taking the location distribution of targets (Eq.(11)) into right expression of ②. So our proof consists of proving ②. We can see that one sufficient condition of ① holds is: $\forall (x, y) \in SF, \frac{E^{(x,y)}(n)}{E^{(\frac{L}{2},\frac{L}{2})}(n)} \geq \frac{p(x,y)}{p^{(\frac{L}{2},\frac{L}{2})}}$. We denote the sufficient condition as Con.1. And one sufficient condition of Con.1 is: if $p(x,y) \ll 1$ ($p(x,y) \leq 0.5$), function $f(p^{(x,y)}) = \frac{E^{(x,y)}(n)}{p^{(x,y)}}$ is a monotone decreasing function of the variable p(x,y), which we denote as Con.2. $p^{(x,y)} \ll 1$ ($p^{(x,y)} \leq 0.5$) is true because the scenario we study is the targets move in the whole SF, not cloud within a small area [1]. Con.2 can be easily proved by getting the exact expression of function$f(p^{(x,y)}) = \frac{E^{(x,y)}(n)}{p^{(x,y)}}$, by which we can get $\frac{\partial f(p)}{\partial p} < 0$ if $p \ll 1$.

Till now, all our analysis is only for $M = 1$. But while designing algorithm for CMOMMT problem, the most hard part is that the UAVs will overlap with each other. Next we will analyze our algorithm still holds in multiple UAVs cases.

Theorem 5: *If there are multiple UAVs conducting the MUTF algorithm, we have:*

$$\frac{E(n^c)}{E(n^{opt})} \geq \frac{E(n_1^c)}{E(n_1^{opt})}. \tag{17}$$

Proof: We prove the simplest case of $M = 2$, and it can be extended to any case of $M \geq 2$. Similar to construct the upper bound for $M = 1$, the two UAVs depart from the densest area of SF. Denote their overlap area as ol^{opt}. In MTUF, denote the overlap area as ol^c. In MUTF, the two UAVs will not always be in the densest area in SF, which means $E(ol^c) \leq E(ol^{opt})$. So $\frac{E(n_2^c)}{E(n_2^{opt})} = \frac{E(n_1^c)(2-\frac{E(ol^c)}{S^*})}{E(n_1^{opt})(2-\frac{E(ol^{opt})}{S^*})} \geq \frac{E(n_1^c)}{E(n_1^{opt})}$.

Theorem 4 and ***Theorem 5*** indicate the MUTF algorithm achieves constant factor to the optimal, only depends on the location distribution of targets, and independent of the UAV number, UAV speed or total number of targets.

V. Evaluation

In this section, we conduct experiments to verify the covered targets number by one UAV with different time-slot lengths (Eq.(8)). The experiment parameters are: $L = 1000m$; $r = 50m$; $v^u = 20m/s$; $\tau = 1s : 1s : 6s$; $N = 300, 500, 700$; $v^{min} = 0m/s$ and $v^{max} = 10m/s$; $\delta = 1m$. We run the simulation for 50,000 time-slots ($T = 50,000$). The CDF of the covered targets number ($N = 700$) is presented in Fig.2, where the X-axis is the covered targets number and the Y-axis is the CDF. Limited by space, only the CDFs of $\tau = 1s$ and $\tau = 6s$ ($[K] = 2$ and $[K] = 12$, respectively) are presented. The overall result is shown in Fig.3. X-axis in Fig.3 is the length of a time-slot in second and Y-axis is the average covered targets number. We can see that the simulation results are very close to the theoretical analysis. Our theoretical analysis can capture the probability distribution of covered targets number. This is because in our scenario, the targets number in unit area in SF can be approximate as discrete discrete independent identically distribution.

VI. Related Works

UAVs can be applied in applications such as smart grids [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37],

[1]In this model, the targets follow the same moving model. In reality, different targets may have priority areas for itself, which means for a given target n, location distribution $p_n^{(x,y)} \ll 1$ may not holds. But if the expected targets number within a small area $\ll N$, it is equivalent to $p^{(x,y)} \ll 1$.

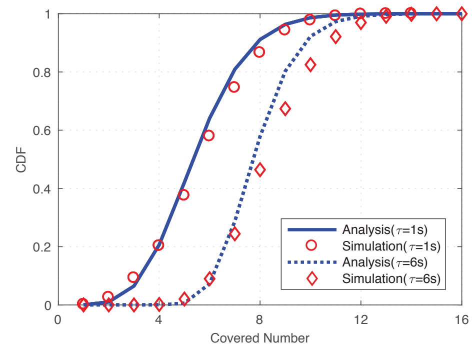

Fig. 2. CDF of the targets number covered by one UAV with different timeslots

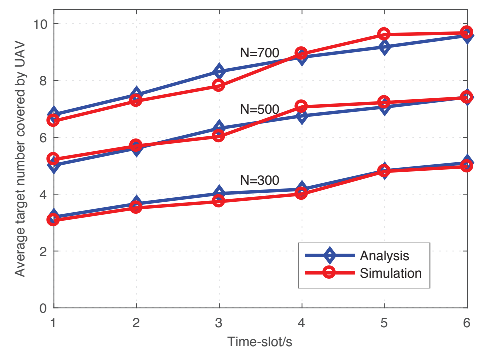

Fig. 3. Average target number covered by one UAV

[38], [39], [40], [41], [42], [43] and smart transportation systems [44], [45], [46], [47], [48], [49], [50], [51]. Single UAV searching stationary targets is studied in [52]. Maximizing the tracking time of a single moving target by a single moving robot is studied in [53]. CMOMMT is first defined in [54] and is NP-hard. [4] [5] introduce distributed methods to solve this problem. [4] uses weighted local force vector to coordinate among robots. Based on the basic coordination idea in [4], a *help* model is added in [5], informing other robots for help tracking the target a UAV is about to lose. [55] proves if the number of UAVs ≥ 3, it is impossible to track all targets while maintain a constant factor of targets' optimal resolution, and a $\frac{1}{2}$ approximation algorithm is proposed to maximize the number of targets or targets resolution. PTZ camera is also studied in the cooperative sensing in mobile robots networkings [10]. Most existing works model targets as a point, but [6] models target as a polygon and PTZ camera is adopted to maximize the total covered boundary number of polygon targets.

VII. Conclusion and Future Work

In this paper, we study the scenario of tracking directional targets. A simple distributed, online algorithm is proposed and our theoretical analysis shows our distributed online algorithm achieves constant factor to optimal, which only is dependent to the location distribution of targets and is independent of UAV number, UAV speed, UAV's FOV, targets number or the direction requirement. In future, we will conduct extensive experiments to further verify the performance of our algorithm.

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