

Q1

1) a) Total = 20 + 5 = 25

ways to select all teachers = 5C4

= 5

ways to form 4 people in group = 25C4

= 12650

probability = $\frac{5}{12650} = \frac{1}{2530} = 0.0003953$

b) ways to select 2 teachers = 5C2

= 10

ways to select 2 parents = 20C2

= 190

probability = $\frac{10 \times 190}{12650} = \frac{38}{253} = 0.1502$

2) Let $A \cap B$ = Kamal will live on campus and buy a new car = 0.37

Let B = Kamal will live on campus = 0.73

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.37}{0.73} = 0.5068$$

3) a) $E = \{(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5), (2,2), (2,4), (2,6), (4,2), (4,4), (4,6),$

b) probability that sum = 9 : $\{(3,6), (4,5), (5,4), (6,3)\} = 4$

probability of all possible outcome : $6 \times 6 = 36$

probability = $\frac{4}{36} = \frac{1}{9} = 0.1111$

c) probability that sum = 7 : $\{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\} = 6$

probability that sum = 8 : $\{(2,6), (6,2), (3,5), (5,3), (4,4)\} = 5$

probability = $\frac{5+6}{36} = 0.3056$

Q2

$$a) P(A) = \text{The computer was purchased from Aime} = \frac{55}{100} = 0.55$$

$$P(D) = \text{The computer was purchased from DatCom} = \frac{16}{100} = 0.16$$

$$P(N) = \text{The computer was purchased from Nuclear} = \frac{35}{100} = 0.35$$

$$b) P(B) = \text{The computer was defective}$$

$$P(B|A) = \frac{1}{100} = 0.01$$

$$P(B) = (0.55)(0.01) + (0.16)(0.03) + (0.35)(0.03) \\ = 0.019$$

$$P(B|D) = \frac{3}{100} = 0.03$$

$$P(B|N) = \frac{3}{100} = 0.03$$

$$2) a) \text{ Let } I = \text{Event of Covid-19 active infection}$$

$$b) P(I'|T) = \frac{P(T|I')P(I')}{P(T)}$$

Let $T = \text{Event of positive RT-PCR test}$

$$P(T)$$

$$= (0.02)(0.85)$$

$$P(I) = 0.15$$

$$0.1595$$

$$P(T|I) = 0.95$$

$$= 0.1066$$

$$P(T|I') = 0.02$$

$$P(I') = 1 - 0.15 = 0.85$$

$$P(I|T) = \frac{P(T|I)P(I)}{P(T)}$$

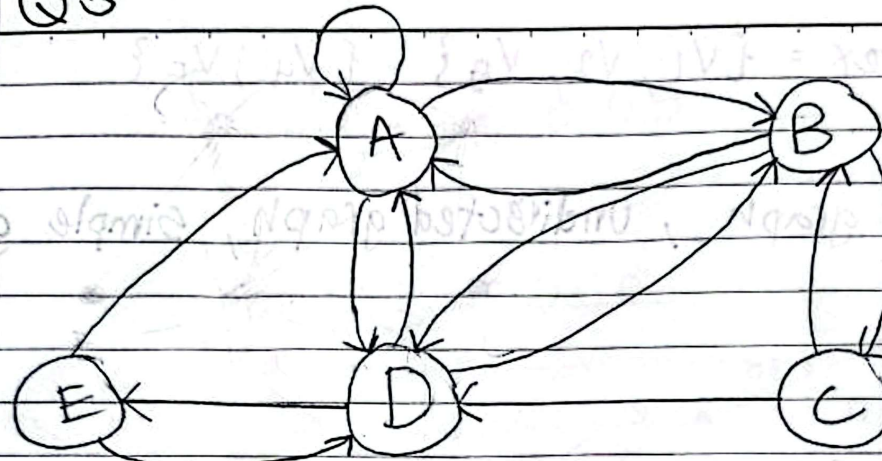
$$P(T) = P(T|I)P(I) + P(T|I')P(I')$$

$$= (0.95)(0.15) + (0.02)(0.85)$$

$$= 0.1595$$

$$P(I|T) = \frac{(0.95)(0.15)}{0.1595} = 0.8934$$

Q3

1.
d)

b)

In-degree

A

B

C

D

E

4

3

1

4

1

out-degree

3

3

2

3

2

total degree

7

6

3

7

3

2)

a) Vertex: is a point where two or more line segment meet. The objects that are drawn as dots.

b) Edge: one of the connections between the vertices

c) Loop: is an edge that connects a vertex to itself

d) Parallel edges: is two or more edges that are incident to the same two vertices.

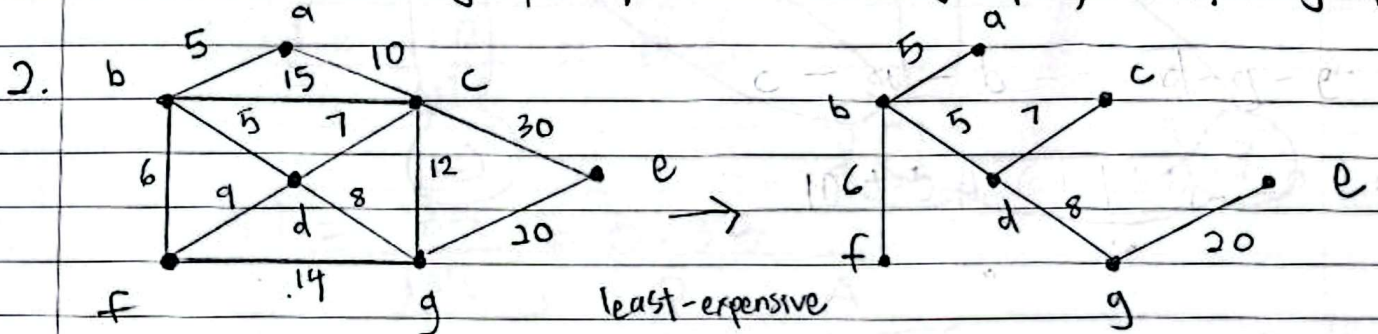
e) Degree of the vertex: is the number of edges that are incident to the vertex.

Q4

1. disjoint vertex = $\{V_1, V_2, V_5\}$, $\{V_3, V_4\}$

a.

b. connected graph, undirected graph, simple graph



$$\text{least expensive} = 5 + 5 + 6 + 7 + 8 + 20 = 51$$

Q5

1. Graph G_1 and G_2 have the same number of vertices and edges. Beside that, graph G_1 and G_2 have same number of degree for corresponding vertices.

$$A_{G_1} = \begin{matrix} & \begin{matrix} a & b & c & d & e & f & g \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \\ f \\ g \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

Q5		G_1	G_2	$f(a_{G_1}) = 1_{G_2}$	$f(e_{G_1}) = 2_{G_2}$
vertices		7	7	$f(b_{G_1}) = 3_{G_2}$	$f(f_{G_1}) = 4_{G_2}$
edges		7	7	$f(c_{G_1}) = 5_{G_2}$	$f(g_{G_1}) = 6_{G_2}$
vertices with 2 degree		7	7	$f(d_{G_1}) = 7_{G_2}$	

$$A_{G_1} = A_{G_2}$$

\therefore So they are isomorphic

$$A_{G_2} = \begin{matrix} & \begin{matrix} 1 & 3 & 5 & 7 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

a) a)

	G_1	G_2
vertices	6	6
edges	9	9
vertices with 3 degree	6	6

$f(A_{G_1}) = 1_{G_1}$

$f(E_{G_1}) = 5_{G_1}$

$f(B_{G_1}) = 2_{G_1}$

$f(F_{G_1}) = 6_{G_1}$

$f(C_{G_1}) = 3_{G_1}$

$f(D_{G_1}) = 4_{G_1}$

	a	b	c	d	e	f
a	0	1	1	0	1	0
b	1	0	1	0	0	1
c	1	1	0	1	0	0
d	0	0	1	0	1	1
e	1	0	0	1	0	1
f	0	1	0	1	1	0

	1	2	3	4	5	6
1	0	1	0	1	1	0
2	1	0	1	0	0	1
3	0	1	0	1	1	0
4	1	0	1	0	0	1
5	1	0	1	0	0	1
6	0	1	0	1	1	0

However, adjacent matrix of G_1 are not equal to G_2 . $f: G_1 \rightarrow G_2$ cannot be defined

it is not isomorphic

adjacent matrix G_1 adjacent matrix G_2 G_1 also has a simple cycle of length 3 but G_2 does not have. For example $a-b-c-a$

b)

	G_1	G_2
vertices	6	6
edges	11	11
vertices with 3 degree	2	2
vertices with 4 degree	4	4

$f(A_{G_1}) = 1_{G_2}$

$f(E_{G_1}) = 5_{G_2}$

$f(B_{G_1}) = 2_{G_2}$

$f(F_{G_1}) = 6_{G_2}$

$f(C_{G_1}) = 3_{G_2}$

$f(D_{G_1}) = 4_{G_2}$

	a	b	c	d	e	f
a	0	1	1	1	1	0
b	1	0	1	0	1	1
c	1	1	0	0	0	1
d	1	0	0	0	1	1
e	1	1	0	1	0	1
f	0	1	1	1	1	0

	1	2	3	4	5	6
1	0	1	1	1	0	0
2	1	0	1	0	1	1
3	1	1	0	0	1	1
4	1	0	0	0	1	1
5	0	1	1	1	0	1
6	0	1	1	1	1	0

The degree of vertex d in G_1 has 4 degree but degree of vertex 1 in G_2 has only 3 degree.

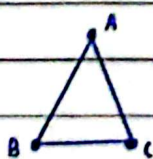
However, adjacent matrix of

 G_1 are not equal to G_2 $f: G_1 \rightarrow G_2$ cannot be defined

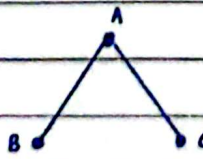
it is not isomorphic

adjacent matrix G_1 adjacent matrix G_2

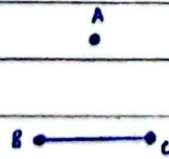
3)



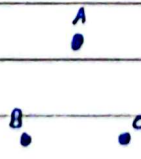
①



②



③



④