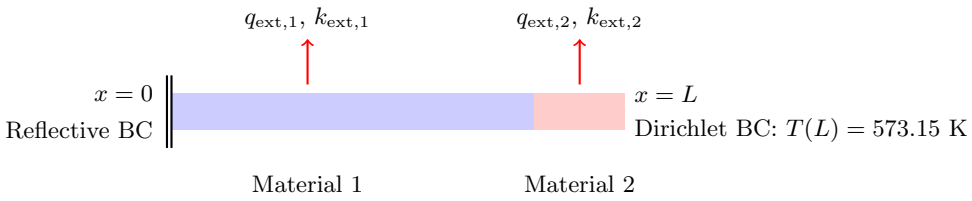
Nonlinear Heat Conduction Problem

Problem Schematic



Problem Setup: We consider a **nonlinear heat conduction problem** defined over a 1D domain with spatial coordinate x, where the system consists of different material properties and external heat sources.

Governing Equation The heat conduction is governed by the nonlinear steady-state heat equation:

$$\frac{d}{dx}\left(k(T)\frac{dT}{dx}\right) + q_{\text{ext}}(T) = 0, \quad x \in [0, 0.5] \tag{1}$$

where:

- $T(x)$ is the temperature distribution,
- $k(T)$ is the thermal conductivity, which varies with the length and temperature,
- $q_{\text{ext}}(T)$ represents the external heat source, which also varies with the length and temperature.

Material and Heat Source Properties We define two different materials in the domain with the following properties:

Thermal Conductivity

1. First material ($0 \leq x \leq 0.4$):

$$k_1(T) = 16 + \mu + \frac{2150}{T - 73.15} \tag{2}$$

2. Second material ($0.4 \leq x \leq 0.5$):

$$k_2(T) = 30 + \mu + 2.09 \times 10^{-2}T - 1.45 \times 10^{-5}T^2 + 7.67 \times 10^{-9}T^3 \quad (3)$$

External Heat Source

1. First material ($0 \leq x \leq 0.4$):

$$q_{\text{ext},1} = \beta + 35000 + \frac{T}{10} \quad (4)$$

2. Second material ($0.4 \leq x \leq 0.5$):

$$q_{\text{ext},2} = 10\beta + 5000 \quad (5)$$

Boundary Conditions The system is subject to the following boundary conditions:

- **At $x = 0$ (left boundary):** Reflective (insulating) boundary condition

$$\frac{dT}{dx} = 0 \quad \text{at} \quad x = 0 \quad (6)$$

- **At $x = 0.5$ (right boundary):** Dirichlet condition

$$T(0.5) = 573.15 \text{ K} \quad (300^\circ C) \quad (7)$$

Objective The goal is to solve for the steady-state temperature distribution $T(x)$ for different parameter values associated with nonlinear material properties and external heat sources.