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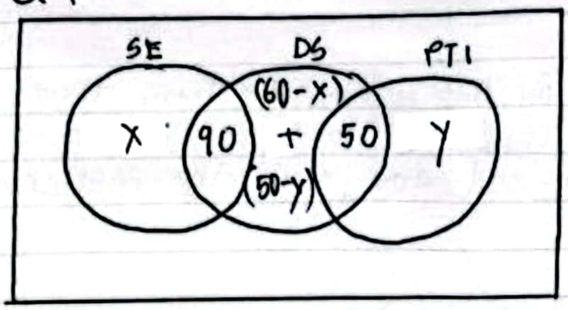
Q1

a.

i)



ii)

~~150 - 90 = 60~~
~~100 - 50 = 50~~ →

~~assume 0 student take DS~~
~~90 + 50 = 140~~
~~(minimum)~~

~~assume 110 student take DS~~
~~140 + 110 = 250~~
~~(maximum)~~

~~∴ 140 ≤ x ≤ 250~~

Student take DS :

$90 + 50 + (60-x) + (50-y)$
$= 250 - x - y$

$\therefore (250 - x - y)$

iii) $90 + 50 = 140$

$\therefore 140$

iv) $(150 - 90) + (100 - 50) = 60 + 50$
$= 110$

$\therefore 110$

b. $A = \{3, 5, 7, 9, 11, 13, 15, 17, 19\}$
$B = \{2, 3, 5, 7, 11, 13, 17, 19\}$
$C = \{5\}$

i) $|A| = 9$
$|B| = 8$
$|C| = 1$

ii) $2^9 = 512$
$512 - 1 = 511$
∴ proper subsets of A is 511

iii) $C \times B = \{(5,2), (5,3), (5,5), (5,7), (5,11), (5,13), (5,17), (5,19)\}$

Q2 a i) Your play table tennis and you miss the midterm test

ii) $\neg(m \vee n) \vee o$

$(\neg m \wedge \neg n) \vee o$ — De Morgan's Law

if you do not play table tennis and don't miss the midterm test, or you pass the subject

b)

a	b	$a \rightarrow b$	$\neg a$	$\neg a \vee b$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$(a \rightarrow b) \equiv (\neg a \vee b)$

c) i) $x \rightarrow y$

ii) $\neg(x \vee z) \rightarrow \neg y$

iii) $y \leftrightarrow (x \wedge z)$

Q3 a) i) $P(5)$: $15/5 = 3$

True, because 15 is divisible by 5

ii) $\forall n P(n)$: False, not all positive integers can divides 15 when $n=2$, 15 is not divisible by 2

iii) $\exists n \neg P(n)$: True, There exists positive integer n which cannot divides 15 for example 2

b) $\exists m \exists n P(m,n)$: True, there exist positive integers m and n where $m \geq n$. For example when $m=5$ and $n=1$.

negation : $\forall m \forall n \neg P(m,n)$: For all positive integers m and n, m is not greater or equal to n

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Q4 direct proof

let $x=1$, $y=2$

$a=\frac{3}{2}=1.5$

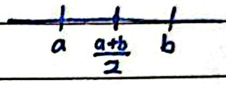
$x<a<y$

$1<\frac{3}{2}<2$ (True)

or

method 2

assume a and b are integers, then $a<b$



$a+a<a+b$

$2a<a+b$

$a<\frac{a+b}{2}$

$a+b<b+b$

$a+b<2b$

$\frac{a+b}{2}<b$

Thus

$a<\frac{a+b}{2}<b$

(prove)

indirect proof

If x and y are real numbers with $x>y$, there exists a rational number a satisfying $x>a>y$

let $x=2$

$y=1$

$a=\frac{3}{2}=1.5$

$x>a>y$

$2>\frac{3}{2}>1$ (True)

contradiction proof

if x and y are real numbers with $x<y$, there exists a rational number a satisfying $x>a>y$

let $x=1$

$y=2$

$a=\frac{3}{2}$

$x>a>y$

$1>\frac{3}{2}>2$ (True)

∴ Thus we conclude that the statement is true

Q5

a. $(m, n) \in R$ if 2 divides $m - n$

2 devides $(1 - 5)$ ✓ $-4 \div 2 = -2$
2 devides $(1 - 4)$
2 devides $(1 - 3)$ ✓ $-2 \div 2 = -1$
2 devides $(1 - 2)$
2 devides $(1 - 1)$

2 devides $(2 - 5)$
2 devides $(2 - 4)$ ✓ $-2 \div 2 = -1$
2 devides $(2 - 3)$
2 devides $(2 - 2)$
2 devides $(2 - 1)$

2 divides (3 − 5) ✓ $-2 \div 2 = -1$
2 divides (3 − 4)
2 divides (3 − 3)
2 divides (3 − 2)
2 divides (3 − 1) ✓ $(2 \div 2 = 1)$

2 divides (4 − 5)
2 divides (4 − 4)
2 divides (4 − 3)
2 divides (4 − 2) ✓ $2 \div 2 = 1$
2 divides (4 − 1)

2 divides (5 − 5)
2 divides (5 − 4)
2 divides (5 − 3) ✓ $2 \div 2 = 1$
2 divides (5 − 2)
2 divides (5 − 1) ✓ $4 \div 2 = 2$

$R = \{(1,5), (1,3), (2,4), (3,5), (3,1), (4,2), (5,3), (5,1),$
$(1,1), (2,2), (3,3), (4,4), (5,5)\}$

$R^{-1} = \{(5,1), (3,1), (4,2), (5,3), (1,3), (2,4), (3,5), (1,5),$
$(1,1), (2,2), (3,3), (4,4), (5,5)\}$

b. $1+5 \le 4$ ✗ $2+5 \le 4$ ✗ $3+5 \le 4$ ✗
$1+4 \le 4$ ✗ $2+4 \le 4$ ✗ $3+4 \le 4$ ✗
$1+3 \le 4$ ✓ $2+3 \le 4$ ✗ $3+3 \le 4$ ✗
$1+2 \le 4$ ✓ $2+2 \le 4$ ✓ $3+2 \le 4$ ✗
$1+1 \le 4$ ✓ $2+1 \le 4$ ✓ $3+1 \le 4$ ✓

$4+5 \le 4$ ✗ $5+5 \le 4$ ✗
$4+4 \le 4$ ✗ $5+4 \le 4$ ✗
$4+3 \le 4$ ✗ $5+3 \le 4$ ✗
$4+2 \le 4$ ✗ $5+2 \le 4$ ✗
$4+1 \le 4$ ✗ $5+1 \le 4$ ✗

$R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$

c. both of them are symmetric
$\forall x, y \in X, (x,y) \in R \rightarrow (y,x) \in R$

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Q5

d.

i. $(x,y) \in R$ if $xy = 1$

$R = \{(1,1)\}$

reflexive ✗
symmetric ✓
antisymmetric ✗
transitive ✓
partial order ✗

$\therefore$ symmetric, transitive

ii. $(x,y) \in R$ if $x = y^2$

$R = \{(1,1), (4,2), (9,3)\}$

reflexive ✗
symmetric ✗
antisymmetric ✓
transitive ✗
partial order ✗

$(x,y) \in R \wedge x \neq y \rightarrow (y,x) \notin R$

$\therefore$ antisymmetric

iii. $(x,y) \in R$ if $x = y$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$

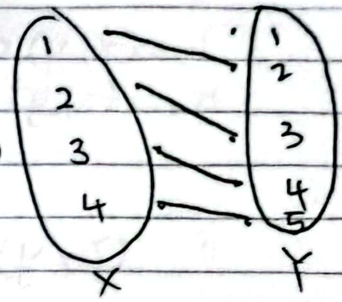
reflexive ✓
symmetric ✓
antisymmetric ✗
transitive ✓
partial order ✗

$\therefore$ reflexive, symmetric, transitive

Q6

a.

i) $f(n) = n+1$



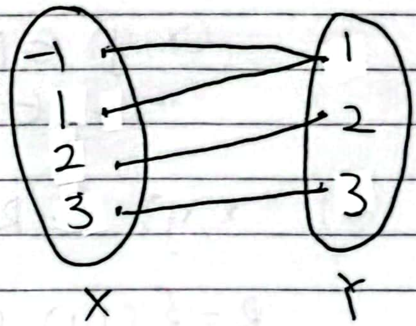
$f(n_1) = f(n_2)$

$n_1 + 1 = n_2 + 1 \quad (-1)$

$n_1 = n_2$

(One to one) each element at Y has at most one arow

ii) $f(n) = |n|$



$f(n_1) = f(n_2)$

$|n_1| = |n_2|$

$n_1 = n_2$

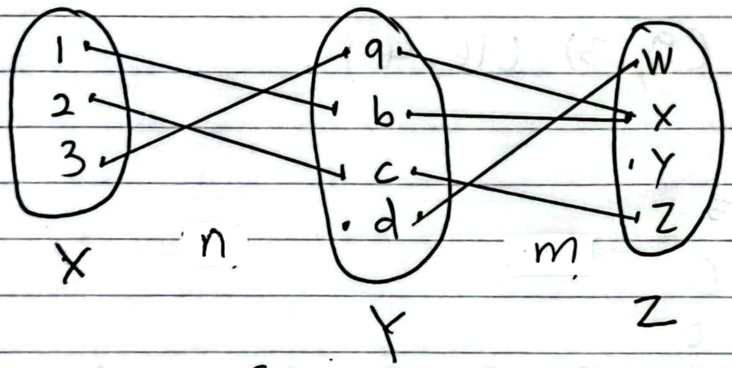
$|-3| = |3|$

$3 = 3$

(onto) each element at Y has at least one arrow

$f(-3) = f(3)$ not one to one

b.



$\therefore m \circ n = \{(1,x), (2,z), (3,x)\}$

c. $g = \{(1,a), (2,c), (3,c)\}$

$g(S) = a$

$g(T) = a, c$

$g^{-1}(U) = 1$

$g^{-1}(V) = 1, 2, 3$