PADP(18CS73)
Unit 1

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Principles of Parallel Algorithm Design

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Chapter Overview: Algorithms and Concurrency

- Introduction to Parallel Algorithms
  - Tasks and Decomposition
  - Processes and Mapping
  - Processes Versus Processors
- Decomposition Techniques
  - Recursive Decomposition
  - Recursive Decomposition
  - Exploratory Decomposition
  - Hybrid Decomposition
- Characteristics of Tasks and Interactions
  - Task Generation, Granularity, and Context
  - Characteristics of Task Interactions.

Chapter Overview: Concurrency and Mapping

- Mapping Techniques for Load Balancing
  - Static and Dynamic Mapping
- Methods for Minimizing Interaction Overheads
  - Maximizing Data Locality
  - Minimizing Contention and Hot-Spots
  - Overlapping Communication and Computations
  - Replication vs. Communication
  - Group Communications vs. Point-to-Point Communication
- Parallel Algorithm Design Models
  - Data-Parallel, Work-Pool, Task Graph, Master-Slave, Pipeline, and Hybrid Models

Preliminaries: Decomposition, Tasks, and Dependency Graphs

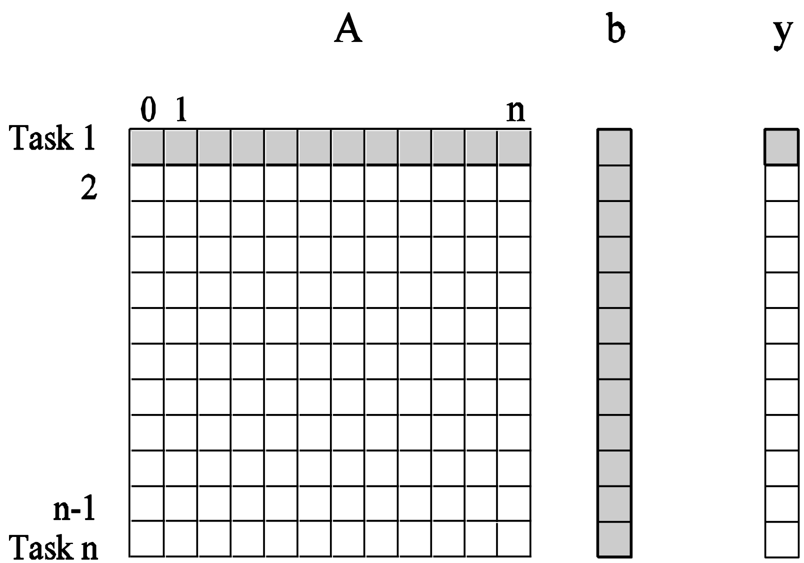
- The first step in developing a parallel algorithm is to decompose the problem into tasks that can be executed concurrently

- A given problem may be docomposed into tasks in many different ways.

- Tasks may be of same, different, or even interminate sizes.

- A decomposition can be illustrated in the form of a directed graph with nodes corresponding to tasks and edges indicating that the result of one task is required for processing the next.
- Such a graph is called a *task dependency graph*.

Example: Multiplying a Dense Matrix with a Vector



Computation of each element of output vector ***y*** is independent of other elements. Based on this, a dense matrix-vector product can be decomposed into ***n*** tasks. The figure highlights the portion of the matrix and vector accessed by Task 1.

Observations: While tasks share data (namely, the vector ***b***), they do not have any control dependencies - i.e., no task needs to wait for the (partial) completion of any other. All tasks are of the same size in terms of number of operations. *Is this the maximum number of tasks we could decompose this problem into?*

Example: Database Query Processing

Consider the execution of the query:

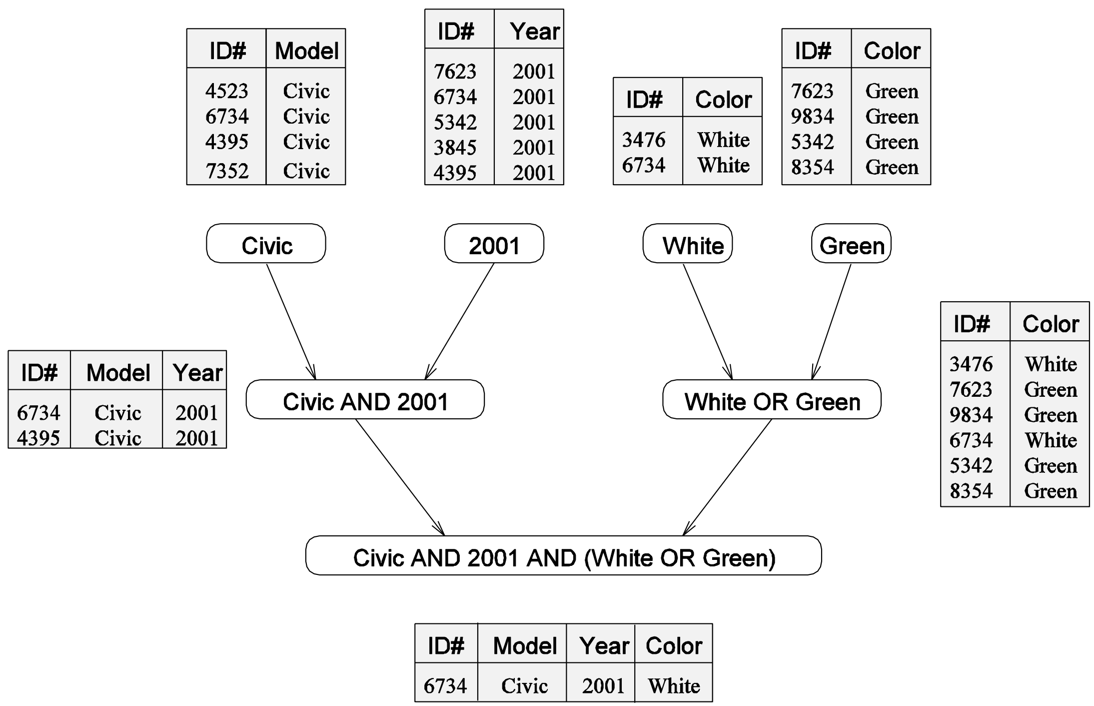
MODEL = ``CIVIC" AND YEAR = 2001 AND
(COLOR = ``GREEN" OR COLOR = ``WHITE)

on the following database:

ID#	Model	Year	Color	Dealer	Price
4523	Civic	2002	Blue	MN	$18,000
3476	Corolla	1999	White	IL	$15,000
7623	Camry	2001	Green	NY	$21,000
9834	Prius	2001	Green	CA	$18,000
6734	Civic	2001	White	OR	$17,000
5342	Altima	2001	Green	FL	$19,000
3845	Maxima	2001	Blue	NY	$22,000
8354	Accord	2000	Green	VT	$18,000
4395	Civic	2001	Red	CA	$17,000
7352	Civic	2002	Red	WA	$18,000

Example: Database Query Processing

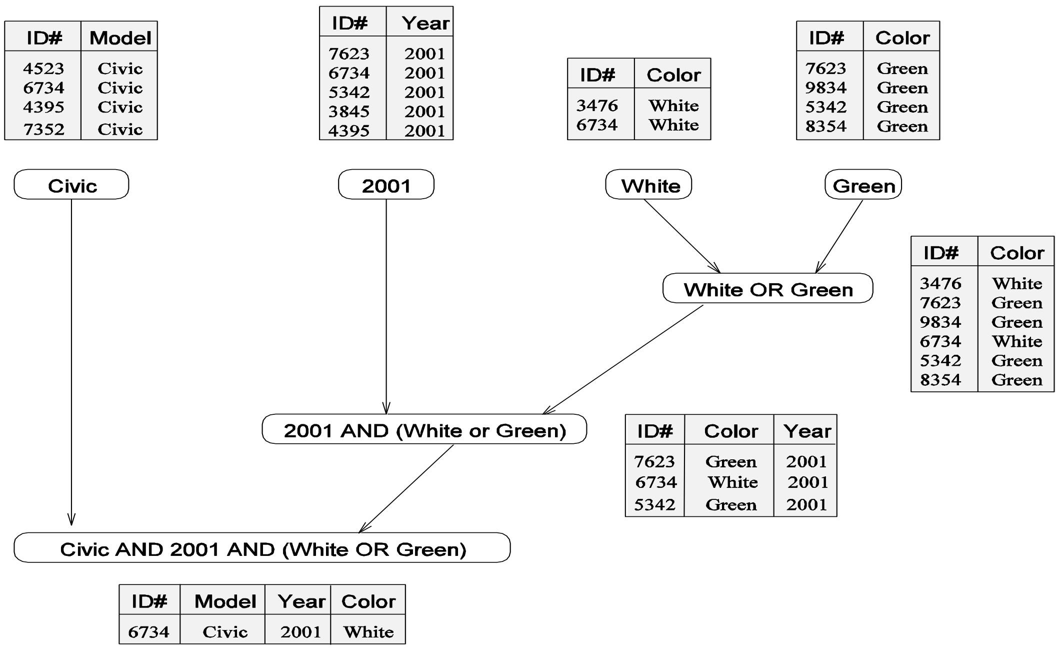
The execution of the query can be divided into subtasks in various ways. Each task can be thought of as generating an intermediate table of entries that satisfy a particular clause.



Decomposing the given query into a number of tasks. Edges in this graph denote that the output of one task is needed to accomplish the next.

Example: Database Query Processing

Note that the same problem can be decomposed into subtasks in other ways as well.

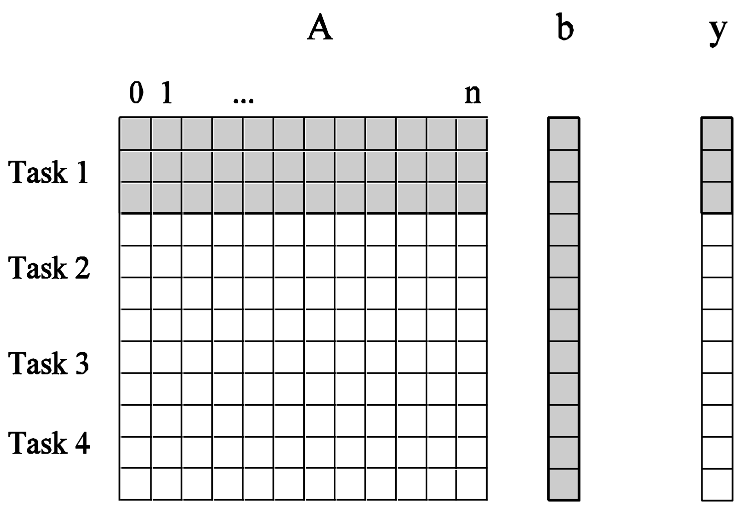


An alternate decomposition of the given problem into subtasks, along with their data dependencies.

Different task decompositions may lead to significant differences with respect to their eventual parallel performance.

Granularity of Task Decompositions

- The number of tasks into which a problem is decomposed determines its granularity.
- Decomposition into a large number of tasks results in fine-grained decomposition and that into a small number of tasks results in a coarse grained decomposition.



A coarse grained counterpart to the dense matrix-vector product example. Each task in this example corresponds to the computation of three elements of the result vector.

Degree of Concurrency

- The number of tasks that can be executed in parallel is the ***degree of concurrency*** of a decomposition.

- Since the number of tasks that can be executed in parallel may change over program execution, the ***maximum degree of concurrency*** is the maximum number of such tasks at any point during execution. *What is the maximum degree of concurrency of the database query examples?*
- The ***average degree of concurrency*** is the average number of tasks that can be processed in parallel over the execution of the program.
- *Assuming that each tasks in the database example takes identical processing time, what is the average degree of concurrency in each decomposition?*
- The degree of concurrency increases as the decomposition becomes finer in granularity and vice versa.

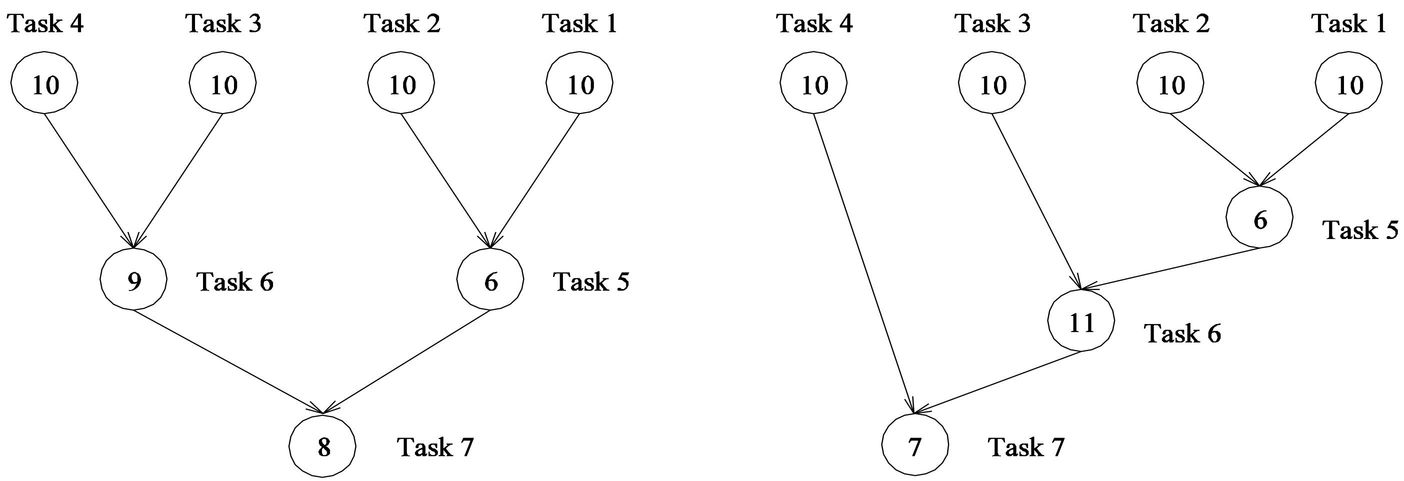
Critical Path Length

- A directed path in the task dependency graph represents a sequence of tasks that must be processed one after the other.

- The longest such path determines the shortest time in which the program can be executed in parallel.
- The length of the longest path in a task dependency graph is called the critical path length.

Critical Path Length

Consider the task dependency graphs of the two database query decompositions:



(a) (b)

What are the critical path lengths for the two task dependency graphs? If each task takes 10 time units, what is the shortest parallel execution time for each decomposition? How many processors are needed in each case to achieve this minimum parallel execution time? What is the maximum degree of concurrency?

maximum degree of concurrency

- In most cases, the maximum degree of concurrency is less than the total number of tasks due to dependencies among the tasks.
- For example, the maximum degree of concurrency in the task-graphs of Figures 3.2 and 3.3 is four. In these task-graphs, maximum concurrency is available right at the beginning when tables for Model, Year, Color Green, and Color White can be computed simultaneously. In general,
- for task dependency graphs that are trees, the maximum degree of concurrency is always equal to the number of leaves in the tree.

average degree of concurrency

- The degree of concurrency also depends on the shape of the task-dependency graph and the same granularity, in general, does not guarantee the same degree of concurrency.
- For example, consider the two task graphs in Figure 3.5, which are abstractions of the task graphs of Figures 3.2 and 3.3, respectively
- The number inside each node represents the amount of work required to complete the task corresponding to that node.
- The average degree of concurrency of the task graph in Figure 3.5(a) is 2.33 and that of the task graph in Figure 3.5(b) is 1.88

average degree of concurrency

Phase 1: Task 1, Task 2, Task 3 and Task 4 can be executed parallelly (if you have more than 3 processors). So the degree of concurrency in this phase is the sum of the weights of those four nodes: 10+10+10+10 = 40.

Phase 2: Task 6 and Task 5 can be executed parallelly (if you have more than 1 processor). So the degree of concurrency in this phase is: 9+6 = 15.

Phase 3: You can execute only Task 7, so the degree of concurrency here is 8.

The maximum number of concurrency is max(40, 15, 8) = 40. The average degree of concurrency is (40+15+8)/(10+9+8) = 63/27.

average degree of concurrency

Graph b:
Phase 1: Task 1, Task 2, Task 3 and Task 4 can be executed parallelly (if you have more than 3 processors). So the degree of concurrency in this phase is: 10+10+10+10 = 40.
Phase 2: You can execute only Task 5, so the degree of concurrency here is 6.
Phase 3: You can execute only Task 6, so the degree of concurrency here is 11.
Phase 4: You can execute only Task 7, so the degree of concurrency here is 7.
The maximum number of concurrency is max(40, 6, 11, 7) = 40. The average degree of concurrency is (40+6+11+7)/(10+6+11+7) = 64/34

average degree of concurrency

- The ratio of the total amount of work to the critical-path length is the average degree of concurrency.
- Therefore, a shorter critical path favors a higher degree of concurrency.
- For example, the critical path length is 27 in the task-dependency graph shown in Figure 3.5(a) and s 34 in the task-dependency graph shown in Figure 3.5(b).
- Since the total amount of work required to solve the problems using the two decompositions is 63 and 64, respectively, the average degree of concurrency of the two task-dependency graphs is 2.33 and 1.88, respectively.

Limits on Parallel Performance

- It would appear that the parallel time can be made arbitrarily small by making the decomposition finer in granularity.

- There is an inherent bound on how fine the granularity of a computation can be. *For example, in the case of multiplying a dense matrix with a vector, there can be no more than* ***(n^2)*** *concurrent tasks.*
- Concurrent tasks may also have to exchange data with other tasks. This results in communication overhead. The tradeoff between the granularity of a decomposition and associated overheads often determines performance bounds.

Task Interaction Graphs

- Subtasks generally exchange data with others in a decomposition. For example, even in the trivial decomposition of the dense matrix-vector product, if the vector is not replicated across all tasks, they will have to communicate elements of the vector.

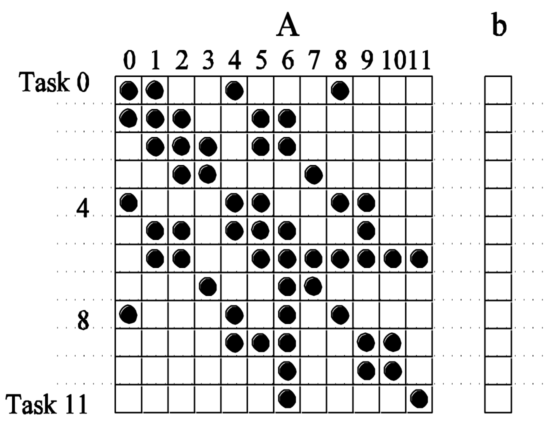
- The graph of tasks (nodes) and their interactions/data exchange (edges) is referred to as a *task interaction graph*.

- Note that *task interaction graphs* represent data dependencies, whereas *task dependency graphs* represent control dependencies.

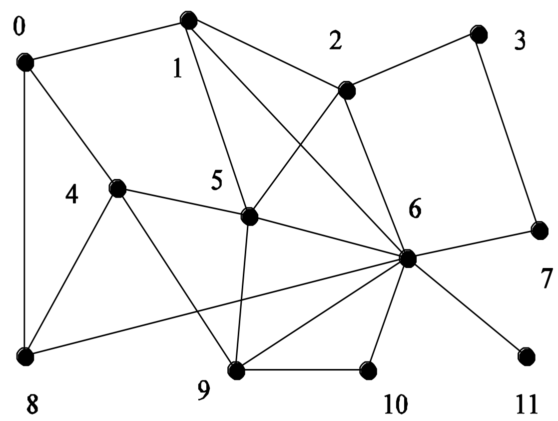
Task Interaction Graphs: An Example

Consider the problem of multiplying a sparse matrix ***A*** with a vector ***b****.* The following observations can be made:

- As before, the computation of each element of the result vector can be viewed as an independent task.
- Unlike a dense matrix-vector product though, only non-zero elements of matrix ***A*** participate in the computation.
- If, for memory optimality, we also partition ***b*** across tasks, then one can see that the task interaction graph of the computation is identical to the graph of the matrix ***A*** (the graph for which ***A*** represents the adjacency structure).



(a)



(b)

Task Interaction Graphs, Granularity, and Communication

In general, if the granularity of a decomposition is finer, the associated overhead (as a ratio of useful work assocaited with a task) increases.

Example: Consider the sparse matrix-vector product example from previous foil. Assume that each node takes unit time to process and each interaction (edge) causes an overhead of a unit time.

Viewing node 0 as an independent task involves a useful computation of one time unit and overhead (communication) of three time units.

Now, if we consider nodes 0, 4, and 5 as one task, then the task has useful computation totaling to three time units and communication corresponding to four time units (four edges). Clearly, this is a more favorable ratio than the former case.

Processes and Mapping

- In general, the number of tasks in a decomposition exceeds the number of processing elements available.

- For this reason, a parallel algorithm must also provide a mapping of tasks to processes.

Note: We refer to the mapping as being from tasks to processes, as opposed to processors. This is because typical programming APIs, as we shall see, do not allow easy binding of tasks to physical processors. Rather, we aggregate tasks into processes and rely on the system to map these processes to physical processors. We use processes, not in the UNIX sense of a process, rather, simply as a collection of tasks and associated data.

Processes and Mapping

- Appropriate mapping of tasks to processes is critical to the parallel performance of an algorithm.
- Mappings are determined by both the task dependency and task interaction graphs.
- Task dependency graphs can be used to ensure that work is equally spread across all processes at any point (minimum idling and optimal load balance).
- Task interaction graphs can be used to make sure that processes need minimum interaction with other processes (minimum communication).

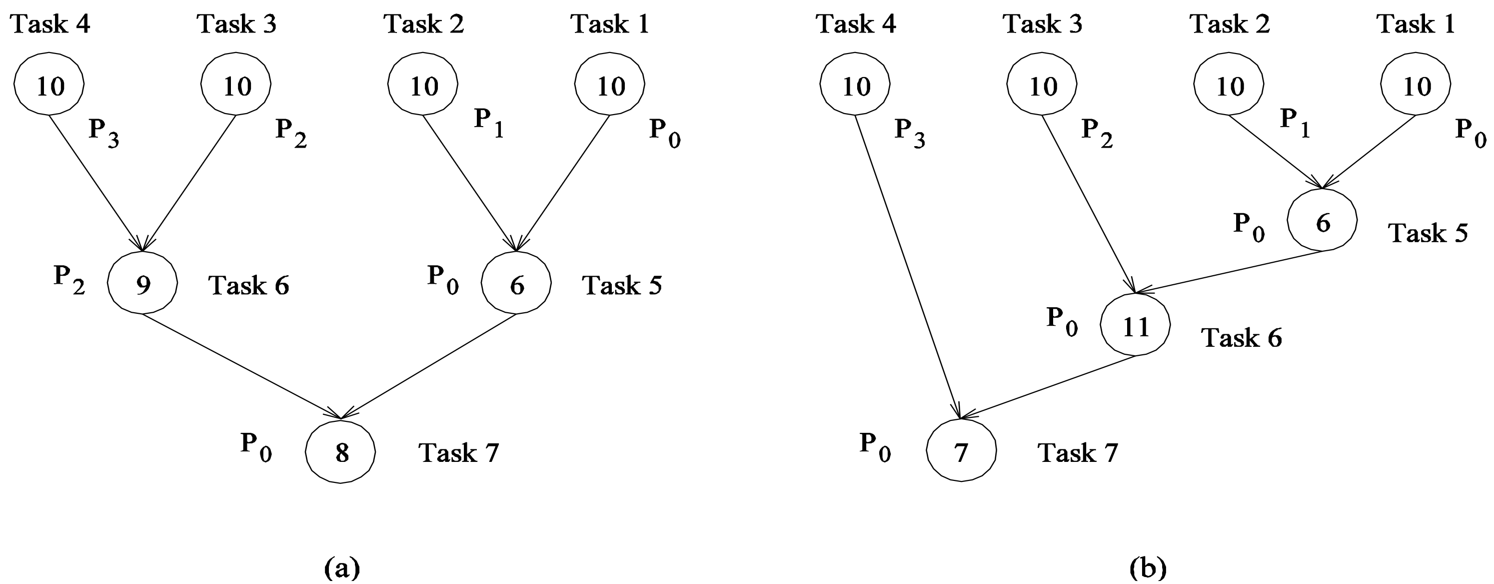
Processes and Mapping

An appropriate mapping must minimize parallel execution time by:

- Mapping independent tasks to different processes.
- Assigning tasks on critical path to processes as soon as they become available.
- Minimizing interaction between processes by mapping tasks with dense interactions to the same process.

Note: These criteria often conflict eith each other. For example, a decomposition into one task (or no decomposition at all) minimizes interaction but does not result in a speedup at all! Can you think of other such conflicting cases?

Processes and Mapping: Example



Mapping tasks in the database query decomposition to processes. These mappings were arrived at by viewing the dependency graph in terms of levels (no two nodes in a level have dependencies). Tasks within a single level are then assigned to different processes.

Decomposition Techniques

So how does one decompose a task into various subtasks?

While there is no single recipe that works for all problems, we present a set of commonly used techniques that apply to broad classes of problems. These include:

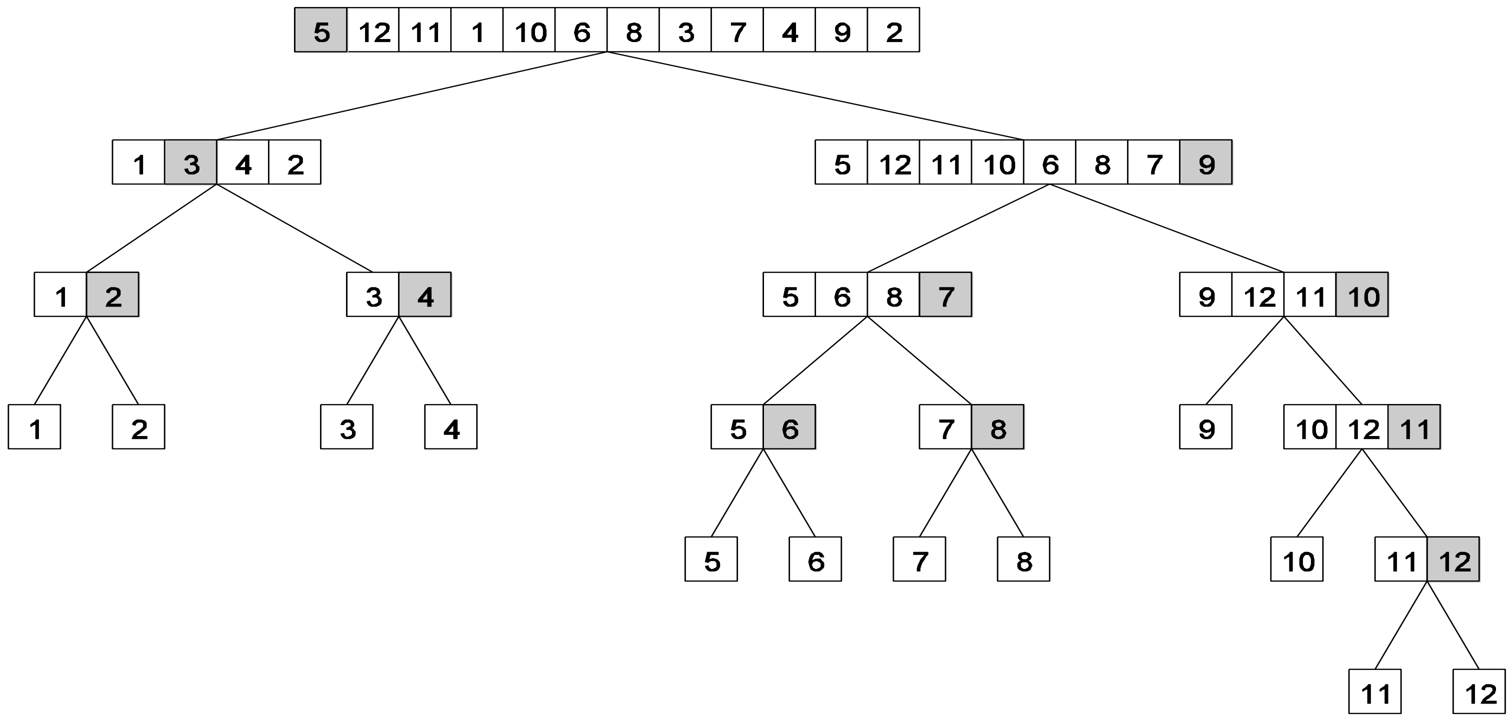
- recursive decomposition
- data decomposition
- exploratory decomposition
- speculative decomposition

Recursive Decomposition

- Generally suited to problems that are solved using the divide-and-conquer strategy.
- A given problem is first decomposed into a set of sub-problems.
- These sub-problems are recursively decomposed further until a desired granularity is reached.

Recursive Decomposition: Example

A classic example of a divide-and-conquer algorithm on which we can apply recursive decomposition is Quicksort.



In this example, once the list has been partitioned around the pivot, each sublist can be processed concurrently (i.e., each sublist represents an independent subtask). This can be repeated recursively.

Recursive Decomposition: Example

The problem of finding the minimum number in a given list (or indeed any other associative operation such as sum, AND, etc.) can be fashioned as a divide-and-conquer algorithm. The following algorithm illustrates this.

We first start with a simple serial loop for computing the minimum entry in a given list:

procedure SERIAL_MIN (*A, n*)

begin

min = A[0];

for *i* := 1 **to** *n* − 1 **do**

    if (*A*[*i*] < *min*) *min* := *A*[*i*];

endfor;

return *min*;

end SERIAL_MIN

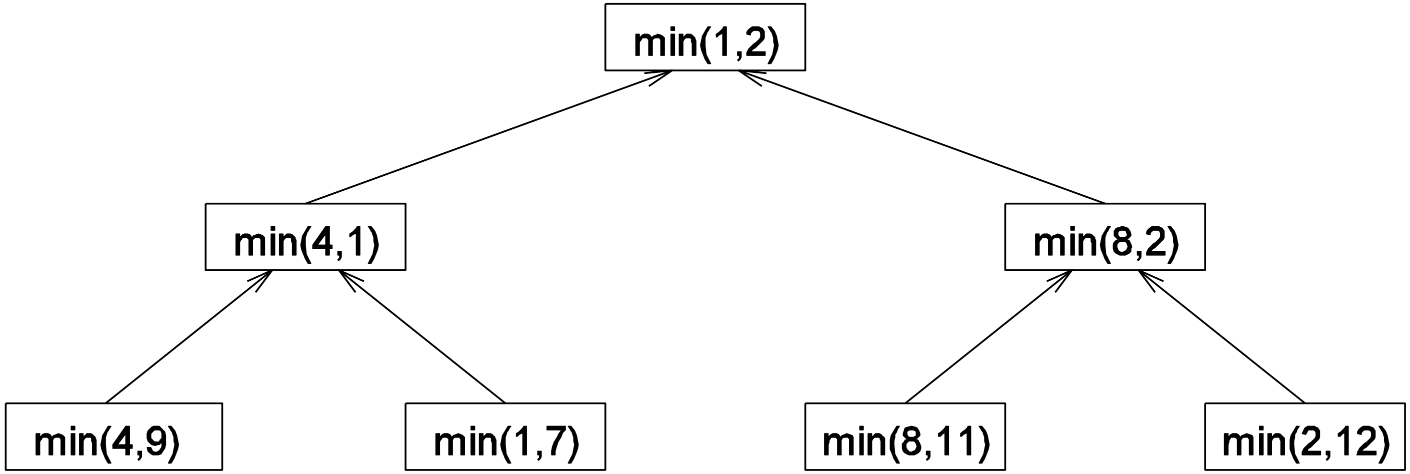
Recursive Decomposition: Example

We can rewrite the loop as follows:

```
procedure RECURSIVE_MIN (A, n)
begin
if ( n = 1 ) then
    min := A [0]  ;
else
    lmin := RECURSIVE_MIN ( A, n/2 );
    rmin := RECURSIVE_MIN (  &(A[n/2]), n - n/2 );
    if (lmin  < rmin) then
            min := lmin;
    else
            min := rmin;
    endelse;
endelse;
return min;
end RECURSIVE_MIN
```

Recursive Decomposition: Example

The code in the previous foil can be decomposed naturally using a recursive decomposition strategy. We illustrate this with the following example of finding the minimum number in the set {4, 9, 1, 7, 8, 11, 2, 12}. The task dependency graph associated with this computation is as follows:



Data Decomposition

- Identify the data on which computations are performed.
- Partition this data across various tasks.
- This partitioning induces a decomposition of the problem.
- Data can be partitioned in various ways - this critically impacts performance of a parallel algorithm.

Data Decomposition: Output Data Decomposition

- Often, each element of the output can be computed independently of others (but simply as a function of the input).
- A partition of the output across tasks decomposes the problem naturally.

Output Data Decomposition: Example

Consider the problem of multiplying two ***n*** x ***n*** matrices ***A*** and ***B*** to yield matrix ***C***. The output matrix ***C*** can be partitioned into four tasks as follows:

$$\begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix} \rightarrow \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

Task 1: $C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1}$

Task 2: $C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2}$

Task 3: $C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1}$

Task 4: $C_{2,2} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2}$

Output Data Decomposition: Example

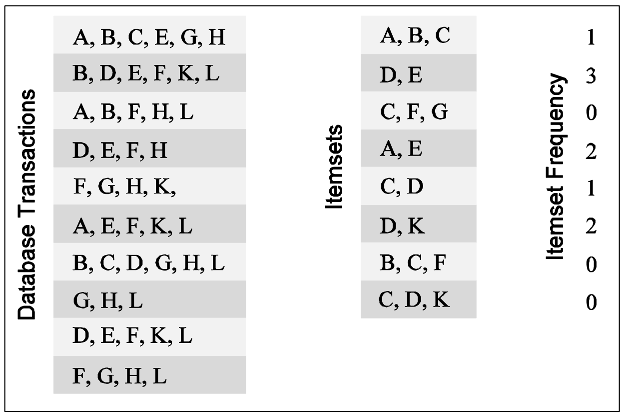
A partitioning of output data does not result in a unique decomposition into tasks. For example, for the same problem as in previus foil, with identical output data distribution, we can derive the following two (other) decompositions:

Decomposition I	**Decomposition II**
Task 1: $C_{1,1} = A_{1,1} B_{1,1}$	Task 1: $C_{1,1} = A_{1,1} B_{1,1}$
Task 2: $C_{1,1} = C_{1,1} + A_{1,2} B_{2,1}$	Task 2: $C_{1,1} = C_{1,1} + A_{1,2} B_{2,1}$
Task 3: $C_{1,2} = A_{1,1} B_{1,2}$	Task 3: $C_{1,2} = A_{1,2} B_{2,2}$
Task 4: $C_{1,2} = C_{1,2} + A_{1,2} B_{2,2}$	Task 4: $C_{1,2} = C_{1,2} + A_{1,1} B_{1,2}$
Task 5: $C_{2,1} = A_{2,1} B_{1,1}$	Task 5: $C_{2,1} = A_{2,2} B_{2,1}$
Task 6: $C_{2,1} = C_{2,1} + A_{2,2} B_{2,1}$	Task 6: $C_{2,1} = C_{2,1} + A_{2,1} B_{1,1}$
Task 7: $C_{2,2} = A_{2,1} B_{1,2}$	Task 7: $C_{2,2} = A_{2,1} B_{1,2}$
Task 8: $C_{2,2} = C_{2,2} + A_{2,2} B_{2,2}$	Task 8: $C_{2,2} = C_{2,2} + A_{2,2} B_{2,2}$

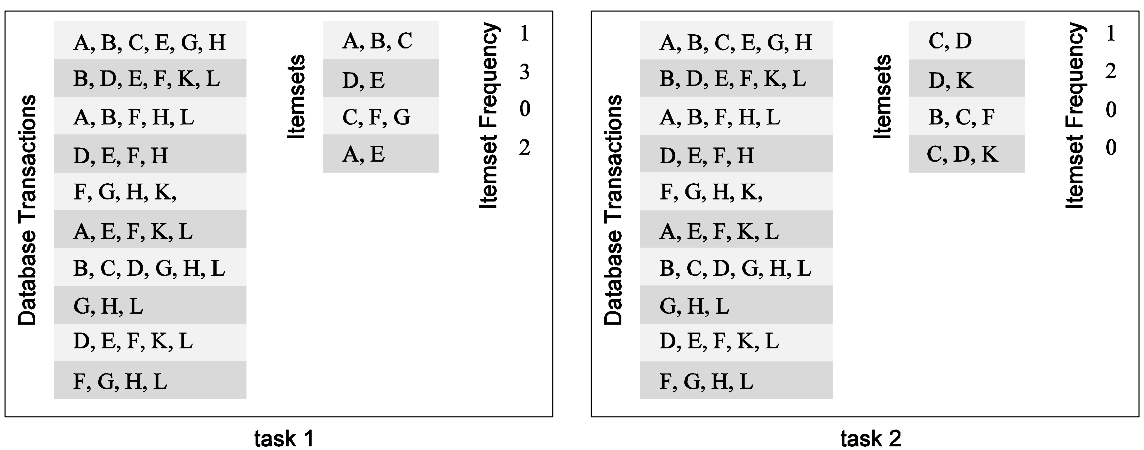
Output Data Decomposition: Example

Consider the problem of counting the instances of given itemsets in a database of transactions. In this case, the output (itemset frequencies) can be partitioned across tasks.

(a) Transactions (input), itemsets (input), and frequencies (output)



(b) Partitioning the frequencies (and itemsets) among the tasks



Output Data Decomposition: Example

From the previous example, the following observations can be made:

- If the database of transactions is replicated across the processes, each task can be independently accomplished with no communication.
- If the database is partitioned across processes as well (for reasons of memory utilization), each task first computes partial counts. These counts are then aggregated at the appropriate task.

Input Data Partitioning

- Generally applicable if each output can be naturally computed as a function of the input.
- In many cases, this is the only natural decomposition because the output is not clearly known a-priori (e.g., the problem of finding the minimum in a list, sorting a given list, etc.).
- A task is associated with each input data partition. The task performs as much of the computation with its part of the data. Subsequent processing combines these partial results.

Input Data Partitioning: Example

In the database counting example, the input (i.e., the transaction set) can be partitioned. This induces a task decomposition in which each task generates partial counts for all itemsets. These are combined subsequently for aggregate counts.

Partitioning the transactions among the tasks

task 1

Database Transactions
A, B, C, E, G, H
B, D, E, F, K, L
A, B, F, H, L
D, E, F, H
F, G, H, K,

Itemsets	Itemset Frequency
A, B, C	1
D, E	2
C, F, G	0
A, E	1
C, D	0
D, K	1
B, C, F	0
C, D, K	0

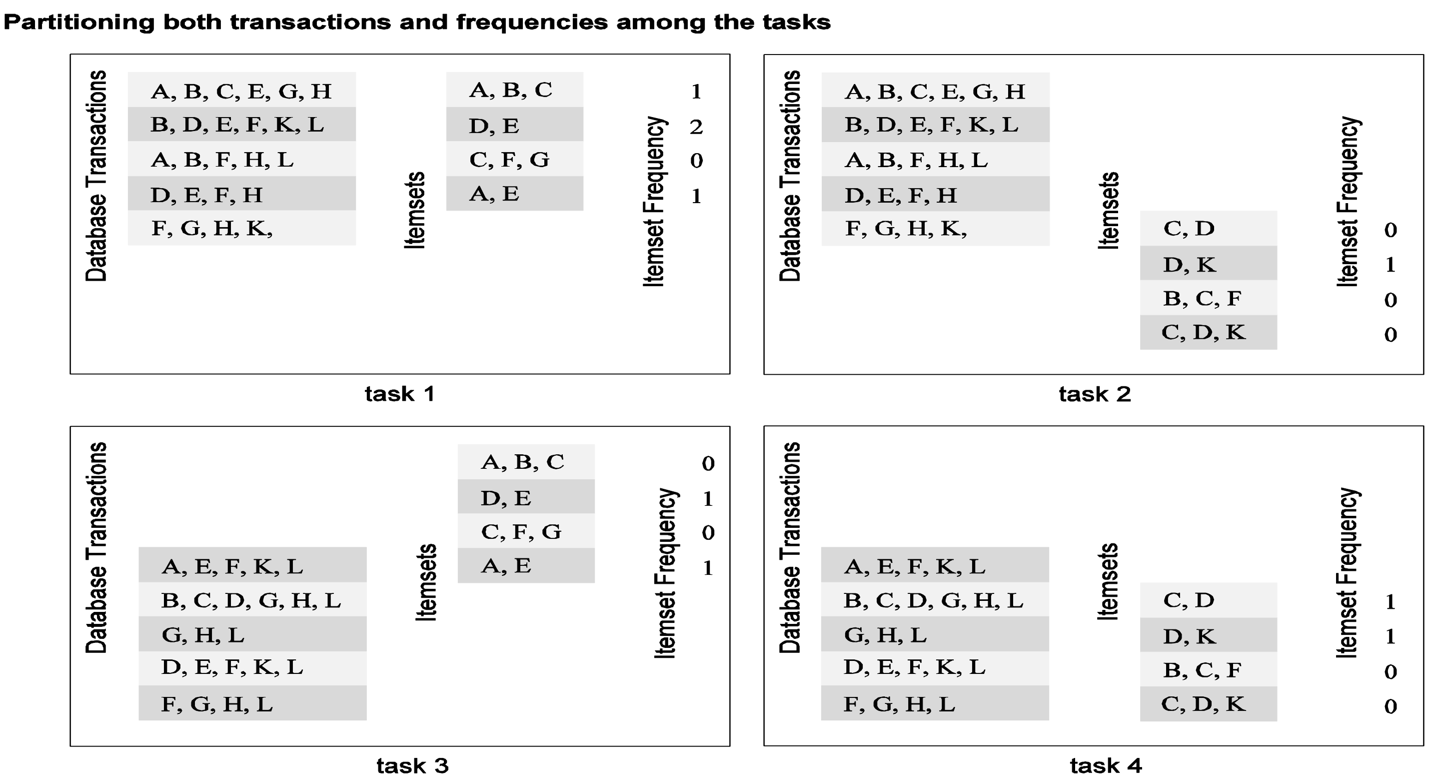
task 2

Database Transactions
A, E, F, K, L
B, C, D, G, H, L
G, H, L
D, E, F, K, L
F, G, H, L

Itemsets	Itemset Frequency
A, B, C	0
D, E	1
C, F, G	0
A, E	1
C, D	1
D, K	1
B, C, F	0
C, D, K	0

Partitioning Input *and* Output Data

Often input and output data decomposition can be combined for a higher degree of concurrency. For the itemset counting example, the transaction set (input) and itemset counts (output) can both be decomposed as follows:

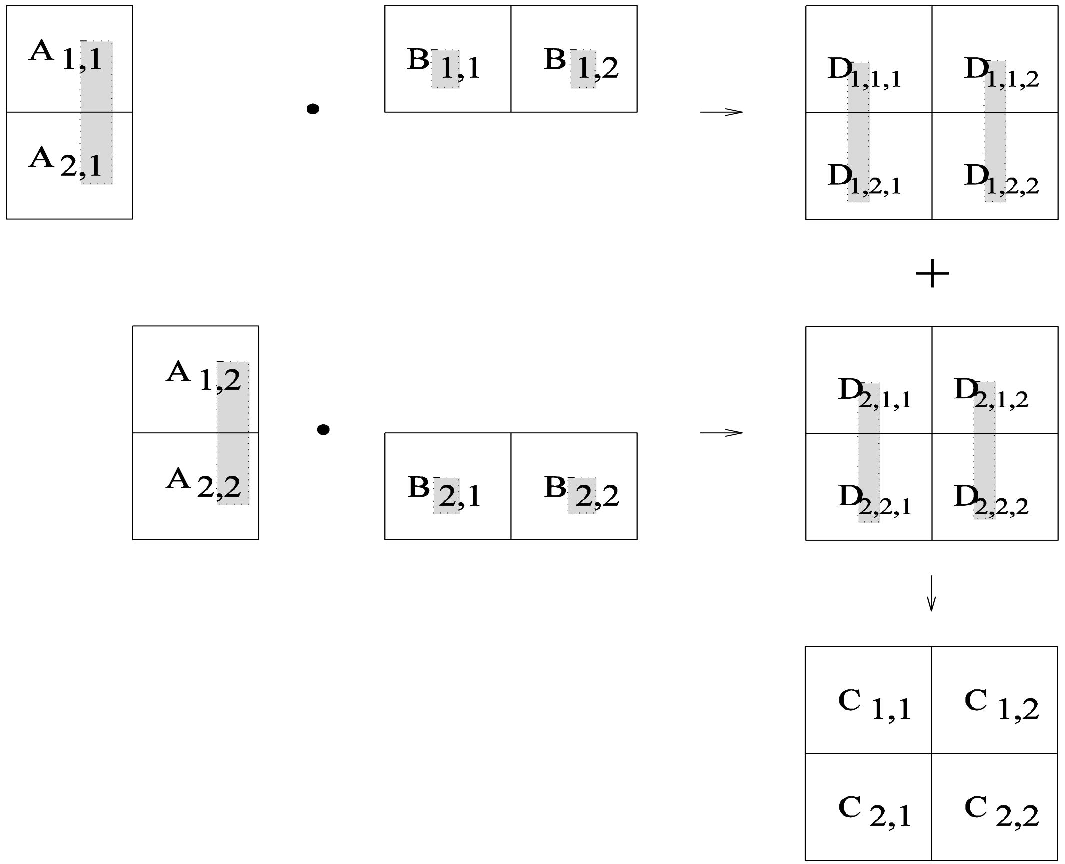


Intermediate Data Partitioning

- Computation can often be viewed as a sequence of transformation from the input to the output data.
- In these cases, it is often beneficial to use one of the intermediate stages as a basis for decomposition.

Intermediate Data Partitioning: Example

Let us revisit the example of dense matrix multiplication. We first show how we can visualize this computation in terms of intermediate matrices ***D***.



Intermediate Data Partitioning: Example

A decomposition of intermediate data structure leads to the following decomposition into 8 + 4 tasks:

Stage I

$$\begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix} \rightarrow \begin{pmatrix} \begin{pmatrix} D_{1,1,1} & D_{1,1,2} \\ D_{1,2,2} & D_{1,2,2} \end{pmatrix} \\ \begin{pmatrix} D_{2,1,1} & D_{2,1,2} \\ D_{2,2,2} & D_{2,2,2} \end{pmatrix} \end{pmatrix}$$

Stage II

$$\begin{pmatrix} D_{1,1,1} & D_{1,1,2} \\ D_{1,2,2} & D_{1,2,2} \end{pmatrix} + \begin{pmatrix} D_{2,1,1} & D_{2,1,2} \\ D_{2,2,2} & D_{2,2,2} \end{pmatrix} \rightarrow \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

Task 01: $\boldsymbol{D}_{1,1,1} = \boldsymbol{A}_{1,1}\,\boldsymbol{B}_{1,1}$

Task 02: $\boldsymbol{D}_{2,1,1} = \boldsymbol{A}_{1,2}\,\boldsymbol{B}_{2,1}$

Task 03: $\boldsymbol{D}_{1,1,2} = \boldsymbol{A}_{1,1}\,\boldsymbol{B}_{1,2}$

Task 04: $\boldsymbol{D}_{2,1,2} = \boldsymbol{A}_{1,2}\,\boldsymbol{B}_{2,2}$

Task 05: $\boldsymbol{D}_{1,2,1} = \boldsymbol{A}_{2,1}\,\boldsymbol{B}_{1,1}$

Task 06: $\boldsymbol{D}_{2,2,1} = \boldsymbol{A}_{2,2}\,\boldsymbol{B}_{2,1}$

Task 07: $\boldsymbol{D}_{1,2,2} = \boldsymbol{A}_{2,1}\,\boldsymbol{B}_{1,2}$

Task 08: $\boldsymbol{D}_{2,2,2} = \boldsymbol{A}_{2,2}\,\boldsymbol{B}_{2,2}$

Task 09: $\boldsymbol{C}_{1,1} = \boldsymbol{D}_{1,1,1} + \boldsymbol{D}_{2,1,1}$

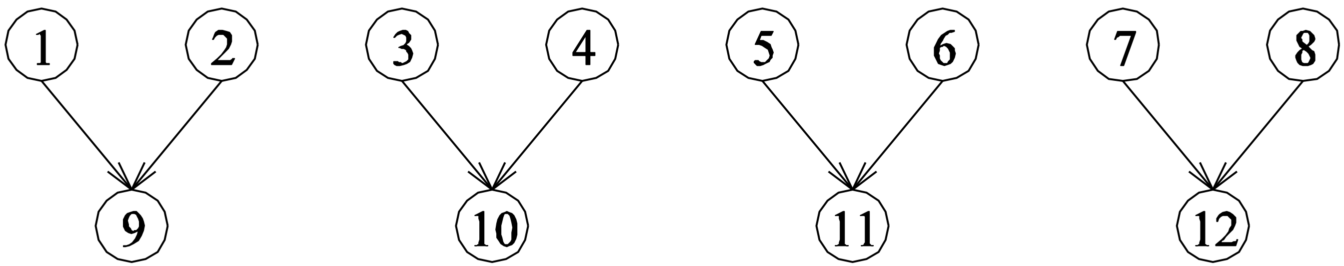
Task 10: $\boldsymbol{C}_{1,2} = \boldsymbol{D}_{1,1,2} + \boldsymbol{D}_{2,1,2}$

Task 11: $\boldsymbol{C}_{2,1} = \boldsymbol{D}_{1,2,1} + \boldsymbol{D}_{2,2,1}$

Task 12: $\boldsymbol{C}_{2,,2} = \boldsymbol{D}_{1,2,2} + \boldsymbol{D}_{2,2,2}$

Intermediate Data Partitioning: Example

The task dependency graph for the decomposition (shown in previous foil) into 12 tasks is as follows:



The Owner Computes Rule

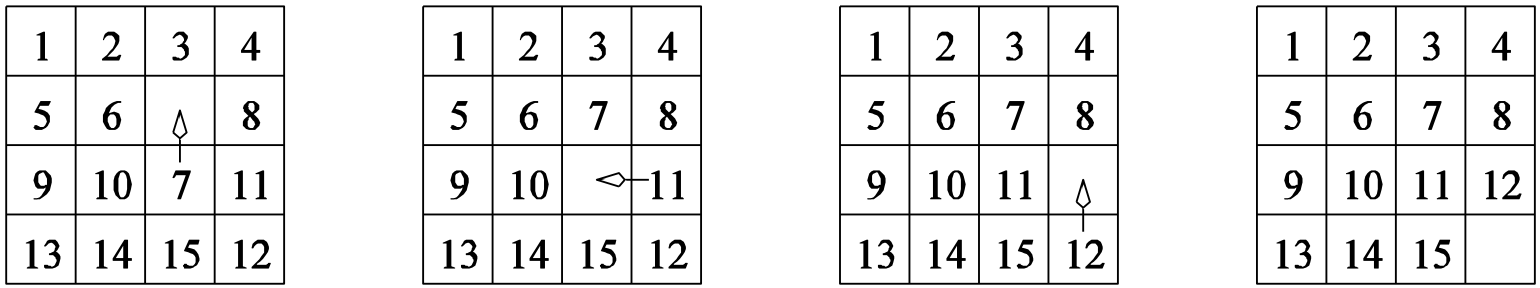
- The *Owner Computes Rule* generally states that the process assined a particular data item is responsible for all computation associated with it.
- In the case of input data decomposition, the owner computes rule imples that all computations that use the input data are performed by the process.
- In the case of output data decomposition, the owner computes rule implies that the output is computed by the process to which the output data is assigned.

Exploratory Decomposition

- In many cases, the decomposition of the problem goes hand-in-hand with its execution.
- These problems typically involve the exploration (search) of a state space of solutions.
- Problems in this class include a variety of discrete optimization problems (0/1 integer programming, QAP, etc.), theorem proving, game playing, etc.

Exploratory Decomposition: Example

A simple application of exploratory decomposition is in the solution to a 15 puzzle (a tile puzzle). We show a sequence of three moves that transform a given initial state (a) to desired final state (d).

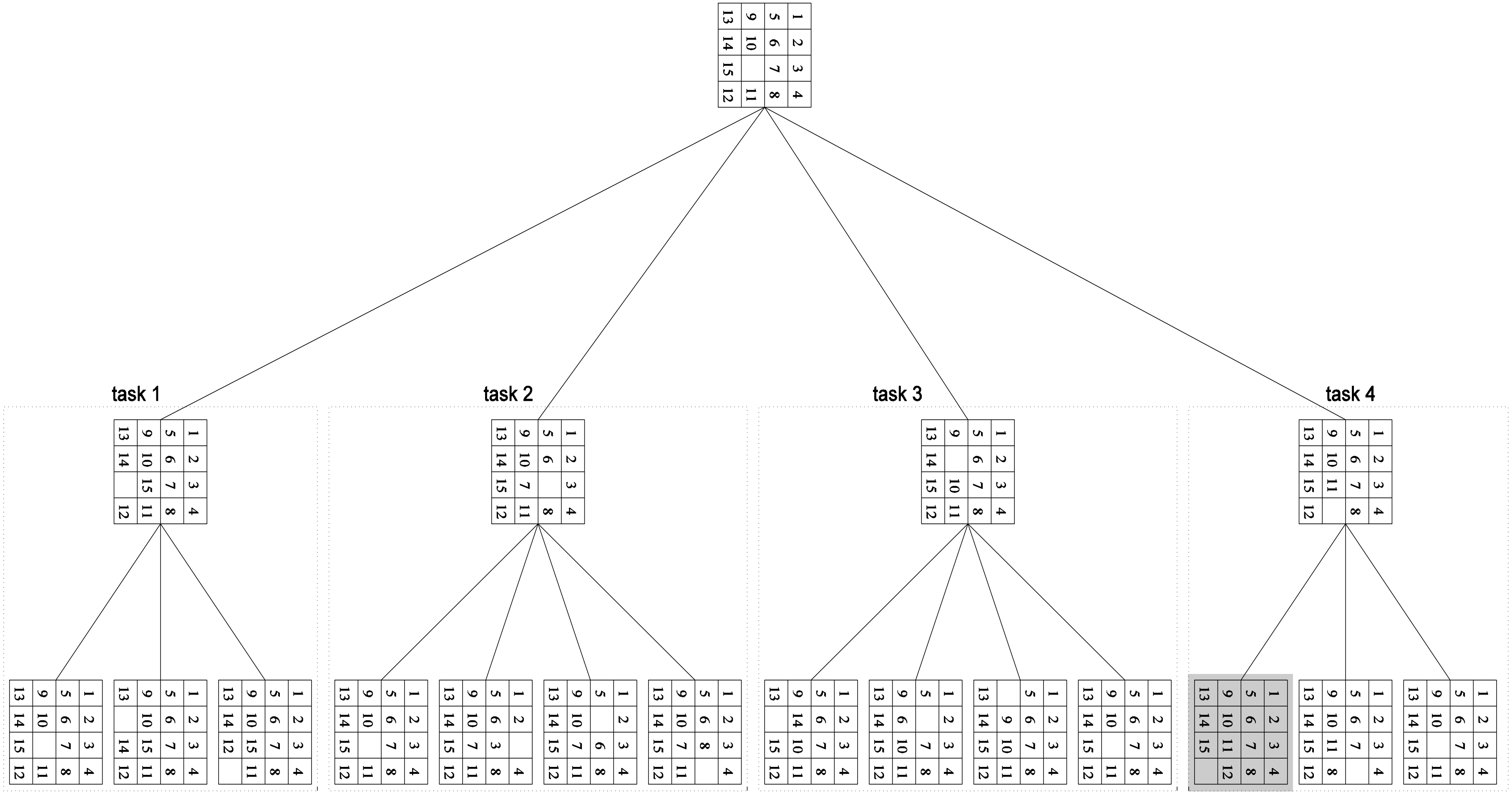


(a) (b) (c) (d)

Of-course, the problem of computing the solution, in general, is much more difficult than in this simple example.

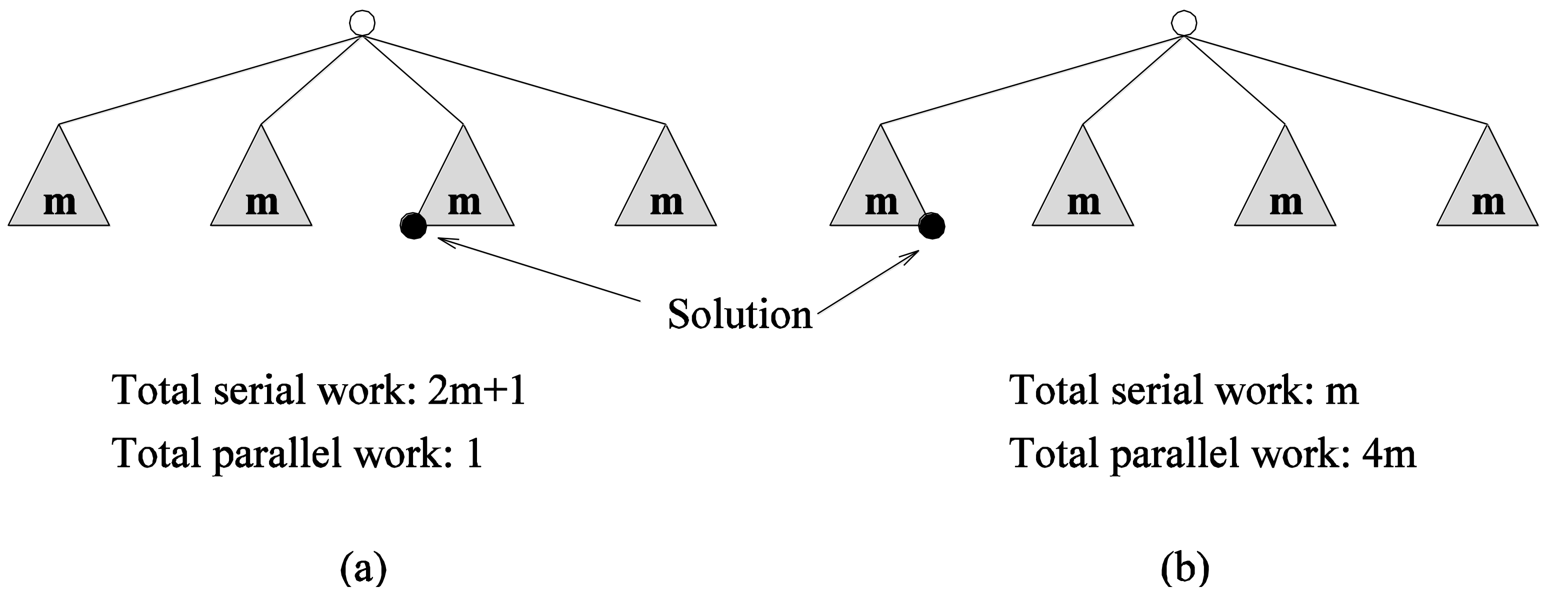
Exploratory Decomposition: Example

The state space can be explored by generating various successor states of the current state and to view them as independent tasks.



Exploratory Decomposition: Anomalous Computations

- In many instances of exploratory decomposition, the decomposition technique may change the amount of work done by the parallel formulation.
- This change results in super- or sub-linear speedups.



Speculative Decomposition

- In some applications, dependencies between tasks are not known a-priori.
- For such applications, it is impossible to identify independent tasks.
- There are generally two approaches to dealing with such applications: conservative approaches, which identify independent tasks only when they are guaranteed to not have dependencies, and, optimistic approaches, which schedule tasks even when they may potentially be erroneous.
- Conservative approaches may yield little concurrency and optimistic approaches may require roll-back mechanism in the case of an error.

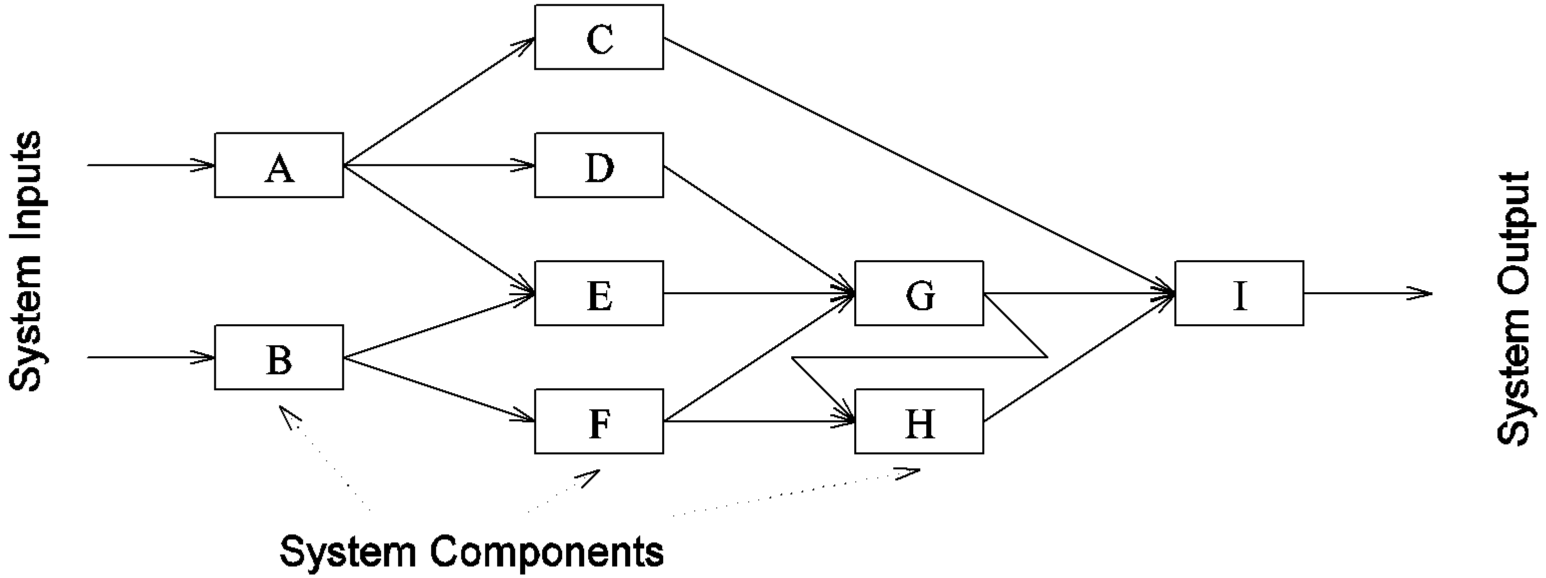
Speculative Decomposition: Example

A classic example of speculative decomposition is in discrete event simulation.

- The central data structure in a discrete event simulation is a time-ordered event list.
- Events are extracted precisely in time order, processed, and if required, resulting events are inserted back into the event list.
- Consider your day today as a discrete event system - you get up, get ready, drive to work, work, eat lunch, work some more, drive back, eat dinner, and sleep.
- Each of these events may be processed independently, however, in driving to work, you might meet with an unfortunate accident and not get to work at all.
- Therefore, an optimistic scheduling of other events will have to be rolled back.

Speculative Decomposition: Example

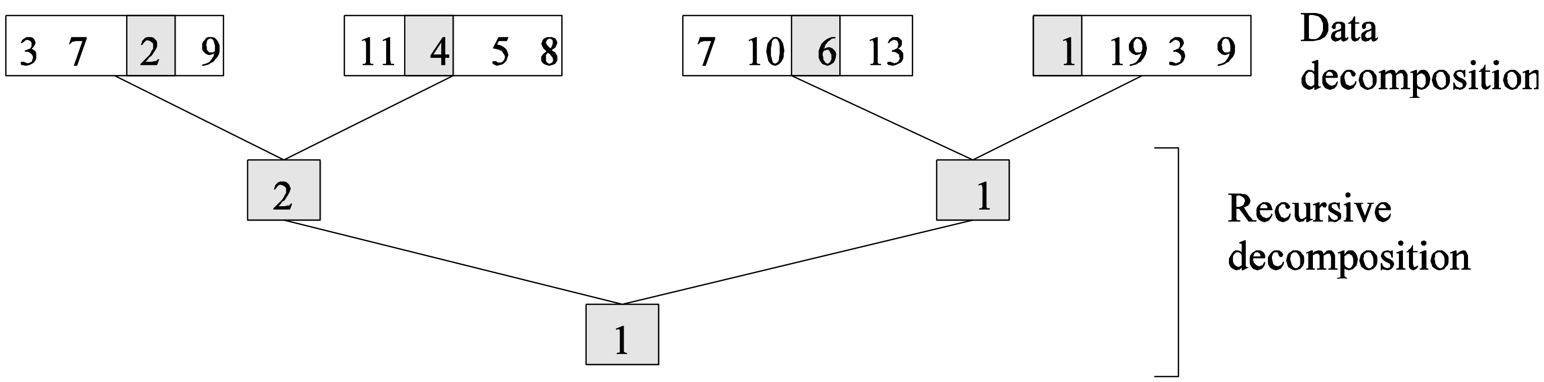
Another example is the simulation of a network of nodes (for instance, an assembly line or a computer network through which packets pass). The task is to simulate the behavior of this network for various inputs and node delay parameters (note that networks may become unstable for certain values of service rates, queue sizes, etc.).



Hybrid Decompositions

Often, a mix of decomposition techniques is necessary for decomposing a problem. Consider the following examples:

- In quicksort, recursive decomposition alone limits concurrency (Why?). A mix of data and recursive decompositions is more desirable.
- In discrete event simulation, there might be concurrency in task processing. A mix of speculative decomposition and data decomposition may work well.
- Even for simple problems like finding a minimum of a list of numbers, a mix of data and recursive decomposition works well.



Characteristics of Tasks

Once a problem has been decomposed into independent tasks, the characteristics of these tasks critically impact choice and performance of parallel algorithms. Relevant task characteristics include:

- Task generation.
- Task sizes.
- Size of data associated with tasks.

Task Generation

- Static task generation: Concurrent tasks can be identified a-priori. Typical matrix operations, graph algorithms, image processing applications, and other regularly structured problems fall in this class. These can typically be decomposed using data or recursive decomposition techniques.
- Dynamic task generation: Tasks are generated as we perform computation. A classic example of this is in game playing - each 15 puzzle board is generated from the previous one. These applications are typically decomposed using exploratory or speculative decompositions.

Task Sizes

- Task sizes may be uniform (i.e., all tasks are the same size) or non-uniform.
- Non-uniform task sizes may be such that they can be determined (or estimated) a-priori or not.
- Examples in this class include discrete optimization problems, in which it is difficult to estimate the effective size of a state space.

Size of Data Associated with Tasks

- The size of data associated with a task may be small or large when viewed in the context of the size of the task.
- A small context of a task implies that an algorithm can easily communicate this task to other processes dynamically (e.g., the 15 puzzle).
- A large context ties the task to a process, or alternately, an algorithm may attempt to reconstruct the context at another processes as opposed to communicating the context of the task (e.g., 0/1 integer programming).

Characteristics of Task Interactions

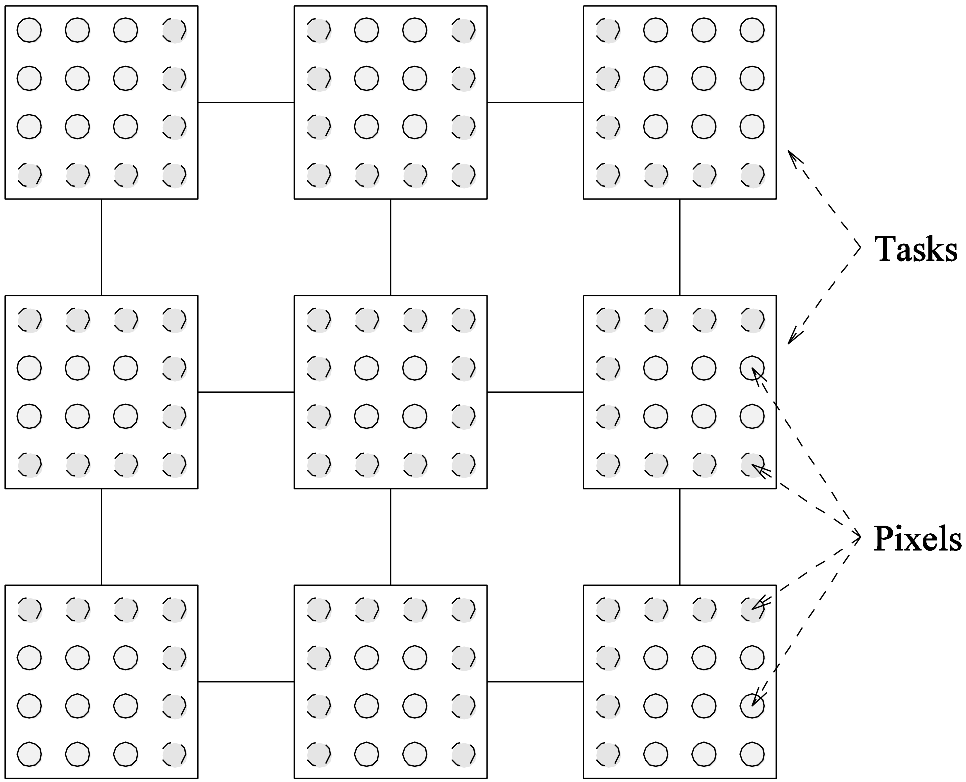
- Tasks may communicate with each other in various ways. The associated dichotomy is:
- Static interactions: The tasks and their interactions are known a-priori. These are relatively simpler to code into programs.
- Dynamic interactions: The timing or interacting tasks cannot be determined a-priori. These interactions are harder to code, especitally, as we shall see, using message passing APIs.

Characteristics of Task Interactions

- Regular interactions: There is a definite pattern (in the graph sense) to the interactions. These patterns can be exploited for efficient implementation.
- Irregular interactions: Interactions lack well-defined topologies.

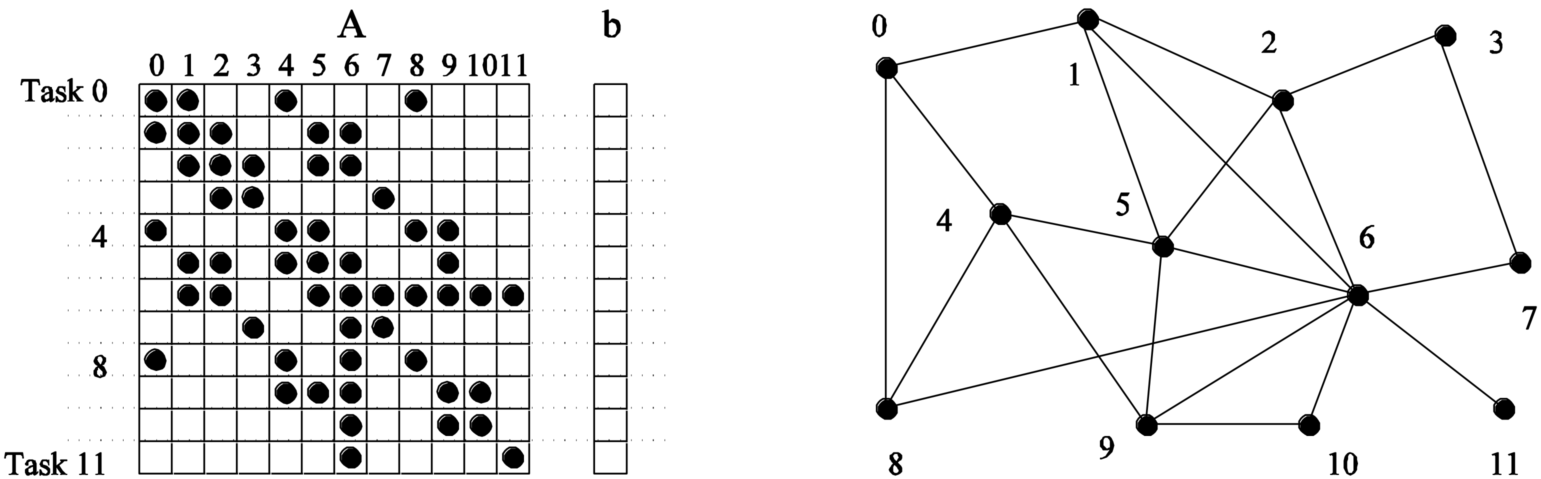
Characteristics of Task Interactions: Example

A simple example of a regular static interaction pattern is in image dithering. The underlying communication pattern is a structured (2-D mesh) one as shown here:



Characteristics of Task Interactions: Example

The multiplication of a sparse matrix with a vector is a good example of a static irregular interaction pattern. Here is an example of a sparse matrix and its associated interaction pattern.



(a) (b)

Characteristics of Task Interactions

- Interactions may be read-only or read-write.
- In read-only interactions, tasks just read data items associated with other tasks.
- In read-write interactions tasks read, as well as modily data items associated with other tasks.
- In general, read-write interactions are harder to code, since they require additional synchronization primitives.

Characteristics of Task Interactions

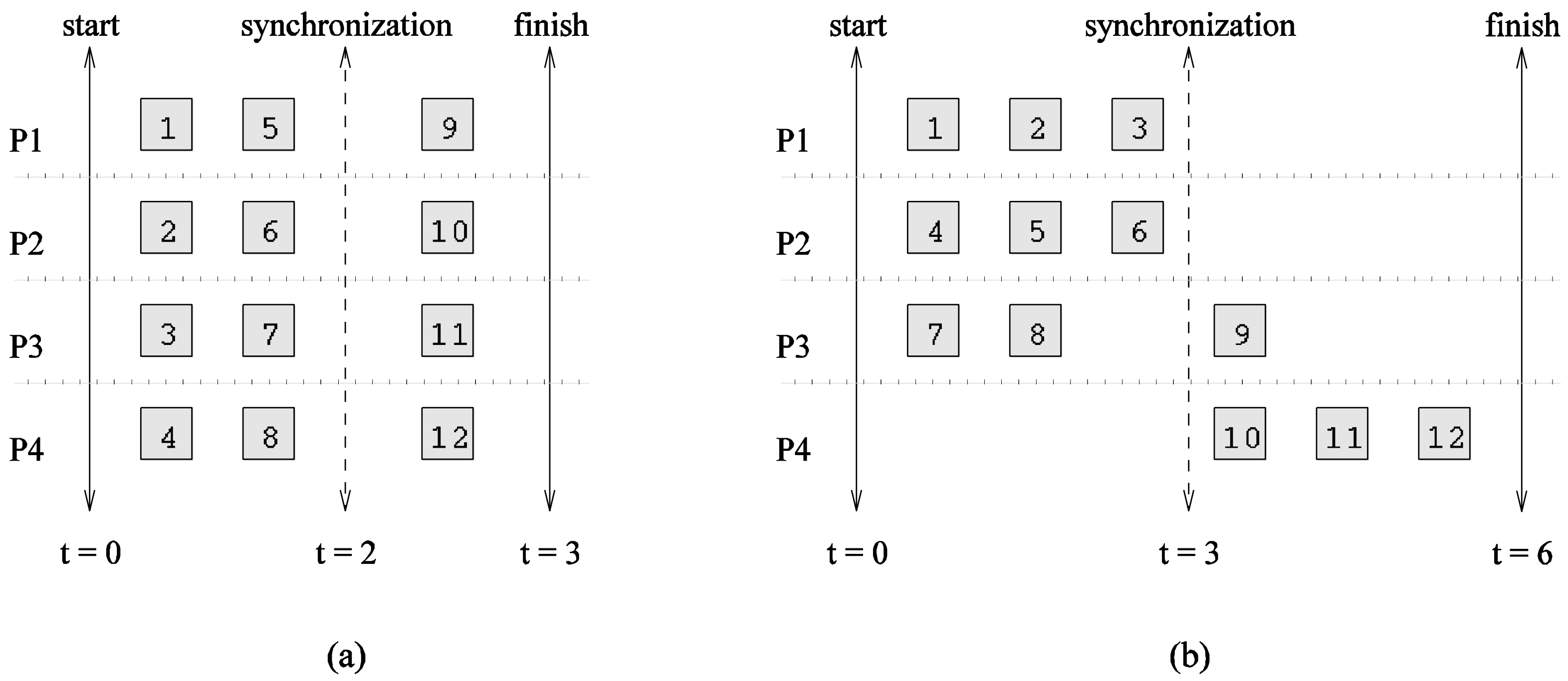
- Interactions may be one-way or two-way.
- A one-way interaction can be initiated and accomplished by one of the two interacting tasks.
- A two-way interaction requires participation from both tasks involved in an interaction.
- One way interactions are somewhat harder to code in message passing APIs.

Mapping Techniques

- Once a problem has been decomposed into concurrent tasks, these must be mapped to processes (that can be executed on a parallel platform).
- Mappings must minimize overheads.
- Primary overheads are communication and idling.
- Minimizing these overheads often represents contradicting objectives.
- Assigning all work to one processor trivially minimizes communication at the expense of significant idling.

Mapping Techniques for Minimum Idling

Mapping must simultaneously minimize idling and load balance. Merely balancing load does not minimize idling.



Mapping Techniques for Minimum Idling

Mapping techniques can be static or dynamic.

- Static Mapping: Tasks are mapped to processes a-priori. For this to work, we must have a good estimate of the size of each task. Even in these cases, the problem may be NP complete.
- Dynamic Mapping: Tasks are mapped to processes at runtime. This may be because the tasks are generated at runtime, or that their sizes are not known.

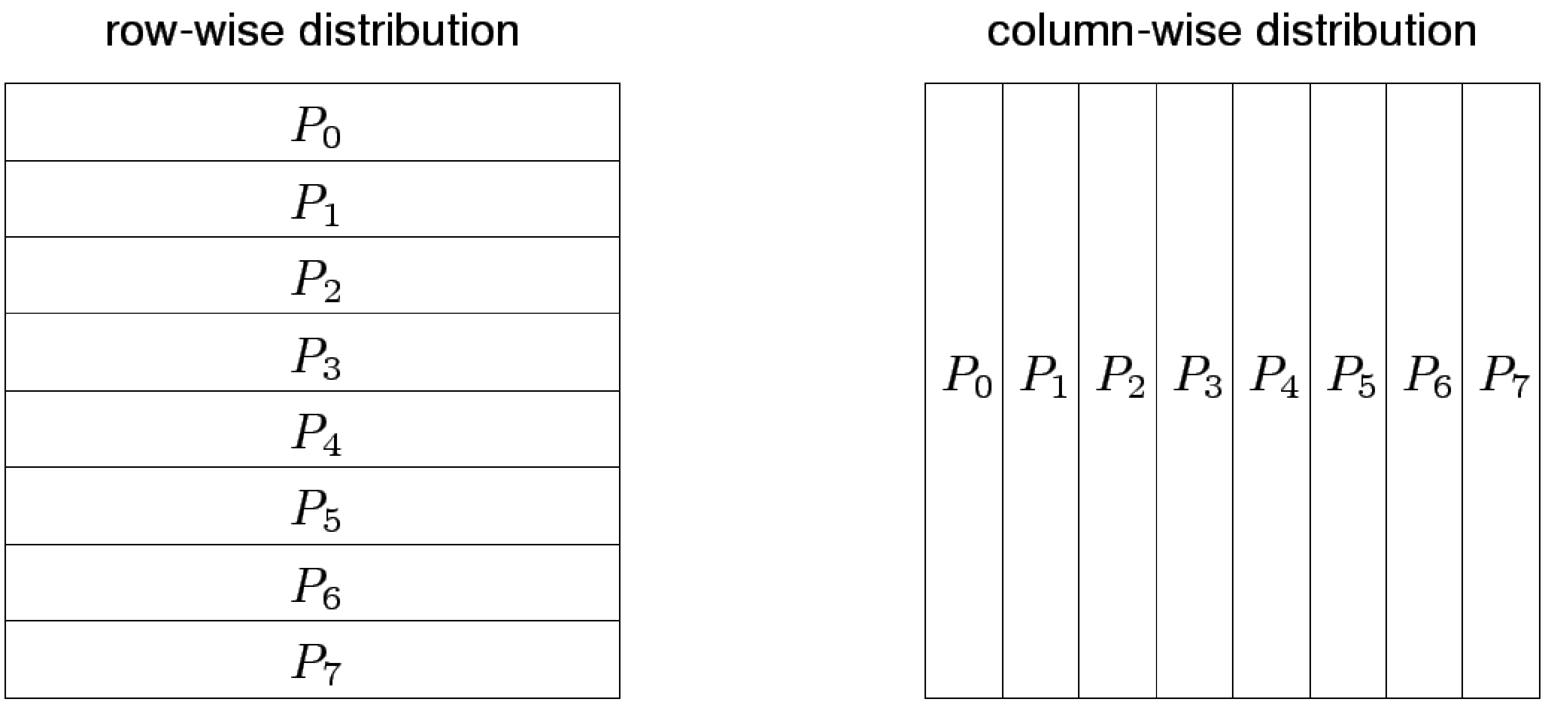
Other factors that determine the choice of techniques include the size of data associated with a task and the nature of underlying domain.

Schemes for Static Mapping

- Mappings based on data partitioning.
- Mappings based on task graph partitioning.
- Hybrid mappings.

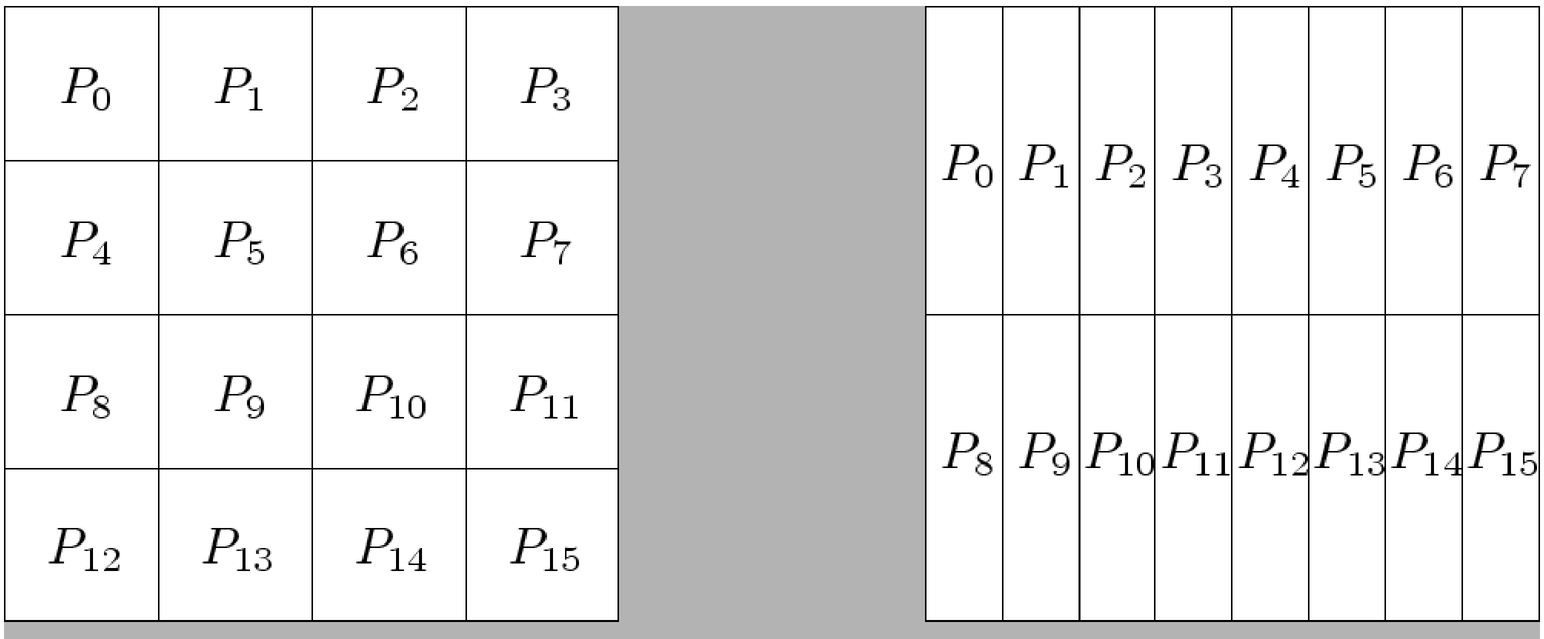
Mappings Based on Data Partitioning

We can combine data partitioning with the ``owner-computes'' rule to partition the computation into subtasks. The simplest data decomposition schemes for dense matrices are 1-D block distribution schemes.



Block Array Distribution Schemes

Block distribution schemes can be generalized to higher dimensions as well.

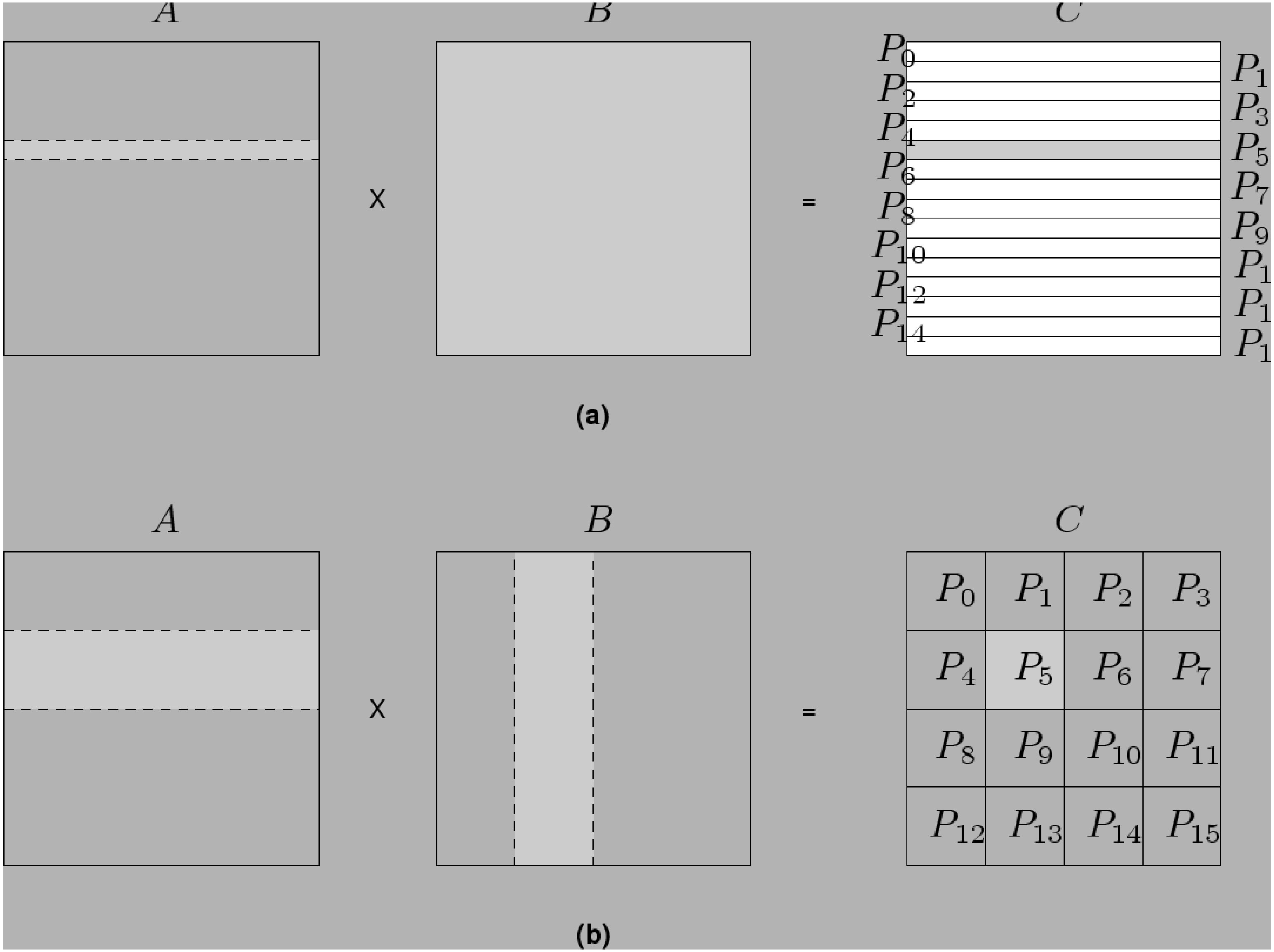


(a) **(b)**

Block Array Distribution Schemes: Examples

- For multiplying two dense matrices ***A*** and ***B***, we can partition the output matrix ***C*** using a block decomposition.
- For load balance, we give each task the same number of elements of ***C***. (Note that each element of ***C*** corresponds to a single dot product.)
- The choice of precise decomposition (1-D or 2-D) is determined by the associated communication overhead.
- In general, higher dimension decomposition allows the use of larger number of processes.

Data Sharing in Dense Matrix Multiplication



(a)

(b)

Cyclic and Block Cyclic Distributions

- If the amount of computation associated with data items varies, a block decomposition may lead to significant load imbalances.
- A simple example of this is in LU decomposition (or Gaussian Elimination) of dense matrices.

LU Factorization of a Dense Matrix

A decomposition of LU factorization into 14 tasks - notice the significant load imbalance.

$$\begin{pmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{pmatrix} \rightarrow \begin{pmatrix} L_{1,1} & 0 & 0 \\ L_{2,1} & L_{2,2} & 0 \\ L_{3,1} & L_{3,2} & L_{3,3} \end{pmatrix} \cdot \begin{pmatrix} U_{1,1} & U_{1,2} & U_{1,3} \\ 0 & U_{2,2} & U_{2,3} \\ 0 & 0 & U_{3,3} \end{pmatrix}$$

1: $A_{1,1} \rightarrow L_{1,1}U_{1,1}$

2: $L_{2,1} = A_{2,1}U_{1,1}^{-1}$

3: $L_{3,1} = A_{3,1}U_{1,1}^{-1}$

4: $U_{1,2} = L_{1,1}^{-1}A_{1,2}$

5: $U_{1,3} = L_{1,1}^{-1}A_{1,3}$

6: $A_{2,2} = A_{2,2} - L_{2,1}U_{1,2}$

7: $A_{3,2} = A_{3,2} - L_{3,1}U_{1,2}$

8: $A_{2,3} = A_{2,3} - L_{2,1}U_{1,3}$

9: $A_{3,3} = A_{3,3} - L_{3,1}U_{1,3}$

10: $A_{2,2} \rightarrow L_{2,2}U_{2,2}$

11: $L_{3,2} = A_{3,2}U_{2,2}^{-1}$

12: $U_{2,3} = L_{2,2}^{-1}A_{2,3}$

13: $A_{3,3} = A_{3,3} - L_{3,2}U_{2,3}$

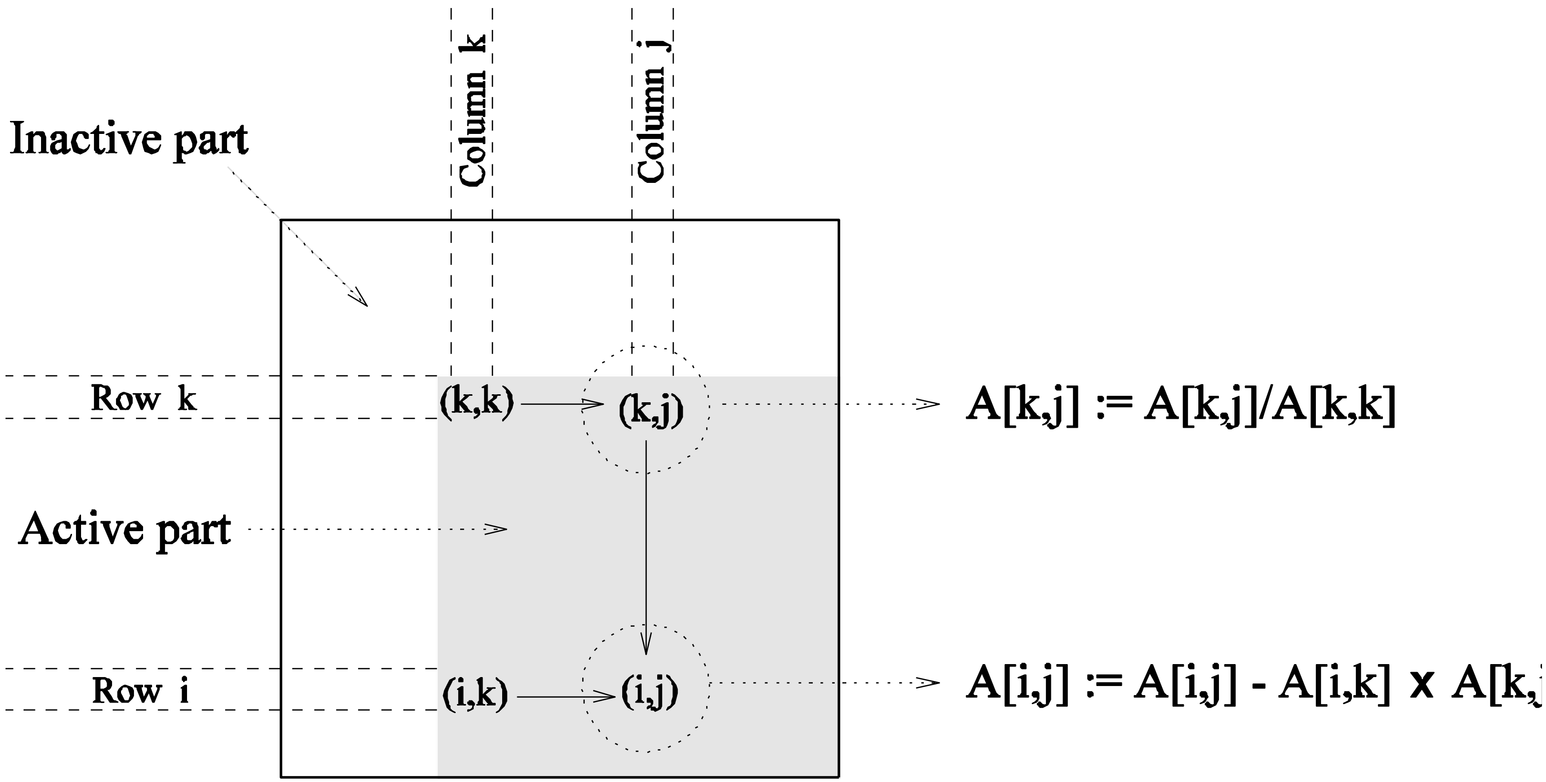
14: $A_{3,3} \rightarrow L_{3,3}U_{3,3}$

Block Cyclic Distributions

- Variation of the block distribution scheme that can be used to alleviate the load-imbalance and idling problems.
- Partition an array into many more blocks than the number of available processes.
- Blocks are assigned to processes in a round-robin manner so that each process gets several non-adjacent blocks.

Block-Cyclic Distribution for Gaussian Elimination

The active part of the matrix in Gaussian Elimination changes. By assigning blocks in a block-cyclic fashion, each processor receives blocks from different parts of the matrix.



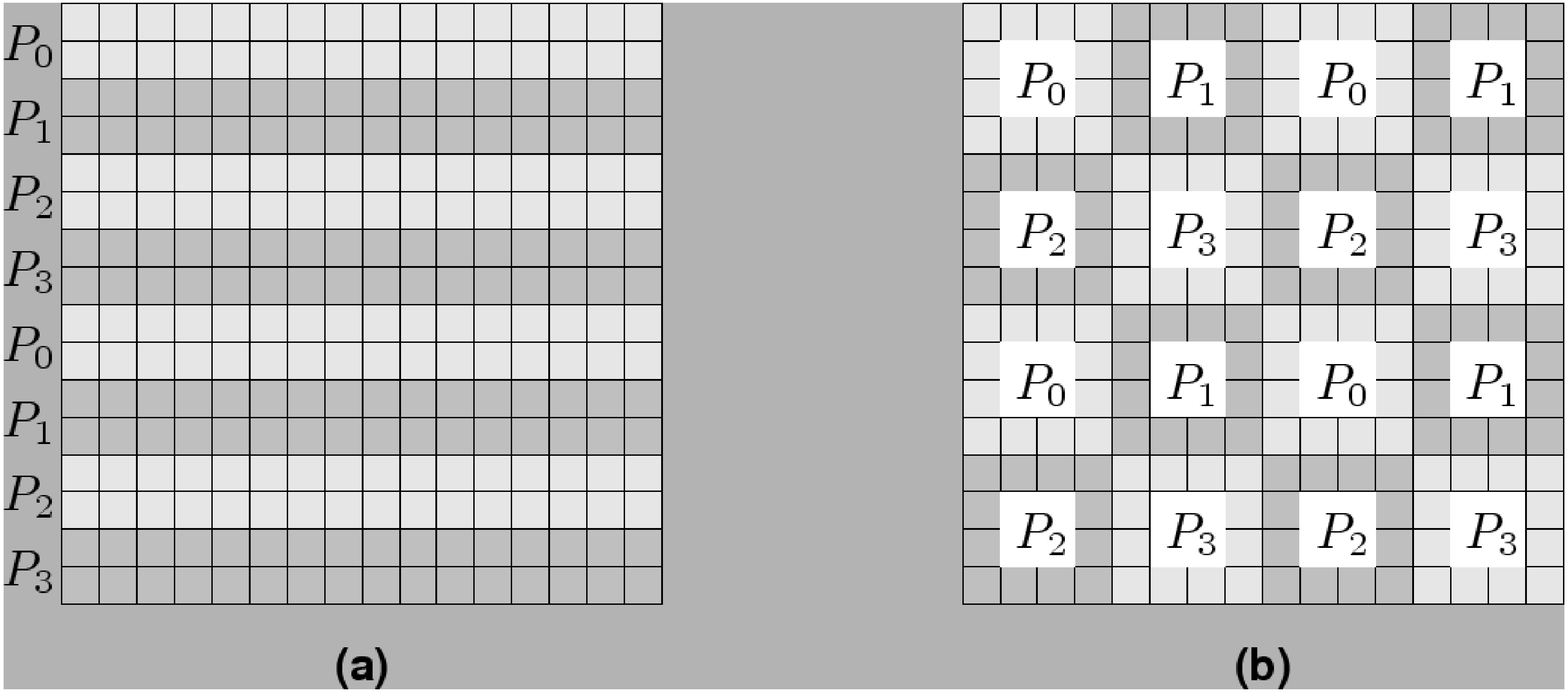
Block-Cyclic Distribution: Examples

One- and two-dimensional block-cyclic distributions among 4 processes.

P_0 T_1	P_3 T_4	P_6 T_5
P_1 T_2	P_4 T_6 T_{10}	P_7 T_8 T_{12}
P_2 T_3	P_5 T_7 T_{11}	P_8 T_9 T_{13} T_{14}

Block-Cyclic Distribution

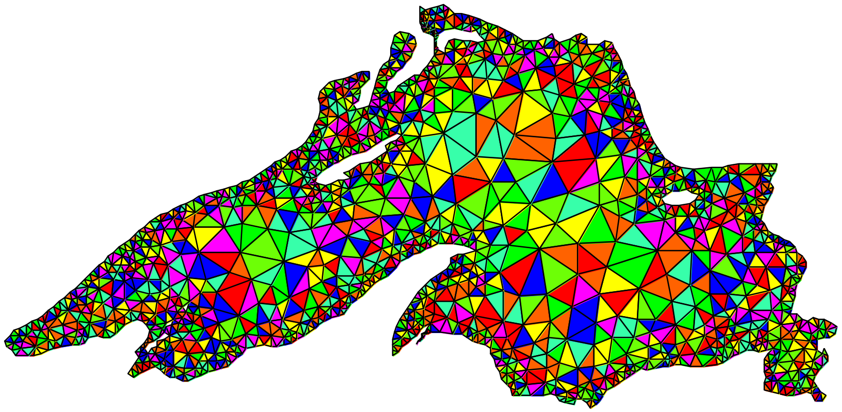
- A cyclic distribution is a special case in which block size is one.
- A block distribution is a special case in which block size is ***n/p*** , where ***n*** is the dimension of the matrix and ***p*** is the number of processes.



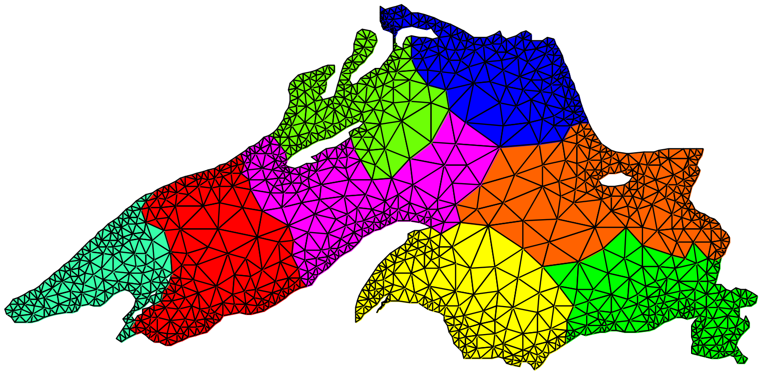
Graph Partitioning Dased Data Decomposition

- In case of sparse matrices, block decompositions are more complex.
- Consider the problem of multiplying a sparse matrix with a vector.
- The graph of the matrix is a useful indicator of the work (number of nodes) and communication (the degree of each node).
- In this case, we would like to partition the graph so as to assign equal number of nodes to each process, while minimizing edge count of the graph partition.

Partitioning the Graph of Lake Superior



Random Partitioning



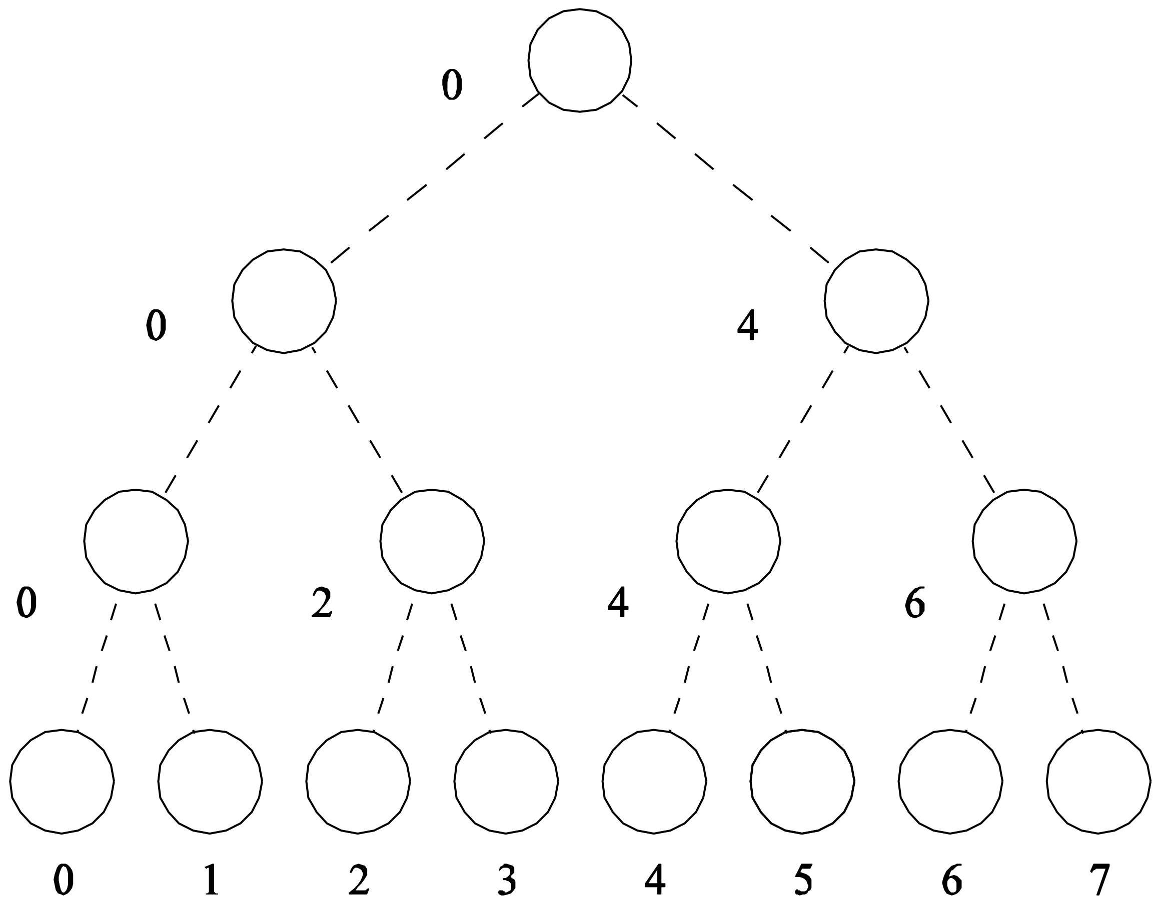
Partitioning for minimum edge-cut.

Mappings Based on Task Paritioning

- Partitioning a given task-dependency graph across processes.
- Determining an optimal mapping for a general task-dependency graph is an NP-complete problem.
- Excellent heuristics exist for structured graphs.

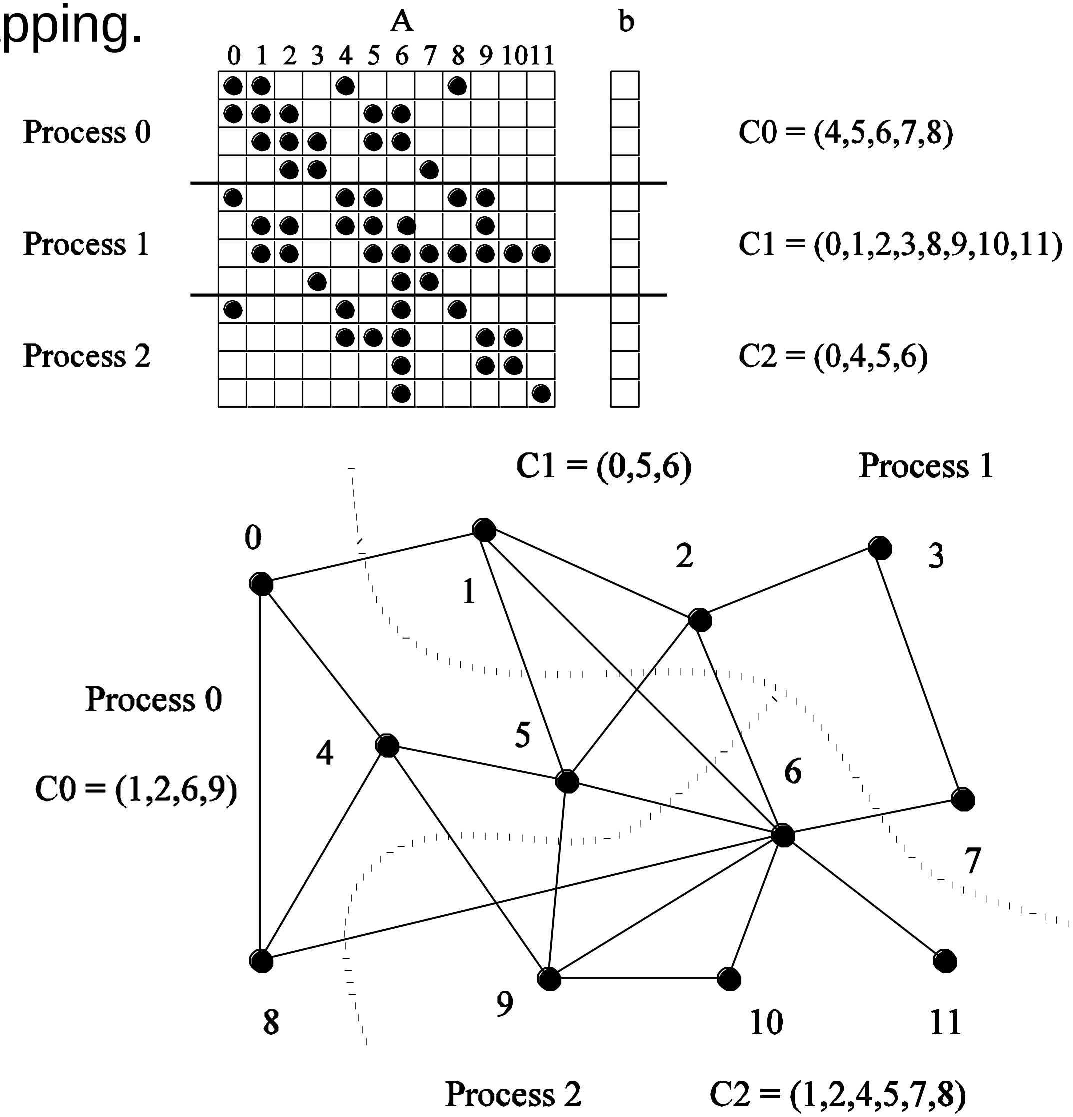
Task Paritioning: Mapping a Binary Tree Dependency Graph

Example illustrates the dependency graph of one view of quick-sort and how it can be assigned to processes in a hypercube.



Task Paritioning: Mapping a Sparse Graph

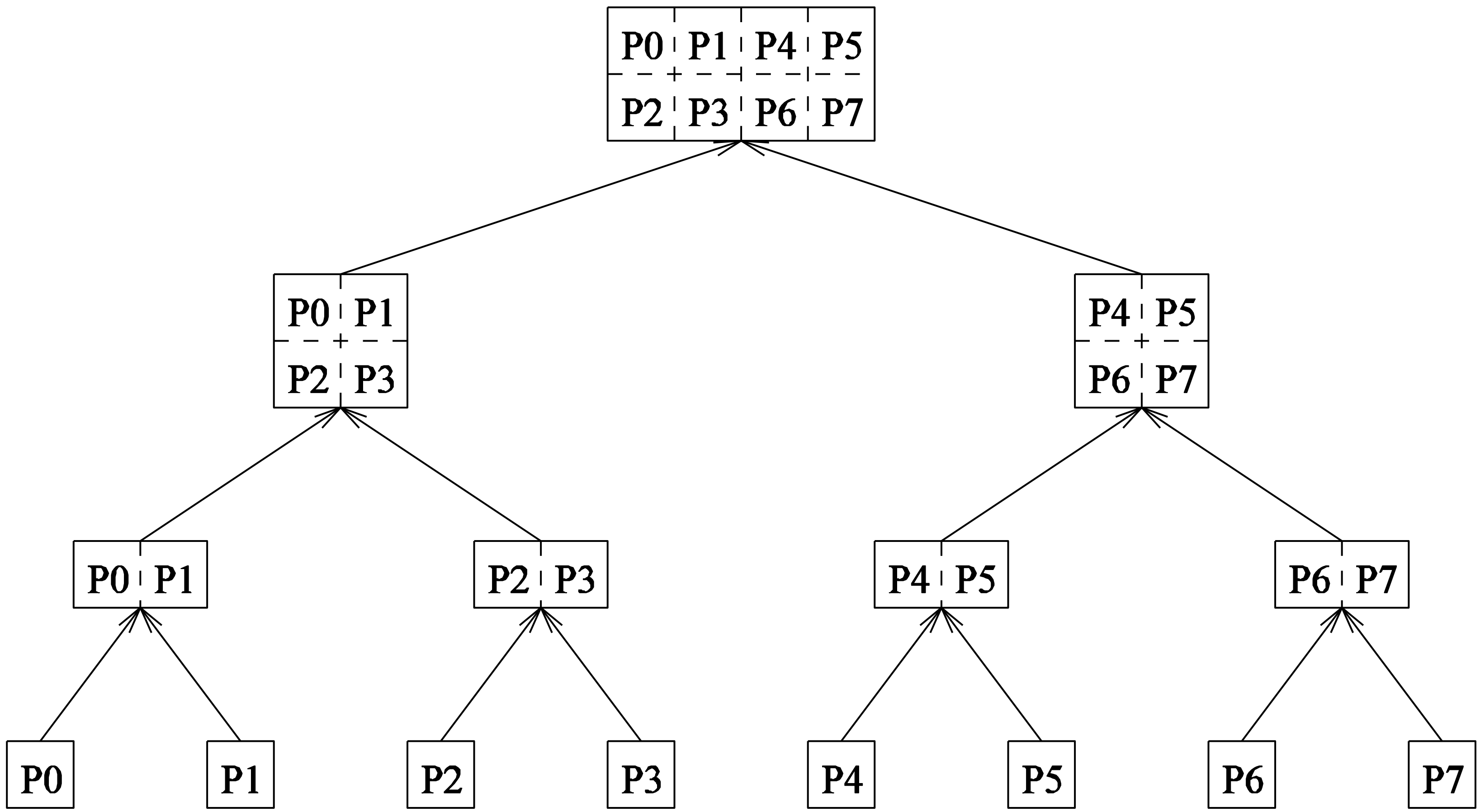
Sparse graph for computing a sparse matrix-vector product and its mapping.



Hierarchical Mappings

- Sometimes a single mapping technique is inadequate.
- For example, the task mapping of the binary tree (quicksort) cannot use a large number of processors.
- For this reason, task mapping can be used at the top level and data partitioning within each level.

An example of task partitioning at top level with data partitioning at the lower level.



Schemes for Dynamic Mapping

- Dynamic mapping is sometimes also referred to as dynamic load balancing, since load balancing is the primary motivation for dynamic mapping.
- Dynamic mapping schemes can be centralized or distributed.

Centralized Dynamic Mapping

- Processes are designated as masters or slaves.
- When a process runs out of work, it requests the master for more work.
- When the number of processes increases, the master may become the bottleneck.
- To alleviate this, a process may pick up a number of tasks (a chunk) at one time. This is called Chunk scheduling.
- Selecting large chunk sizes may lead to significant load imbalances as well.
- A number of schemes have been used to gradually decrease chunk size as the computation progresses.

Distributed Dynamic Mapping

- Each process can send or receive work from other processes.
- This alleviates the bottleneck in centralized schemes.
- There are four critical questions: how are sensing and receiving processes paired together, who initiates work transfer, how much work is transferred, and when is a transfer triggered?
- Answers to these questions are generally application specific. We will look at some of these techniques later in this class.

Minimizing Interaction Overheads

- Maximize data locality: Where possible, reuse intermediate data. Restructure computation so that data can be reused in smaller time windows.
- Minimize volume of data exchange: There is a cost associated with each word that is communicated. For this reason, we must minimize the volume of data communicated.
- Minimize frequency of interactions: There is a startup cost associated with each interaction. Therefore, try to merge multiple interactions to one, where possible.
- Minimize contention and hot-spots: Use decentralized techniques, replicate data where necessary.

Minimizing Interaction Overheads (continued)

- Overlapping computations with interactions: Use non-blocking communications, multithreading, and prefetching to hide latencies.
- Replicating data or computations.
- Using group communications instead of point-to-point primitives.
- Overlap interactions with other interactions.

Parallel Algorithm Models

An algorithm model is a way of structuring a parallel algorithm by selecting a decomposition and mapping technique and applying the appropriate strategy to minimize interactions.

- Data Parallel Model: Tasks are statically (or semi-statically) mapped to processes and each task performs similar operations on different data.
- Task Graph Model: Starting from a task dependency graph, the interrelationships among the tasks are utilized to promote locality or to reduce interaction costs.

Parallel Algorithm Models (continued)

- Master-Slave Model: One or more processes generate work and allocate it to worker processes. This allocation may be static or dynamic.
- Pipeline / Producer-Comsumer Model: A stream of data is passed through a succession of processes, each of which perform some task on it.
- Hybrid Models: A hybrid model may be composed either of multiple models applied hierarchically or multiple models applied sequentially to different phases of a parallel algorithm.

Programming Using the Using Message Passing Paradigm
Unit 4

Dr. Minal Moharir

message-passing programming paradigm

- Numerous programming languages and libraries have been developed for explicit parallel programming.
- These differ in their view of the address space that they make available to the programmer, the degree of synchronization imposed on concurrent activities, and the multiplicity of programs.
- The ***message-passing programming paradigm*** is one of the oldest and most widely used approaches for programming parallel computers.

Principles of
Message-Passing Programming

- The logical view of a machine supporting the message-passing paradigm consists of p processes, each with its own exclusive address space.
- Each data element must belong to one of the partitions of the space; hence, data must be explicitly partitioned and placed.
- All interactions (read-only or read/write) require cooperation of two processes - the process that has the data and the process that wants to access the data.
- These two constraints, while onerous, make underlying costs very explicit to the programmer.

Principles of Message-Passing Programming

- Message-passing programs are often written using the *asynchronous* or *loosely synchronous* paradigms.
- In the asynchronous paradigm, all concurrent tasks execute asynchronously.
- In the loosely synchronous model, tasks or subsets of tasks synchronize to perform interactions. Between these interactions, tasks execute completely asynchronously.
- Most message-passing programs are written using the *single program multiple data* (SPMD) model.

The Building Blocks: Send and Receive Operations

- The prototypes of these operations are as follows:

```
send(void *sendbuf, int nelems, int dest)
receive(void *recvbuf, int nelems, int source)
```

- Consider the following code segments:

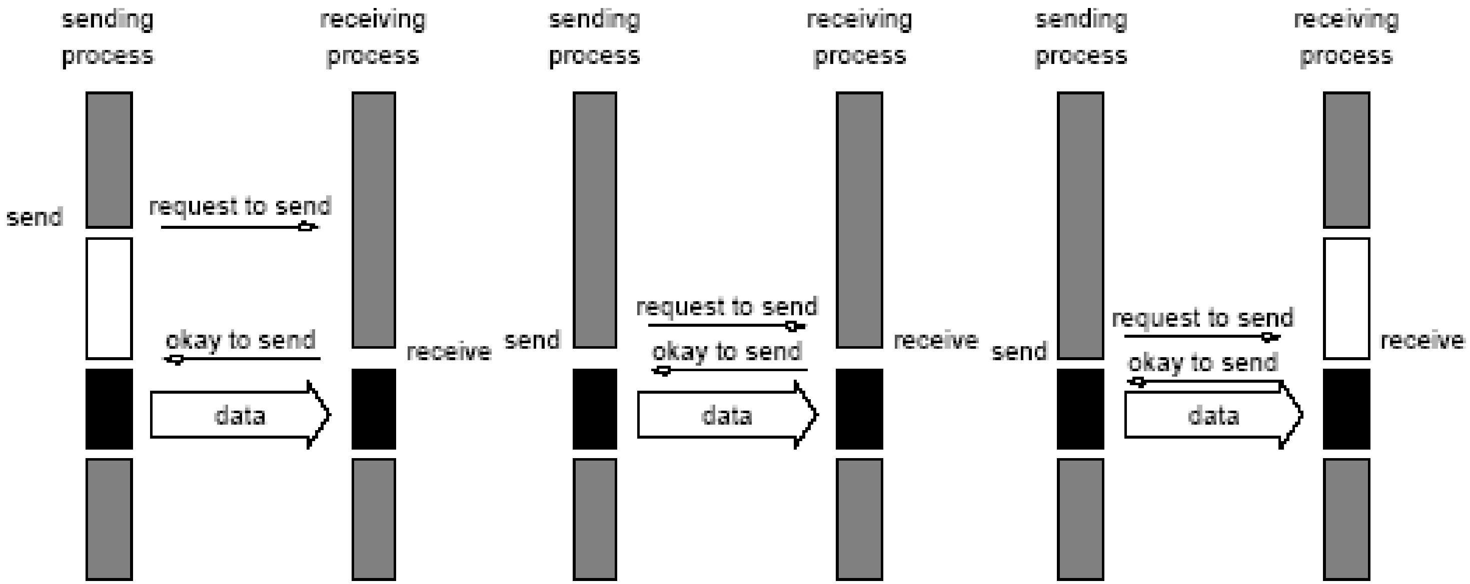
```
P0                      P1
a = 100;                receive(&a, 1, 0)
send(&a, 1, 1);         printf("%d\n", a);
a = 0;
```

- The semantics of the send operation require that the value received by process P1 must be 100 as opposed to 0.
- This motivates the design of the send and receive protocols.

Non-Buffered Blocking Message Passing Operations

- A simple method for forcing send/receive semantics is for the send operation to return only when it is safe to do so.
- In the non-buffered blocking send, the operation does not return until the matching receive has been encountered at the receiving process.
- Idling and deadlocks are major issues with non-buffered blocking sends.
- In buffered blocking sends, the sender simply copies the data into the designated buffer and returns after the copy operation has been completed.
- The data is copied at a buffer at the receiving end as well.
- Buffering alleviates idling at the expense of copying overheads.

Non-Buffered Blocking
Message Passing Operations



(a) Sender comes first; idling at sender

(b) Sender and receiver come at about the same time; idling minimized

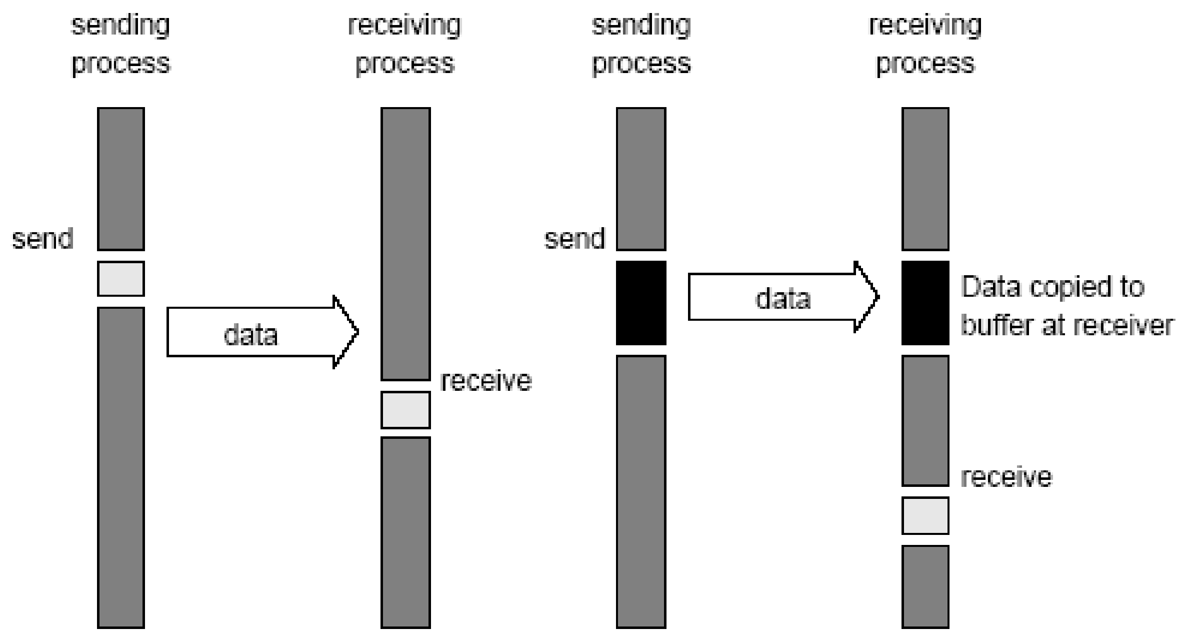
(c) Receiver comes first; idling at receiver

Handshake for a blocking non-buffered send/receive operation. It is easy to see that in cases where sender and receiver do not reach communication point at similar times, there can be considerable idling overheads.

Buffered Blocking Message Passing Operations

- A simple solution to the idling and deadlocking problem outlined above is to rely on buffers at the sending and receiving ends.
- The sender simply copies the data into the designated buffer and returns after the copy operation has been completed.
- The data must be buffered at the receiving end as well.
- Buffering trades off idling overhead for buffer copying overhead.

Buffered Blocking
Message Passing Operations



Blocking buffered transfer protocols: (a) in the presence of communication hardware with buffers at send and receive ends; and (b) in the absence of communication hardware, sender interrupts receiver and deposits data in buffer at receiver end.

Buffered Blocking
Message Passing Operations

Bounded buffer sizes can have signicant impact on performance.

```
P0
for (i = 0; i < 1000; i++){
    produce_data(&a);
    send(&a, 1, 1);
  }
```

```
P1
for (i = 0; i < 1000; i++){
    receive(&a, 1, 0);
    consume_data(&a);
}
```

What if consumer was much slower than producer?

Buffered Blocking Message Passing Operations

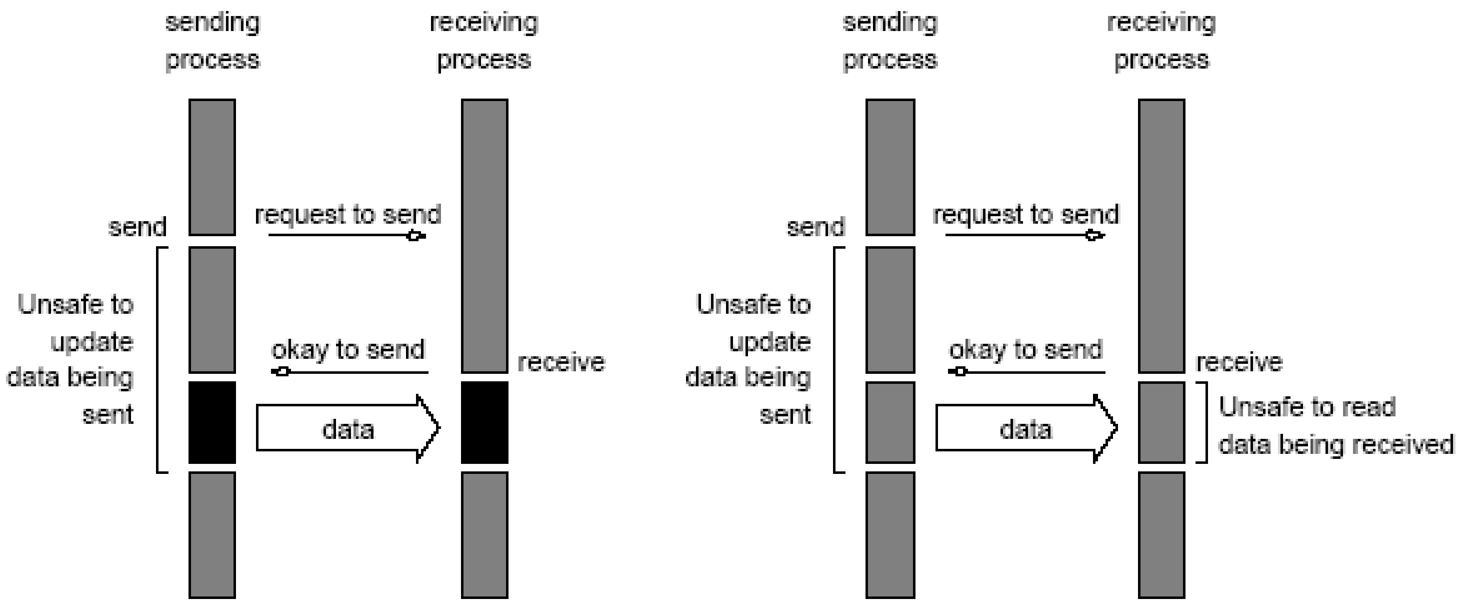
Deadlocks are still possible with buffering since receive operations block.

P0	P1
`receive(&a, 1, 1);`	`receive(&a, 1, 0);`
`send(&b, 1, 1);`	`send(&b, 1, 0);`

Non-Blocking Message Passing Operations

- The programmer must ensure semantics of the send and receive.
- This class of non-blocking protocols returns from the send or receive operation before it is semantically safe to do so.
- Non-blocking operations are generally accompanied by a check-status operation.
- When used correctly, these primitives are capable of overlapping communication overheads with useful computations.
- Message passing libraries typically provide both blocking and non-blocking primitives.

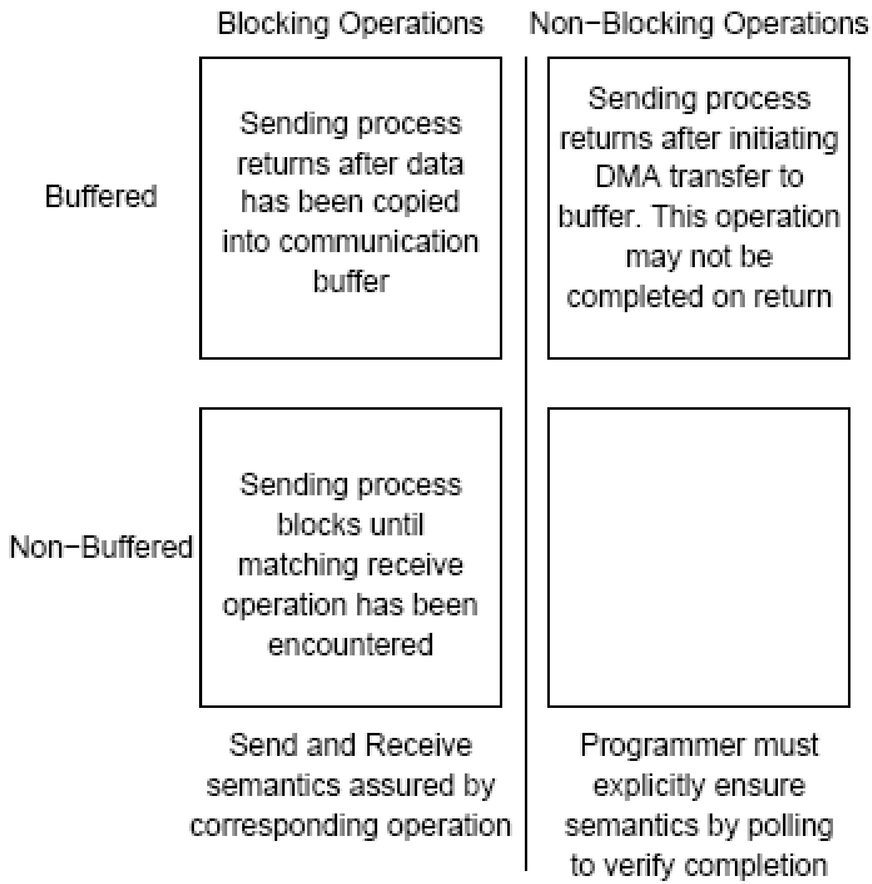
Non-Blocking Message Passing Operations



(a) Without hardware support (b) With hardware support

Non-blocking non-buffered send and receive operations (a) in absence of communication hardware; (b) in presence of communication hardware.

Send and Receive Protocols



Space of possible protocols for send and receive operations.

MPI: the Message Passing Interface

- MPI defines a standard library for message-passing that can be used to develop portable message-passing programs using either C or Fortran.
- The MPI standard defines both the syntax as well as the semantics of a core set of library routines.
- Vendor implementations of MPI are available on almost all commercial parallel computers.
- It is possible to write fully-functional message-passing programs by using only the six routines.

MPI: the Message Passing Interface

The minimal set of MPI routines.

`MPI_Init`	Initializes MPI.
`MPI_Finalize`	Terminates MPI.
`MPI_Comm_size`	Determines the number of processes.
`MPI_Comm_rank`	Determines the label of calling process.
`MPI_Send`	Sends a message.
`MPI_Recv`	Receives a message.

Starting and Terminating the MPI Library

- `MPI_Init` is called prior to any calls to other MPI routines. Its purpose is to initialize the MPI environment.
- `MPI_Finalize` is called at the end of the computation, and it performs various clean-up tasks to terminate the MPI environment.
- The prototypes of these two functions are:

```
int MPI_Init(int *argc, char ***argv)
int MPI_Finalize()
```

- `MPI_Init` also strips off any MPI related command-line arguments.
- All MPI routines, data-types, and constants are prefixed by "`MPI_`". The return code for successful completion is `MPI_SUCCESS`.

Communicators

- A communicator defines a *communication domain* - a set of processes that are allowed to communicate with each other.
- Information about communication domains is stored in variables of type `MPI_Comm`.
- Communicators are used as arguments to all message transfer MPI routines.
- A process can belong to many different (possibly overlapping) communication domains.
- MPI defines a default communicator called `MPI_COMM_WORLD` which includes all the processes.

Querying Information

- The `MPI_Comm_size` and `MPI_Comm_rank` functions are used to determine the number of processes and the label of the calling process, respectively.
- The calling sequences of these routines are as follows:

```
int MPI_Comm_size(MPI_Comm comm, int *size)
int MPI_Comm_rank(MPI_Comm comm, int *rank)
```

- The rank of a process is an integer that ranges from zero up to the size of the communicator minus one.

Our First MPI Program

```
#include <mpi.h>

main(int argc, char *argv[])
{
        int npes, myrank;
        MPI_Init(&argc, &argv);
        MPI_Comm_size(MPI_COMM_WORLD, &npes);
        MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
        printf("From process %d out of %d, Hello World!\n",
                myrank, npes);
        MPI_Finalize();
}
```

Sending and Receiving Messages

- The basic functions for sending and receiving messages in MPI are the `MPI_Send` and `MPI_Recv`, respectively.
- The calling sequences of these routines are as follows:

```
int MPI_Send(void *buf, int count, MPI_Datatype
        datatype, int dest, int tag, MPI_Comm comm)
int MPI_Recv(void *buf, int count, MPI_Datatype
        datatype, int source, int tag,
        MPI_Comm comm, MPI_Status *status)
```

- MPI provides equivalent datatypes for all C datatypes. This is done for portability reasons.
- The datatype `MPI_BYTE` corresponds to a byte (8 bits) and `MPI_PACKED` corresponds to a collection of data items that has been created by packing non-contiguous data.
- The message-tag can take values ranging from zero up to the MPI defined constant `MPI_TAG_UB`.

MPI Datatypes

MPI Datatype	C Datatype
MPI_CHAR	signed char
MPI_SHORT	signed short int
MPI_INT	signed int
MPI_LONG	signed long int
MPI_UNSIGNED_CHAR	unsigned char
MPI_UNSIGNED_SHORT	unsigned short int
MPI_UNSIGNED	unsigned int
MPI_UNSIGNED_LONG	unsigned long int
MPI_FLOAT	float
MPI_DOUBLE	double
MPI_LONG_DOUBLE	long double
MPI_BYTE	
MPI_PACKED	

Sending and Receiving Messages

- MPI allows specification of wildcard arguments for both source and tag.
- If source is set to `MPI_ANY_SOURCE`, then any process of the communication domain can be the source of the message.
- If tag is set to `MPI_ANY_TAG`, then messages with any tag are accepted.
- On the receive side, the message must be of length equal to or less than the length field specified.

Sending and Receiving Messages

- On the receiving end, the status variable can be used to get information about the `MPI_Recv` operation.
- The corresponding data structure contains:

```
typedef struct MPI_Status {
 int MPI_SOURCE;
 int MPI_TAG;
 int MPI_ERROR; };
```

- The MPI_Get_count function returns the precise count of data items received.

```
int MPI_Get_count(MPI_Status *status, MPI_Datatype
                  datatype, int *count)
```

Avoiding Deadlocks

Consider:

```
int a[10], b[10], myrank;
MPI_Status status;
...
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
if (myrank == 0) {
    MPI_Send(a, 10, MPI_INT, 1, 1, MPI_COMM_WORLD);
    MPI_Send(b, 10, MPI_INT, 1, 2, MPI_COMM_WORLD);
}
else if (myrank == 1) {
    MPI_Recv(b, 10, MPI_INT, 0, 2, MPI_COMM_WORLD);
    MPI_Recv(a, 10, MPI_INT, 0, 1, MPI_COMM_WORLD);
}
...
```

If MPI_Send is blocking, there is a deadlock.

Avoiding Deadlocks

- Consider the following piece of code, in which process i sends a message to process *i* + 1 (modulo the number of processes) and receives a message from process *i* - 1 (module the number of processes).

- `int a[10], b[10], npes, myrank;`
- `MPI_Status status;`
- `...`
- `MPI_Comm_size(MPI_COMM_WORLD, &npes);`
- `MPI_Comm_rank(MPI_COMM_WORLD, &myrank);`
- `MPI_Send(a, 10, MPI_INT, (myrank+1)%npes, 1, MPI_COMM_WORLD);`
- `MPI_Recv(b, 10, MPI_INT, (myrank-1+npes)%npes, 1, MPI_COMM_WORLD);`
- `...`

- Once again, we have a deadlock if MPI_Send is blocking.

Avoiding Deadlocks

- We can break the circular wait to avoid deadlocks as follows:

```
• int a[10], b[10], npes, myrank;
• MPI_Status status;
• ...
• MPI_Comm_size(MPI_COMM_WORLD, &npes);
• MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
• if (myrank%2 == 1) {
•       MPI_Send(a, 10, MPI_INT, (myrank+1)%npes, 1,
        MPI_COMM_WORLD);
•       MPI_Recv(b, 10, MPI_INT, (myrank-1+npes)%npes, 1,
        MPI_COMM_WORLD);
• }
• else {
•       MPI_Recv(b, 10, MPI_INT, (myrank-1+npes)%npes, 1,
        MPI_COMM_WORLD);
•       MPI_Send(a, 10, MPI_INT, (myrank+1)%npes, 1,
        MPI_COMM_WORLD);
• }
• ...
```

Sending and Receiving Messages Simultaneously

- To exchange messages, MPI provides the following function:

```
int MPI_Sendrecv(void *sendbuf, int sendcount,
  MPI_Datatype senddatatype, int dest, int sendtag, void *recvbuf, int recvcount, MPI_Datatype recvdatatype, int source, int recvtag, MPI_Comm comm, MPI_Status *status)
```

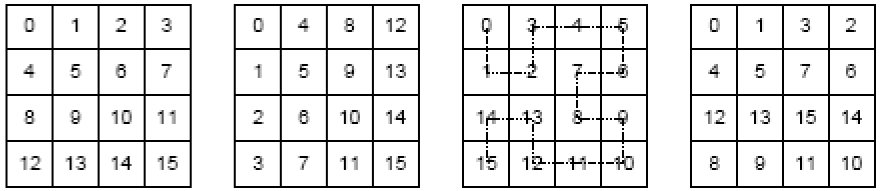
- The arguments include arguments to the send and receive
- functions. If we wish to use the same buffer for both send and
- receive, we can use:

```
int MPI_Sendrecv_replace(void *buf, int count,
  MPI_Datatype datatype, int dest, int sendtag,
  int source, int recvtag, MPI_Comm comm,
  MPI_Status *status)
```

Topologies and Embeddings

- MPI allows a programmer to organize processors into logical k-d meshes.
- The processor ids in `MPI_COMM_WORLD` can be mapped to other communicators (corresponding to higher-dimensional meshes) in many ways.
- The goodness of any such mapping is determined by the interaction pattern of the underlying program and the topology of the machine.
- MPI does not provide the programmer any control over these mappings

Topologies and Embeddings



(a) Row−major mapping (b) Column−major mapping (c) Space−filling curve mapping (d) Hypercube mapping

Different ways to map a set of processes to a two-dimensional grid. (a) and (b) show a row- and column-wise mapping of these processes, (c) shows a mapping that follows a space-lling curve (dotted line), and (d) shows a mapping in which neighboring processes are directly connected in a hypercube.

Creating and Using Cartesian Topologies

- We can create cartesian topologies using the function:

```
int MPI_Cart_create(MPI_Comm comm_old, int ndims,
                    int *dims, int *periods, int reorder,
                    MPI_Comm *comm_cart)
```

  This function takes the processes in the old communicator and creates a new communicator with dims dimensions.
- Each processor can now be identified in this new cartesian topology by a vector of dimension dims.

Creating and Using Cartesian Topologies

- Since sending and receiving messages still require (one-dimensional) ranks, MPI provides routines to convert ranks to cartesian coordinates and vice-versa.

```
int MPI_Cart_coord(MPI_Comm comm_cart, int rank, int maxdims,
                   int *coords)
```

```
int MPI_Cart_rank(MPI_Comm comm_cart, int *coords, int *rank)
```

- The most common operation on cartesian topologies is a shift. To determine the rank of source and destination of such shifts, MPI provides the following function:

```
int MPI_Cart_shift(MPI_Comm comm_cart, int dir, int s_step,
                   int *rank_source, int *rank_dest)
```

Overlapping Communication with Computation

- In order to overlap communication with computation, MPI provides a pair of functions for performing non-blocking send and receive operations.

```
int MPI_Isend(void *buf, int count, MPI_Datatype datatype,
              int dest, int tag, MPI_Comm comm,
              MPI_Request *request)
int MPI_Irecv(void *buf, int count, MPI_Datatype datatype,
              int source, int tag, MPI_Comm comm,
              MPI_Request *request)
```

- These operations return before the operations have been completed. Function `MPI_Test` tests whether or not the non-blocking send or receive operation identified by its request has finished.

```
int MPI_Test(MPI_Request *request, int *flag,
    MPI_Status *status)
```

- `MPI_Wait` waits for the operation to complete.

```
int MPI_Wait(MPI_Request *request, MPI_Status *status)
```

Avoiding Deadlocks

Using non-blocking operations remove most deadlocks. Consider:

```
int a[10], b[10], myrank;
MPI_Status status;
...
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
if (myrank == 0) {
    MPI_Send(a, 10, MPI_INT, 1, 1, MPI_COMM_WORLD);
    MPI_Send(b, 10, MPI_INT, 1, 2, MPI_COMM_WORLD);
}
else if (myrank == 1) {
    MPI_Recv(b, 10, MPI_INT, 0, 2, &status, MPI_COMM_WORLD); MPI_Recv(a, 10,
    MPI_INT, 0, 1, &status, MPI_COMM_WORLD);
}
...
```

Replacing either the send or the receive operations with non-blocking counterparts fixes this deadlock.

Collective Communication and Computation Operations

- MPI provides an extensive set of functions for performing common collective communication operations.
- Each of these operations is defined over a group corresponding to the communicator.
- All processors in a communicator must call these operations.

Collective Communication Operations

- The barrier synchronization operation is performed in MPI using:

```
int MPI_Barrier(MPI_Comm comm)
```

  The one-to-all broadcast operation is:

```
int MPI_Bcast(void *buf, int count, MPI_Datatype datatype,
        int source, MPI_Comm comm)
```

- The all-to-one reduction operation is:

```
int MPI_Reduce(void *sendbuf, void *recvbuf, int count,
        MPI_Datatype datatype, MPI_Op op, int target,
        MPI_Comm comm)
```

Predefined Reduction Operations

Operation	Meaning	Datatypes
`MPI_MAX`	Maximum	C integers and floating point
`MPI_MIN`	Minimum	C integers and floating point
`MPI_SUM`	Sum	C integers and floating point
`MPI_PROD`	Product	C integers and floating point
`MPI_LAND`	Logical AND	C integers
`MPI_BAND`	Bit-wise AND	C integers and byte
`MPI_LOR`	Logical OR	C integers
`MPI_BOR`	Bit-wise OR	C integers and byte
`MPI_LXOR`	Logical XOR	C integers
`MPI_BXOR`	Bit-wise XOR	C integers and byte
`MPI_MAXLOC`	max-min value-location	Data-pairs
`MPI_MINLOC`	min-min value-location	Data-pairs

Collective Communication Operations

- The operation `MPI_MAXLOC` combines pairs of values (v_i, l_i) and returns the pair (v, l) such that v is the maximum among all v_i 's and l is the corresponding l_i (if there are more than one, it is the smallest among all these l_i 's).
- `MPI_MINLOC` does the same, except for minimum value of v_i.

Value	15	17	11	12	17	11
Process	0	1	2	3	4	5

```
MinLoc(Value, Process) = (11, 2)
MaxLoc(Value, Process) = (17, 1)
```

An example use of the `MPI_MINLOC` and `MPI_MAXLOC` operators.

Collective Communication Operations

MPI datatypes for data-pairs used with the MPI_MAXLOC and MPI_MINLOC reduction operations.

MPI Datatype	C Datatype
MPI_2INT	pair of ints
MPI_SHORT_INT	short and int
MPI_LONG_INT	long and int
MPI_LONG_DOUBLE_INT	long double and int
MPI_FLOAT_INT	float and int
MPI_DOUBLE_INT	double and int

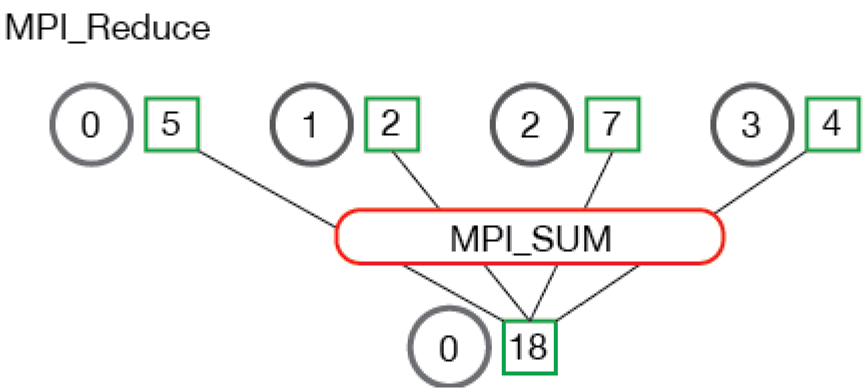
MPI_Reduce

- Reduce is a classic concept from functional programming.
- Data reduction involves reducing a set of numbers into a smaller set of numbers via a function.
- For example, let’s say we have a list of numbers [1, 2, 3, 4, 5]. Reducing this list of numbers with the sum function would produce sum([1, 2, 3, 4, 5]) = 15.
- Similarly, the multiplication reduction would yield multiply([1, 2, 3, 4, 5]) = 120

MPI_Reduce

- MPI_Reduce(void* send_data, void* recv_data, int count, MPI_Datatype datatype, MPI_Op op, int root, MPI_Comm communicator)

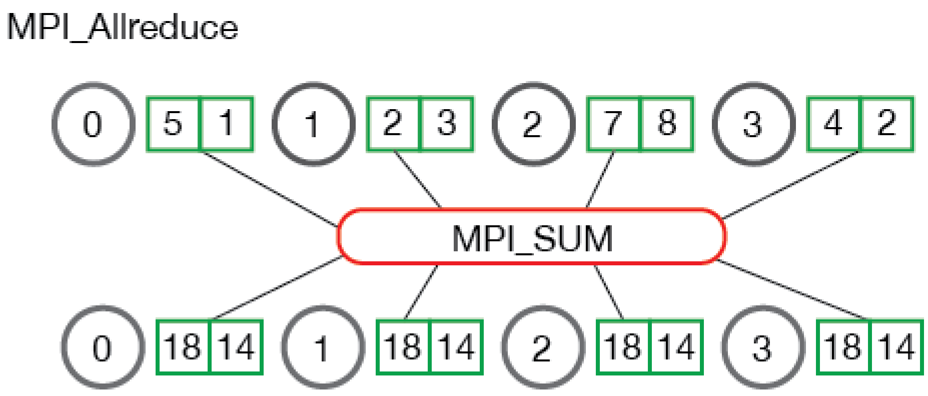
Below is an illustration of the communication pattern of `MPI_Reduce` .



MPI_Allreduce

If the result of the reduction operation is needed by all processes, MPI provides:

```
int MPI_Allreduce(void *sendbuf, void *recvbuf,
             int count, MPI_Datatype datatype,
   MPI_Op op,              MPI_Comm comm)
```



Collective Communication Operations

- If the result of the reduction operation is needed by all processes, MPI provides:

```
int MPI_Allreduce(void *sendbuf, void *recvbuf,
        int count, MPI_Datatype datatype, MPI_Op op,
        MPI_Comm comm)
```

- To compute prefix-sums, MPI provides:

```
int MPI_Scan(void *sendbuf, void *recvbuf, int count,
        MPI_Datatype datatype, MPI_Op op,
        MPI_Comm comm)
```

Collective Communication Operations

- The gather operation is performed in MPI using:

```
int MPI_Gather(void *sendbuf, int sendcount,
         MPI_Datatype senddatatype, void *recvbuf,
         int recvcount, MPI_Datatype recvdatatype,
         int target, MPI_Comm comm)
```

- MPI also provides the MPI_Allgather function in which the data are gathered at all the processes.

```
int MPI_Allgather(void *sendbuf, int sendcount,
         MPI_Datatype senddatatype, void *recvbuf,
         int recvcount, MPI_Datatype recvdatatype,
         MPI_Comm comm)
```

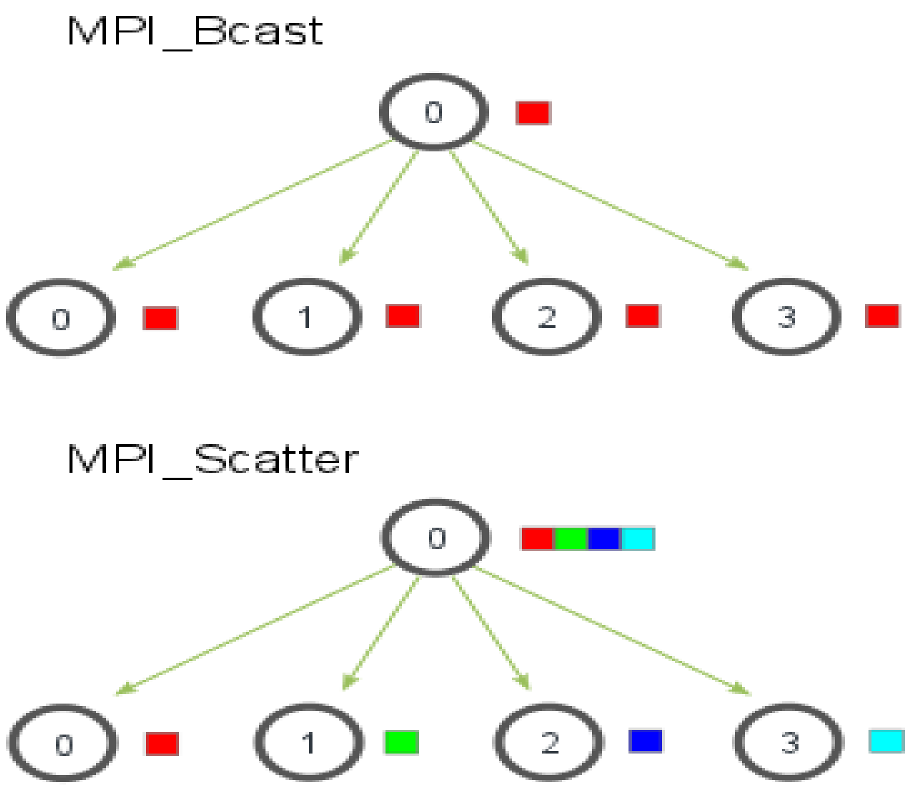
- The corresponding scatter operation is:

```
int MPI_Scatter(void *sendbuf, int sendcount,
         MPI_Datatype senddatatype, void *recvbuf,
         int recvcount, MPI_Datatype recvdatatype,
         int source, MPI_Comm comm)
```

MPI_Scatter

- MPI_Scatter is a collective routine that is very similar to MPI_Bcast .
- MPI_Scatter involves a designated root process sending data to all processes in a communicator.
- The primary difference between MPI_Bcast and MPI_Scatter is small but important.
- MPI_Bcast sends the *same* piece of data to all processes while MPI_Scatter sends *chunks of an array* to different processes. Check out the illustration below for further clarification.

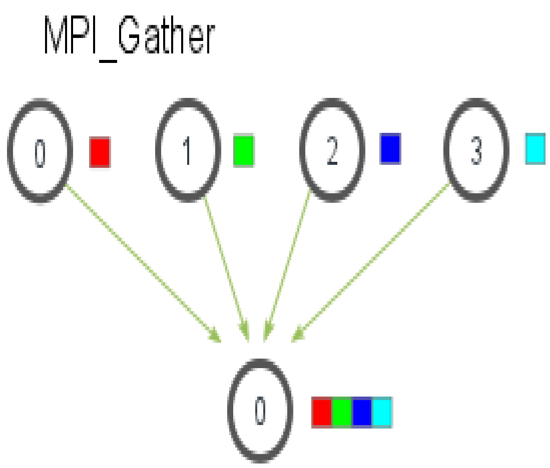
MPI_Scatter



MPI_Gather

- MPI_Gather is the inverse of MPI_Scatter. Instead of spreading elements from one process to many processes, MPI_Gather takes elements from many processes and gathers them to one single process.
- This routine is highly useful to many parallel algorithms, such as parallel sorting and searching. Below is a simple illustration of this algorithm.

MPI_Gather



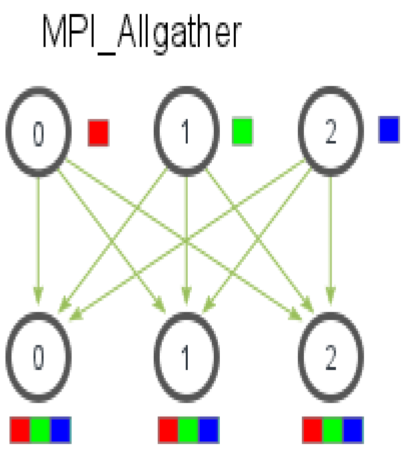
Similar to `MPI_Scatter`, `MPI_Gather` takes elements from each process and gathers them to the root process. The elements are ordered by the rank of the process from which they were received. The function prototype for `MPI_Gather` is identical to that of `MPI_Scatter`.

MPI_Allgather

- So far, we have covered two MPI routines that perform *many-to-one* or *one-to-many* communication patterns, which simply means that many processes send/receive to one process.
- Oftentimes it is useful to be able to send many elements to many processes (i.e. a *many-to-many* communication pattern).
- MPI_Allgather has this characteristic.

MPI_Allgather

Given a set of elements distributed across all processes, `MPI_Allgather` will gather all of the elements to all the processes. In the most basic sense, `MPI_Allgather` is an `MPI_Gather` followed by an `MPI_Bcast`. The illustration below shows how data is distributed after a call to `MPI_Allgather`.



Collective Communication Operations

- The all-to-all personalized communication operation is performed by:

```
int MPI_Alltoall(void *sendbuf, int sendcount,
        MPI_Datatype senddatatype, void *recvbuf,
        int recvcount, MPI_Datatype recvdatatype,
        MPI_Comm comm)
```

- Using this core set of collective operations, a number of programs can be greatly simplified.

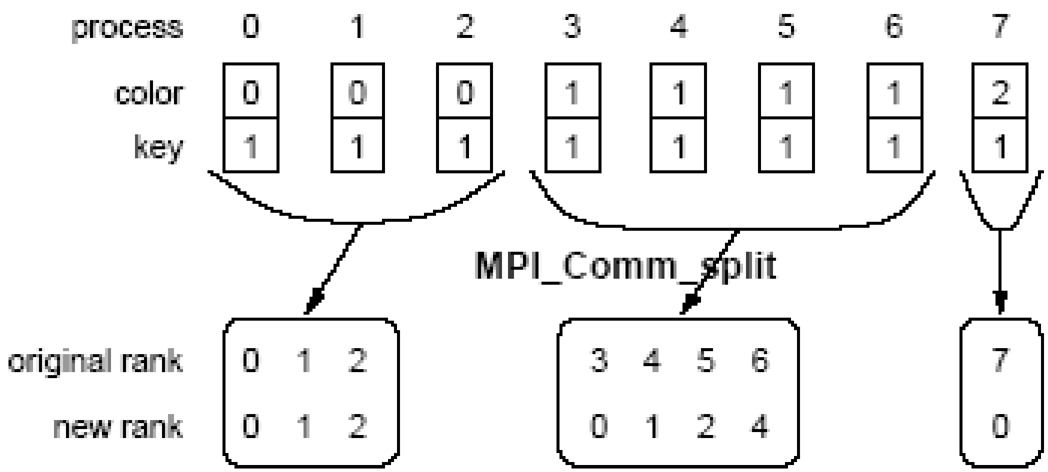
Groups and Communicators

- In many parallel algorithms, communication operations need to be restricted to certain subsets of processes.
- MPI provides mechanisms for partitioning the group of processes that belong to a communicator into subgroups each corresponding to a different communicator.
- The simplest such mechanism is:

```
int MPI_Comm_split(MPI_Comm comm, int color, int key,
                   MPI_Comm *newcomm)
```

- This operation groups processors by color and sorts resulting groups on the key.

Groups and Communicators



Using `MPI_Comm_split` to split a group of processes in a communicator into subgroups.

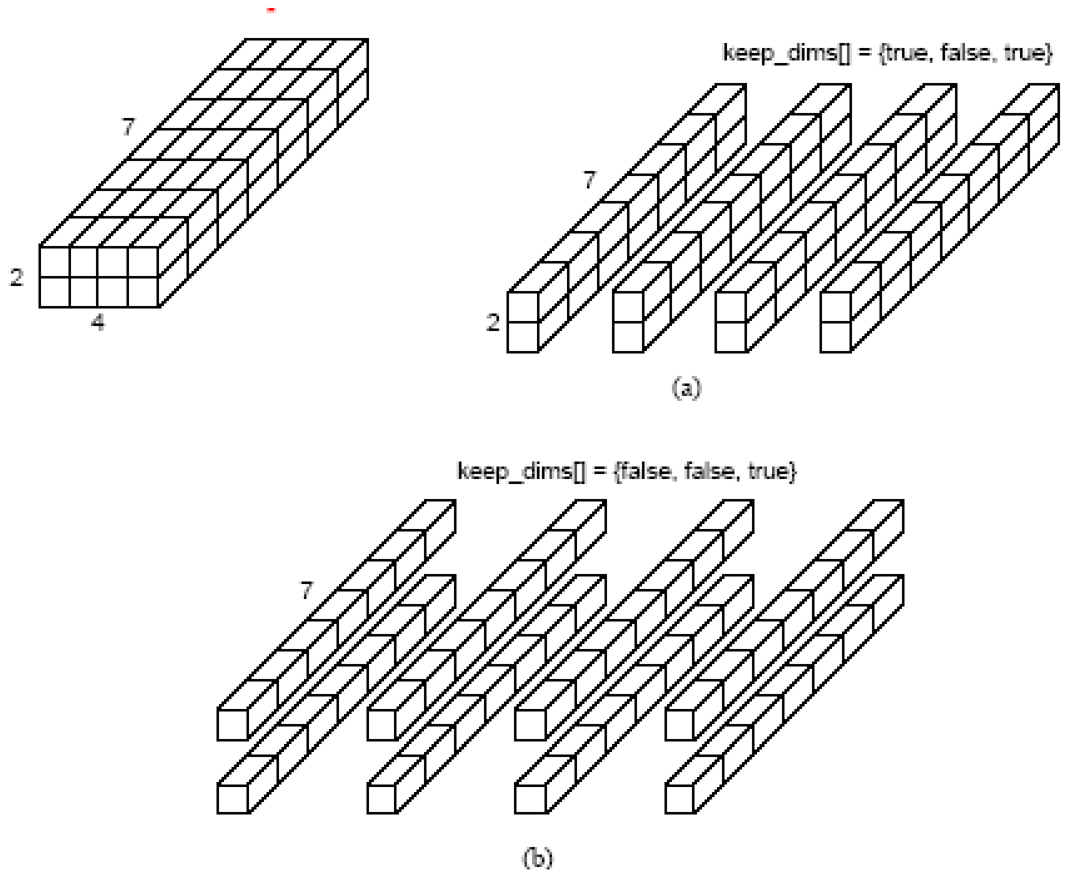
Groups and Communicators

- In many parallel algorithms, processes are arranged in a virtual grid, and in different steps of the algorithm, communication needs to be restricted to a different subset of the grid.
- MPI provides a convenient way to partition a Cartesian topology to form lower-dimensional grids:

```
int MPI_Cart_sub(MPI_Comm comm_cart, int *keep_dims,
        MPI_Comm *comm_subcart)
```

- If `keep_dims[i]` is true (non-zero value in C) then the `i`th dimension is retained in the new sub-topology.
- The coordinate of a process in a sub-topology created by `MPI_Cart_sub` can be obtained from its coordinate in the original topology by disregarding the coordinates that correspond to the dimensions that were not retained.

Groups and Communicators



Splitting a Cartesian topology of size 2 x 4 x 7 into (a) four subgroups of size 2 x 1 x 7, and (b) eight subgroups of size 1 x 1 x 7.