Estimating Pi using the Monte Carlo Method

PADP LAB

Monte Carlo Method

- One method to estimate the value of π(3.141592...) is by using a Monte Carlo method.
- The "Monte Carlo Method" is a method of solving problems using statistics.

Monte Carlo Method

- Given the probability, P, that an event will occur in certain conditions, a computer can be used to generate those conditions repeatedly.

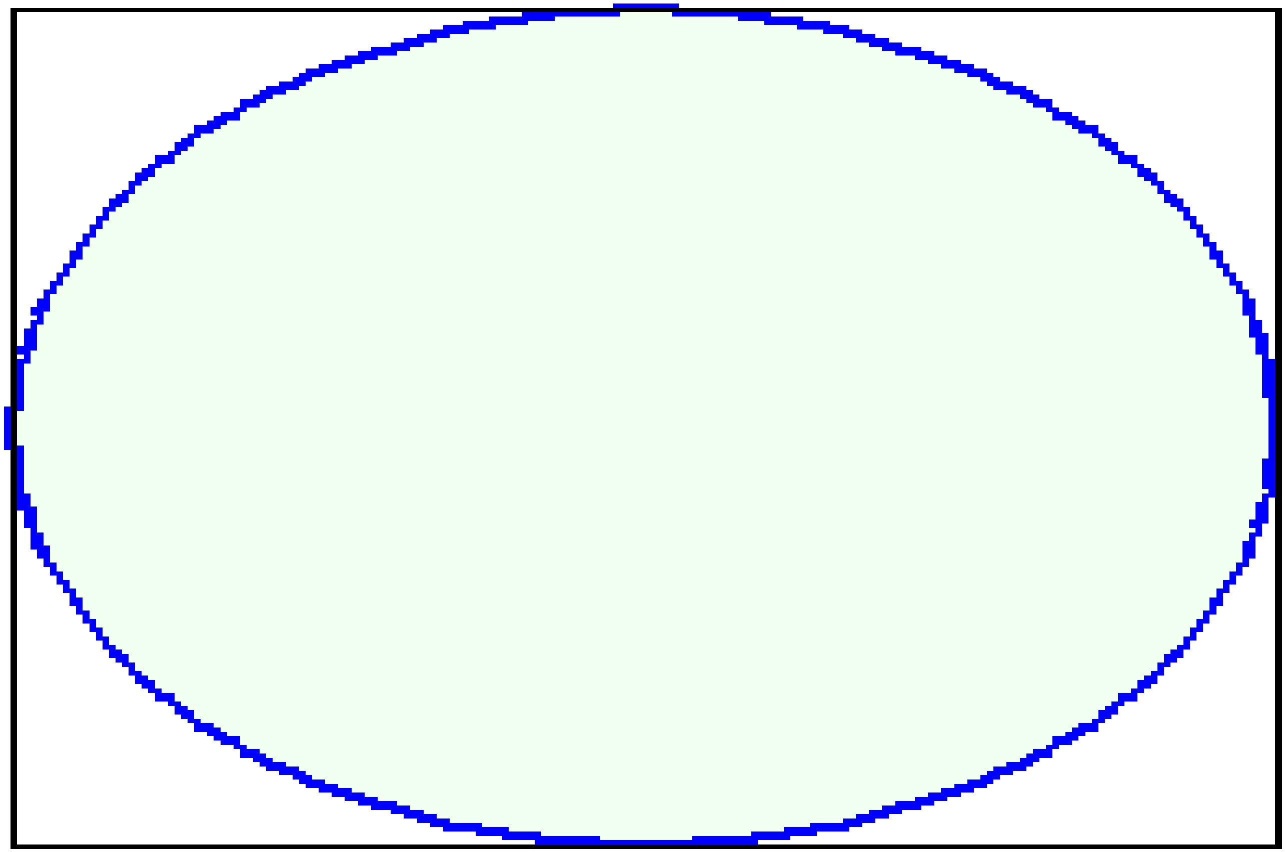
- The number of times the event occurs divided by the number of times the conditions are generated should be approximately equal to P.

Monte Carlo Method

- If a circle of radius R is inscribed inside a square with side length 2R, then the area of the circle will be pi*R^2 and the area of the square will be (2R)^2.

- So the ratio of the area of the circle to the area of the square will be pi/4.

Monte Carlo Method



Monte Carlo Method

- This program picks points at random inside the square.
- It then checks to see if the point is inside the circle (it knows it's inside the circle if $x^2 + y^2 < R^2$, where x and y are the coordinates of the point and R is the radius of the circle).
- The program keeps track of how many points it's picked so far (N_{TOTAL}) and how many of those points fell inside the circle (N_{INNER}).

Monte Carlo Method

- we should get a value that is an approximation of the ratio of the areas we calculated above, $\pi/4$.
- In other words,

  - $\pi/4 \approx N_{INNER}/N_{TOTAL}$

  - $\pi \approx 4 * N_{INNER}/N_{TOTAL}$

- https://academo.org/demos/estimating-pi-monte-carlo/
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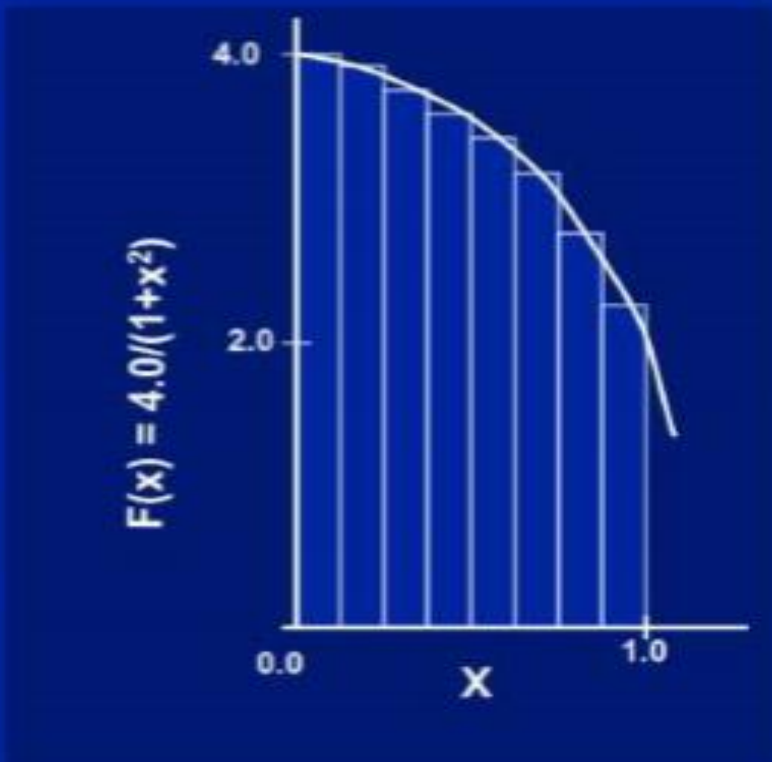
Integral Method: Estimating Pi

Numerical Integration



Mathematically, we know that:

$$\int_0^1 \frac{4.0}{(1+x^2)}\,dx = \pi$$

We can approximate the integral as a sum of rectangles:

$$\sum_{i=0}^{N} F(x_i)\Delta x \approx \pi$$

Where each rectangle has width Δx and height $F(x_i)$ at the middle of interval i.