<u>**Unit-I**</u>

Inequalities on LMVT

Ex.1 : Prove that (if $0 < a < b < 1$) , $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$

and hence show that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$

Ex.2.: Prove that , $1 - \frac{a}{b} < \log\frac{a}{b} < \frac{b}{a} - 1$ and hence show that $\frac{1}{6} < \log\frac{6}{5} < \frac{1}{5}$

Ex.3 : If $f(x) = \sin^{-1} x, 0 < a < b < 1$ then use mean value theorem to prove that

$\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}$. Hence show that $\frac{\pi}{6} - \frac{1}{2\sqrt{3}} < \sin^{-1}\frac{1}{4} < \frac{\pi}{6} - \frac{1}{\sqrt{15}}$

Taylor's series

Ex.1 : Expand using Taylor's theorem, $(x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5$ in powers of x.

Ex.2 : By Taylor's series expand $7 + (x+2) + 3(x+2)^3 + (x+2)^4$ in ascending power of x.

Ex.3 : Expand $x^3 + 7x^2 + x - 6$ in power of $(x-3)$.

Ex.4.: Using Taylor's theorem expand $2x^3 + 3x^2 - 8x + 7$ in powers of $(x-2)$.

Ex.5 : Use Taylor's theorem express $3x^3 - 2x^2 + x - 4$ in powers of $(x+2)$.

<u>**Unit-II**</u>

Directionl derivative

Ex.1 : Find directional derivative of $\phi = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of

vector $2\bar{i} - \bar{j} - 2\bar{k}$.

Ex.2 : Find the directional derivative of $xy^2 + yz^3$ at $(2, -1, 1)$ along the line $2(x-2) = y+1 = z-1$

Ex.3 : Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2z^2x$ at the point $(1, 1, 1)$ in the direction

of the line. $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$

Ex.4 : Find directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at point $(2, -1, 2)$ along a line equally inclined

with co-ordinate axes

Ex.5 : Find the directional derivative of the function $\phi = e^{2x-y-z}$ at $(1,1,1)$ in the direction of the tangent to the curve $x = e^{-t}$, $y = 2\sin t + 1$, $z = t - \cos t$ at $t = 0$.

Vector Integration

Ex.1: Evaluate $\iint_S curl\ \bar{F}\cdot\hat{n}\,ds$ for $\bar{F} = (2x - y)\,\bar{i} - y z^2\,\bar{j} - y^2 z\,\bar{k}$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = 1$ above the XY plane.

Ex.2: Evaluate $\iint_S (\nabla \times \bar{F})\cdot\hat{n}\,ds$ where s is the curved surface of the parabolid : $x^2 + y^2 = 2z$ bounded by the plane $z = 2$ where $\bar{F} = 3(x - y)\,\bar{i} + 2xz\,\bar{j} + xy\,\bar{k}$

Ex.3: Evaluate $\iint curl\ \bar{F}\cdot\hat{n}\,ds$ for the surface of paraboloid $z = 9 - (x^2 + y^2)$ above the plane $z = 0$ and $\bar{F} = (x^2 + y - 4)\,\bar{i} + 3xy\,\bar{j} + (2xz + z^2)\,\bar{k}$

Ex.4: Evaluate $\int_c \bar{F}\cdot d\bar{r}$ by using Stoke's theorem for $\bar{F} = -4y\,\bar{i} + 4x\,\bar{j} + 3\,\bar{k}$. where, S is a disk of radius 1 lying on the plane $z = 1$ and C is its boundary.

Unit-III

Method of variation of parameters

Solve following differential equations by Method of variation of parameters

1) $\dfrac{d^2y}{dx^2} + y = \operatorname{cosec} x$

2) $(D^2 - 4D + 4)y = \dfrac{e^{2x}}{x^2}$

3) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = e^{2x}\sec^2 x$

4) $(D^2 + 3D + 2)y = \sin e^x$

Solve following differential Equations

1) $y^{-3}\dfrac{dy}{dx} - x\,y^{-2} = x^3$

2) $e^y\dfrac{dy}{dx} + e^x e^y = -e^{2x}$

3) $\sec y \tan y\dfrac{dy}{dx} - \sec y = -x$

4) $\dfrac{dy}{dx} - xy = -y^3 e^{-x^2}$