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Q1

i) $a_n = 6a_{n-1} - 9a_{n-2}$ where $a_0 = 1, a_1 = 6$

$a_2 = 6a_1 - 9a_0$
$= 6(6) - 9(1)$
$= 27$

$a_3 = 6a_2 - 9a_1$
$= 6(27) - 9(6)$
$= 108$

$a_4 = 6a_3 - 9a_2$
$= 6(108) - 9(27)$
$= 405$

$a_5 = 6a_4 - 9a_3$
$= 6(405) - 9(108)$
$= 1458$

$a_6 = 6a_5 - 9a_4$
$= 6(1458) - 9(405)$
$= 5103$

27, 108, 405, 1458, 5103.....

ii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$
where $a_0 = 2, a_1 = 5, a_2 = 15$

$a_3 = 6a_2 - 11a_1 + 6a_0$
$= 6(15) - 11(5) + 6(2)$
$= 47$

$a_4 = 6a_3 - 11a_2 + 6a_1$
$= 6(47) - 11(15) + 6(5)$
$= 147$

$a_5 = 6a_4 - 11a_3 + 6a_2$
$= 6(147) - 11(47) + 6(15)$
$= 455$

$a_6 = 6a_5 - 11a_4 + 6a_3$
$= 6(455) - 11(147) + 6(47)$
$= 1395$

$a_7 = 6a_6 - 11a_5 + 6a_4$
$= 6(1395) - 11(455) + 6(147)$
$= 4247$

47, 147, 455, 1395, 4247.....

iii) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
where $a_0 = 1, a_1 = -2, a_2 = -1$

$a_3 = -3a_2 - 3a_1 + a_0$
$= -3(-1) - 3(-2) + 1$
$= 10$

$a_4 = -3a_3 - 3a_2 + a_1$
$= -3(10) - 3(-1) + (-2)$
$= -29$

$a_5 = -3a_4 - 3a_3 + a_2$
$= -3(-29) - 3(10) + (-1)$
$= 56$

$a_6 = -3a_5 - 3a_4 + a_3$
$= -3(56) - 3(-29) + 10$
$= -71$

$a_7 = -3a_6 - 3a_5 + a_4$
$= -3(-71) - 3(56) + (-29)$
$= 16$

10, -29, 56, -71, 16.....

2) $a_{n+1} = 5a_n - 3$ where $a_1 = k$

i) $a_2 = 5a_1 - 3$
$= 5(k) - 3$
$= 5k - 3$

$a_3 = 5a_2 - 3$
$= 5(5k-3) - 3$
$= 25k - 15 - 3$
$= 25k - 18$

$a_4 = 5a_3 - 3$
$= 5(25k - 18) - 3$
$= 125k - 90 - 3$
$= 125k - 93$

ii) $a_4 = 7$
$7 = 125k - 93$
$100 = 125k$
$k = 0.8$

Q2

1a) ways to arrange computer science books = 5 ways
ways to arrange mathematics books = 3 ways
ways to arrange art books = 2 ways

total ways = 5+3+2 = 10 ways

10! = 3628800

b) 5! x 3! x 2! x 3! = 8640

c) Z = 10 identical books
Y = 10 different books
total = 20 books

no of ways = (0Z x 10C10) + (1Z x 10C9) + (2Z x 10C8) + (3Z x 10C7)
+ (4Z x 10C6) + (5Z x 10C5) + (6Z x 10C4) + (7Z x 10C3)
+ (8Z x 10C2) + (9Z x 10C1) + (10Z x 10C0)
= 1024 ways

(as all Z are identical books so it will always be 1)

2a) 120 - 5 + 1 = 196 numbers

b) 200/5 = 40 numbers

c) one digit = {7} = 1

two digit = {17, 27, 37, 47, 57, 67, 77, 87, 97} = 9
= {70, 71, 72, 73, 74, 75, 76, 77, 78, 79} = 10 - 1 (as 77 already exists)
= 9

three digit = {107, 117, 127, 137, 147, 157, 167, 177, 187, 197} = 10
= {170, 171, 172, 173, 174, 175, 176, 177, 178, 179} = 10 - 1 (as 177 already exists)
= 9

total = 1 + 9 + 9 + 10 + 9
= 38 numbers

d) one digit = {5, 6, 7, 8, 9} = 5

two digit = {12, 13, 14, 15, 16, 17, 18, 19} = 8
{23, 24, 25, 26, 27, 28, 29} = 7
{34, 35, 36, 37, 38, 39} = 6
{45, 46, 47, 48, 49} = 5
{56, 57, 58, 59} = 4
{67, 68, 69} = 3
{78, 79} = 2
{89} = 1

three digit (excluded) = 101 - 10 - 10 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - = 28
{100, 101, 102, 103, 104, 105, 106, 107, 108, 109} = 10
{110, 111, 112, 113, 114, 115, 116, 117, 118, 119} = 10
{120, 121, 122} = 3
{130, 131, 132, 133} = 4
{140, 141, 142, 143, 144} = 5
{150, 151, 152, 153, 154, 155} = 6
{160, 161, 162, 163, 164, 165, 166} = 7
{170, 171, 172, 173, 174, 175, 176, 177} = 8
{180, 181, 182, 183, 184, 185, 186, 187, 188} = 9
{190, 191, 192, 193, 194, 195, 196, 197, 198, 199} = 10
{200} = 1

total numbers = 5 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 28 = 69 numbers

Q3

1 a) ACE , B , D = 3!
(ACE: single block)
= 6

b) contain AE = 4!
= 24

contain EA = 4!
= 24

AE or EA = 24 + 24
= 48

c) $MJ_1MJ_2MJ_3MJ_4MJ_5MJ_6MJ_7MJ_8M$

8! × 9P5 = 609638400

d) 10! (5 Jovians and 5 Martians)
= 3628800

2) $11C3 = \frac{11!}{3!8!} = 165$

3) specialty pizza = 4
0 topping = 1
1 topping = 17
2 topping = 17C2 = 136
3 topping = 17C3 = 680

total = 4 + 1 + 17 + 136 + 680
= 838

4) $4C3 = \frac{4!}{3!1!} = 4$

which is {a, b, c}, {a, c, d}, {a, b, d}
{b, c, d}

Q4

1) pigeonholes : 0 - 100
= 101 possible

as pigeons > pigeonholes

Thus pigeons > 101
= 102

so, at least 102 students are required in a class

2) pigeonholes = {A, B, C, D, F} = 5

maximum student per grade = 5 × 5
before reached 6 = 25

Thus adding 1 more student will cause 6 students get the same grade

At least 26 students are required

3) pigeons = 35
pigeonholes = 26

$\frac{35}{26} = 1.35 = 2$ (proved)

as pigeons > pigeonholes so that at least two of them have first names that start with the same letter.

4) first name = {Dennis, Evita, Ferdinand} = 3
last name = {Oh, Pietro, Quine, Rostenkowski} = 4
pigeonholes = 3 × 4 = 12
pigeons = 13 (shown)

as pigeons > pigeonholes so that at least 2 of them have the same first and last names