# FEM_2D

October 18, 2023

[ ]:
```python
# Restart the kernel
import os
current_dir ='../../'
os.chdir(current_dir)
```

[ ]:
```python
from src.codes.basic import *
from src.codes.utils import *
from src.codes.base_classes import Base_class_fem_heat_conduction
from src.codes.reductor.rom_class import FEM_solver_rom_ecsw
from src.codes.algorithms.ecsw import ecsw_red
```

#### 0.0.1 class for data (geometry, material property, mesh)

[ ]:
```python
nref= [10,2]
L = [20,20]
mat_layout = np.zeros((5,2),dtype=int)
src_layout = np.zeros((5,2),dtype=int)
```

[ ]:
```python
fdict = {}

cond_list = []
cond_list.append( lambda T,mu: mu*T + 100. + 0.*T )
fdict["cond"] = cond_list

dcond_list = []
dcond_list.append( lambda T,mu: mu + 0. + 0.*T )
fdict["dcond"] = dcond_list

qext_list = []
qext_list.append( lambda T,mu: 0.0+1.0 + 0.*T )
fdict["qext"] = qext_list
```

[ ]:
```python
bc = {}
bc['x_min']={'type':'dirichlet','value':10.0}
bc['x_max']={'type':'dirichlet','value':10.0}
bc['y_min']={'type':'dirichlet','value':10.0}
bc['y_max']={'type':'refl','value':np.nan}
```

[ ]:
```python
class probdata:

    def __init__(self, bc, cond_layout, qext_layout, fdict, nref, L, mu,␣
 ↪pb_dim=2):

        self.dim_ = pb_dim
        # refine the mesh and update material and source layouts
        repeats = np.asarray(nref, dtype=int)
        self.cell2mat_layout = self.repeat_array(mat_layout,repeats)
        self.cell2src_layout = self.repeat_array(src_layout,repeats)

        ## change this mapping if needed.

        self.fdict = fdict

        # mesh data
        # cells
        self.ncells = [None] * pb_dim
        self.npts = [None] * pb_dim
        self.deltas = [None] * pb_dim
        self.xi=[]
        for i in range(pb_dim):
            self.ncells[i] = self.cell2mat_layout.shape[i]
            self.npts[i] = self.ncells[i]+1
            self.xi.append(np.linspace(0,L[i],self.npts[i]))
            self.deltas[i] = L[i]/self.ncells[i]

        self.n_verts = np.prod(np.array(self.npts))

        # Create nodal connectivity for the continuous Finite Element Method␣
 ↪(cFEM)
        self.connectivity()

        # Store parameter value
        self.mu = mu

        # Store the dirichlet nodes if any
        handle_boundary_conditions(self, bc)

        # Determining the global equation numbers based on dirichlet nodes and␣
 ↪storing in class
        get_glob_node_equation_id(self, self.dir_nodes)

        # Get global node numbers and equation IDs for the current element
        self.glob_node_eqnId = []
        self.glob_node_nonzero_eqnId = []
        self.local_node_nonzero_eqnId = []
```

```
        self.Le = []
        self.global_indices = []
        self.local_indices = []

        for i in range(self.n_cells):
            get_element_global_nodes_and_nonzero_eqnId(self, i, self.node_eqnId)



    def repeat_array(self, arr,repeats):
        for dim,n in enumerate(repeats):
            arr = np.repeat(arr,n,axis=dim)
        return arr


    def connectivity(self):
        """
        Define nodal connectivity for each cell in the mesh.
        """

        # Initialize the connectivity array
        self.n_cells = np.prod(np.array(self.ncells))

        self.gn = np.zeros((self.n_cells,2**self.dim_),dtype=int)

        # compute node ID from (i,j) cell identifiers
        # def node(*args):
        #     index = 0
        #     multiplier = 1
        #     for i, n in enumerate(args):
        #         index += n * multiplier
        #         if i < len(self.npts) - 1:
        #             multiplier *= self.npts[i]
        #     return index

        node = lambda i,j: i+j*self.npts[0]

        # Loop over all cells to define their nodal connectivity
        iel = 0
        for j in range(self.ncells[1]):
            for i in range(self.ncells[0]):
                # counter-clockwise
                self.gn[iel,0] = node(i  ,j  )
                self.gn[iel,1] = node(i+1,j  )
                self.gn[iel,2] = node(i+1,j+1)
                self.gn[iel,3] = node(i  ,j+1)
                iel += 1
```

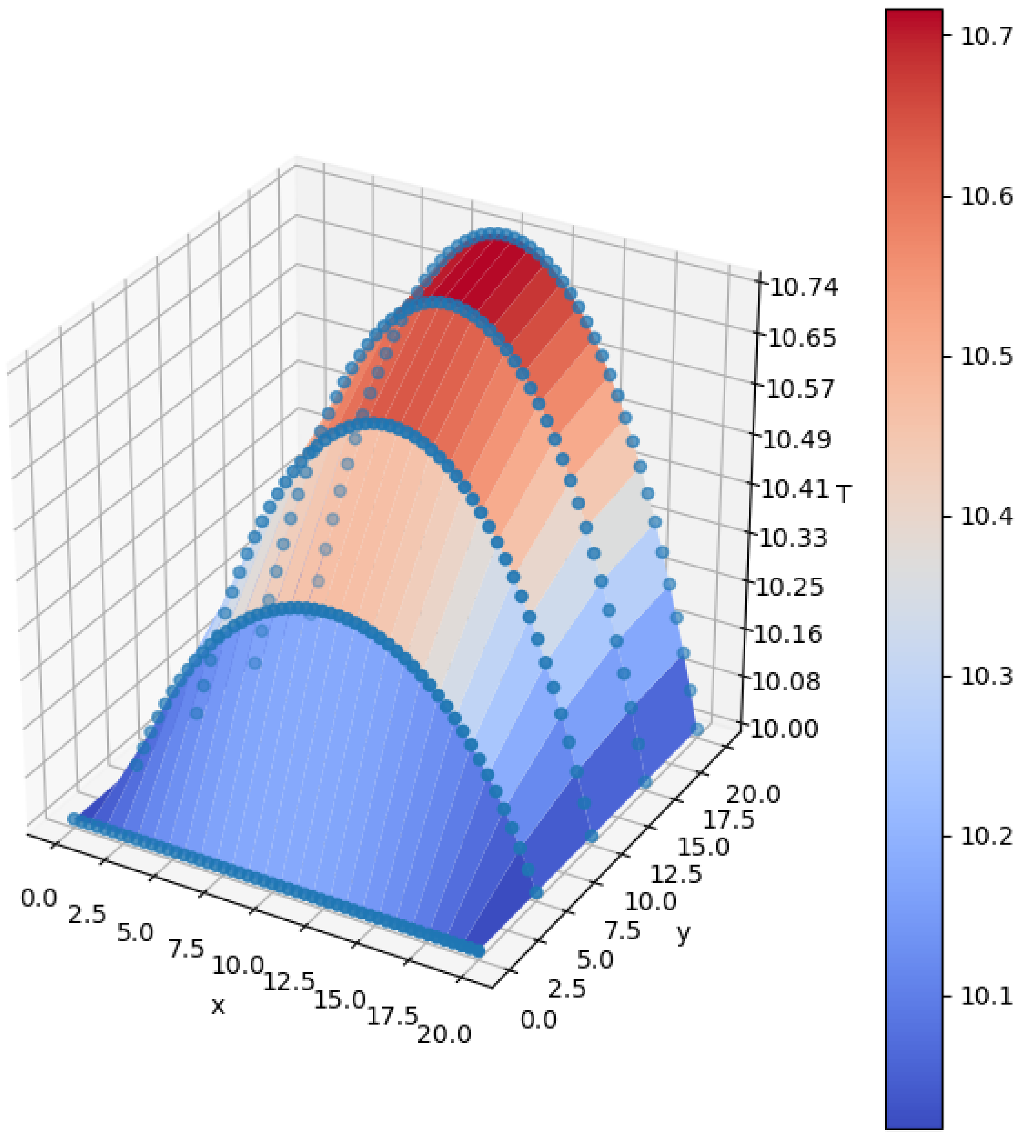
```

```

### 0.0.2 Simulate FOS

```
[ ]: random.seed(25)
params = np.r_[1.:4.0:0.01]
quad_deg = 3
N_snap = 15 # Training Snapshots
NL_solutions = []
param_list = []
K_mus = []
q_mus = []
#
```

```
[ ]: for i in range(N_snap):
    print(f"\n\n\n Snap {i} \n\n\n")
    param = random.choice(params) # Choose from parameter list
    param_list.append(param)

    if i==0:
        d = probdata(bc, mat_layout, src_layout, fdict, nref, L, param,␣
 ↪pb_dim=2)
        FOS = Base_class_fem_heat_conduction(d,quad_deg)
    else:
        FOS.mu = param
    T_init = np.zeros(d.n_verts) + 2.0
    NL_solution_p, Ke, rhs_e, mask = solve_fos(FOS, T_init)
    NL_solutions.append(NL_solution_p.flatten())
    K_mus.append(Ke)
    q_mus.append(rhs_e)
    plot2D(d.xi[0], d.xi[1], NL_solution_p)
```

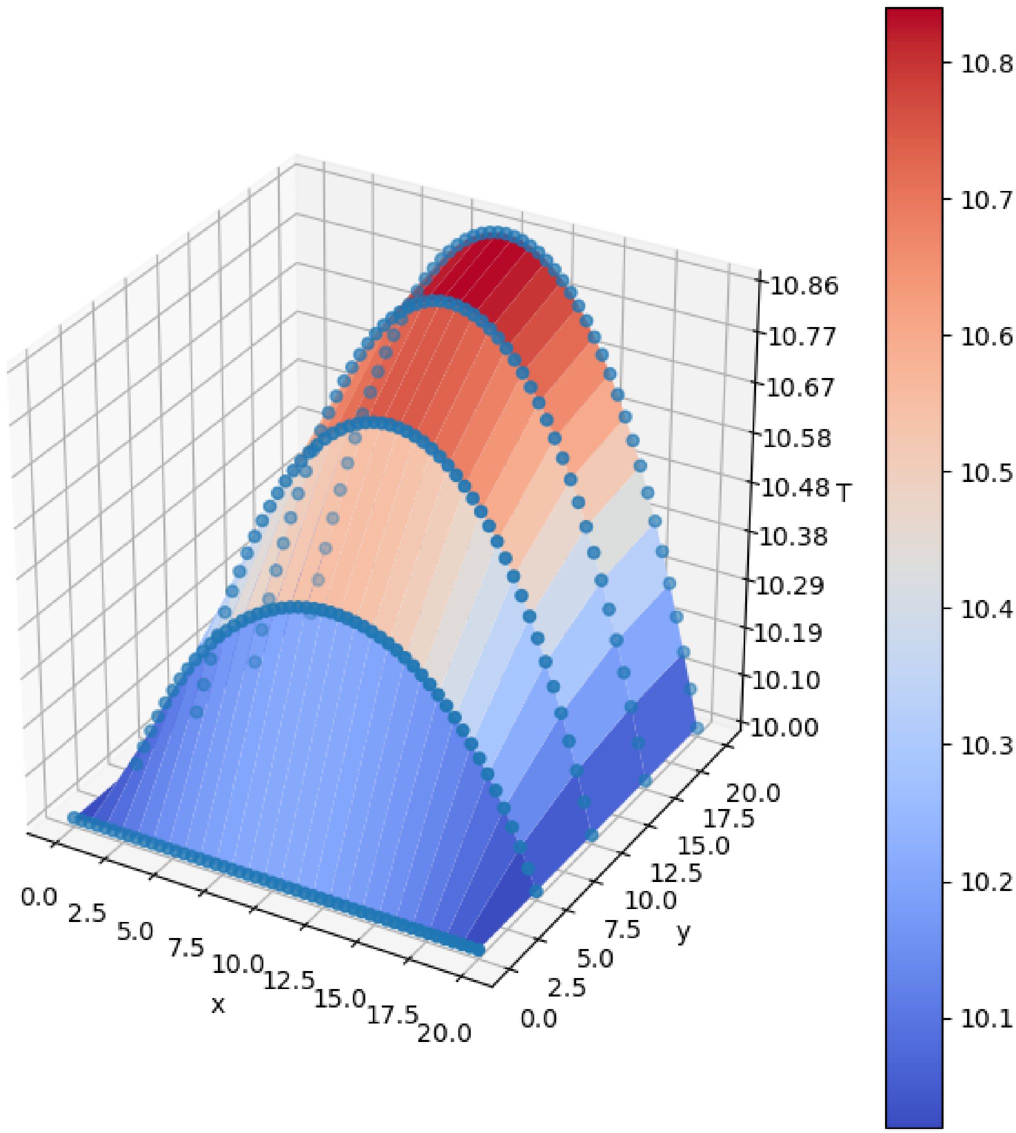
```


 Snap 0



initial residual = 11986.651822042377

iter 0, NL residual=1324.6513464306388, delta=9.770837155448827
iter 1, NL residual=11.299873829035011, delta=-0.7630796870436435
iter 2, NL residual=0.0009297289458881234, delta=-0.00597025279632086
iter 3, NL residual=5.863596350429197e-11, delta=-3.8601837583283444e-07
Convergence !!!
```




Snap 1

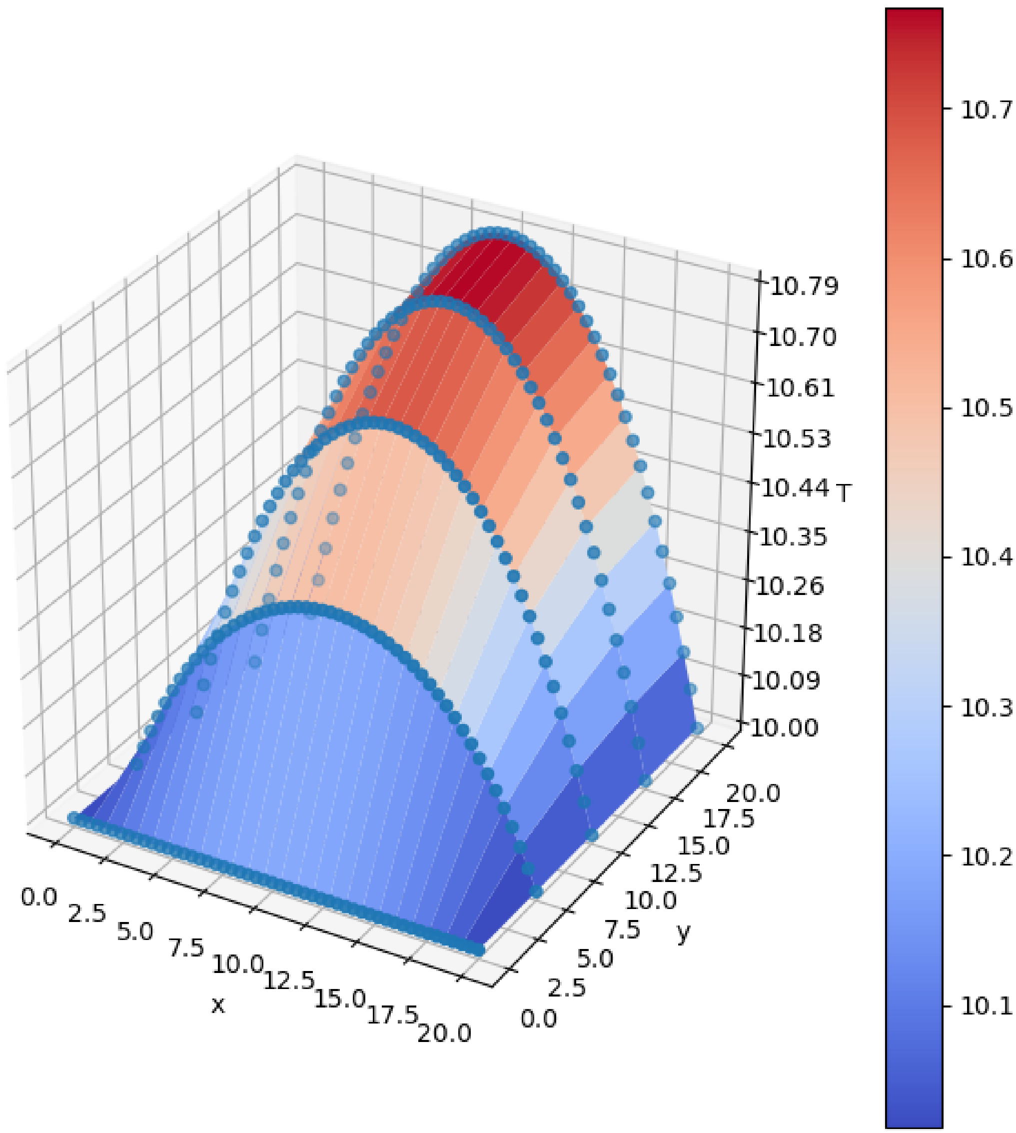
```
initial residual = 10796.002612555554

iter 0, NL residual=430.1273081220602, delta=9.267220150665285
iter 1, NL residual=0.6083380775937282, delta=-0.29541303410816255
iter 2, NL residual=1.3310436522078134e-06, delta=-0.0003811418342108528
```

```
Convergence !!!
```




```
 Snap 2


initial residual = 11448.905882745215

iter 0, NL residual=897.9573942240792, delta=9.548205887620298
iter 1, NL residual=4.293851645158141, delta=-0.5589006266791802
```

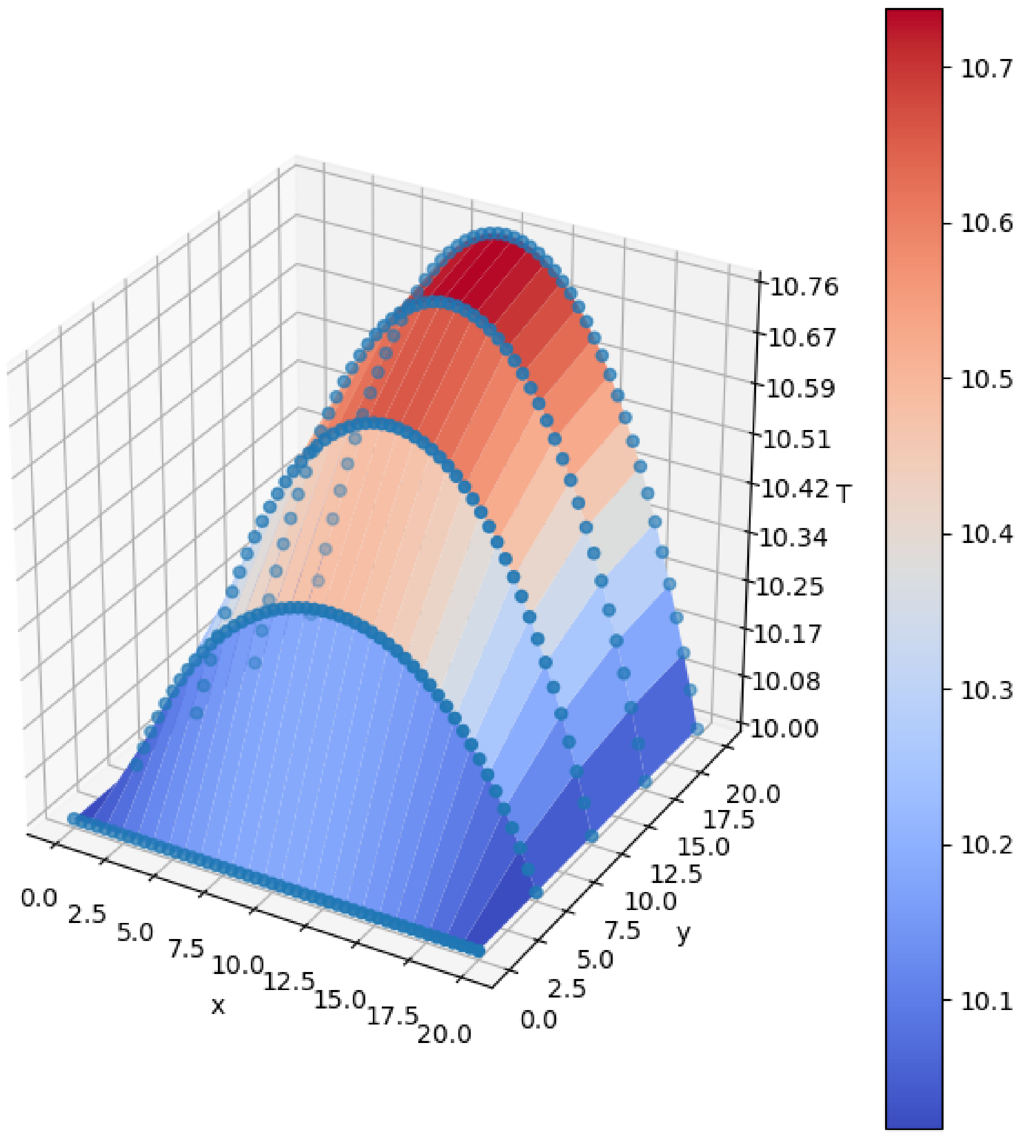
```
iter 2, NL residual=0.0001091457821580371, delta=-0.0024418888587483434
iter 3, NL residual=4.55201871090765e-11, delta=-4.9178207860296254e-08
Convergence !!!
```




Snap 3

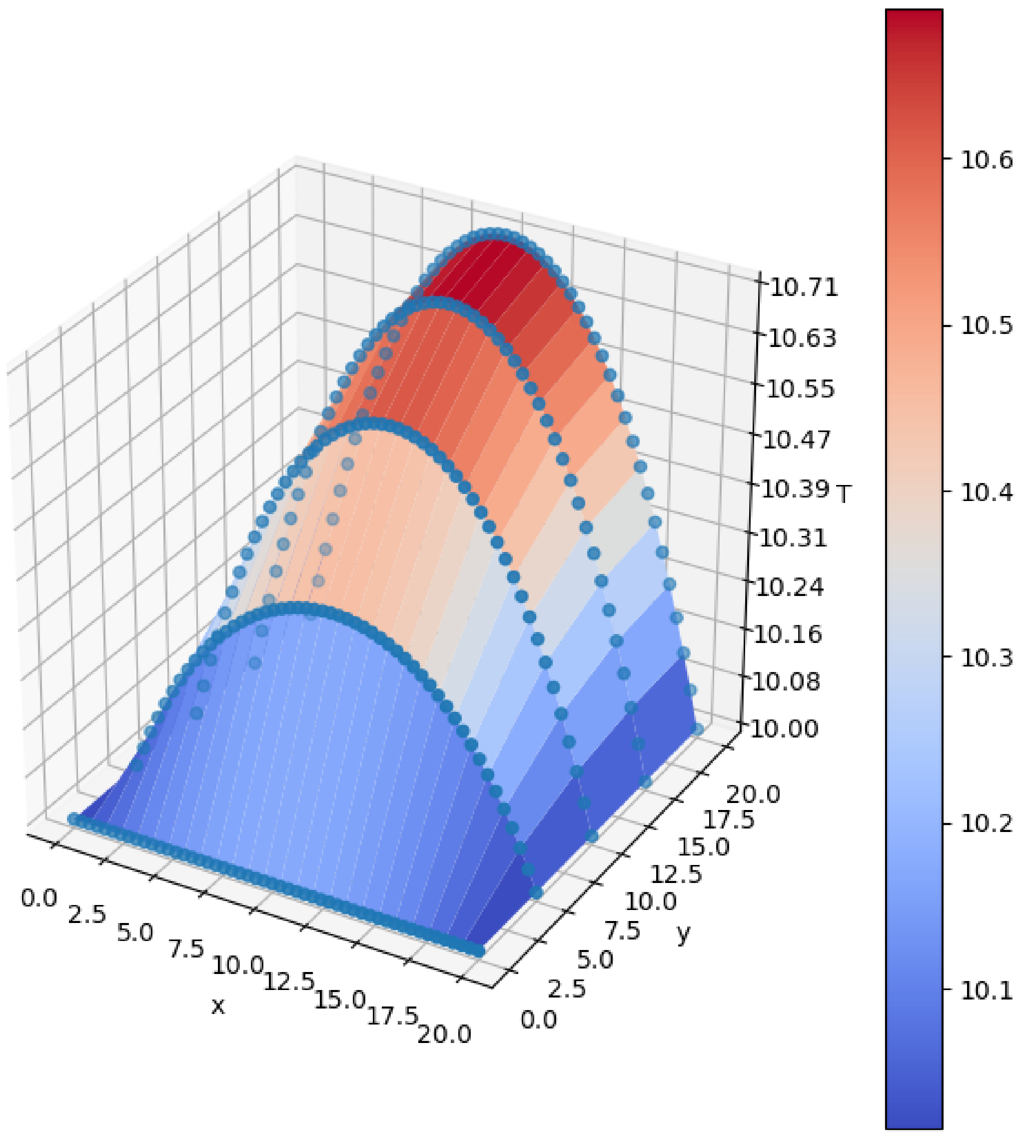
```
initial residual = 11749.78131183665
```

```
iter 0, NL residual=1132.1482252709548, delta=9.673720095303679
iter 1, NL residual=7.691518785349875, delta=-0.6745323304166354
iter 2, NL residual=0.00039827846930358065, delta=-0.004195038886152979
iter 3, NL residual=5.76955941131255e-11, delta=-1.7129744545449967e-07
Convergence !!!
```



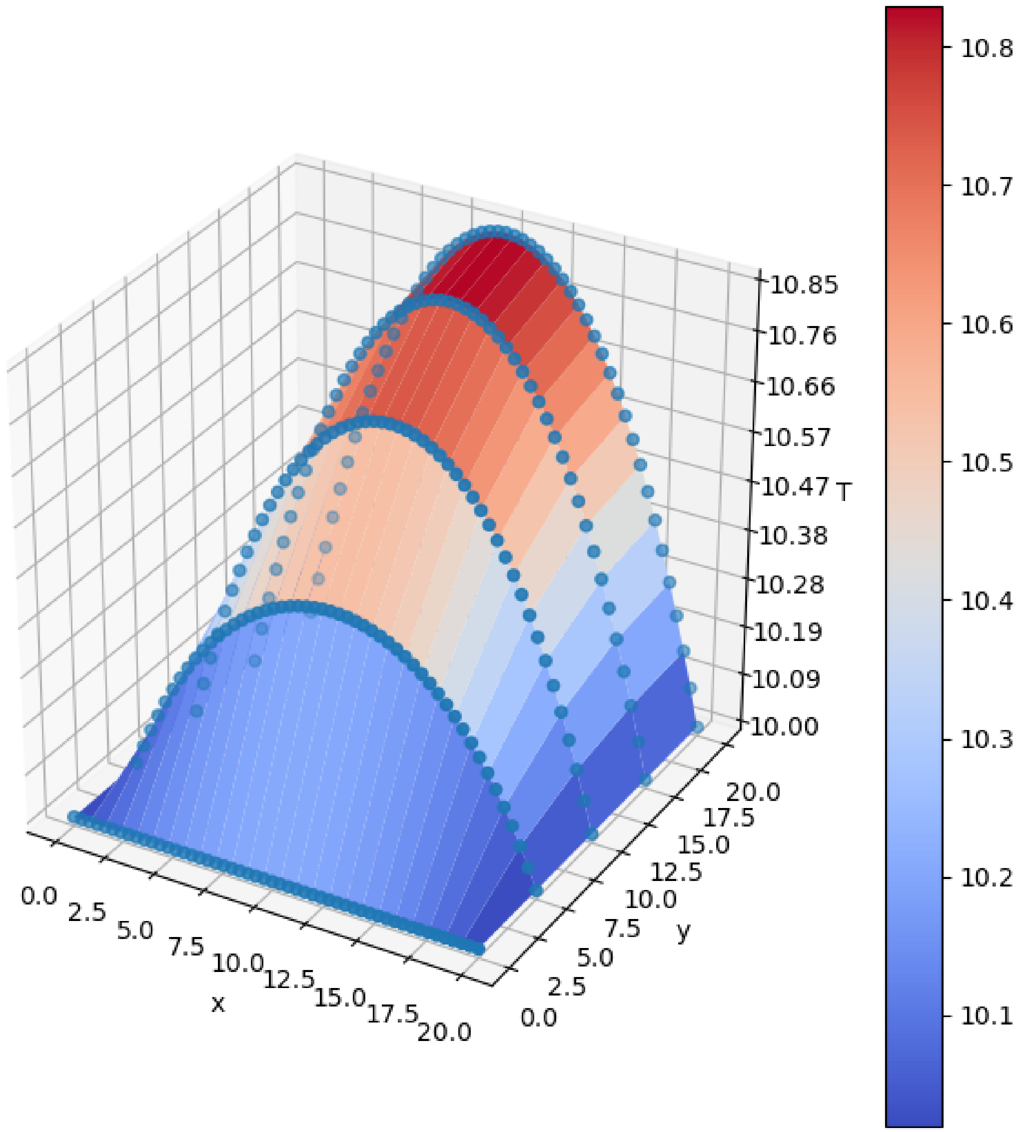

Snap 4

```
initial residual = 12300.359246313501

iter 0, NL residual=1590.500199072437, delta=9.897218008177278
iter 1, NL residual=17.490484433739706, delta=-0.8770728266283285
iter 2, NL residual=0.002417843817180866, delta=-0.00887262198503041
iter 3, NL residual=7.296817667895163e-11, delta=-9.597222523700614e-07
Convergence !!!
```



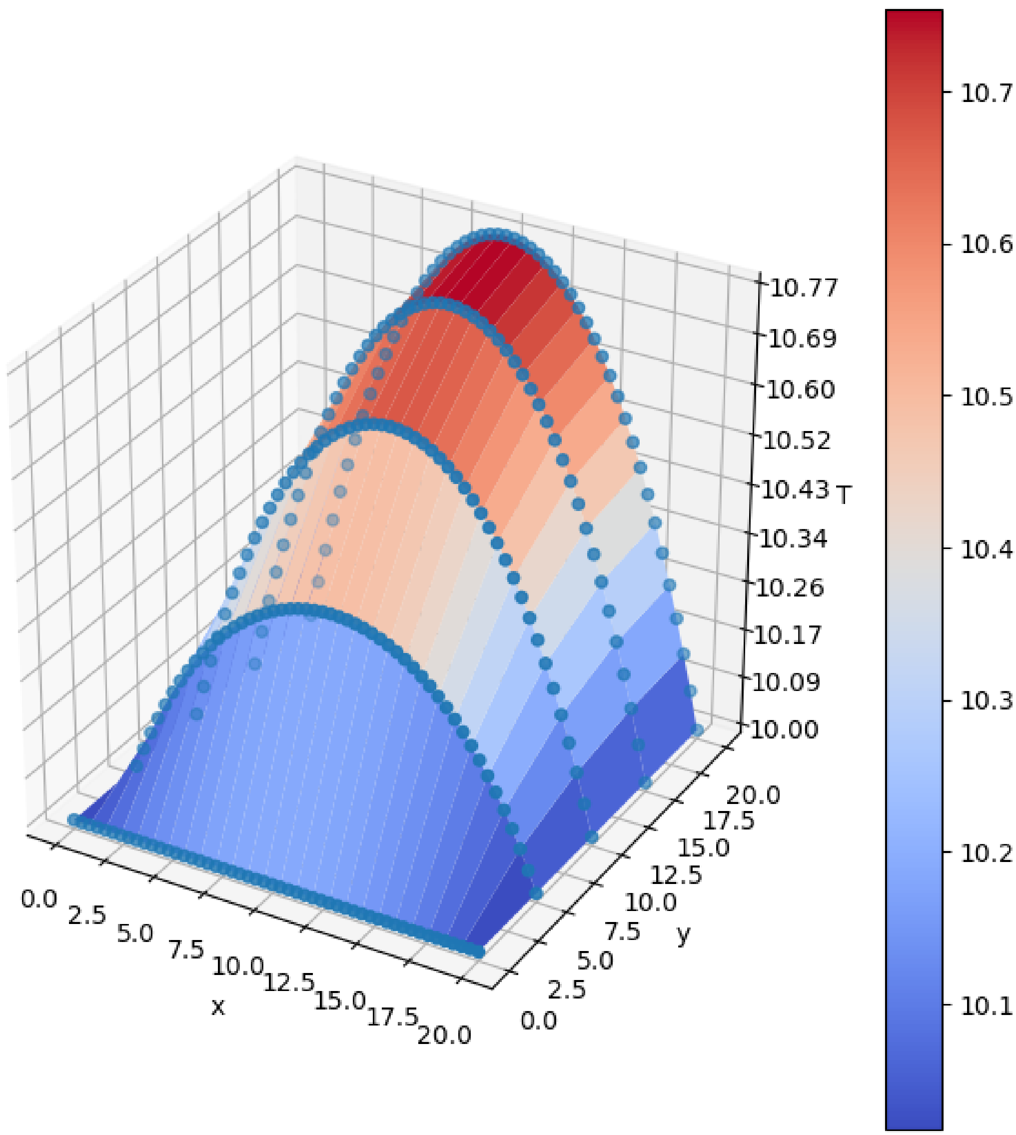

```
Snap 5
```

```
initial residual = 10885.611499624636

iter 0, NL residual=491.0283663493382, delta=9.306502851811977
iter 1, NL residual=0.8733120002638732, delta=-0.3326148382031104
iter 2, NL residual=3.0272816895873947e-06, delta=-0.0005396394097674344
Convergence !!!
```



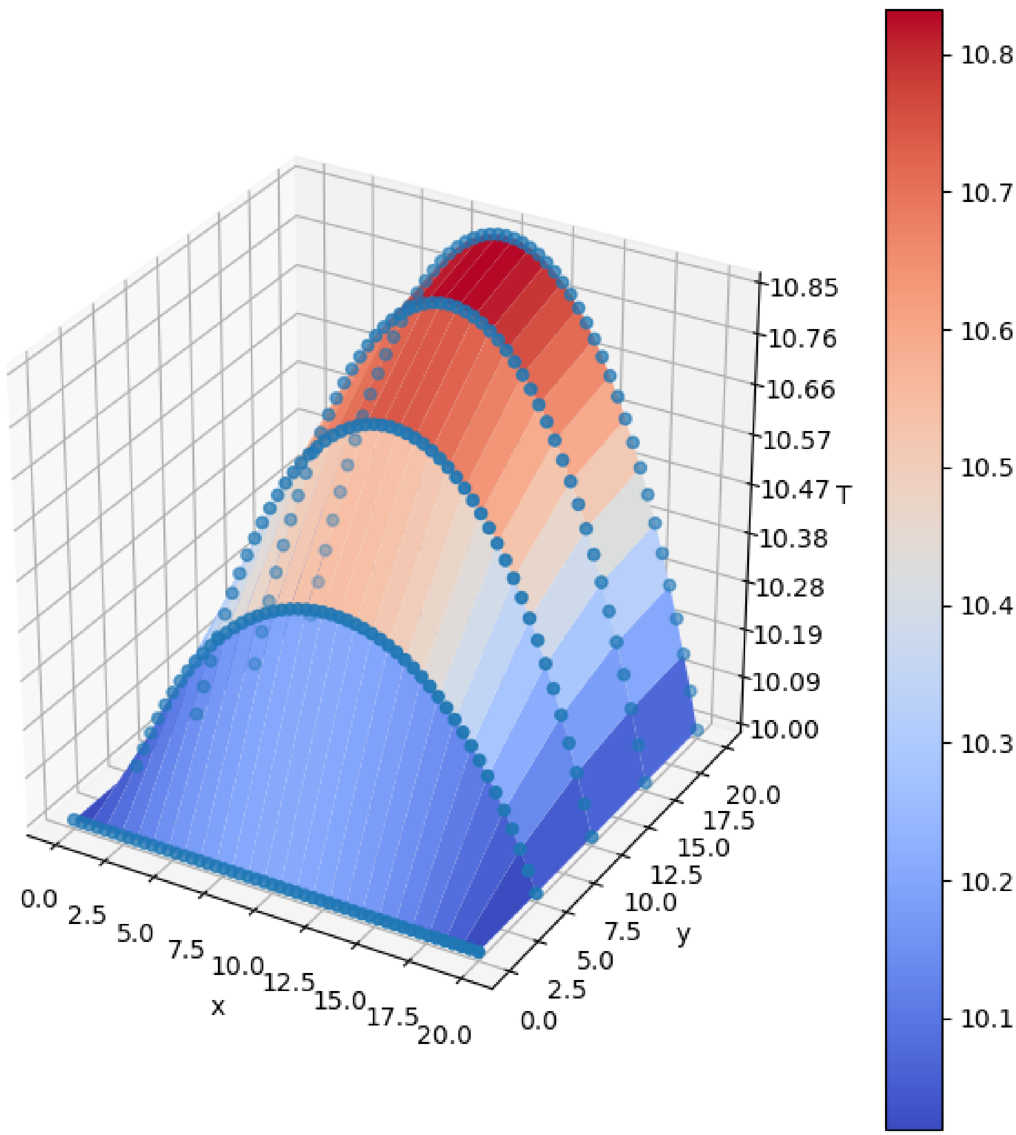

Snap 6

```
initial residual = 11583.337561782488

iter 0, NL residual=1001.1582842065261, delta=9.604587739539669
iter 1, NL residual=5.657694313410112, delta=-0.6110072666295774
iter 2, NL residual=0.00020166032680943424, delta=-0.0031573139218115556
iter 3, NL residual=5.631711341751676e-11, delta=-8.897178890772488e-08
Convergence !!!
```

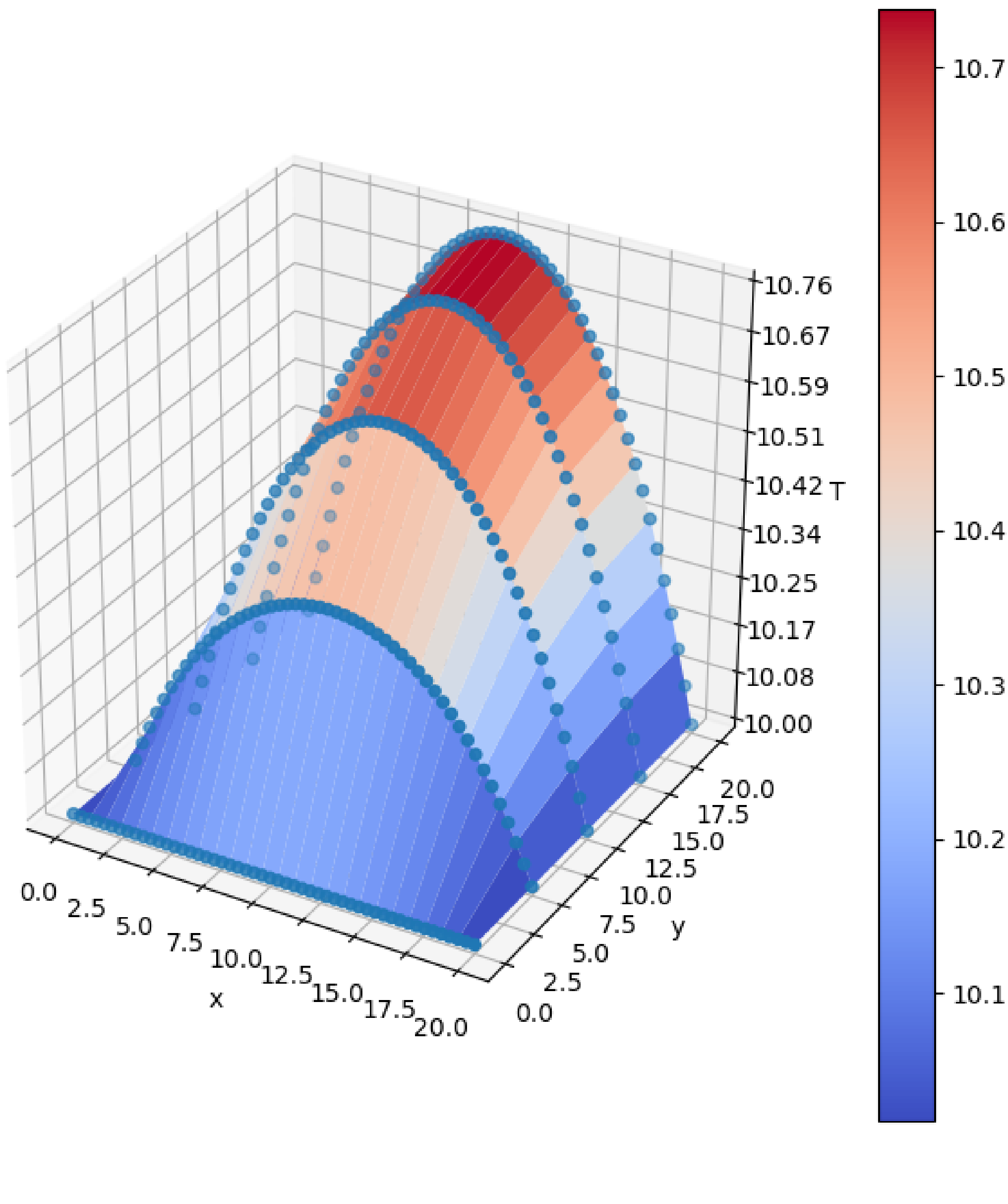


Snap 7

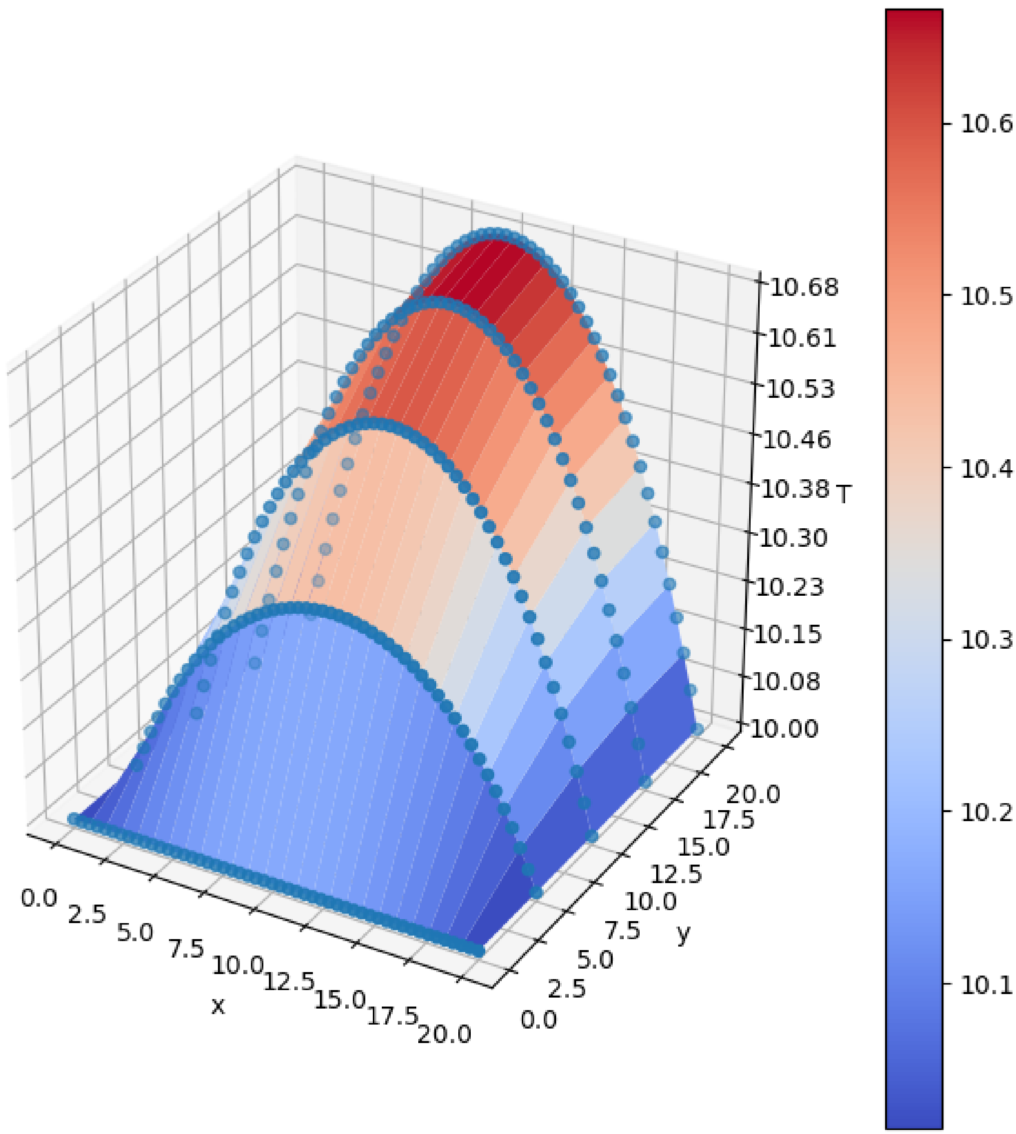
```
initial residual = 10860.008779488995

iter 0, NL residual=473.51981633681385, delta=9.295302912700691
iter 1, NL residual=0.7911917502548279, delta=-0.32201977420425854
iter 2, NL residual=2.419302912087643e-06, delta=-0.0004908223594698444
Convergence !!!
```



```
 Snap 8


initial residual = 11749.78131183665

iter 0, NL residual=1132.1482252709548, delta=9.673720095303679
iter 1, NL residual=7.691518785349875, delta=-0.6745323304166354
iter 2, NL residual=0.00039827846930358065, delta=-0.004195038886152979
iter 3, NL residual=5.76955941131255e-11, delta=-1.7129744545449967e-07
Convergence !!!
```



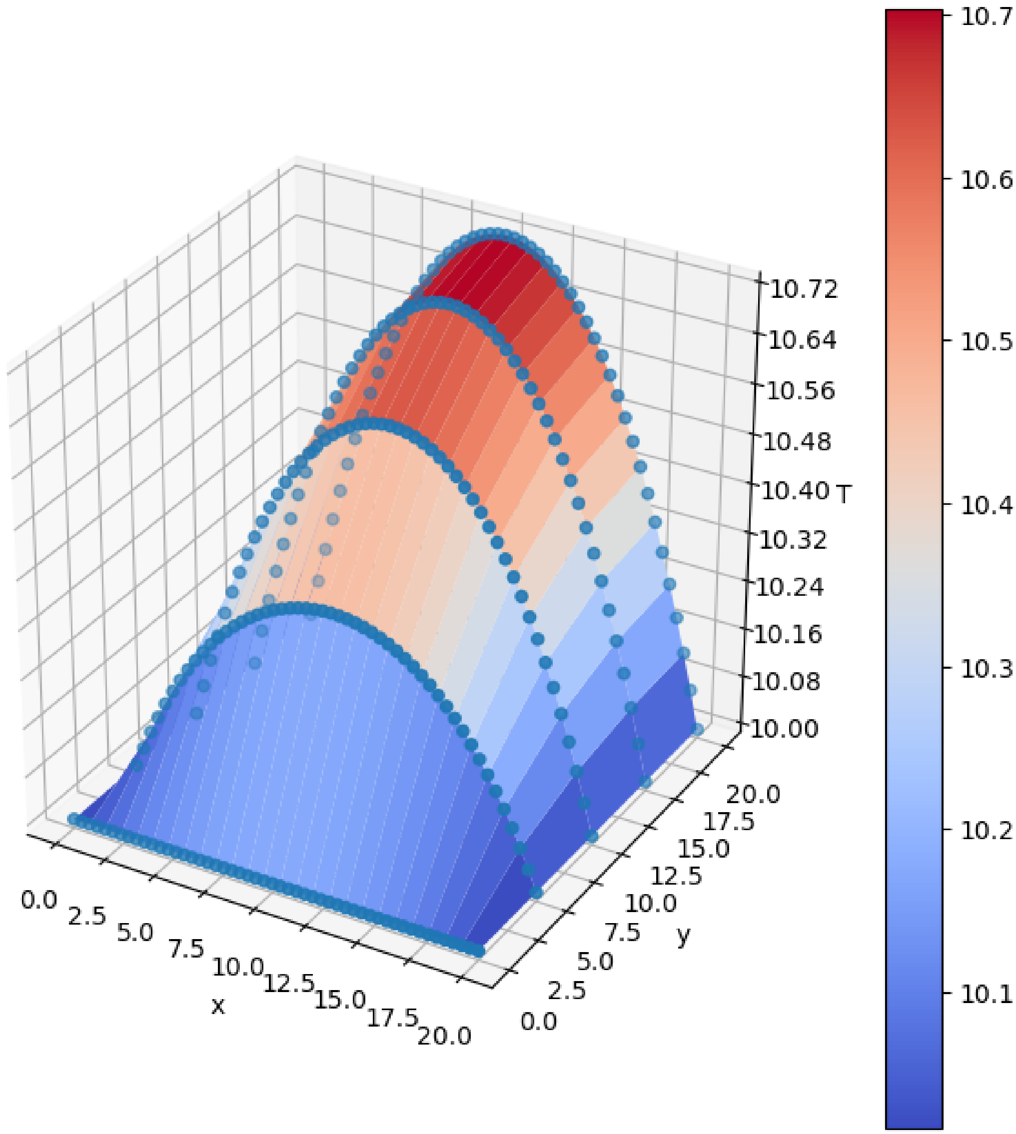
```
 Snap 9


initial residual = 12601.276308862416

iter 0, NL residual=1857.0462564585055, delta=10.016111397172208
iter 1, NL residual=25.076307937586055, delta=-0.9830147165117947
iter 2, NL residual=0.005284775733179206, delta=-0.012251129648249393
iter 3, NL residual=2.802775082267266e-10, delta=-2.012560741461733e-06
Convergence !!!
```




```
Snap 10
```

```
initial residual = 12140.302502454553

iter 0, NL residual=1453.3175388282611, delta=9.833052144904274
iter 1, NL residual=14.12219359089362, delta=-0.8193726717310355
iter 2, NL residual=0.0015157108549731686, delta=-0.0073127839566327485
```

```
iter 3, NL residual=6.074580407401862e-11, delta=-6.154569538683099e-07
Convergence !!!
```

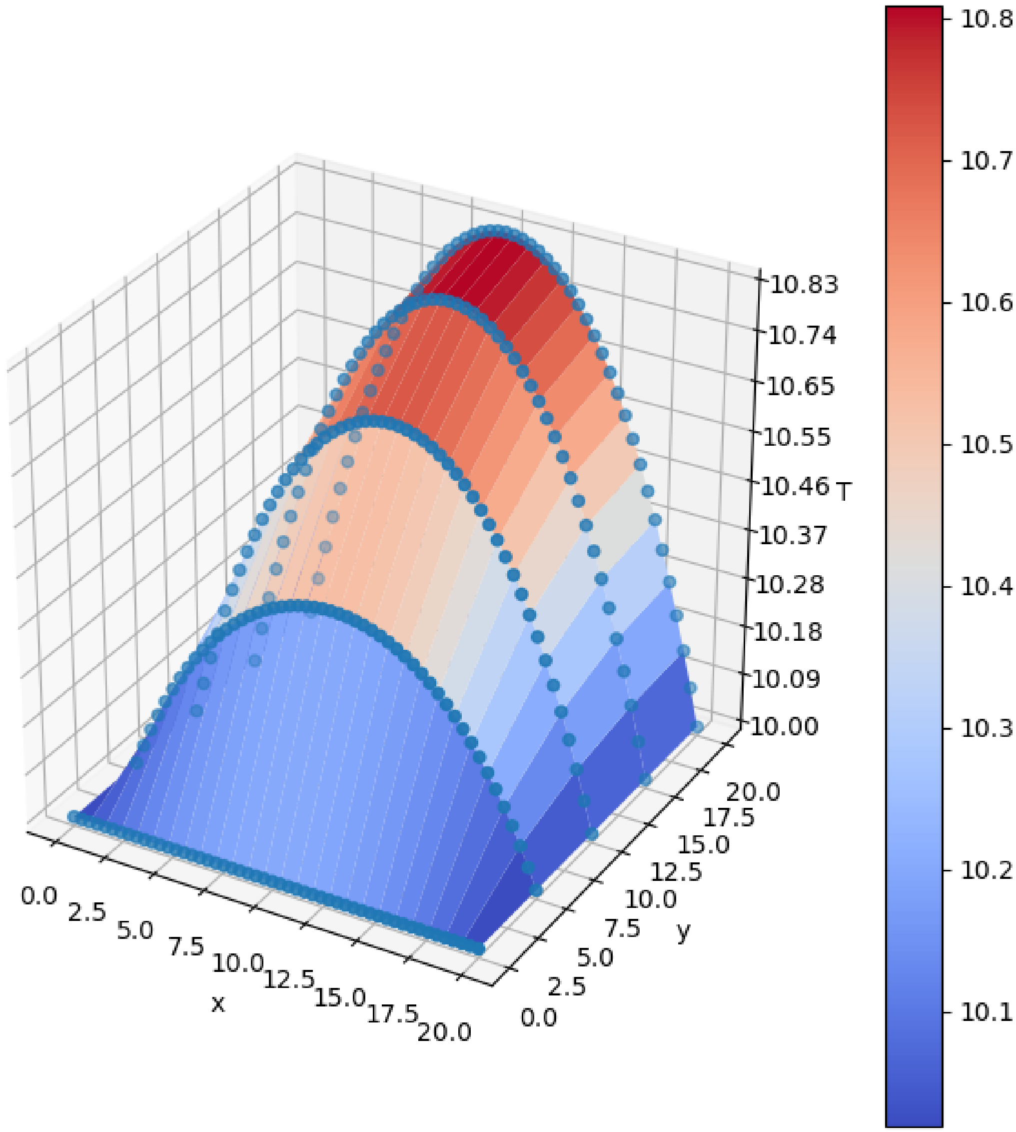



```
Snap 11
```

```
initial residual = 11064.834495879742
```

```
iter 0, NL residual=616.0053869417208, delta=9.384376348132012
```

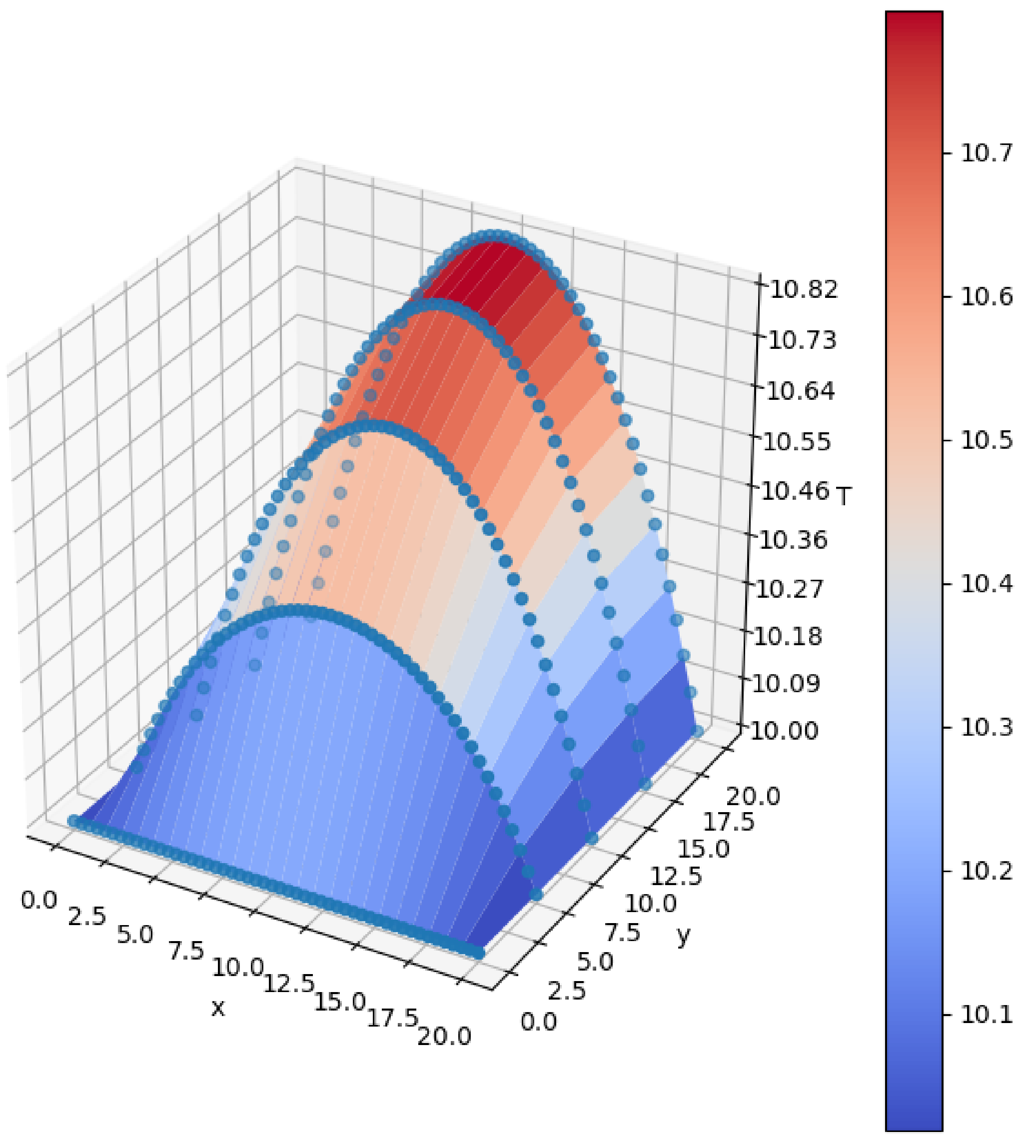
```
iter 1, NL residual=1.606759904362956, delta=-0.4060214096605569
iter 2, NL residual=1.202870103291367e-05, delta=-0.0009662780627629353
iter 3, NL residual=3.573747145199104e-11, delta=-5.7689925979903725e-09
Convergence !!!
```




```
 Snap 12
```

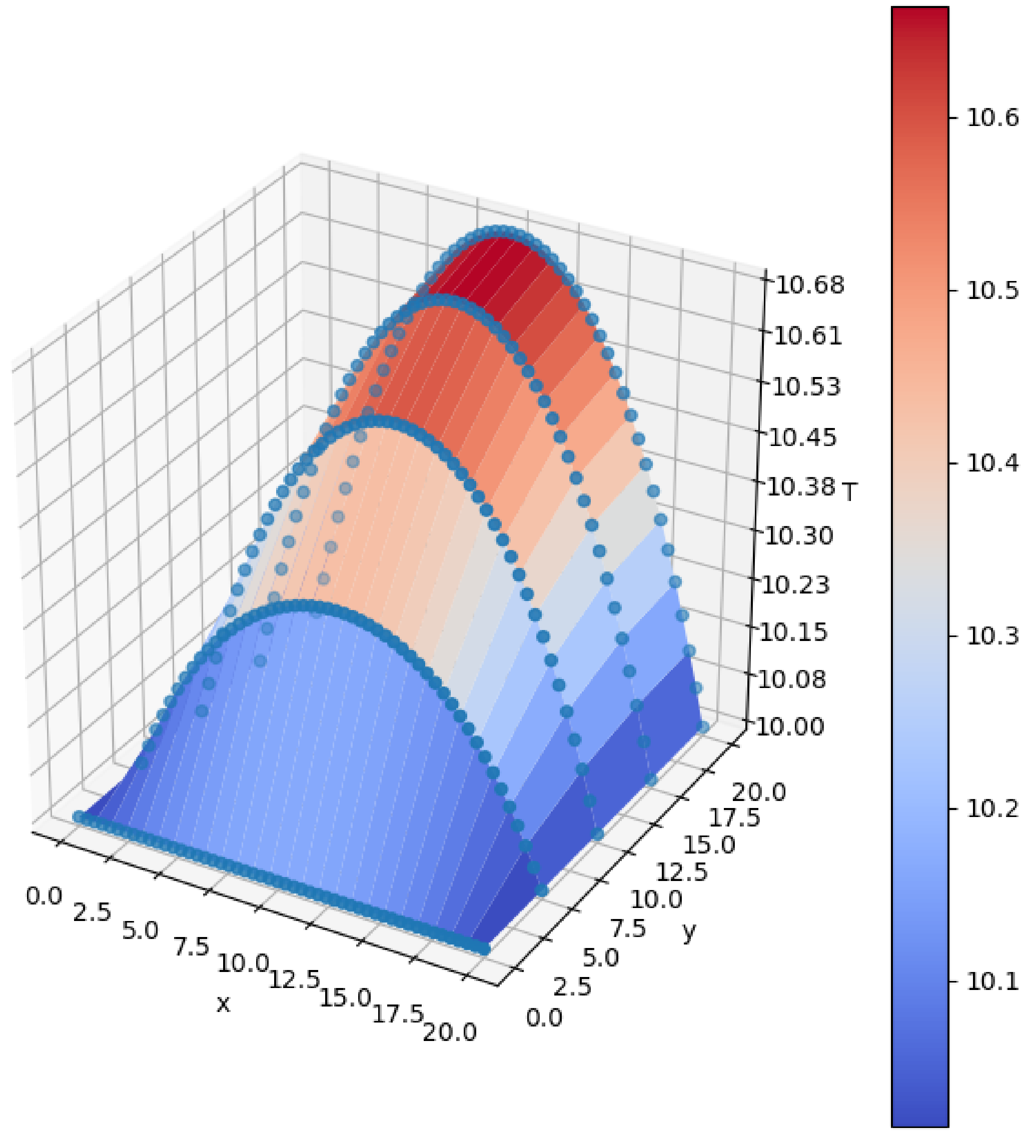
```
initial residual = 11154.448521159917
```

```
iter 0, NL residual=680.074652092181, delta=9.422971109045507
iter 1, NL residual=2.0879360440243087, delta=-0.44222931991353054
iter 2, NL residual=2.170384471683257e-05, delta=-0.0012390495023096384
iter 3, NL residual=3.674459898869056e-11, delta=-1.0255342003648238e-08
Convergence !!!
```



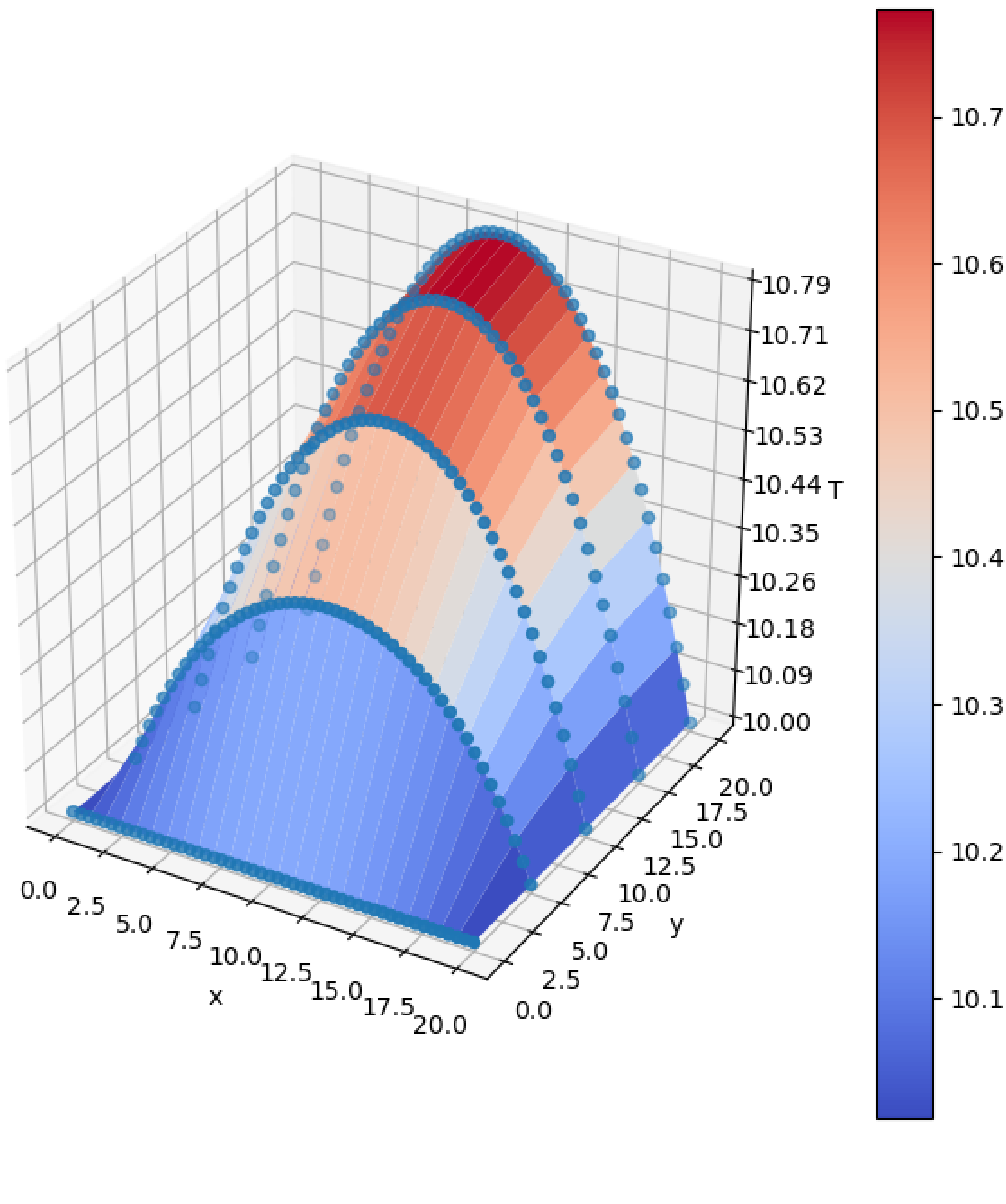

```
Snap 13
```

```
initial residual = 12633.289529023508

iter 0, NL residual=1886.0616636341792, delta=10.028628083981312
iter 1, NL residual=25.9835885640026, delta=-0.9940940284622881
iter 2, NL residual=0.005706734879676642, delta=-0.012644653410365936
iter 3, NL residual=3.18430318026223e-10, delta=-2.1638957469904146e-06
Convergence !!!
```




```
Snap 14
```

```
initial residual = 11391.29333804649

iter 0, NL residual=854.4426007791867, delta=9.523891675152585
iter 1, NL residual=3.781049519870575, delta=-0.5363486845869734
iter 2, NL residual=8.217370874450559e-05, delta=-0.002167959253620812
iter 3, NL residual=4.040347681664785e-11, delta=-3.736525535080068e-08
Convergence !!!
```



[ ]:
```
NLS = np.asarray(NL_solutions)
np.shape(NLS)
```

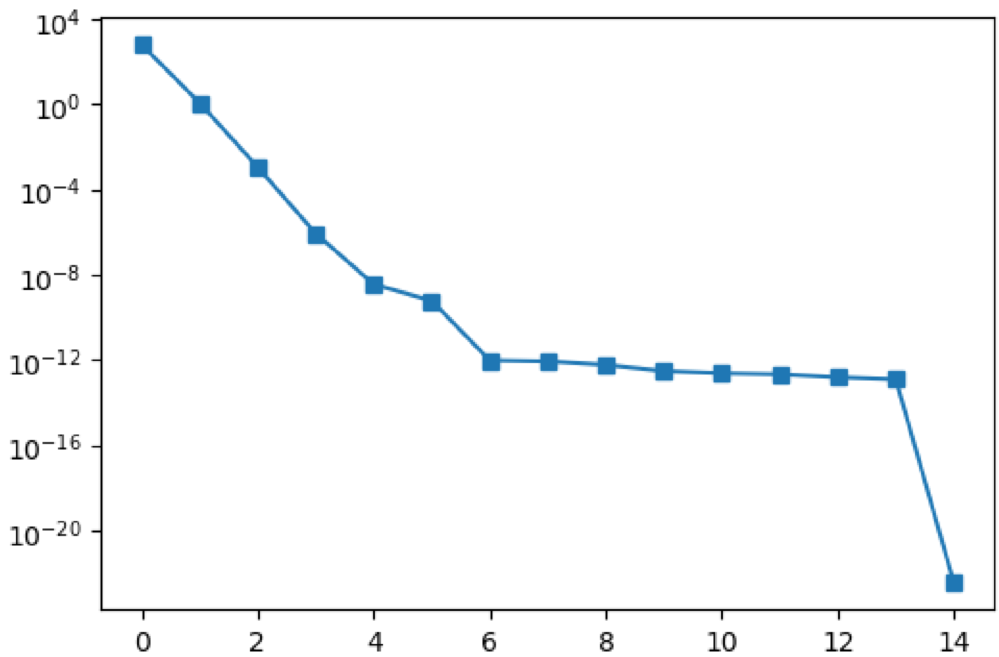
[ ]: (15, 255)

### 0.0.3 ECSW Hyper-reduction

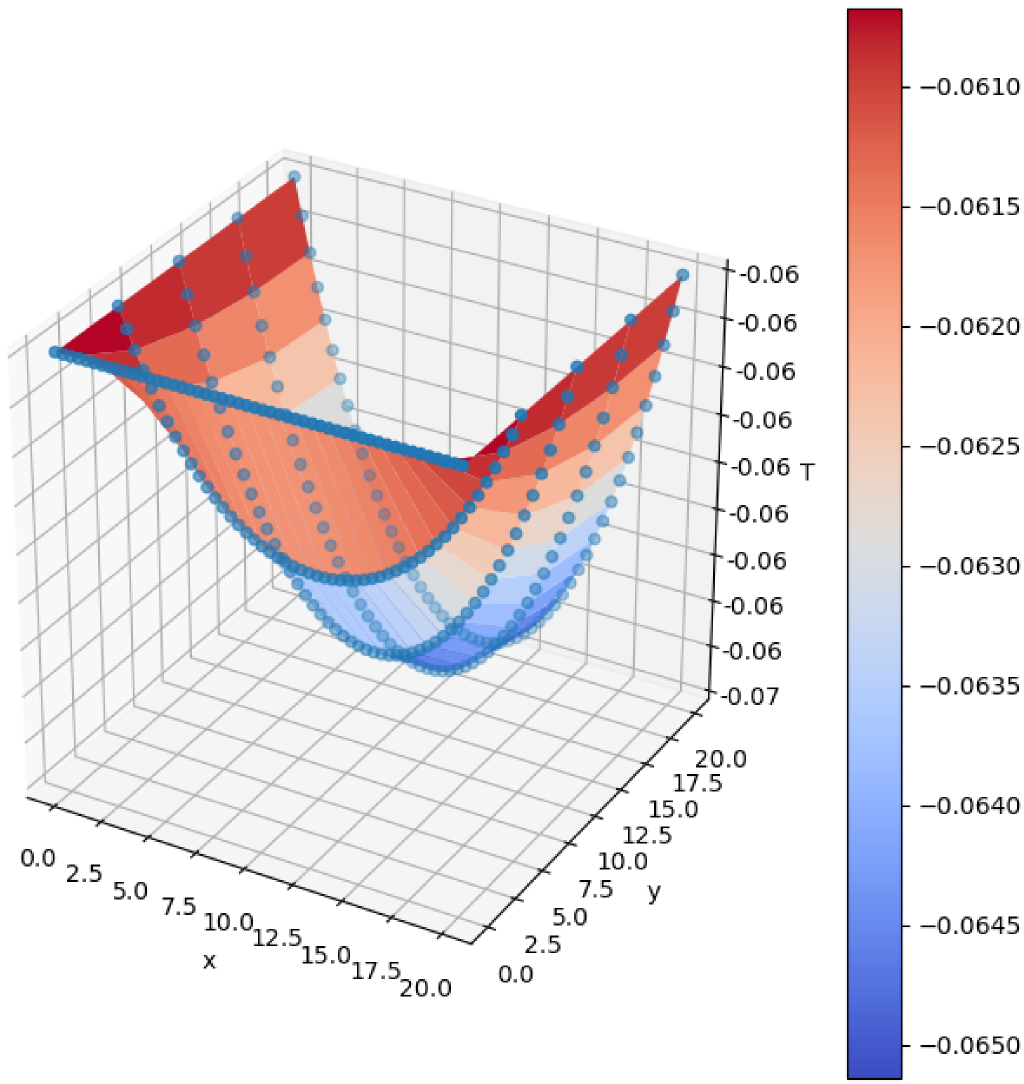
**Step 1: Perform SVD on the snapshots (calculate $\mathbb{V}(=\mathbb{W})$):**

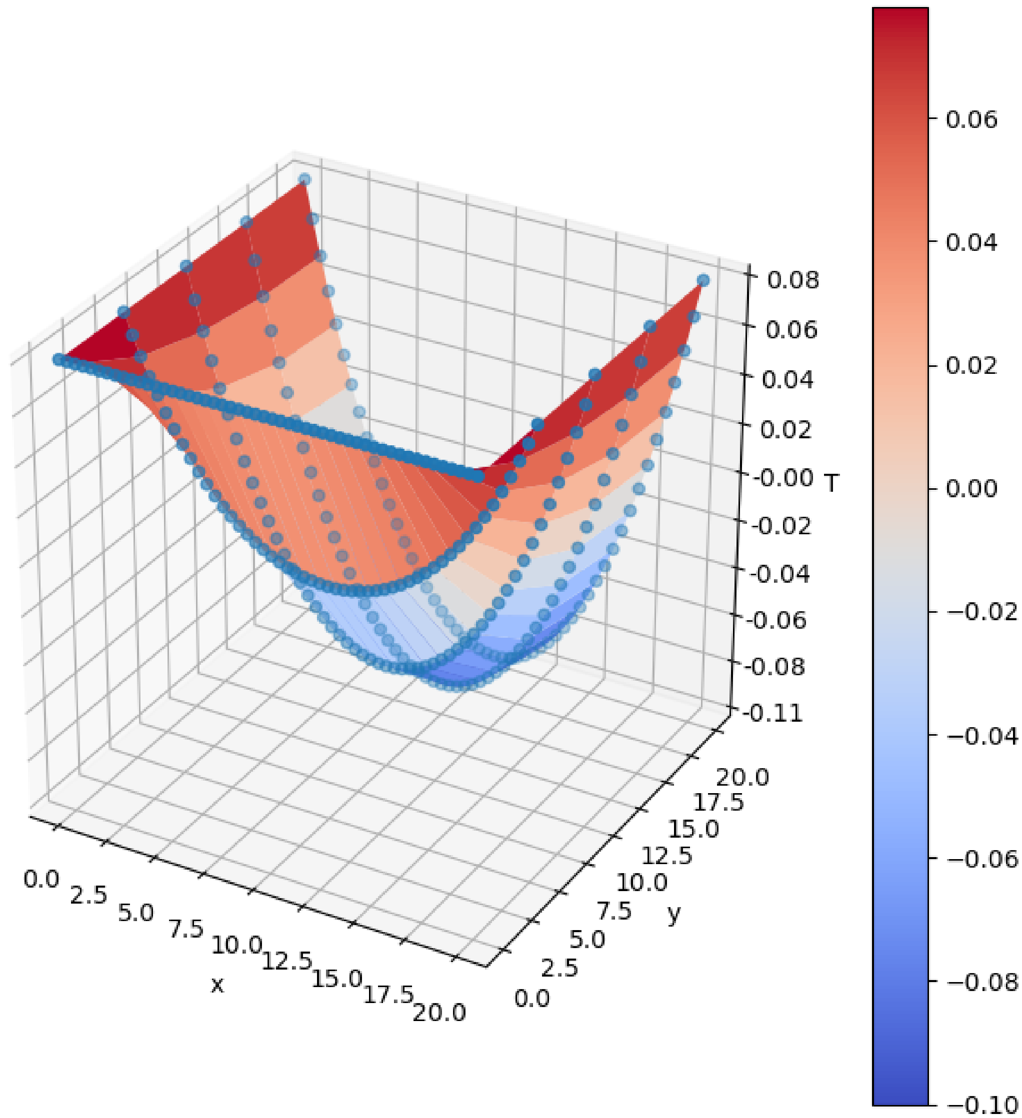
[ ]:
```
n_sel = 4
U, S, Vt = np.linalg.svd(np.transpose(NLS), full_matrices=False)
V_sel = U[:, :n_sel]
P_sel = V_sel[mask,:]@np.transpose(V_sel[mask,:])
```

[ ]:
```
plt.figure(figsize = (6,4))
plt.semilogy(S,'s-')
plt.show()
```

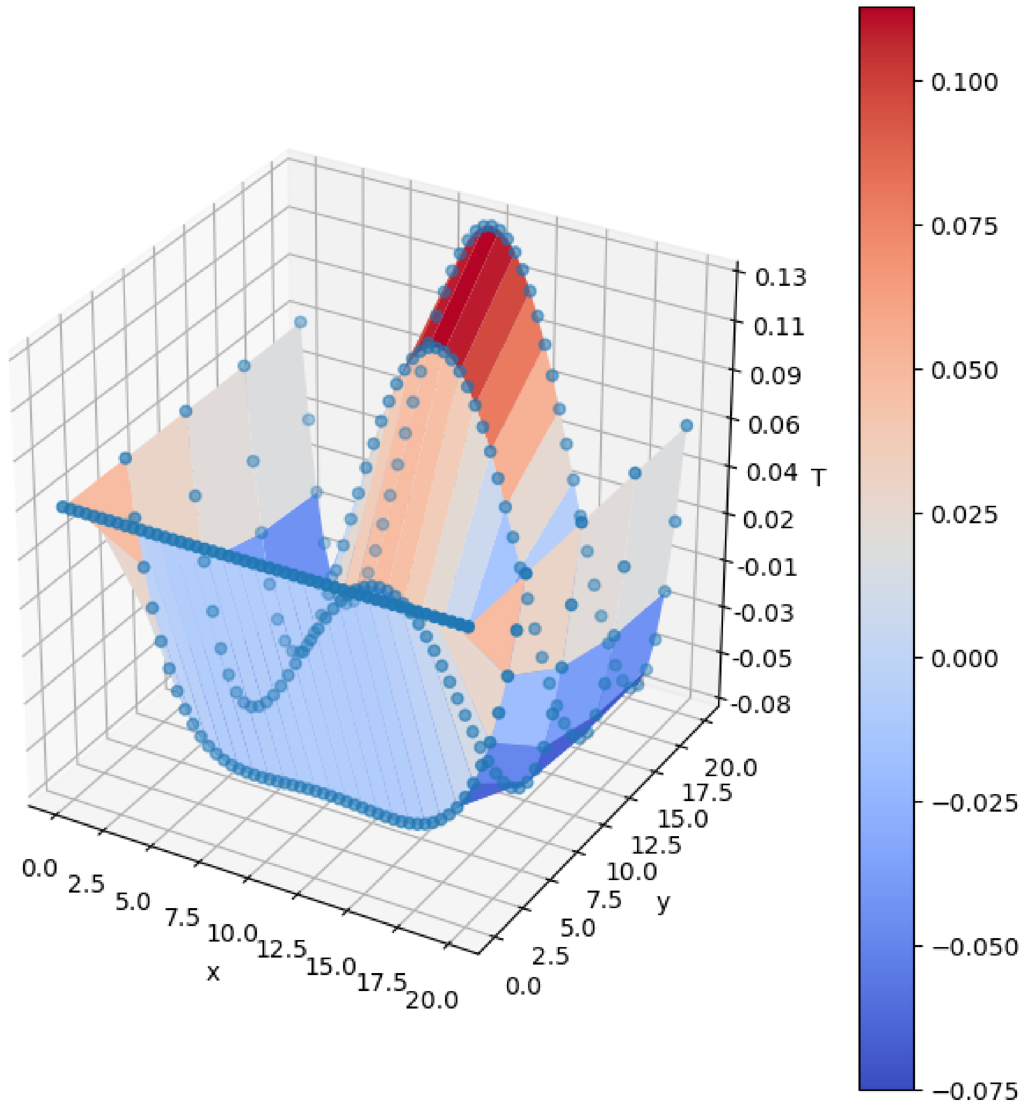



[ ]:
```
for i in range(n_sel):
    plot2D(d.xi[0], d.xi[1], V_sel[:,i])
```

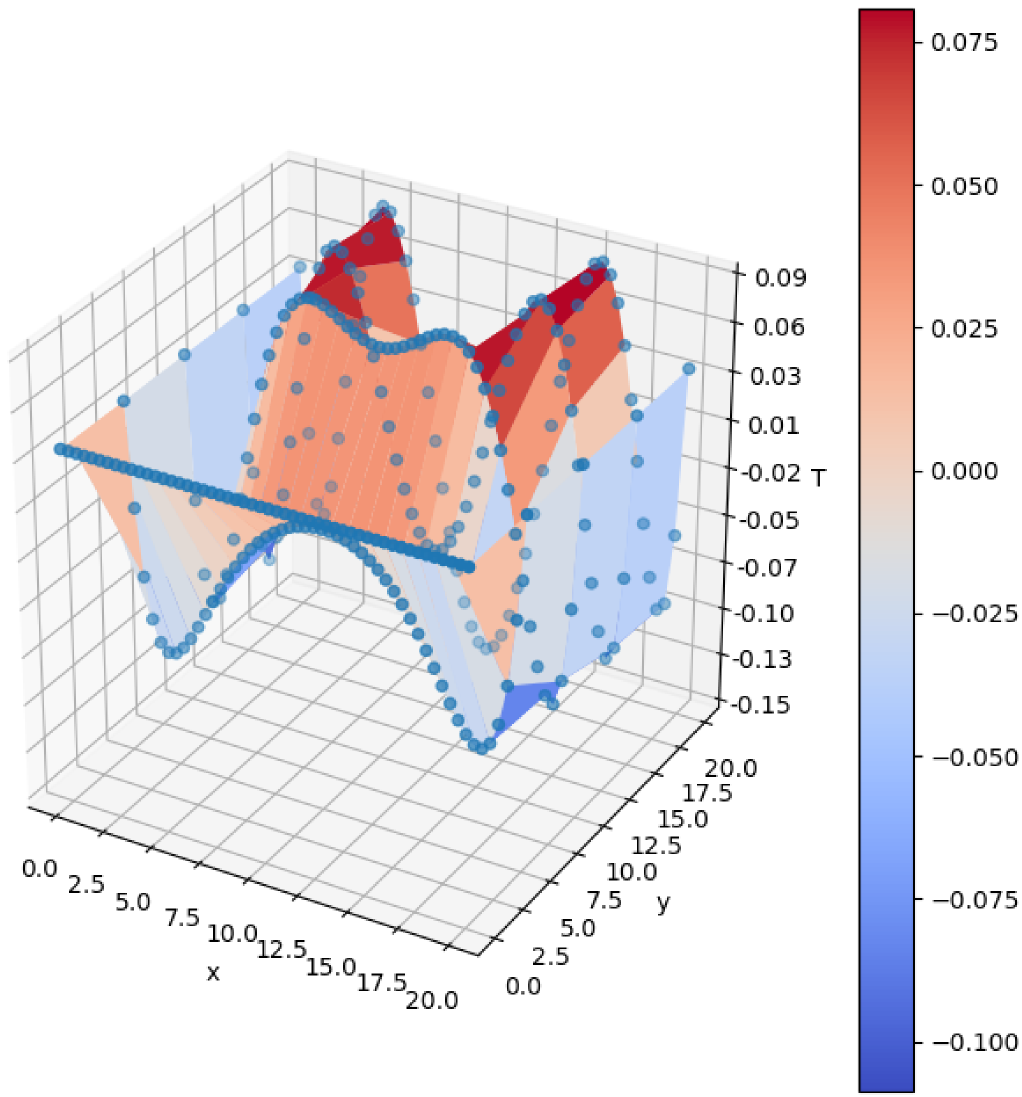

-0.06
-0.06
-0.06
-0.06
-0.06
-0.06
-0.06
-0.06
-0.06
-0.07
T
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
x
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
y
−0.0610
−0.0615
−0.0620
−0.0625
−0.0630
−0.0635
−0.0640
−0.0645
−0.0650



0.08
0.06
0.04
0.02
-0.00
-0.02
-0.04
-0.06
-0.08
-0.11
T
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
x
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
y
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10



x
y
T
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
0.13
0.11
0.09
0.06
0.04
0.02
-0.01
-0.03
-0.05
-0.08
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075




**ECSW**

```
[ ]: tic_h_setup_b = time.time()
     tol = 1e-11
     xi, residual = ecsw_red(d, V_sel, d.Le, K_mus, q_mus, n_sel, N_snap,
       ↪mask,NL_solutions, tol=tol)
     toc_h_setup_b = time.time()
```

```
[ ]: print(f"this is the residual from fnnls: {residual}")
```

```
this is the residual from fnnls: 7.477797790548108e-07
```

```
[ ]: colors = ['red' if value > 0 else 'blue' for value in xi]
     sizes = [15 if value > 0 else 1 for value in xi]
```

```
[ ]: plot2D(np.arange(d.ncells[0]),np.arange(d.ncells[1]), xi, scattr=True, clr=colors, sz=sizes)
```


```
[ ]: print(f"Fraction of total elements active in the ROM: {len(xi[xi>0])*100/len(xi)}%")
```

```
Fraction of total elements active in the ROM: 3.0%
```

### 0.0.4 ROM Simulation

```
[ ]: params_rm = params[~np.isin(params,param_list)]
     param_rom = random.choice(params_rm)
```

```
[ ]: # Define the data-class

     d_test = probdata(bc, mat_layout, src_layout, fdict, nref, L, param_rom, pb_dim=2)
     FOS_test = Base_class_fem_heat_conduction(d_test,quad_deg)
     ROM = FEM_solver_rom_ecsw(d_test, quad_deg)
```

```
[ ]: # Initial guess
```

```
T_init_fos = np.zeros(FOS_test.n_nodes) + 2.
T_init_rom = np.transpose(V_sel)@T_init_fos # crucial to ensure the initial␣
↪guess is contained in the reduced subspace
```

[ ]:
```
# Time taken to perform a FO simulation with the current parameter value

tic_fos = time.time()
NL_solution_p_fos_test, _, _, _, = solve_fos(FOS_test,T_init_fos)
toc_fos = time.time()
```

```
initial residual = 11839.406876314039

iter 0, NL residual=1204.1482272617636, delta=9.710640238988155
iter 1, NL residual=8.954715747533228, delta=-0.7082907108304918
iter 2, NL residual=0.0005571970098416532, delta=-0.004825068882310571
iter 3, NL residual=5.400782938625764e-11, delta=-2.3643937605583957e-07
Convergence !!!
```

[ ]:
```
# Time taken to simulate a ROM without hyper-reduction

tic_rom_woh = time.time()
NL_solution_p_reduced_woh = ROM.solve_rom(T_init_rom,np.ones_like(xi),V_sel)
toc_rom_woh = time.time()
```

```
initial residual = 3093.588819067262

iter 0, NL residual=5.025653611012314, delta=-2.7721799682761377e-05
```

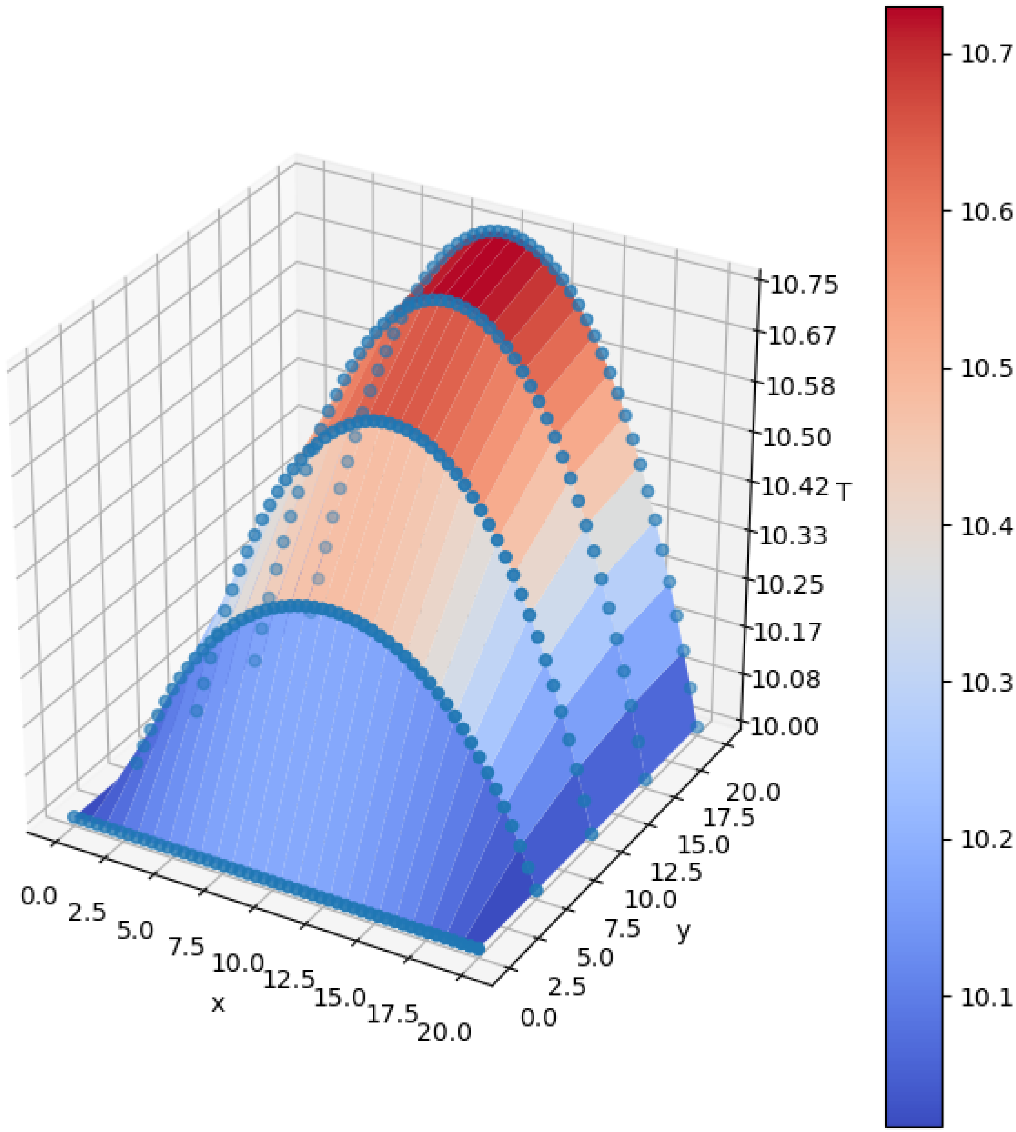
```
c:\Users\supar\anaconda3\lib\site-
packages\scipy\sparse\linalg\_dsolve\linsolve.py:229: SparseEfficiencyWarning:
spsolve requires A be CSC or CSR matrix format
  warn('spsolve requires A be CSC or CSR matrix format',
```

```
iter 1, NL residual=0.00818001715848066, delta=1.1170040801051897
iter 2, NL residual=2.4726416060799482e-08, delta=0.0012758049890004186
Convergence !!!
```

[ ]:
```
# Time taken to simulate a ROM *with* hyper-reduction

tic_rom = time.time()
NL_solution_p_reduced = ROM.solve_rom(T_init_rom,xi,V_sel)
toc_rom = time.time()
```

```
initial residual = 0.7554641684911987

iter 0, NL residual=0.0009875092084944347, delta=-9.958626647212375e-05
iter 1, NL residual=2.8966893628913547e-06, delta=1.1172459578712572
Convergence !!!
```

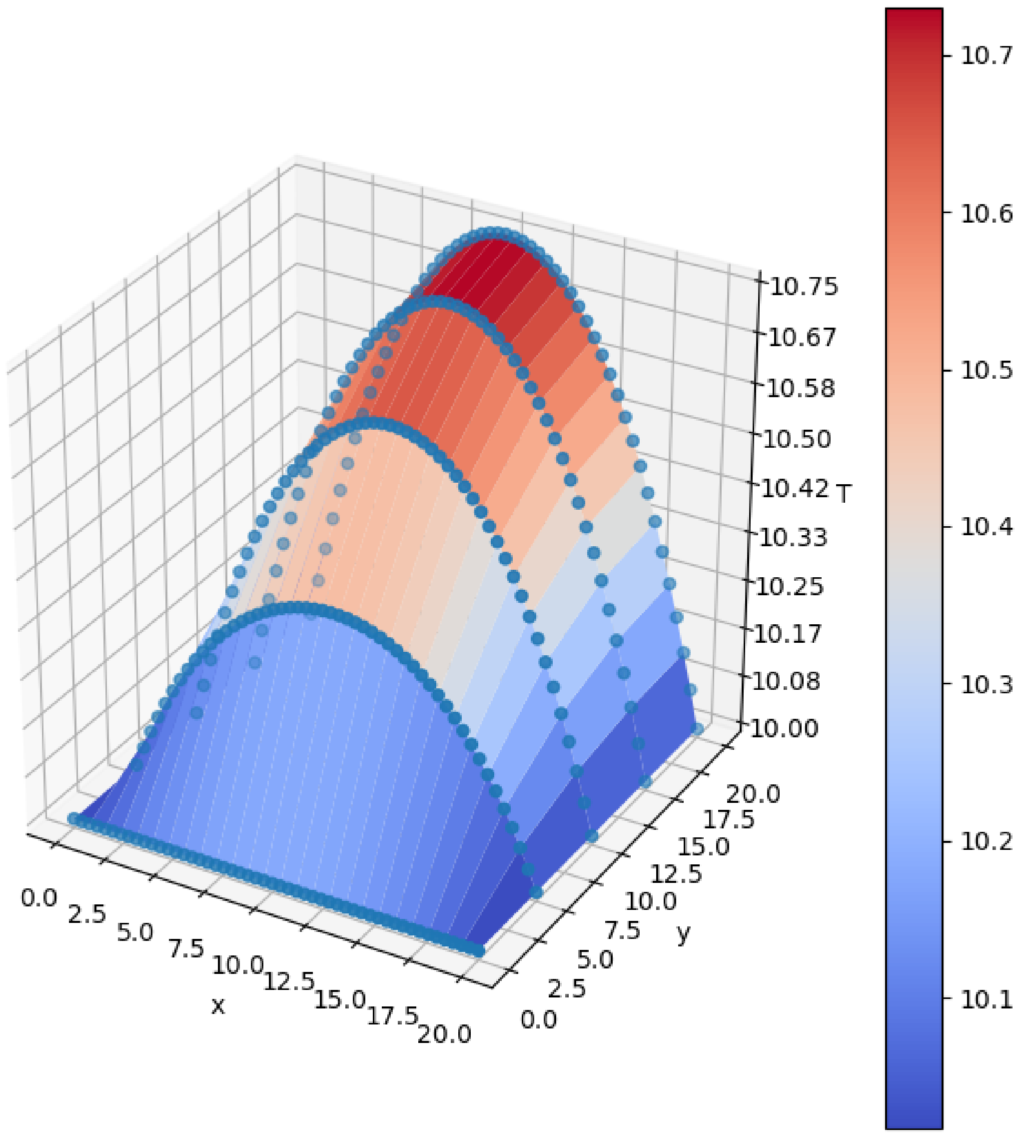
[ ]:
```
sol_red = V_sel@NL_solution_p_reduced.reshape(-1,1)  #+pca.mean_.reshape(-1,1)
plot2D(d_test.xi[0], d_test.xi[1], sol_red)

print(f"RMS_error is {np.linalg.norm(sol_red-NL_solution_p_fos_test.
↪reshape(-1,1))*100/np.linalg.norm(NL_solution_p_fos_test.reshape(-1,1))} %")
```




```
RMS_error is 0.0010584873085830936 %
```

[ ]:
```
plot2D(d_test.xi[0], d_test.xi[1], NL_solution_p_fos_test)
```

```
print(f"\n\nROM Error without hyperreduction is {np.linalg.
↪norm(V_sel@NL_solution_p_reduced_woh.reshape(-1,1)-NL_solution_p_fos_test.
↪reshape(-1,1))*100/np.linalg.norm(NL_solution_p_fos_test.reshape(-1,1))} %")
```




```
ROM Error without hyperreduction is 2.3253909682701527e-09 %
```

**Speedups**

[ ]:
```
fos_sim_time = toc_fos - tic_fos
rom_sim_time_woh = toc_rom_woh - tic_rom_woh
rom_sim_time = toc_rom - tic_rom
```

[ ]:
```python
print(f"speedup without hyperreduction:{fos_sim_time/rom_sim_time_woh}")
print(f"speedup with hyperreduction:{fos_sim_time/(rom_sim_time)}")
# h_total_setup_time = (toc_h_setup_b+toc_h_setup_a) -
↪(tic_h_setup_b+tic_h_setup_a) #this is one time
```

```
speedup without hyperreduction:1.269505461880573
speedup with hyperreduction:38.030205349612096
```