Introduction to the basics of AI

Session 7

Z. TAIA-ALAOUI

Outline

- Definitions
- SVM / MLP
- Shallow Neural Network
- Implementation

Statistical Tools - Dataset

- **Sample**

$$X_{k \in [1,N]} \in R^p$$

- **Set of samples**

$$\mathbf{X} = \{X_k \in \mathrm{R}^p\}_{k \in [1,\mathrm{N}]}$$

IRIS DATASET

p variables

N samples

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
1	5.1	3.5	1.4	0.2
2	4.9	3.0	1.4	0.2
3	[illegible]	[illegible]	[illegible]	[illegible]
4	4.6	3.1	1.5	0.2
5	5.0	3.6	1.4	0.2
6	5.4	3.9	1.7	0.4
7	4.6	3.4	1.4	0.3
8	5.0	3.4	1.5	0.2

Row = Sample = Observation = Measurement

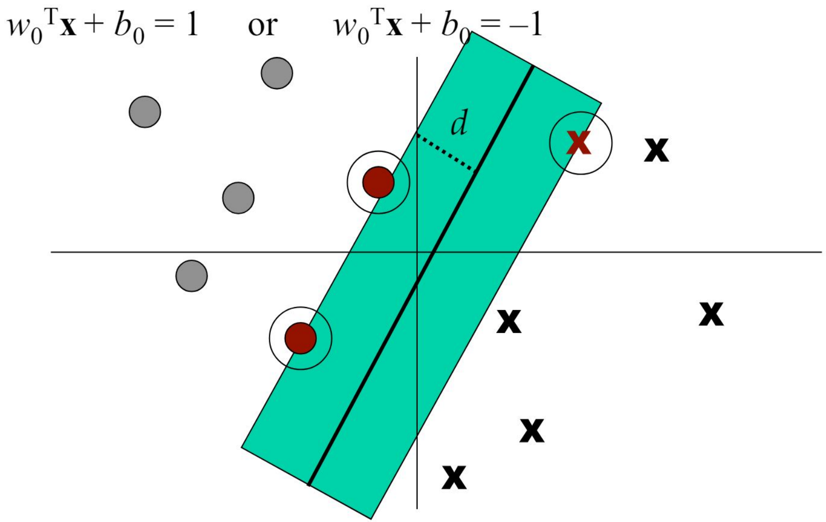
Statistical Tools - Dataset

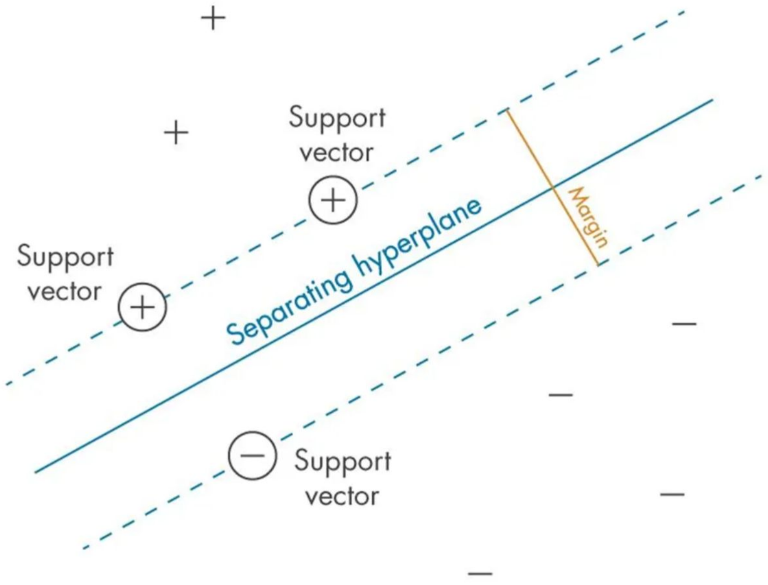
- **Set of N samples expressed in a space of p Variables**

$$\mathbf{X} = \begin{array}{c c} & \begin{array}{ccccc} & & & V_j & \end{array} \\ \begin{array}{c} \\ \\ \\ X_i \\ \\ \\ \end{array} & \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1p} \\ x_{21} & x_{22} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ x_{N1} & . & . & . & x_{Np} \end{pmatrix} \end{array} = [V_1, V_2, \ldots, V_p] = \begin{bmatrix} X_1^{\mathrm{T}} \\ X_2^{\mathrm{T}} \\ . \\ . \\ . \\ X_N^{\mathrm{T}} \end{bmatrix}$$

$$X_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ . \\ . \\ . \\ x_{ip} \end{bmatrix} \qquad V_j = \begin{bmatrix} x_{1j} \\ x_{2j} \\ . \\ . \\ . \\ x_{Nj} \end{bmatrix}$$

SVM - Definitions





SVM - Definitions

- **Hyperplane:** Hyperplane is the decision boundary that is used to separate the data points of different classes in a feature space.
- **Support Vectors:** Support vectors are the closest data points to the hyperplane.
- **Margin**: Margin is the distance between the support vector and hyperplane.
- **Kernel**: Kernel is the mathematical function, which is used in SVM to map the original input data points into high-dimensional feature spaces.
- **C:** Margin maximisation and misclassification fines are balanced by the regularisation parameter C in SVM. A stricter penalty is imposed with a greater value of C, which results in a smaller margin and perhaps fewer misclassifications

SVM - Definitions

$$\hat{y} = \begin{cases} 1 & : \ w^T x + b \geq 0 \\ 0 & : \ w^T x + b < 0 \end{cases}$$

Hard Margin

$$\underset{w,b}{\text{minimize}} \frac{1}{2} w^T w = \underset{W,b}{\text{minimize}} \frac{1}{2} \left\| w \right\|^2$$

$$\text{subject to } y_i(w^T x_i + b) \geq 1 \ for \ i = 1, 2, 3, \cdots, m$$

Soft Margin

$$\underset{w,b}{\text{minimize}} \ \frac{1}{2} w^T w + C \sum_{i=1}^{m} \zeta_i$$

$$\text{subject to } y_i(w^T x_i + b) \geq 1 - \zeta_i \ \ and \ \zeta_i \geq 0 \ \ for \ i = 1, 2, 3, \cdots, m$$

SVM - Definitions

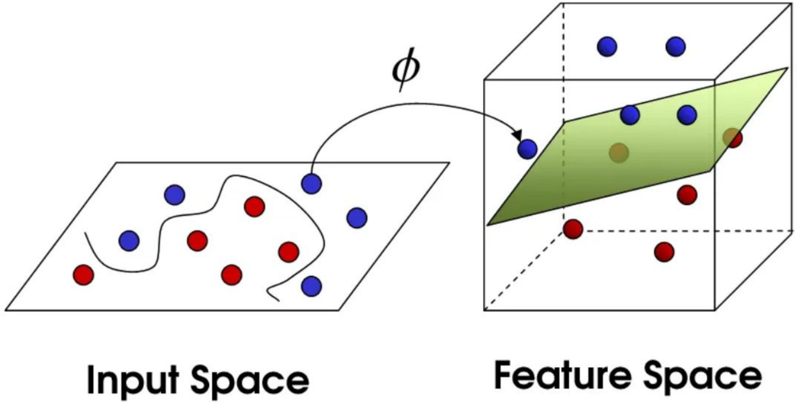
- Non-linear case

$$\text{Linear}: \; K(w,b) = w^T x + b$$

$$\text{Polynomial}: \; K(w,x) = (\gamma w^T x + b)^N$$

$$\text{Gaussian RBF}: K(w,x) = \exp(-\gamma ||x_i - x_j||^n$$

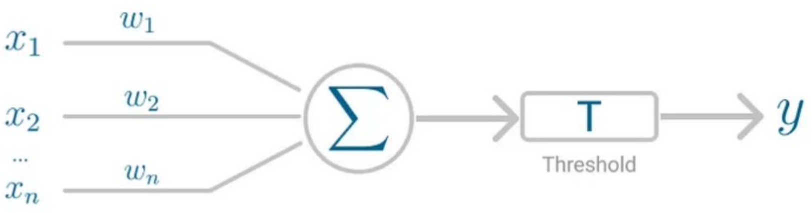
$$\text{Sigmoid}: K(x_i,x_j) = \tanh(\alpha x_i^T x_j + b)$$



Multi-Layer Perceptron - Definitions

- The perceptron imitates human perception

$$\underbrace{f(x,w)}_{\text{output}} = \underbrace{x_1}_{\text{inputs}} w_1 + \cdots + x_n \underbrace{w_n}_{\text{weights}}$$



$$\hat{y} = sign(\, w^T x + b)$$

- T is called activation function

Multi-Layer Perceptron - Definitions

- Classification with MLP

$$L_{\text{Perceptron}}(w,b) = \sum_{i=1}^{M} \max(0, -y^{(i)}(w^T x^{(i)} + b))$$

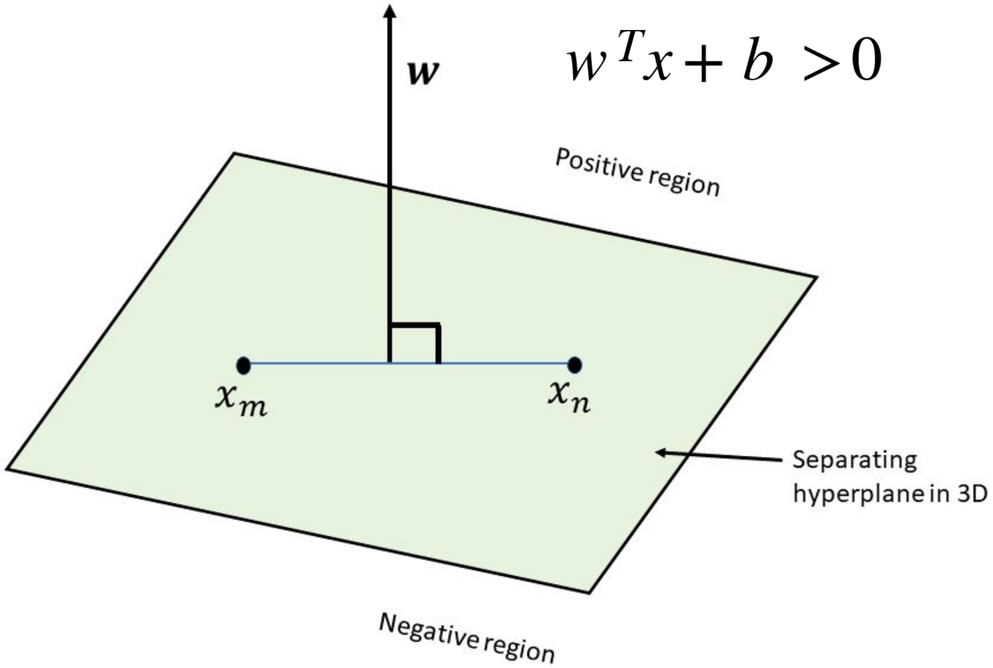
$$w^T x + b < 0 \qquad\qquad w^T x + b > 0$$

Mis-classified sample

Well classified sample

Multi-Layer Perceptron - Definitions

- Classification with MLP: Geometrical Interpretation



Multi-Layer Perceptron - Definitions

- Classification with MLP: Learning

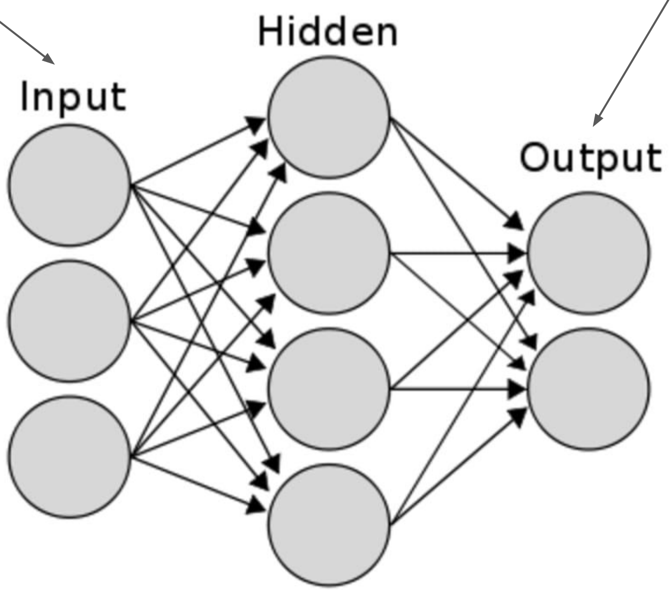
Data: Training Data:(x_i, y_i); $\forall i \in \{1, 2, \ldots, N\}$, Learning Rate: η
Result: Separating Hyper-plane coefficients : $\mathbf{w}^*$
Initialize $\mathbf{w} \leftarrow \mathbf{0}$;
repeat
    get example (x_i, y_i);
    $\hat{y}_i \leftarrow w^T x_i$;
    if $\hat{y}_i y_i \leq 0$ **then**
        $w \leftarrow w + \eta y_i x_i$
until *convergence*;

Vanilla (shallow) NN

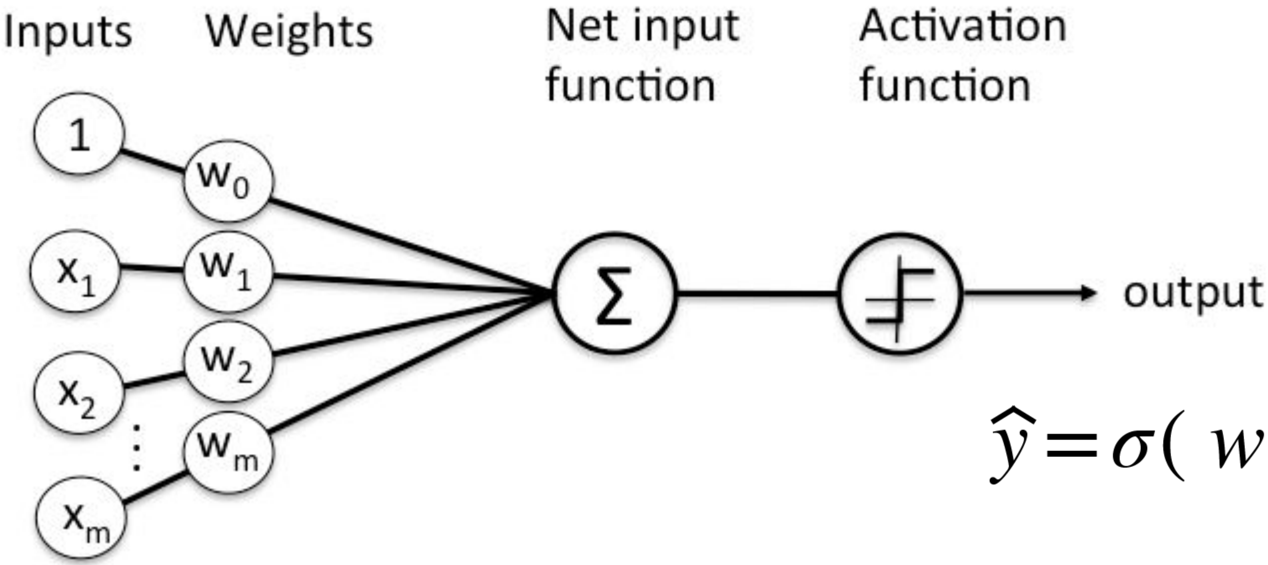
Hidden Feature Space (Latent space)

Your measurements

Your desired outputs

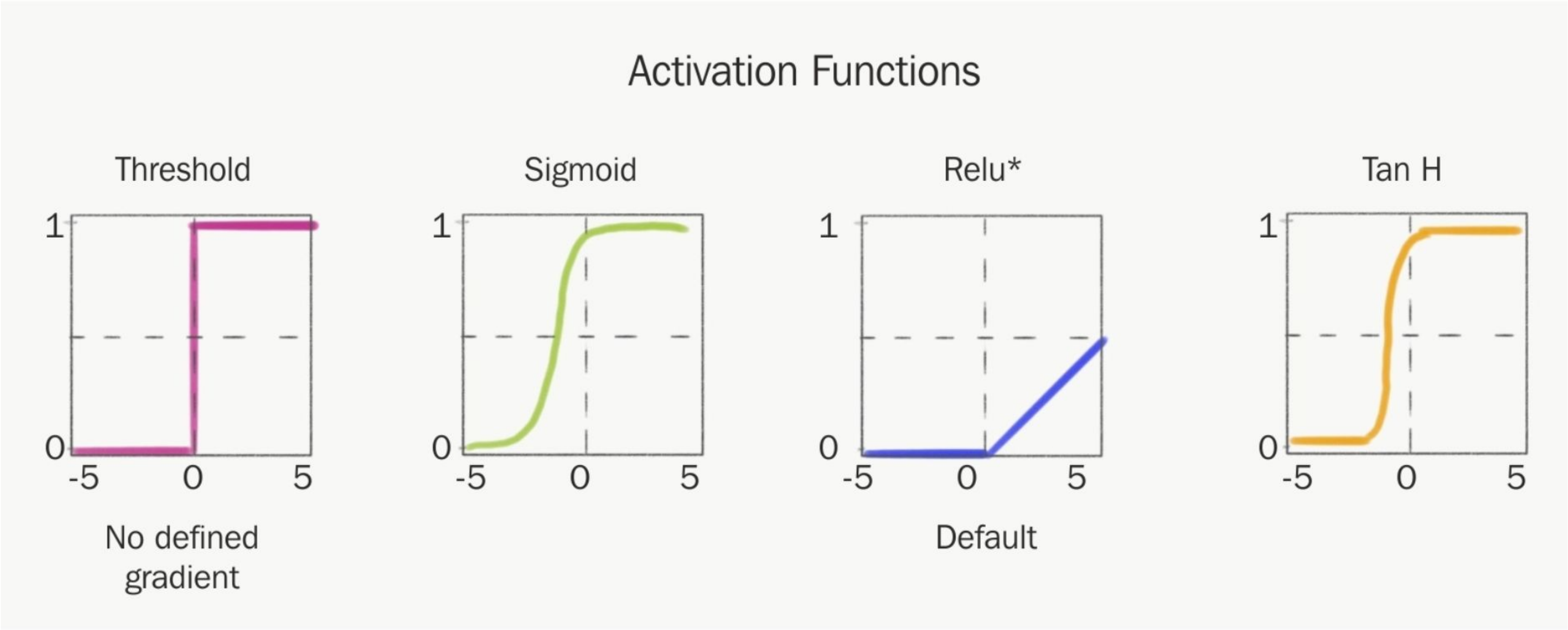


ANN

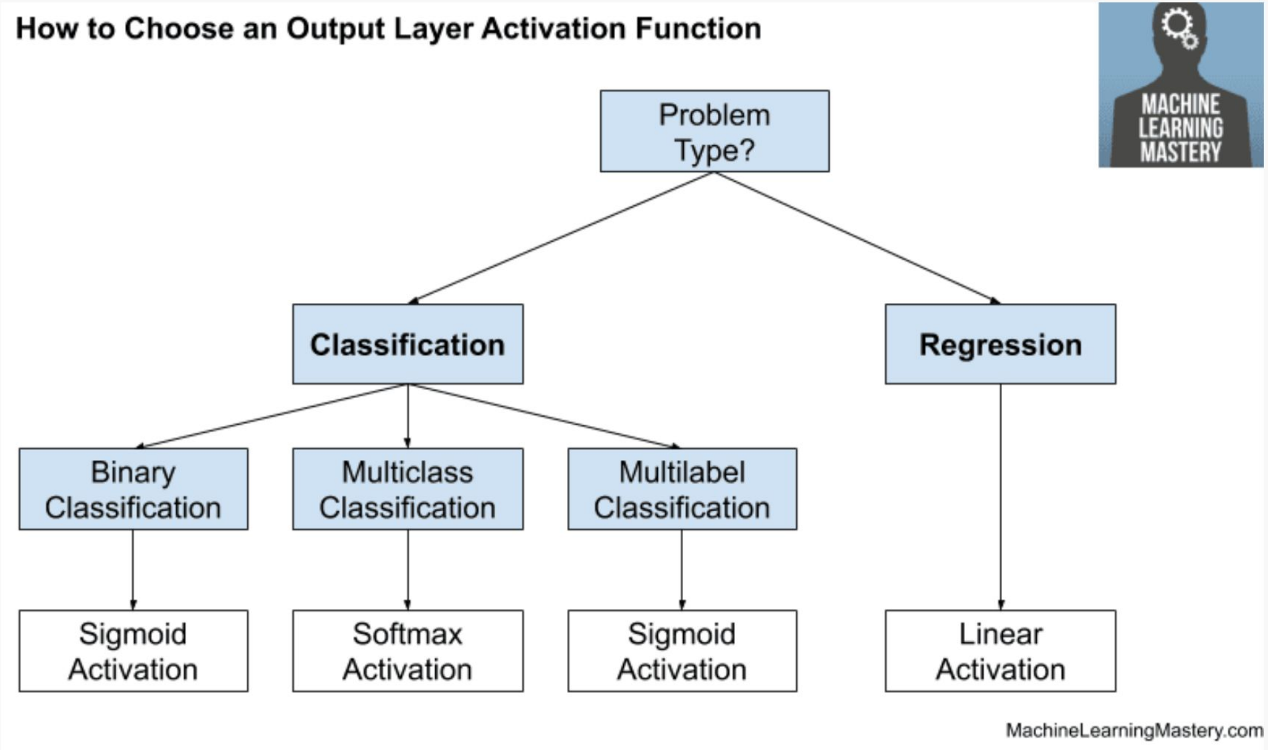


$$\hat{y} = \sigma(w^T x + b)$$

Activation Functions



Activation Functions



How to Choose an Output Layer Activation Function

Activation Functions

Sigmoid

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Softmax

$$s\left(x_i\right) = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

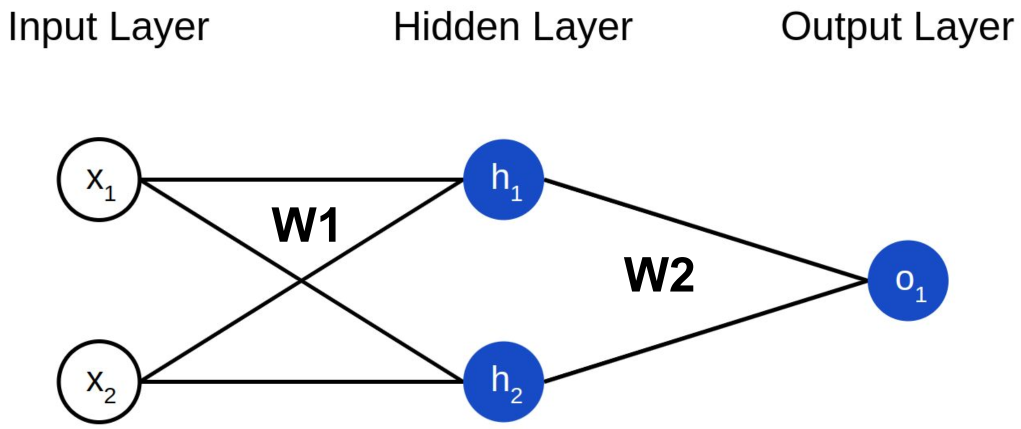
Learning

- Forward Propagation

$$\mathbf{z} = \mathbf{W}^{(1)}\mathbf{x}$$

$$\mathbf{h} = \phi(\mathbf{z})$$

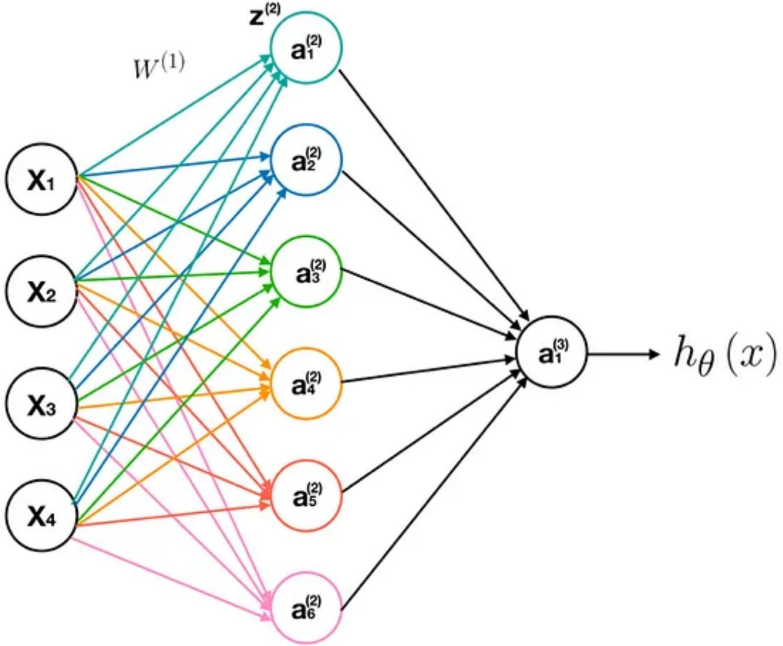
$$\mathbf{o} = \mathbf{W}^{(2)}\mathbf{h}$$



Learning

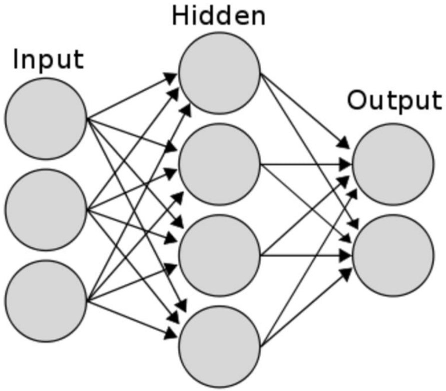
- Forward Propagation

$$W^TX = \begin{bmatrix} \theta_{11}{}^{(1)} & \theta_{12}{}^{(1)} & \theta_{13}{}^{(1)} & \theta_{14}{}^{(1)} \\ \theta_{21}{}^{(1)} & \theta_{22}{}^{(1)} & \theta_{23}{}^{(1)} & \theta_{24}{}^{(1)} \\ \theta_{31}{}^{(1)} & \theta_{32}{}^{(1)} & \theta_{33}{}^{(1)} & \theta_{34}{}^{(1)} \\ \theta_{41}{}^{(1)} & \theta_{42}{}^{(1)} & \theta_{43}{}^{(1)} & \theta_{44}{}^{(1)} \\ \theta_{51}{}^{(1)} & \theta_{52}{}^{(1)} & \theta_{53}{}^{(1)} & \theta_{54}{}^{(1)} \\ \theta_{61}{}^{(1)} & \theta_{62}{}^{(1)} & \theta_{63}{}^{(1)} & \theta_{64}{}^{(1)} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} z_1{}^{(1)} \\ z_2{}^{(1)} \\ z_3{}^{(1)} \\ z_4{}^{(1)} \\ z_5{}^{(1)} \\ z_6{}^{(1)} \end{bmatrix} = Z^{(2)}$$



Learning

- Backward Propagation



$$\mathbf{z} = \mathbf{W}^{(1)}\mathbf{x}$$

$$\mathbf{h} = \phi(\mathbf{z})$$

$$\mathbf{o} = \mathbf{W}^{(2)}\mathbf{h}$$

Loss

$$L = l(\mathbf{o}, y)$$

$$\hat{o} = \sigma(w^T x + b)$$

$$w_{ij}^{(k)} = w_{ij}^{(k)} - \eta \frac{\partial L}{\partial w_{ij}^{(k)}}$$

$$b_i^{(k)} = b_i^{(k)} - \eta \frac{\partial L}{\partial b_i^{(k)}}$$

Vocabulary

- Batch Learning: for each epoch, a batch of samples is select to train the model
- Epoch: complete sequence of iterations in a batch
- Iteration: one backward pass
- Learning rate: speed at which weights are updated

Sources

https://towardsdatascience.com/multilayer-perceptron-explained-with-a-real-life-example-and-python-code-sentiment-analysis-cb408ee93141

https://web.mit.edu/6.034/wwwbob/svm-notes-long-08.pdf

https://course.ccs.neu.edu/cs6140sp15/2_GD_REG_pton_NN/lecture_notes/lectureNotes_Perceptron.pdf

https://towhttps://web.mit.edu/6.034/wwwbob/svm-notes-long-08.pdf

https://towardsdatascience.com/the-kernel-trick-c98cdbcaeb3f