

Ice Caps

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```
library(tidyverse)
```

```
## — Attaching core tidyverse packages — tidyverse 2.0.0 —
## ✓ dplyr      1.1.3      ✓ readr      2.1.4
## ✓ forcats    1.0.0      ✓ stringr    1.5.0
## ✓ ggplot2    3.4.4      ✓ tibble     3.2.1
## ✓ lubridate  1.9.3      ✓ tidyr      1.3.0
## ✓ purrr      1.0.2
## — Conflicts — tidyverse_conflicts() —
## ✖ dplyr::filter() masks stats::filter()
## ✖ dplyr::lag()     masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(dplyr)
library(lubridate)
library(zoo)
```

```
##
## Attaching package: 'zoo'
##
## The following objects are masked from 'package:base':
##
##      as.Date, as.Date.numeric
```

```
library(ggplot2)
library(tseries)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method             from
##   as.zoo.data.frame zoo
```

```
library(forecast)
library(fpp)
```

```
## Loading required package: fma
## Loading required package: expsmooth
## Loading required package: lmtest
```

```
library(vars)
```

```
## Loading required package: MASS
##
## Attaching package: 'MASS'
##
## The following objects are masked from 'package:fma':
##
##      cement, housing, petrol
##
## The following object is masked from 'package:dplyr':
##
##      select
##
## Loading required package: strucchange
## Loading required package: sandwich
##
## Attaching package: 'strucchange'
##
## The following object is masked from 'package:stringr':
##
##      boundary
##
## Loading required package: urca
```

```
library(TSA)
```

```
## Registered S3 methods overwritten by 'TSA':  
##   method      from  
##   fitted.Arima forecast  
##   plot.Arima   forecast  
##  
## Attaching package: 'TSA'  
##  
## The following object is masked from 'package:readr':  
##  
##     spec  
##  
## The following objects are masked from 'package:stats':  
##  
##     acf, arima  
##  
## The following object is masked from 'package:utils':  
##  
##     tar
```

Combining and Preparing Data: North

```

icecaps_N <- data.frame()

# Loop through the 12 CSV files
for (i in 1:12) {
  if (i < 10){
    df <- read.csv(paste0("N_0", i, "_extent_v3.0.csv"))
  }
  if (i >= 10){
    df <- read.csv(paste0("N_", i, "_extent_v3.0.csv"))
  }
  icecaps_N <- rbind(icecaps_N, df)
}

# Combine the year and month columns into a new "date" column and convert to a year-month format
icecaps_N <- icecaps_N %>%
  mutate(date = ym(paste(year, str_pad(mo, 2, pad = "0"), sep = "-"))) %>%
  mutate(date = as.yearmon(date)) %>%
  dplyr::select(date, extent, area) %>% # Select only the date and extent columns
  arrange(date) %>% # Sort by the date column
  mutate(extent = replace(extent, extent == -9999.00, NA), # Replace -9999.00 with NA
         in the extent column
         area = replace(area, area == -9999.00, NA)) # Replace -9999.00 with NA in the area column

head(icecaps_N)

```

```

##      date extent  area
## 1 Nov 1978  11.65  9.04
## 2 Dec 1978  13.67 10.90
## 3 Jan 1979  15.41 12.41
## 4 Feb 1979  16.18 13.18
## 5 Mar 1979  16.34 13.21
## 6 Apr 1979  15.45 12.53

```

Combining and Preparing Data: South

```

icecaps_S <- data.frame()

# Loop through the 12 CSV files
for (i in 1:12) {
  if (i < 10){
    df <- read.csv(paste0("S_0", i, "_extent_v3.0.csv"))
  }
  if (i >= 10){
    df <- read.csv(paste0("S_", i, "_extent_v3.0.csv"))
  }
  icecaps_S <- rbind(icecaps_S, df)
}

# Combine the year and month columns into a new "date" column and convert to a year-month format
icecaps_S <- icecaps_S %>%
  mutate(date = ym(paste(year, str_pad(mo, 2, pad = "0"), sep = "-"))) %>%
  mutate(date = as.yearmon(date)) %>%
  dplyr::select(date, extent, area) %>% # Select only the date and extent columns
  arrange(date) %>% # Sort by the date column
  mutate(extent = replace(extent, extent == -9999.00, NA), # Replace -9999.00 with NA
         in the extent column
         area = replace(area, area == -9999.00, NA)) # Replace -9999.00 with NA in the area column

head(icecaps_S)

```

```

##      date extent  area
## 1 Nov 1978  15.90 11.69
## 2 Dec 1978  10.40  6.97
## 3 Jan 1979   5.40  3.47
## 4 Feb 1979   3.14  2.11
## 5 Mar 1979   4.00  2.66
## 6 Apr 1979   7.49  5.45

```

Handling Missing Data for North and South

```

# Function that imputates missing data by replacing missing values with the average of
# all values of the same month

fill_missing <- function(df) {
  for (i in 1:nrow(df)){
    if (is.na(df[i, "extent"])){ # if there is a missing value in extent column
      my_sum <- 0
      month_count <- 0
      df_month <- month(df[i,"date"]) # store the month of missing value
      for (j in 1:nrow(df)){
        if (i != j){
          # take the average of extents of all same months
          if (month(df[j,"date"]) == df_month){
            my_sum <- my_sum + (df[j,"extent"])
            month_count <- month_count + 1
          }
        }
      }
      # replace missing value with calculated average
      df[i, "extent"] <- (my_sum/month_count)
    }

    if (is.na(df[i, "area"])){ # if there is a missing value in area column
      my_sum <- 0
      month_count <- 0
      df_month <- month(df[i,"date"]) # store the month of missing value
      for (j in 1:nrow(df)){
        if (i != j){
          # take the average of areas of all same months
          if (month(df[j,"date"]) == df_month){
            my_sum <- my_sum + (df[j,"area"])
            month_count <- month_count + 1
          }
        }
      }
      # replace missing value with calculated average
      df[i, "area"] <- (my_sum/month_count)
    }
  }
  df
}

```

```

icecaps_N <- fill_missing(icecaps_N)
icecaps_S <- fill_missing(icecaps_S)

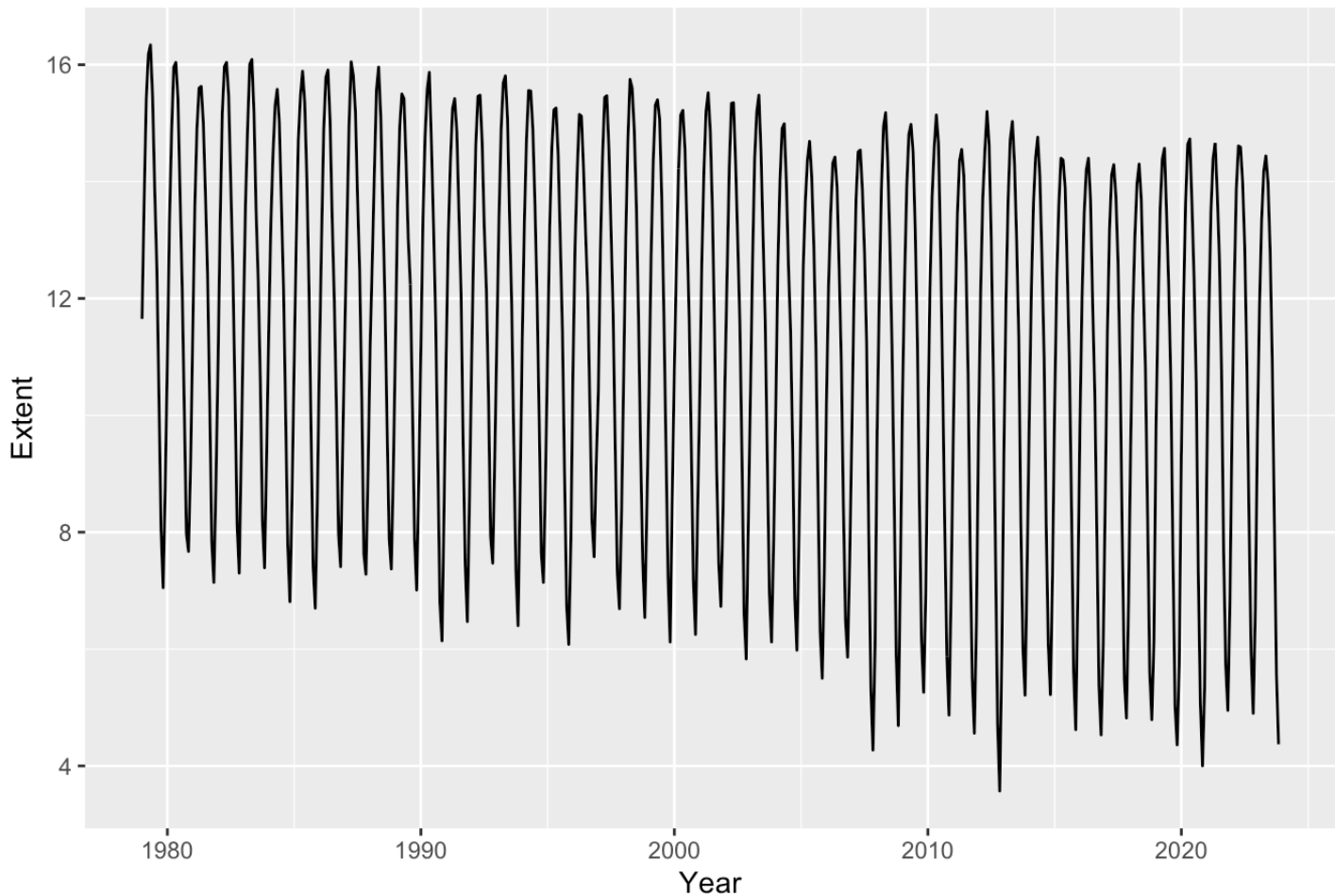
```

Plotting the Data

```
ts_extent_N <- ts(icecaps_N$extent, start=c(1979,1), frequency=12)
ts_area_N <- ts(icecaps_N$area, start=c(1979,1), frequency=12)
```

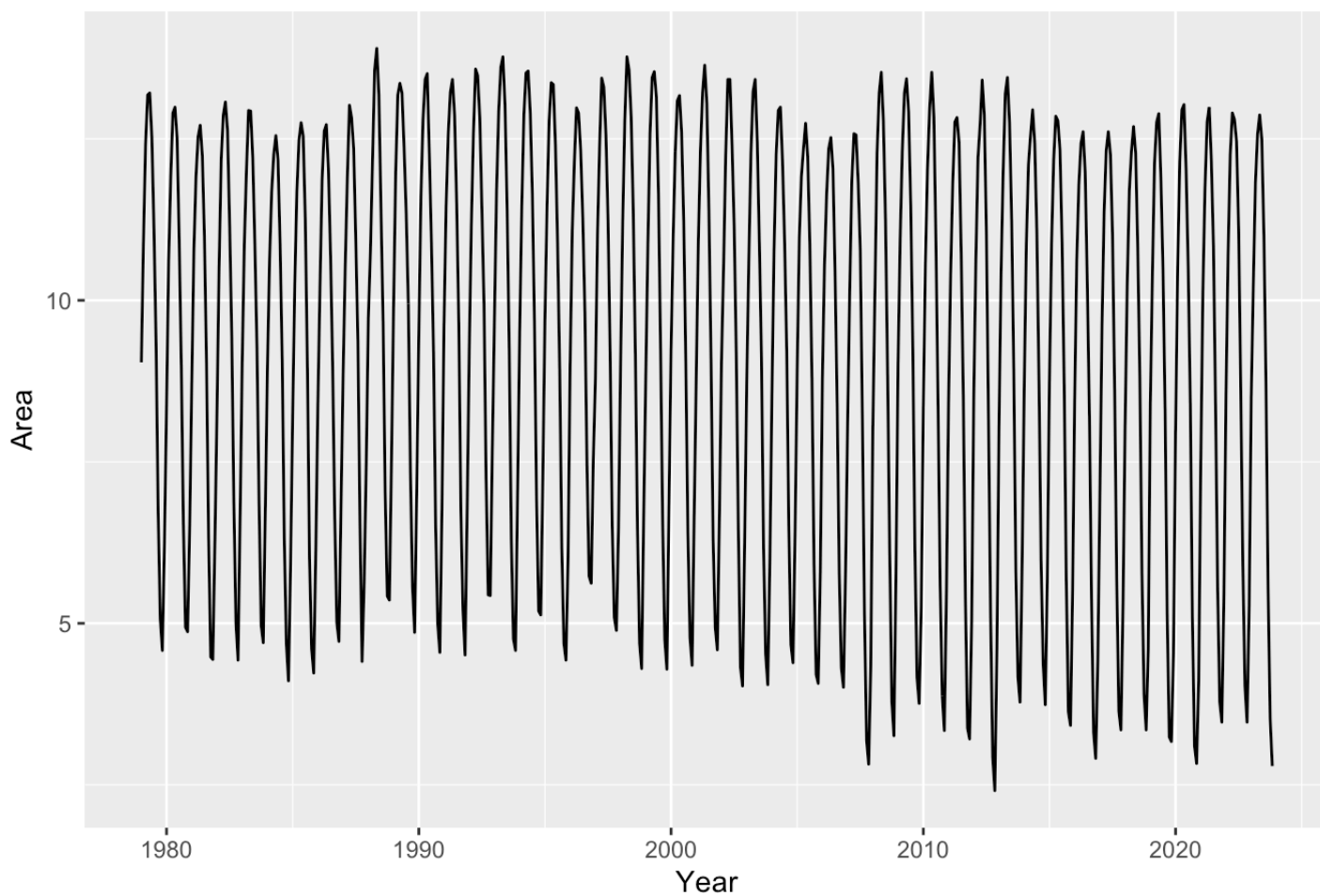
```
autoplot(ts_extent_N, xlab = "Year", ylab = "Extent", main = "Original Time Series Ex  
tent (North)")
```

Original Time Series Extent (North)



```
autoplot(ts_area_N, xlab = "Year", ylab = "Area", main = "Original Time Series Area (North)")
```

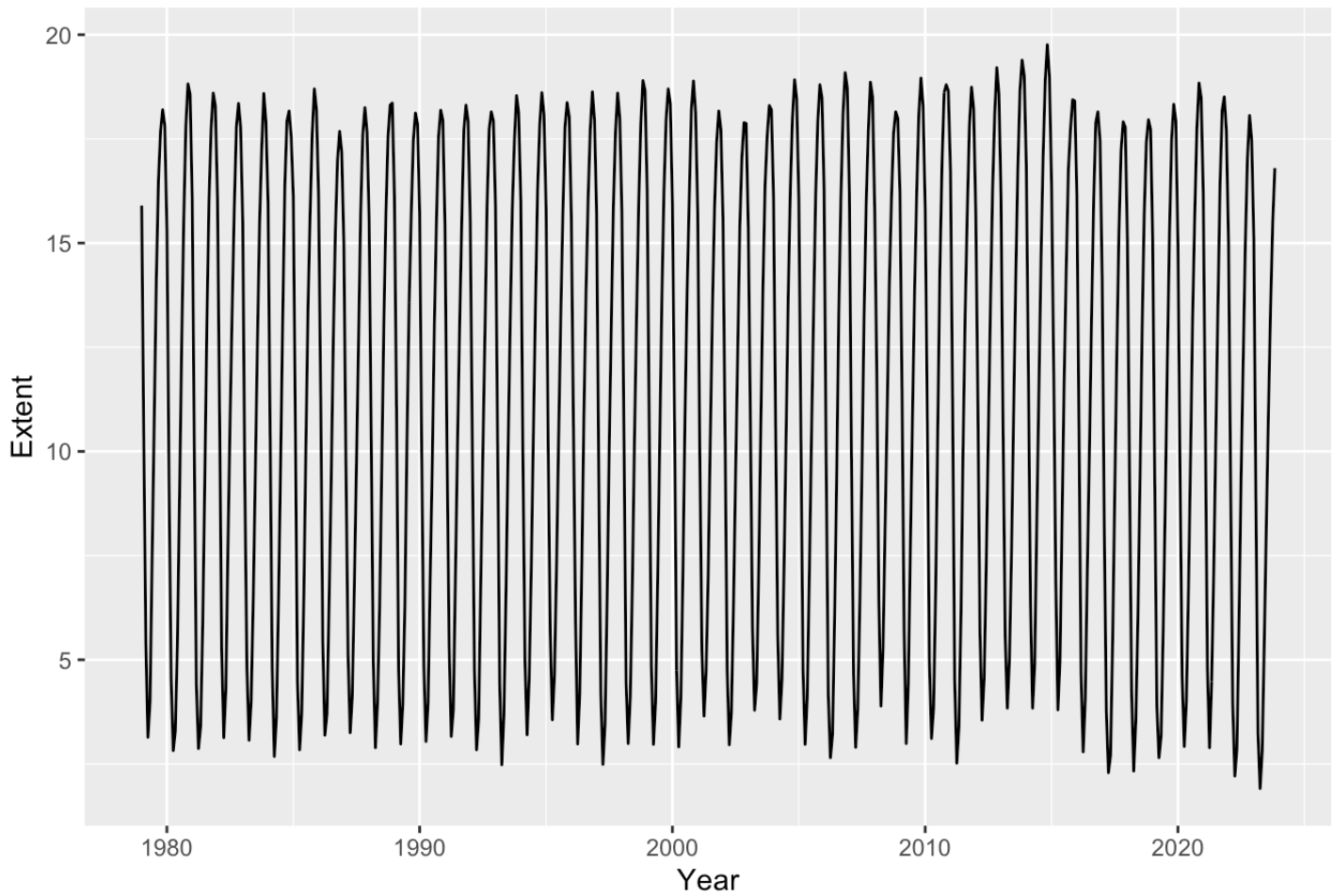
Original Time Series Area (North)



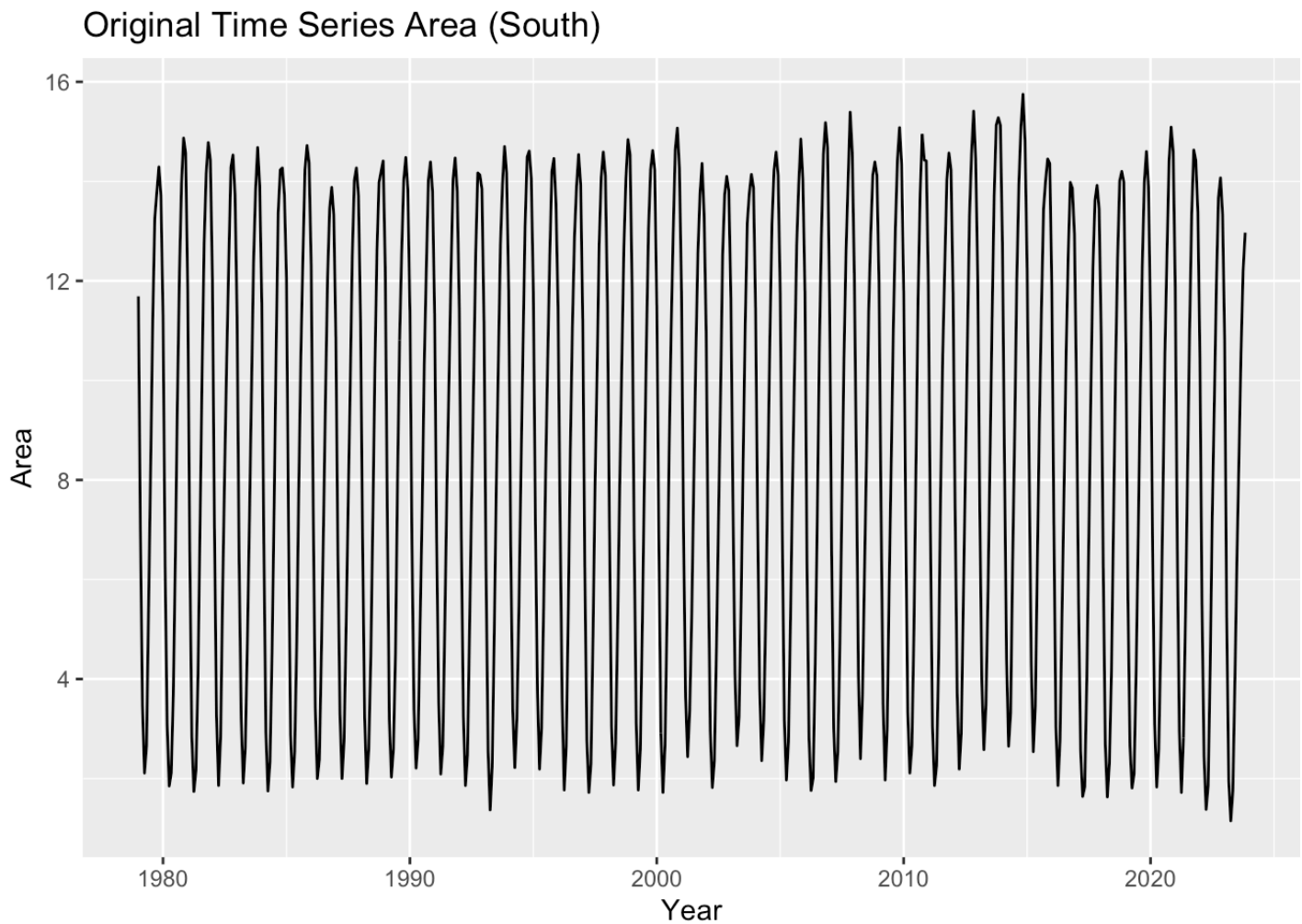
```
ts_extent_S <- ts(icecaps_S$extent, start=c(1979,1), frequency=12)
ts_area_S <- ts(icecaps_S$area, start=c(1979,1), frequency=12)

autoplot(ts_extent_S, xlab = "Year", ylab = "Extent", main = "Original Time Series Ex
tent (South)")
```


Original Time Series Extent (South)



```
autoplot(ts_area_S, xlab = "Year", ylab = "Area", main = "Original Time Series Area (South)")
```



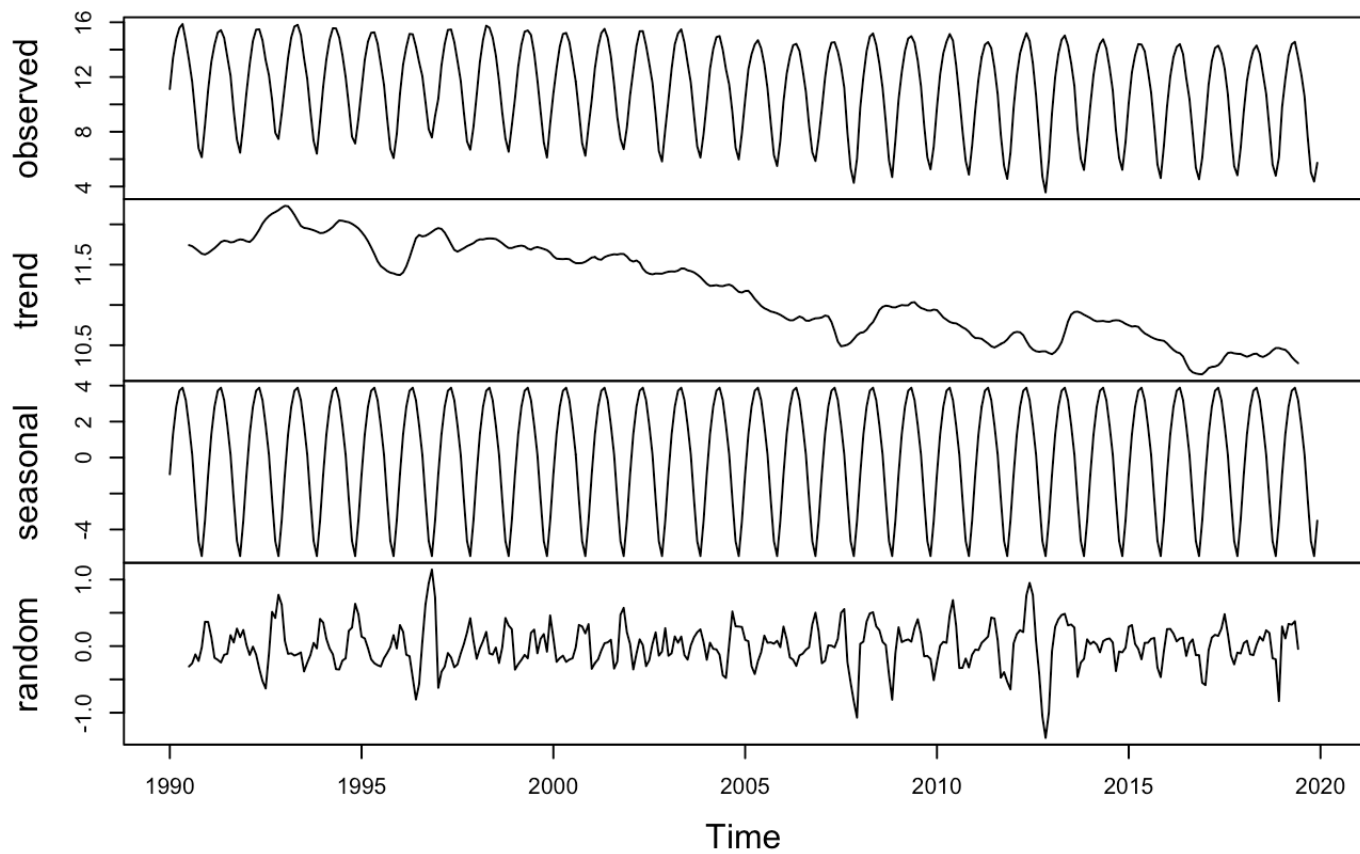
Getting train and test data

```
train_extent_N <- window(ts_extent_N, start = c(1990, 1), end = c(2019, 12))  
test_extent_N <- window(ts_extent_N, start = c(2020, 1))
```

```
train_area_N <- window(ts_area_N, start = c(1990, 1), end = c(2019, 12))  
test_area_N <- window(ts_area_N, start = c(2020, 1))
```

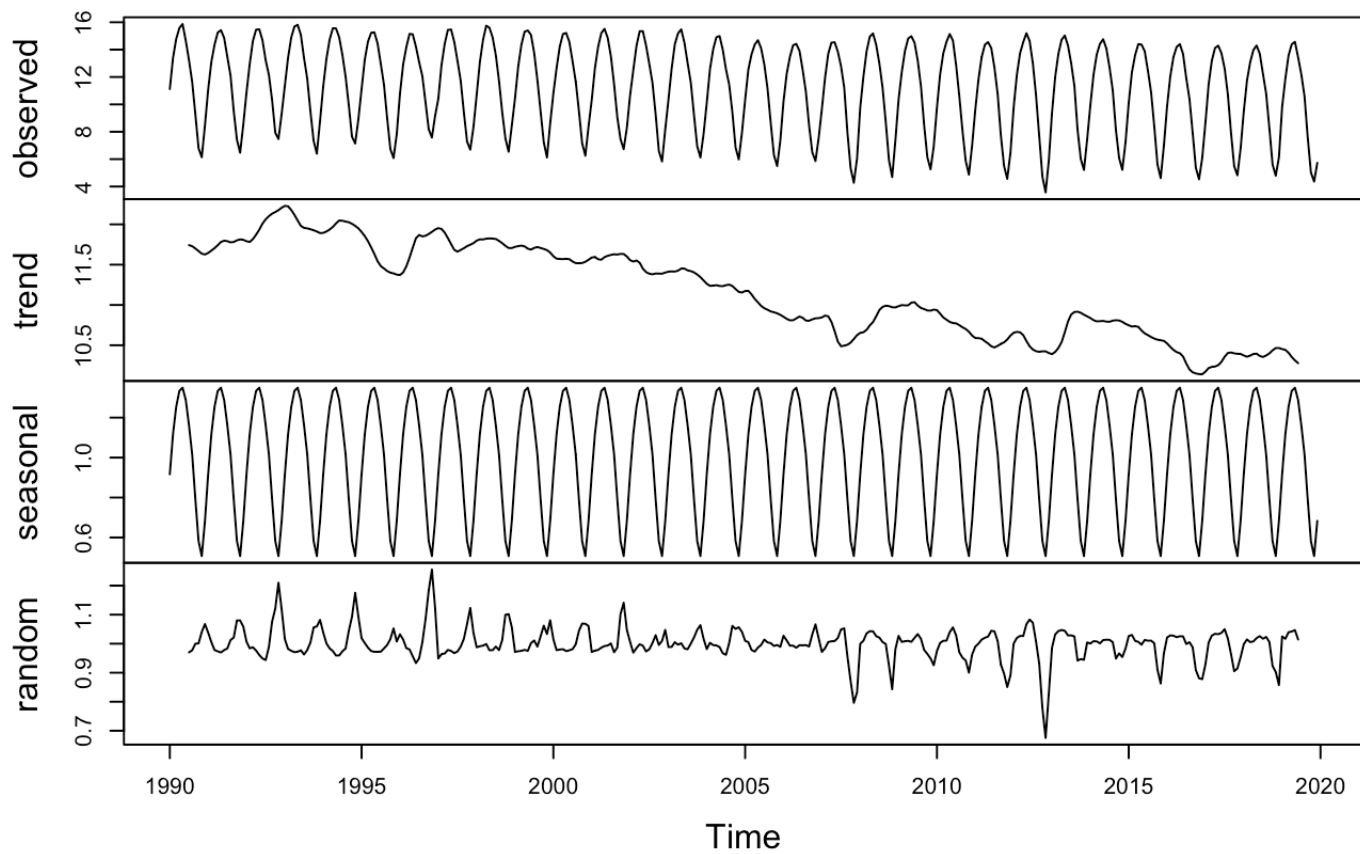
```
#exponential smoothing  
fit_add <- decompose(train_extent_N, type="additive")  
plot(fit_add)
```

Decomposition of additive time series



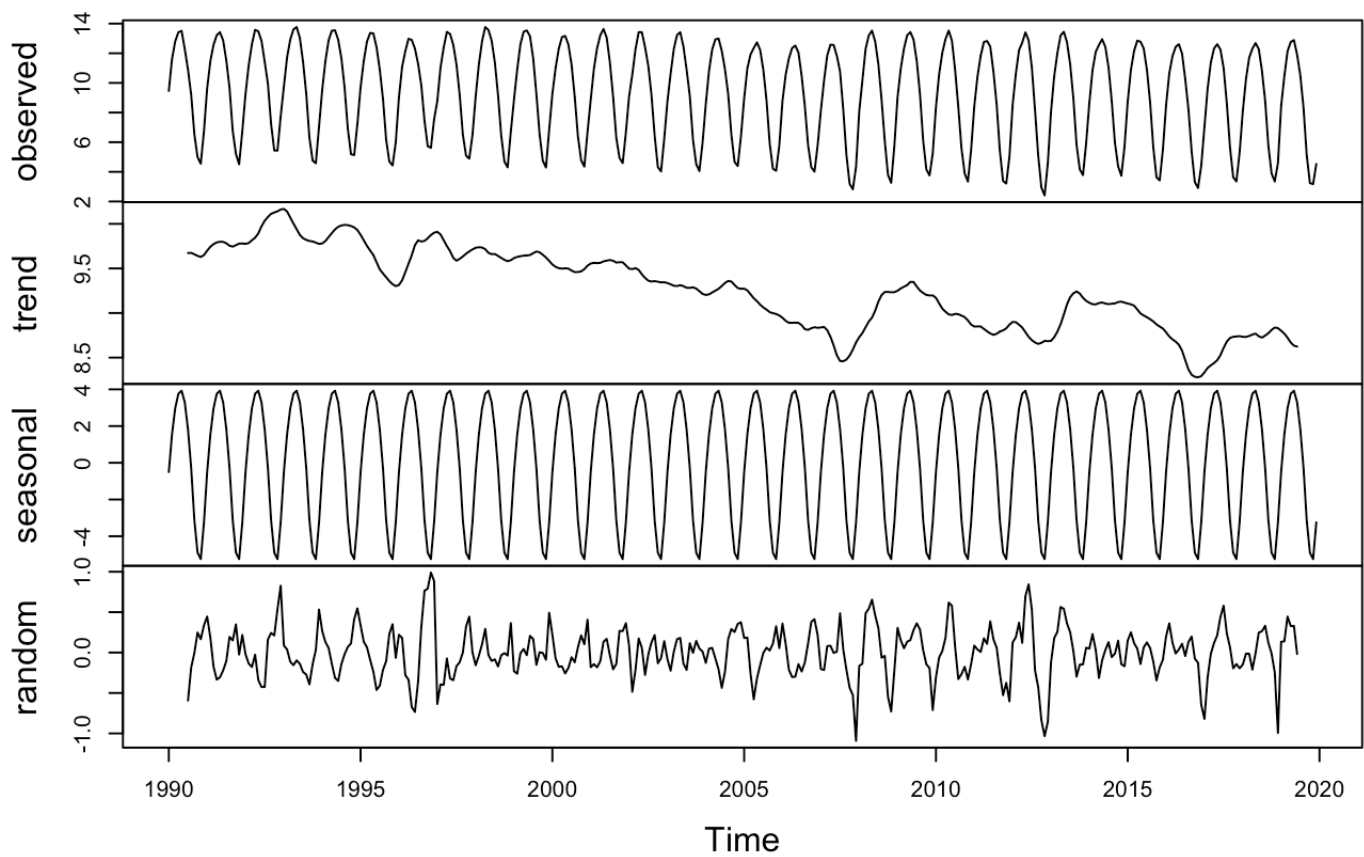
```
fit_mult <- decompose(train_extent_N, type="multiplicative")  
plot(fit_mult)
```

Decomposition of multiplicative time series



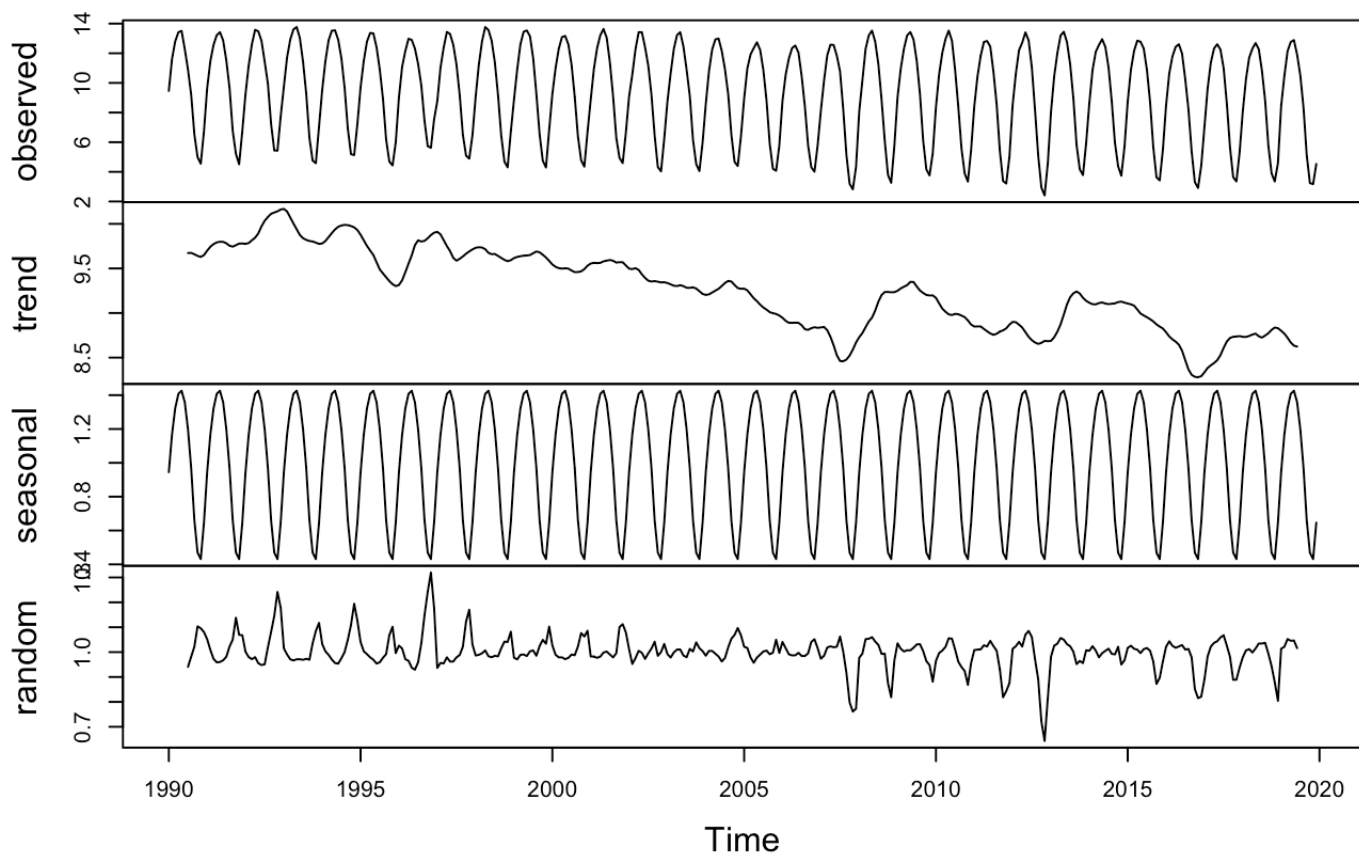
```
fit_add <- decompose(train_area_N, type="additive")  
plot(fit_add)
```

Decomposition of additive time series



```
fit_mult <- decompose(train_area_N, type="multiplicative")  
plot(fit_mult)
```

Decomposition of multiplicative time series



```
#holt winters additive
fit_hw_add <- hw(train_extent_N, h=length(test_extent_N), seasonal="add")
summary(fit_hw_add)
```

```
##
## Forecast method: Holt-Winters' additive method
##
## Model Information:
## Holt-Winters' additive method
##
## Call:
## hw(y = train_extent_N, h = length(test_extent_N), seasonal = "add")
##
## Smoothing parameters:
##   alpha = 0.8553
##   beta  = 0.0244
##   gamma = 0.1446
##
## Initial states:
```

```

##      l = 12.003
##      b = -0.0651
##      s = -3.115 -5.2471 -4.5906 -2.5271 -0.2114 1.33
##              2.9469 3.8459 3.8495 2.9995 1.4686 -0.7493
##
##      sigma: 0.2988
##
##      AIC      AICc      BIC
## 1266.877 1268.667 1332.941
##
## Error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 0.004451767 0.2920815 0.2137866 -0.3050211 2.274539 0.6206018
##              ACF1
## Training set 0.192518
##
## Forecasts:
##      Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## Jan 2020      8.919274 8.53635046 9.302198 8.333642910 9.504905
## Feb 2020      11.076380 10.56637746 11.586382 10.296398538 11.856361
## Mar 2020      12.482069 11.86567314 13.098464 11.539373061 13.424764
## Apr 2020      13.222261 12.51069392 13.933828 12.134013089 14.310509
## May 2020      13.411099 12.61145596 14.210742 12.188150553 14.634047
## Jun 2020      12.679263 11.79641434 13.562111 11.329062705 14.029462
## Jul 2020      11.249293 10.28675575 12.211830 9.777219286 12.721367
## Aug 2020      9.436336 8.39673332 10.475939 7.846400877 11.026271
## Sep 2020      6.582164 5.46749742 7.696831 4.877428416 8.286900
## Oct 2020      4.155141 2.96695840 5.343323 2.337972663 5.972309
## Nov 2020      3.493723 2.23323425 4.754212 1.565971631 5.421475
## Dec 2020      5.417641 4.08579155 6.749490 3.380753104 7.454529
## Jan 2021      8.606855 7.18606732 10.027642 6.433947808 10.779762
## Feb 2021      10.763961 9.27398818 12.253933 8.485244442 13.042677
## Mar 2021      12.169649 10.61085048 13.728448 9.785672163 14.553627
## Apr 2021      12.909842 11.28247555 14.537208 10.420999942 15.398683
## May 2021      13.098680 11.40292293 14.794436 10.505243523 15.692116
## Jun 2021      12.366843 10.60280277 14.130884 9.668976093 15.064711
## Jul 2021      10.936874 9.10459679 12.769151 8.134647928 13.739100
## Aug 2021      9.123917 7.22339987 11.024434 6.217326974 12.030507
## Sep 2021      6.269745 4.30094094 8.238549 3.258718987 9.280771
## Oct 2021      3.842722 1.80554497 5.879898 0.727128851 6.958314
## Nov 2021      3.181304 1.07563697 5.286971 -0.039035888 6.401644
## Dec 2021      5.105222 2.93091686 7.279526 1.779909439 8.430534
## Jan 2022      8.294436 6.03707679 10.551795 4.842103173 11.746768
## Feb 2022      10.451541 8.12560055 12.777482 6.894321872 14.008761
## Mar 2022      11.857230 9.46247032 14.251990 8.194761012 15.519699
## Apr 2022      12.597422 10.13359088 15.061254 8.829317224 16.365528
## May 2022      12.786260 10.25309087 15.319430 8.912111895 16.660409

```

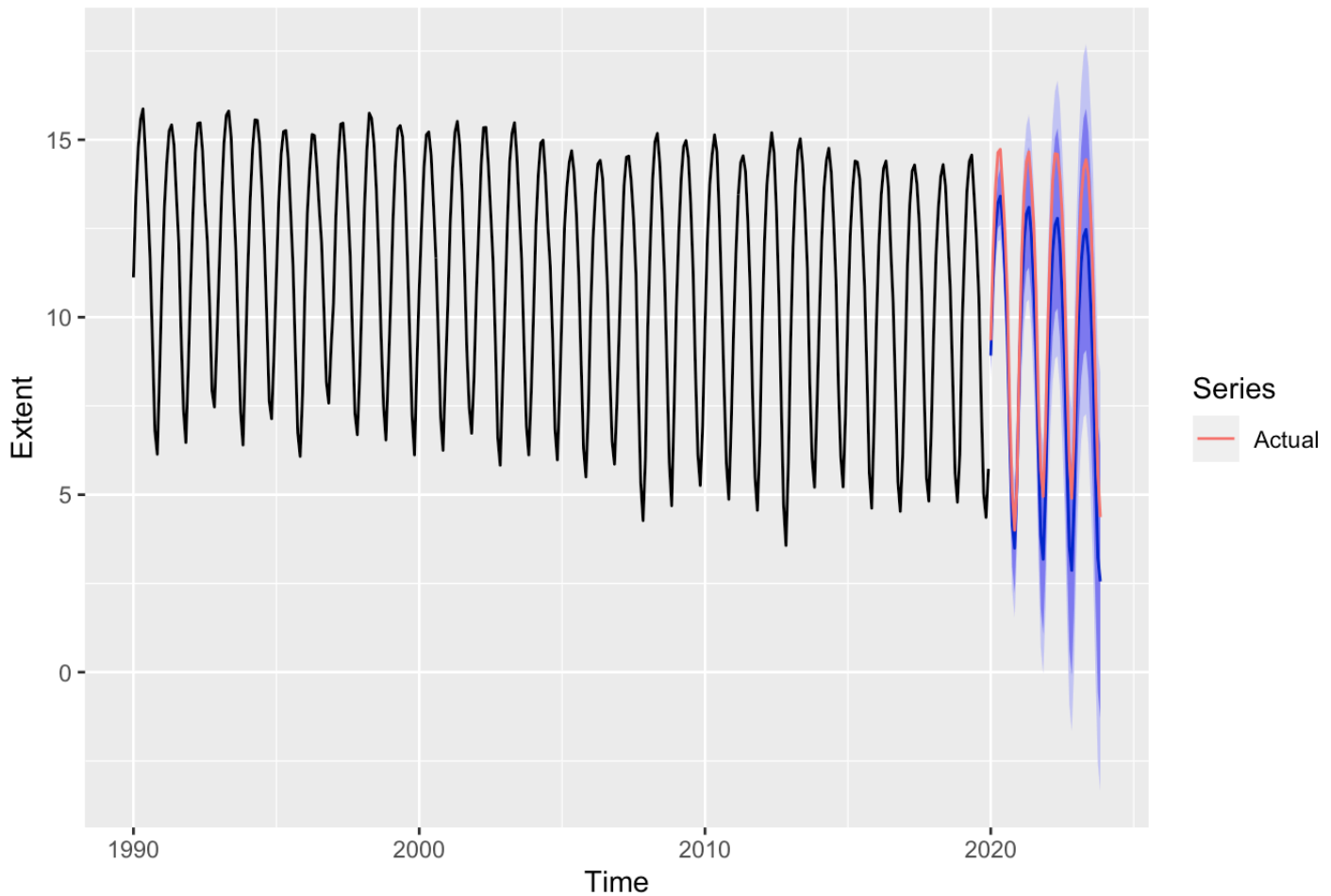
## Jun 2022	12.054424	9.45163814	14.657210	8.073806470	16.035042
## Jul 2022	10.624455	7.95176303	13.297146	6.536925577	14.711984
## Aug 2022	8.811498	6.06860166	11.554394	4.616600273	13.006395
## Sep 2022	5.957326	3.14391826	8.770733	1.654590245	10.260061
## Oct 2022	3.530302	0.64606810	6.414536	-0.880753250	7.941358
## Nov 2022	2.868885	-0.08649759	5.824267	-1.650982577	7.388752
## Dec 2022	4.792802	1.76594389	7.819661	0.163621773	9.421983
## Jan 2023	7.982016	4.87103010	11.093003	3.224173402	12.739859
## Feb 2023	10.139122	6.95626788	13.321976	5.271366637	15.006878
## Mar 2023	11.544811	8.28973559	14.799886	6.566602765	16.523019
## Apr 2023	12.285003	8.95735028	15.612656	7.195797208	17.374209
## May 2023	12.473841	9.07325151	15.874431	7.273088046	17.674594
## Jun 2023	11.742005	8.26811684	15.215893	6.429151585	17.054858
## Jul 2023	10.312035	6.76448530	13.859585	4.886525731	15.737545
## Aug 2023	8.499078	4.87750079	12.120656	2.960353427	14.037803
## Sep 2023	5.644907	1.94893449	9.340879	-0.007594972	11.297408
## Oct 2023	3.217883	-0.55285203	6.988618	-2.548958613	8.984725
## Nov 2023	2.556466	-1.28940190	6.402333	-3.325281212	8.438213

```

autoplot(fit_hw_add) +
  autolayer(test_extent_N, series = "Actual") +
  xlab("Time") +
  ylab("Extent") +
  ggtitle("Holt-Winters Forecast and Actual Values") +
  guides(color = guide_legend(title = "Series"))

```


Holt-Winters Forecast and Actual Values



```
#Red- Actual
# blue line predicted
# blue shade confidence interval
```

```
#holt winters additive damping
fit_hw_add_damp <- hw(train_extent_N, h=length(test_extent_N), seasonal="add",
                      damped=TRUE)
summary(fit_hw_add_damp)
```

```
##
## Forecast method: Damped Holt-Winters' additive method
##
## Model Information:
## Damped Holt-Winters' additive method
##
## Call:
## hw(y = train_extent_N, h = length(test_extent_N), seasonal = "add",
##
```

```

## Call:
##       damped = TRUE)
##
## Smoothing parameters:
##       alpha = 0.9999
##       beta  = 0.0059
##       gamma = 1e-04
##       phi   = 0.9645
##
## Initial states:
##       l = 11.4974
##       b = 0.0245
##       s = -3.4534 -5.408 -4.6346 -2.3524 0.1138 1.7213
##           3.1181 3.8539 3.6908 2.8529 1.3501 -0.8525
##
##       sigma: 0.2875
##
##       AIC      AICc      BIC
## 1240.114 1242.120 1310.064
##
## Error measures:
##
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -0.00758449 0.2806433 0.2026014 -0.2644521 2.16215 0.5881322
##
##           ACF1
## Training set 0.1013728
##
## Forecasts:
##           Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## Jan 2020      8.324067  7.9556034  8.692531  7.7605505  8.887584
## Feb 2020     10.520270  9.9977183 11.042821  9.7210964 11.319443
## Mar 2020     12.016694 11.3749254 12.658463 11.0351936 12.998194
## Apr 2020     12.848605 12.1055428 13.591667 11.7121895 13.985020
## May 2020     13.005724 12.1727550 13.838693 11.7318078 14.279641
## Jun 2020     12.264377 11.3495479 13.179207 10.8652665 13.663488
## Jul 2020     10.862066  9.8714479 11.852684  9.3470464 12.377085
## Aug 2020      9.249157  8.1875401 10.310773  7.6255542 10.872759
## Sep 2020      6.777849  5.6491381  7.906561  5.0516344  8.504065
## Oct 2020      4.490675  3.2981303  5.683220  2.6668351  6.314516
## Nov 2020      3.712565  2.4589643  4.966165  1.7953482  5.629781
## Dec 2020      5.662783  4.3505314  6.975034  3.6558675  7.669698
## Jan 2021      8.259228  6.8904242  9.628033  6.1658230 10.352634
## Feb 2021     10.457732  9.0342557 11.881208  8.2807130 12.634751
## Mar 2021     11.956376 10.4799013 13.432850  9.6983030 14.214448
## Apr 2021     12.790427 11.2624650 14.318389 10.4536107 15.127244
## May 2021     12.949611 11.3715341 14.527688 10.5361506 15.363072
## Jun 2021     12.210256 10.5833194 13.837192  9.7220713 14.698440
## Jul 2021     10.809865  9.1352242 12.484506  8.2487230 13.371007

```

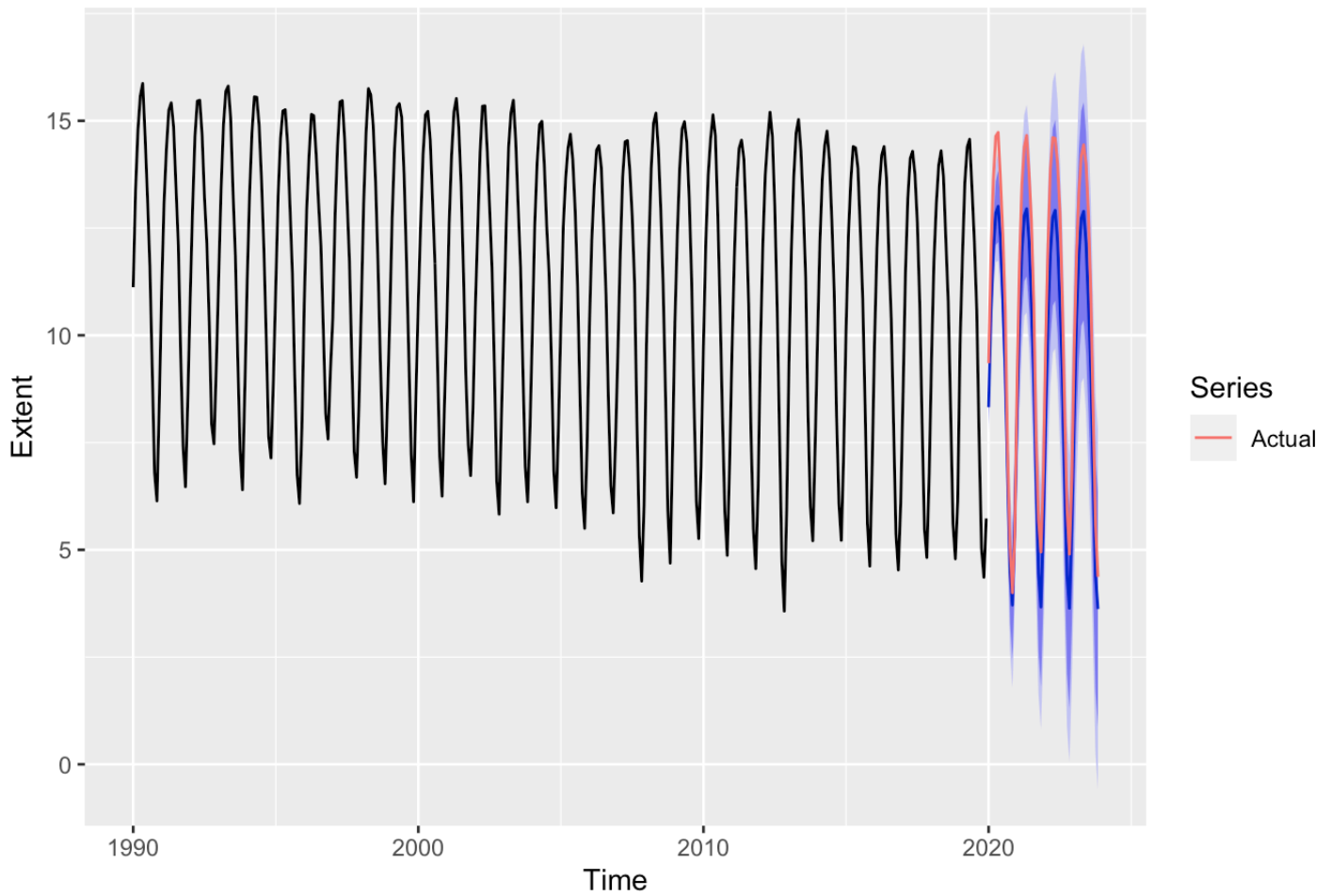
## Aug 2021	9.198808	7.4775306	10.920086	6.5663412	11.831276
## Sep 2021	6.729288	4.9623644	8.496212	4.0270115	9.431565
## Oct 2021	4.443837	2.6321916	6.255483	1.6731643	7.214510
## Nov 2021	3.667389	1.8118856	5.522892	0.8296414	6.505136
## Dec 2021	5.619210	3.7206605	7.517760	2.7156290	8.522791
## Jan 2022	8.217202	6.2763624	10.158042	5.2489438	11.185461
## Feb 2022	10.417197	8.4347964	12.399598	7.3853768	13.449018
## Mar 2022	11.917280	9.8940005	13.940559	8.8229413	15.011618
## Apr 2022	12.752718	10.6892090	14.816228	9.5968530	15.908584
## May 2022	12.913241	10.8101168	15.016365	9.6967902	16.129692
## Jun 2022	12.175176	10.0330243	14.317328	8.8990377	15.451315
## Jul 2022	10.776030	8.5954110	12.956650	7.4410608	14.111000
## Aug 2022	9.166174	6.9476230	11.384726	5.7731928	12.559156
## Sep 2022	6.697812	4.4418418	8.953783	3.2476032	10.148021
## Oct 2022	4.413479	2.1205810	6.706376	0.9067944	7.920163
## Nov 2022	3.638108	1.3087555	5.967460	0.0756710	7.200544
## Dec 2022	5.590968	3.2256159	7.956320	1.9734741	9.208462
## Jan 2023	8.189963	5.7890414	10.590884	4.5180705	11.861854
## Feb 2023	10.390924	7.9548626	12.826986	6.6652895	14.116559
## Mar 2023	11.891939	9.4211441	14.362734	8.1131844	15.670693
## Apr 2023	12.728277	10.2231425	15.233412	8.8970043	16.559550
## May 2023	12.889667	10.3505728	15.428761	9.0064575	16.772876
## Jun 2023	12.152439	9.5797534	14.725124	8.2178559	16.087022
## Jul 2023	10.754100	8.1481799	13.360020	6.7686891	14.739511
## Aug 2023	9.145022	6.5062133	11.783831	5.1093121	13.180733
## Sep 2023	6.677411	4.0060484	9.348773	2.5919145	10.762907
## Oct 2023	4.393801	1.6902110	7.097392	0.2590167	8.528586
## Nov 2023	3.619129	0.8836271	6.354630	-0.5644600	7.802717

```

autoplot(fit_hw_add_damp) +
  autolayer(test_extent_N, series = "Actual") +
  xlab("Time") +
  ylab("Extent") +
  ggtitle("Holt-Winters Forecast and Actual Values") +
  guides(color = guide_legend(title = "Series"))

```

Holt-Winters Forecast and Actual Values



```
# Comparing RMSE Train  
print(accuracy(fit_hw_add)[2])
```

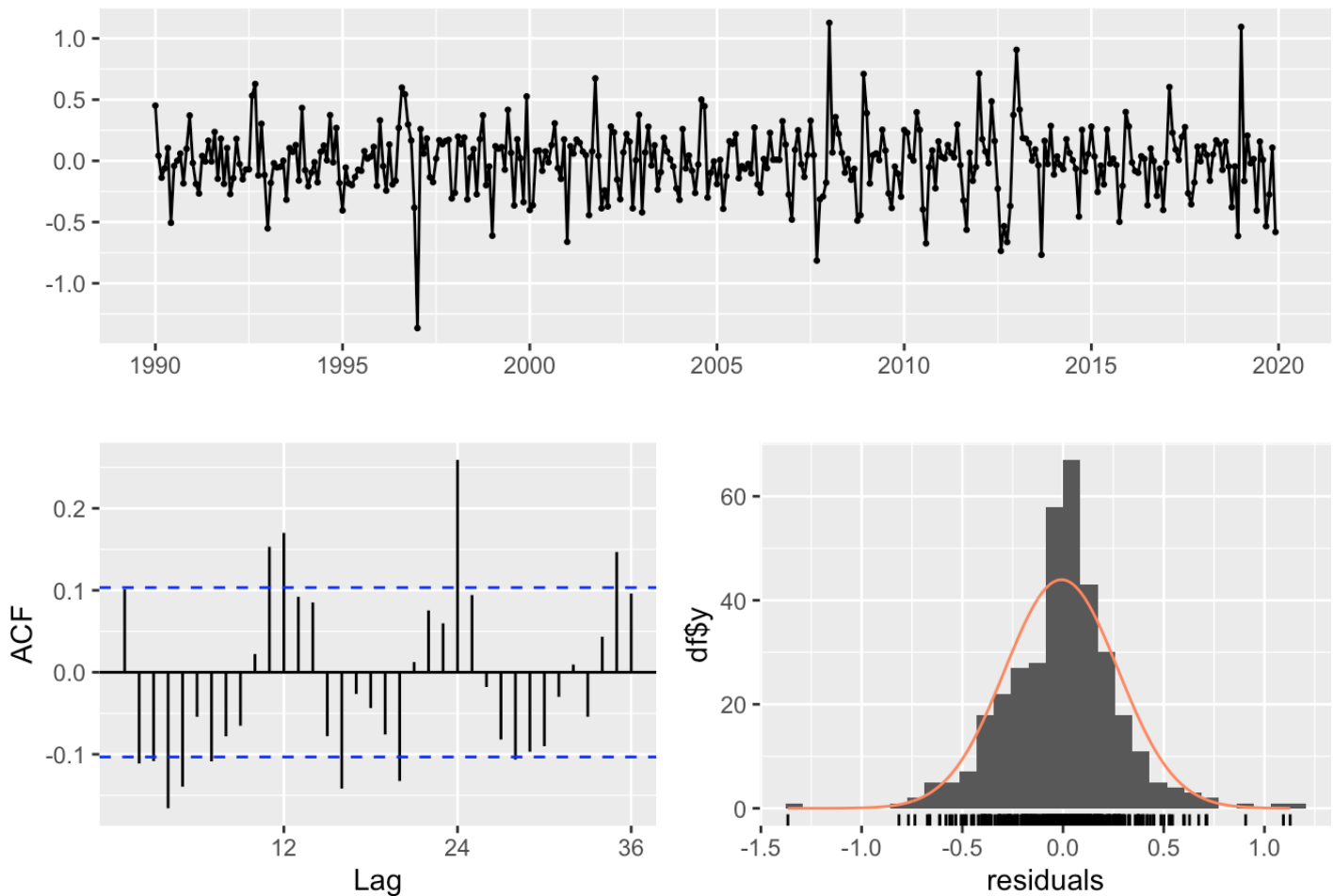
```
## [1] 0.2920815
```

```
print(accuracy(fit_hw_add_damp)[2])
```

```
## [1] 0.2806433
```

```
#  
checkresiduals(fit_hw_add_damp)
```

Residuals from Damped Holt-Winters' additive method



```
##
## Ljung-Box test
##
## data: Residuals from Damped Holt-Winters' additive method
## Q* = 114.26, df = 24, p-value = 1.008e-13
##
## Model df: 0. Total lags used: 24
```

```
accuracy_hw_add_damp <- accuracy(fit_hw_add_damp, test_extent_N)
accuracy_hw_add_damp
```

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -0.00758449 0.2806433 0.2026014 -0.2644521 2.16215 0.5881322
## Test set      1.40202303 1.4790112 1.4161840 13.7836164 14.04930 4.1110446
##           ACF1 Theil's U
## Training set 0.1013728      NA
## Test set      0.7192263 0.6200398
```

Auto Arima

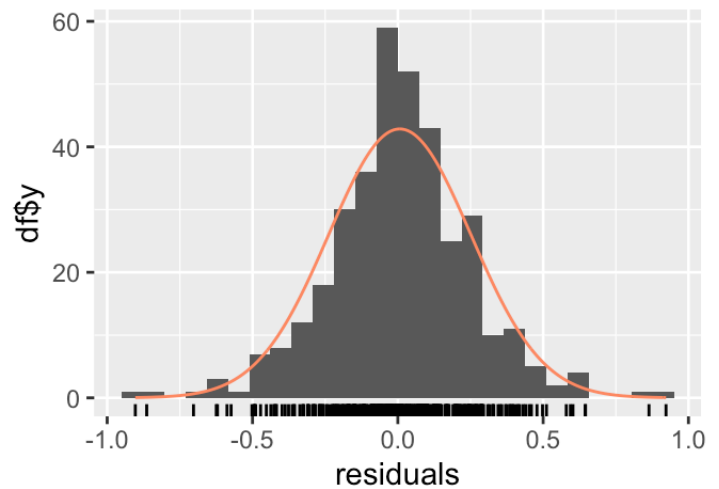
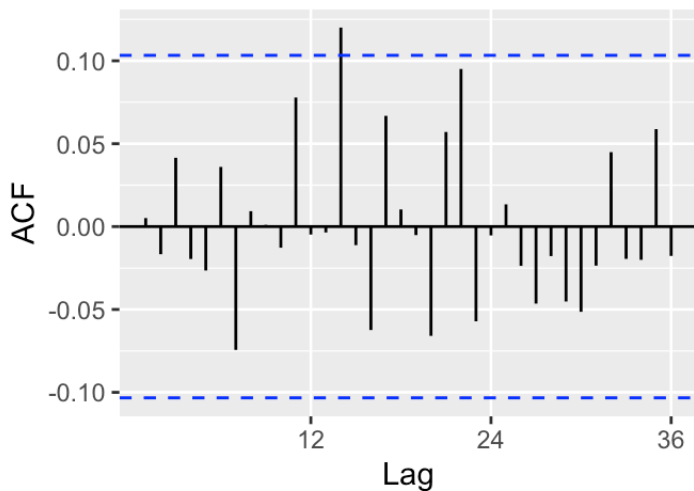
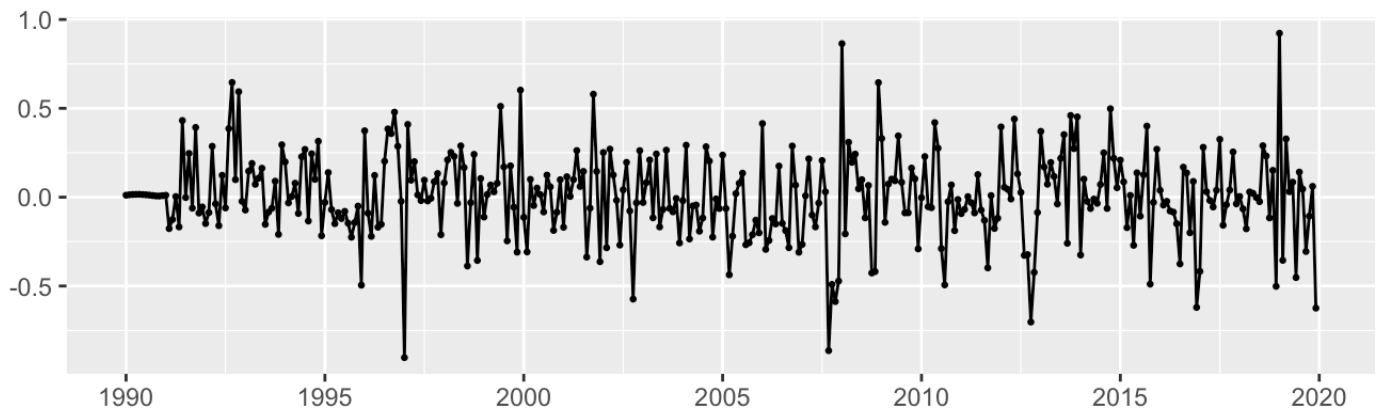
```
arima_model <- auto.arima(train_extent_N, seasonal=TRUE)
```

```
arima_model
```

```
## Series: train_extent_N
## ARIMA(2,0,0)(2,1,2)[12] with drift
##
## Coefficients:
##          ar1      ar2      sar1      sar2      sma1      sma2      drift
##          0.8705  -0.1789  -0.6624   0.1020  -0.1334  -0.5246  -0.0051
## s.e.    0.0539   0.0538   0.2918   0.0973   0.2916   0.2024   0.0009
##
## sigma^2 = 0.06315:  log likelihood = -15.89
## AIC=47.78   AICc=48.2   BIC=78.59
```

```
checkresiduals(arima_model$residuals)
```

Residuals

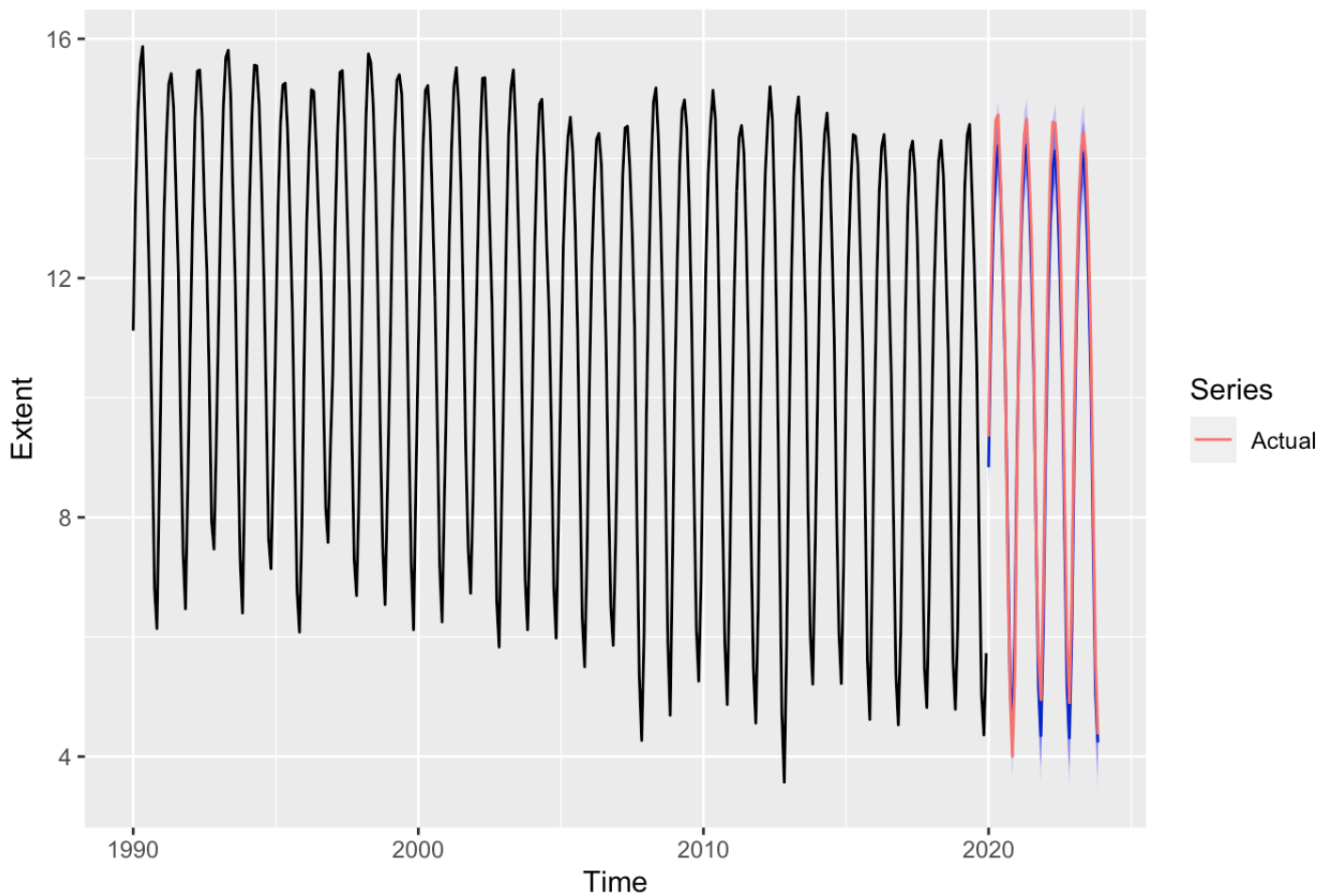


```
##
##  Ljung-Box test
##
## data:  Residuals
## Q* = 22.408, df = 24, p-value = 0.5549
##
## Model df: 0.   Total lags used: 24
```

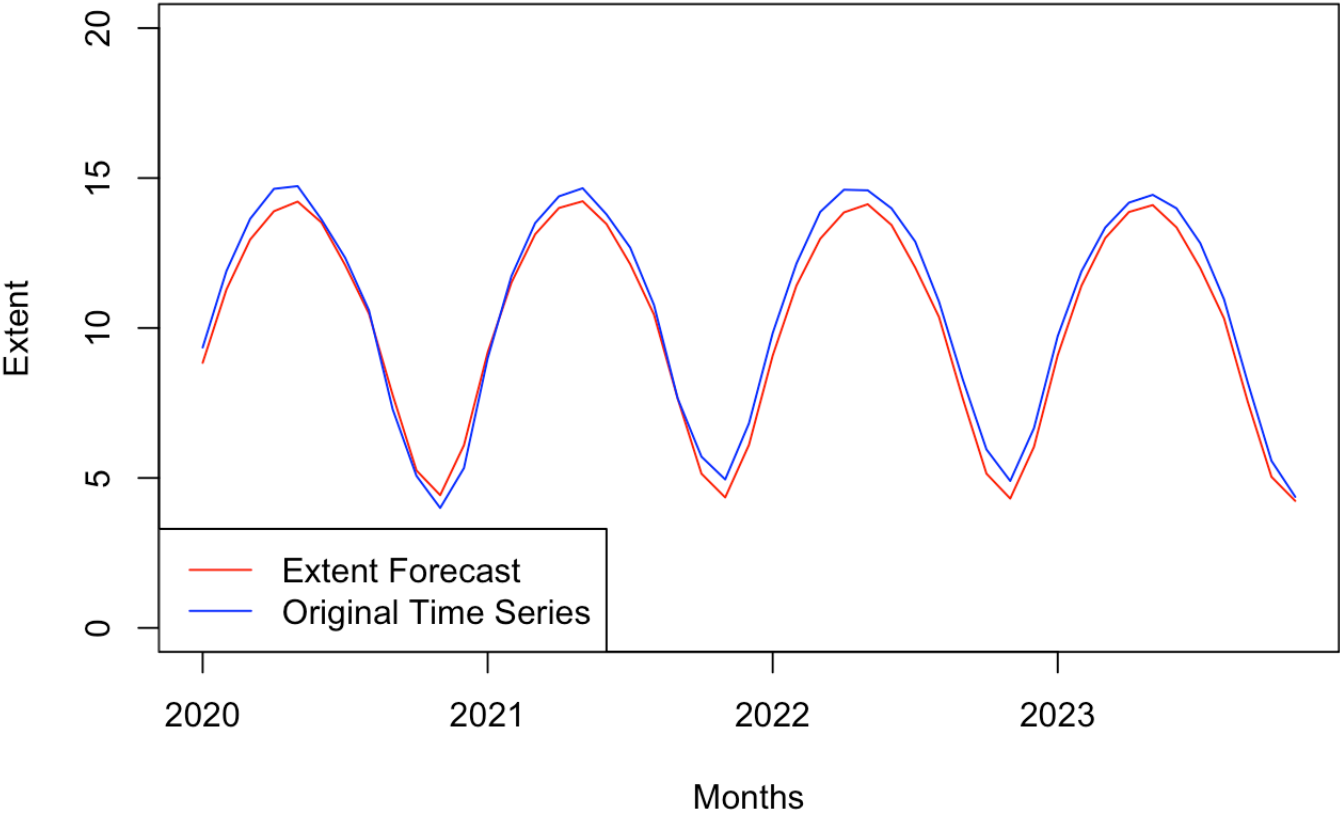
```
arima_forecast <- forecast(arima_model, h = length(test_extnt_N))

autoplot(arima_forecast) +
  autolayer(test_extnt_N, series = "Actual") +
  xlab("Time") +
  ylab("Extent") +
  ggtitle("Forecast and Actual Values") +
  guides(color = guide_legend(title = "Series"))
```

Forecast and Actual Values



```
plot(arima_forecast$mean, xlab = "Months", ylab = "Extent", col = "red", ylim=c(0,20))
lines(test_extent_N, col = "blue")
legend("bottomleft", legend = c("Extent Forecast", "Original Time Series"), col = c("red", "blue"), lty = c(1,1))
```

```
accuracy(arima_forecast, test_extent_N)
```

##		ME	RMSE	MAE	MPE	MAPE	MASE
##	Training set	0.006692764	0.2445760	0.1808994	-0.1175827	1.982333	0.5251335
##	Test set	0.419576368	0.5518514	0.5065326	3.9381654	5.523027	1.4704150
##		ACF1 Theil's U					
##	Training set	0.005160017	NA				
##	Test set	0.761791620	0.2848214				