Draft on procedural textures

Scott TALLEC

June 18, 2023

Contents

1 Interpolation

1.1 Linear interpolation

Linear interpolation function between a and b:

$$\begin{array}{rccl} f & : & [0;1] & \longrightarrow [a;b] \\ & & x & \longmapsto a \cdot (1-x) + b \cdot x \end{array}$$

1.2 Bilinear interpolation

1.2.1 Non-linear interpolation function

Cosine interpolation between a and b:

$$\begin{array}{rccl} f & : & [0;1] & \longrightarrow [a;b] \\ & & x & \longmapsto a \cdot \frac{1+\cos(x \cdot \pi)}{2} + b \cdot \frac{1-\cos(x \cdot \pi)}{2} \end{array}$$

The classic polynomial interpolation is not the fastest. Besides it suffers from a well-known side-effect called the Runge's phenomenon. Spline functions (e.g. cubic splines) are faster without any side-effect.

Figure 1: Grayscale linear interpolation

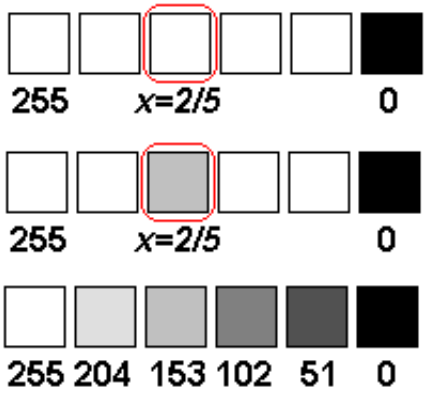


Figure 2: Bilinear interpolation process

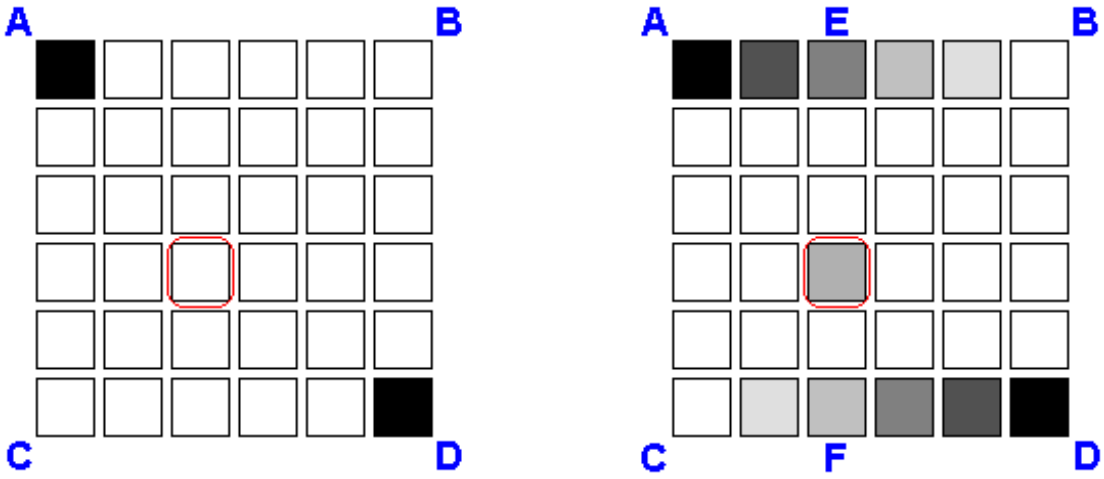
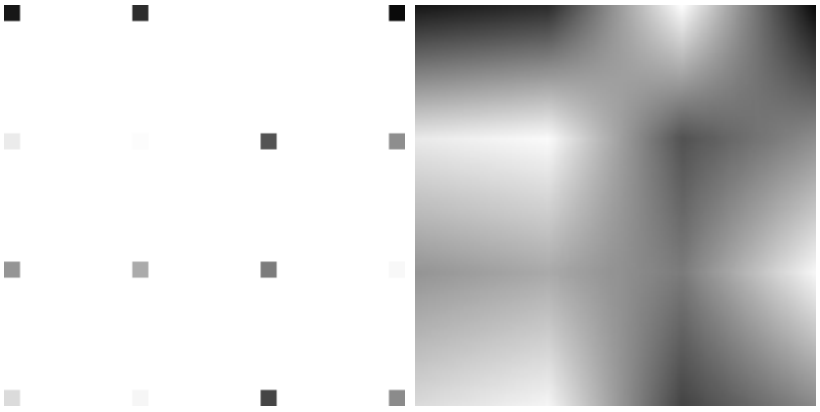


Figure 3: Bilinear interpolation



(a) Interpolation points in 2D (b) Applying the bilinear interpolation

Figure 4: Cosine interpolation

2 Fusion of various interpolations

2.1 Definitions

Frequency Number of pixels used as interpolation point (on one dimension). These pixels are split by a fixed interval called *step*, which depends on the frequency and the size of the picture.

Octave Virtual layer drawn in parallel to the reference layer, at a given frequency. Octaves are drawn in sequence, where the nth octave has a higher frequency than the $n-1$th. The value of the new frequency is typically the square of the previous value. In the end the resulting layers are merged together.

Persistance Weight factor for every octave that provides more or less impact on the final result.

Note: since the layer values must remain in $[0; 255] \cap \mathbb{N}$, we must take care that the sum does not overflow. Limiting the persistence to 0.5 makes sure we do not sum over 255 when octave persistences are powers of the original one.

$$\sum_{k=1}^{\infty} p^k = \frac{p}{1-p} \leqslant 1 \Rightarrow p \leqslant 0.5$$

Texture parameters:

- **seed** $\in \mathbb{N}$
- **frequency** $\in \{2^i; i \in \mathbb{N}\}$
- **octaves** $\in \mathbb{N}$
- **persistence** $\in [0, 1]$

Figure 5: Frequency