DSP Lab. Week 9
DCT

Kyuheon Kim
Media Lab. Rm567
kyuheonkim@khu.ac.kr
Last update : September 2, 2019

- Discrete Cosine Transform (DCT)
  - which is able to perform decorrelation of the input signal in a data-inde pendent manner.
  - Spatial frequency shows how frequently pixel values are changed in an image domain.
  - DCT represents this spatial frequency based on macroblock, and decor relates DC and AC components.

$$F(u,v) = \frac{2\,C(u)\,C(v)}{\sqrt{MN}} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \cos\frac{(2i+1)\cdot u\pi}{2M} \cdot \cos\frac{(2j+1)\cdot v\pi}{2N} \cdot f(i,j) \tag{8.15}$$

where i, II = 0, 1, ... , M − 1, j, v = 0, l, ... ,N − 1, and the constants C(u) and C(v) are determ ined by

$$C(\xi) = \begin{cases} \frac{\sqrt{2}}{2} & if \quad \xi = 0, \\ 1 & otherwise. \end{cases} \tag{8.16}$$

- 2D DCT

$$F(u,v) = \frac{C(u)\,C(v)}{4} \sum_{i=0}^{7}\sum_{j=0}^{7} \cos\frac{(2i+1)u\pi}{16} \cos\frac{(2j+1)v\pi}{16} f(i,j) \qquad (8.17)$$

- 2D IDCT

$$\tilde{f}(i,j) = \sum_{u=0}^{7}\sum_{v=0}^{7} \frac{C(u)\,C(v)}{4} \cos\frac{(2i+1)u\pi}{16} \cos\frac{(2j+1)v\pi}{16} F(u,v) \qquad (8.18)$$

- 1D DCT

$$F(u) = \frac{C(u)}{2} \sum_{i=0}^{7} \cos\frac{(2i+1)u\pi}{16} f(i) \qquad (8.19)$$

- 1D IDCT

$$\tilde{f}(i) = \sum_{u=0}^{7} \frac{C(u)}{2} \cos\frac{(2i+1)u\pi}{16} F(u) \qquad (8.20)$$

- One-dimensional DCT
  - Any signal can be expressed as a sum of multiple signals that are sine or cosine waveforms at various amplitudes and frequencies. This is kn own as Fourier analysis.
  - The terms DC and AC are carried over to describe these components o f a signal (usually) composed of one DC and several AC components.
  - The process of determining the amplitudes of the AC and DC compone nts of the signal is called a Cosine Transform. The role of the IDCT is t o reconstruct (recompose) the signal.
  - The DCT and IDCT use the same set of cosine functions; they are kno wn as basis functions.
  - Suppose f(i) represents a signal that changes with time i. The ID DCT transforms f(i), which is in the time domain, to F(u), which is in the fr equency domain. The coefficients F(u) are known as the frequency res ponses and form the frequency spectrum of f(i).

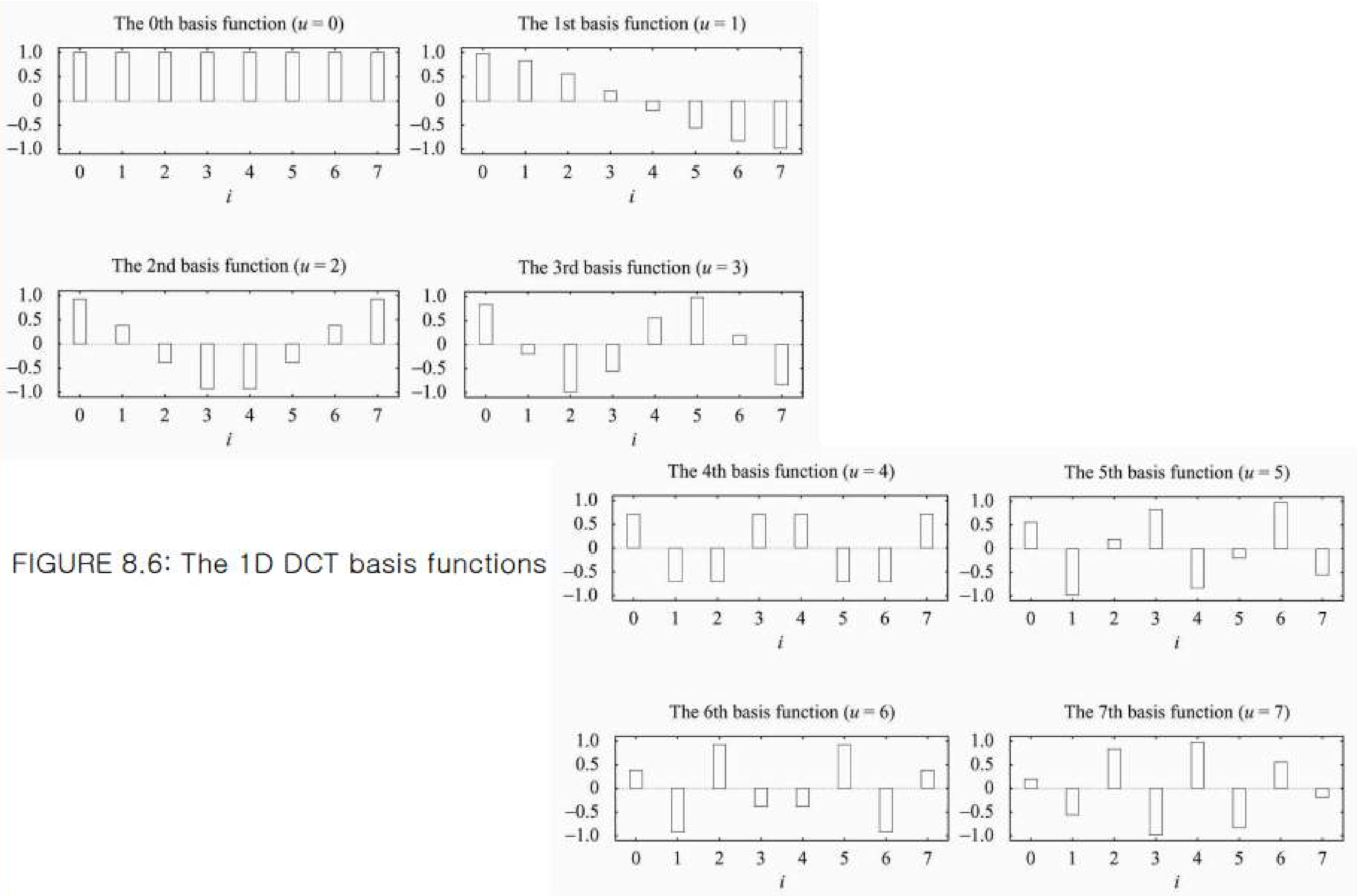


FIGURE 8.6: The 1D DCT basis functions

- Example 8.1~8.4

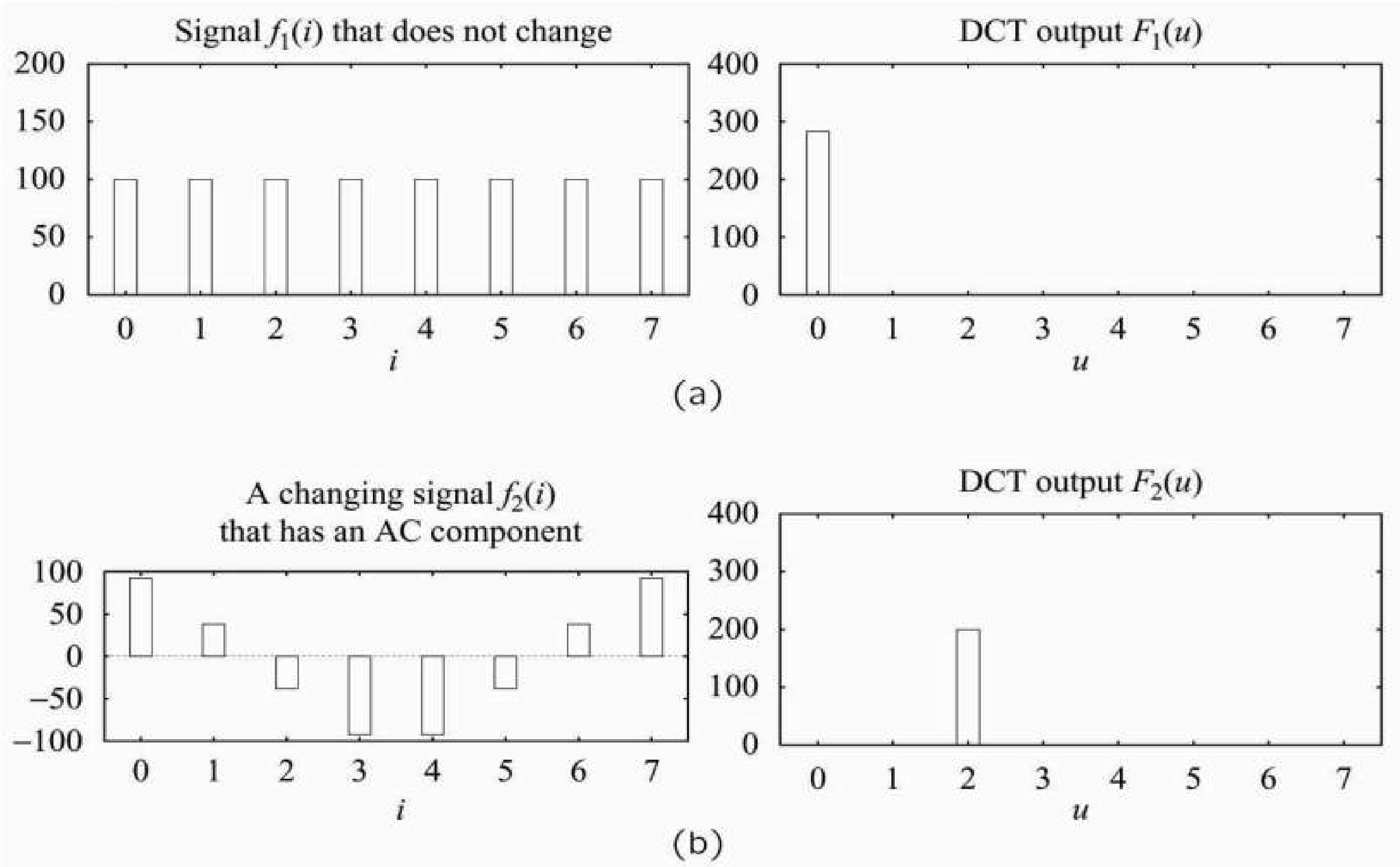


FIGURE 8.7: Examples of ID Discrete Cosine Transform: (a) a DC signal; (b) an AC signal

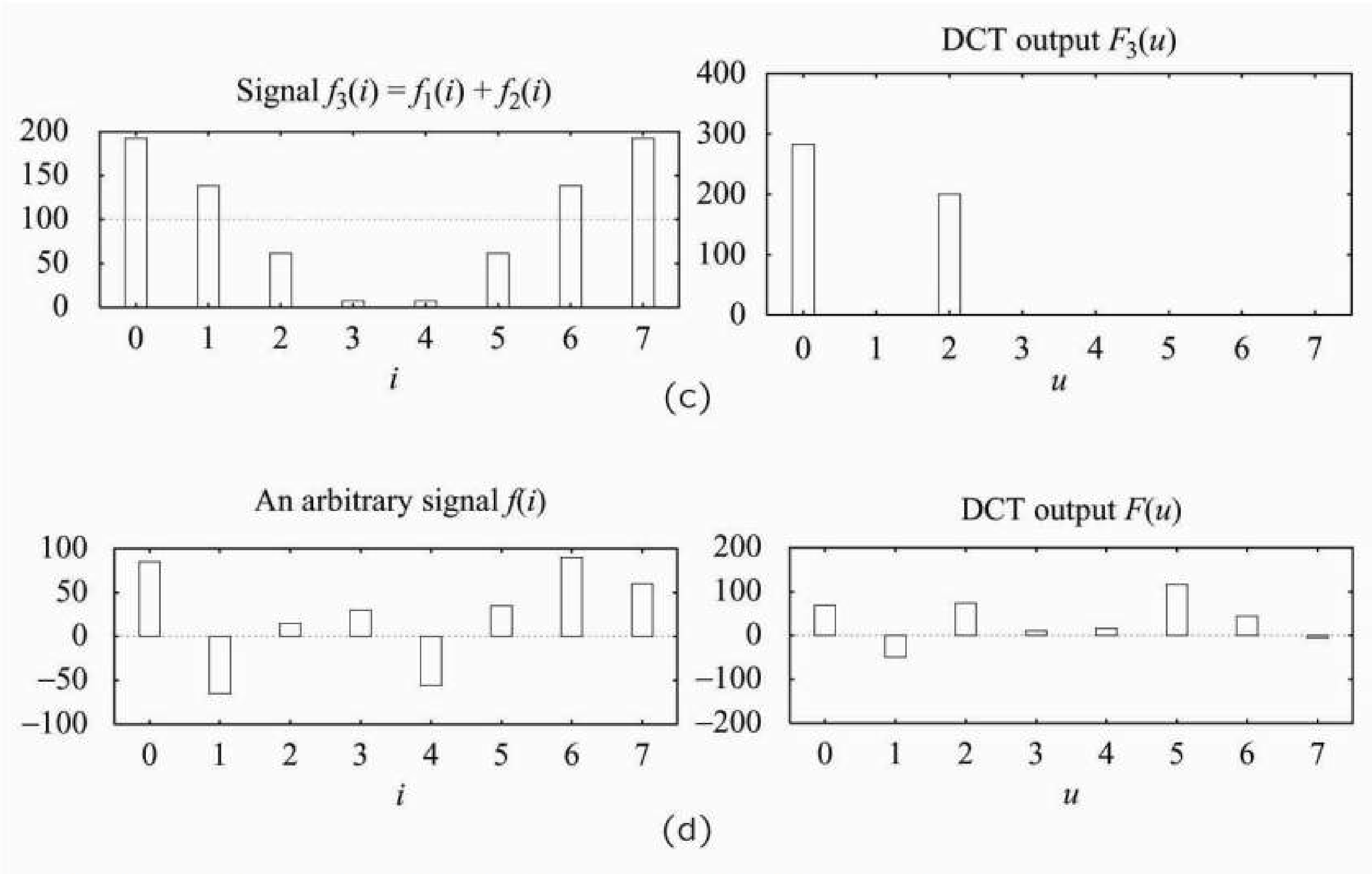


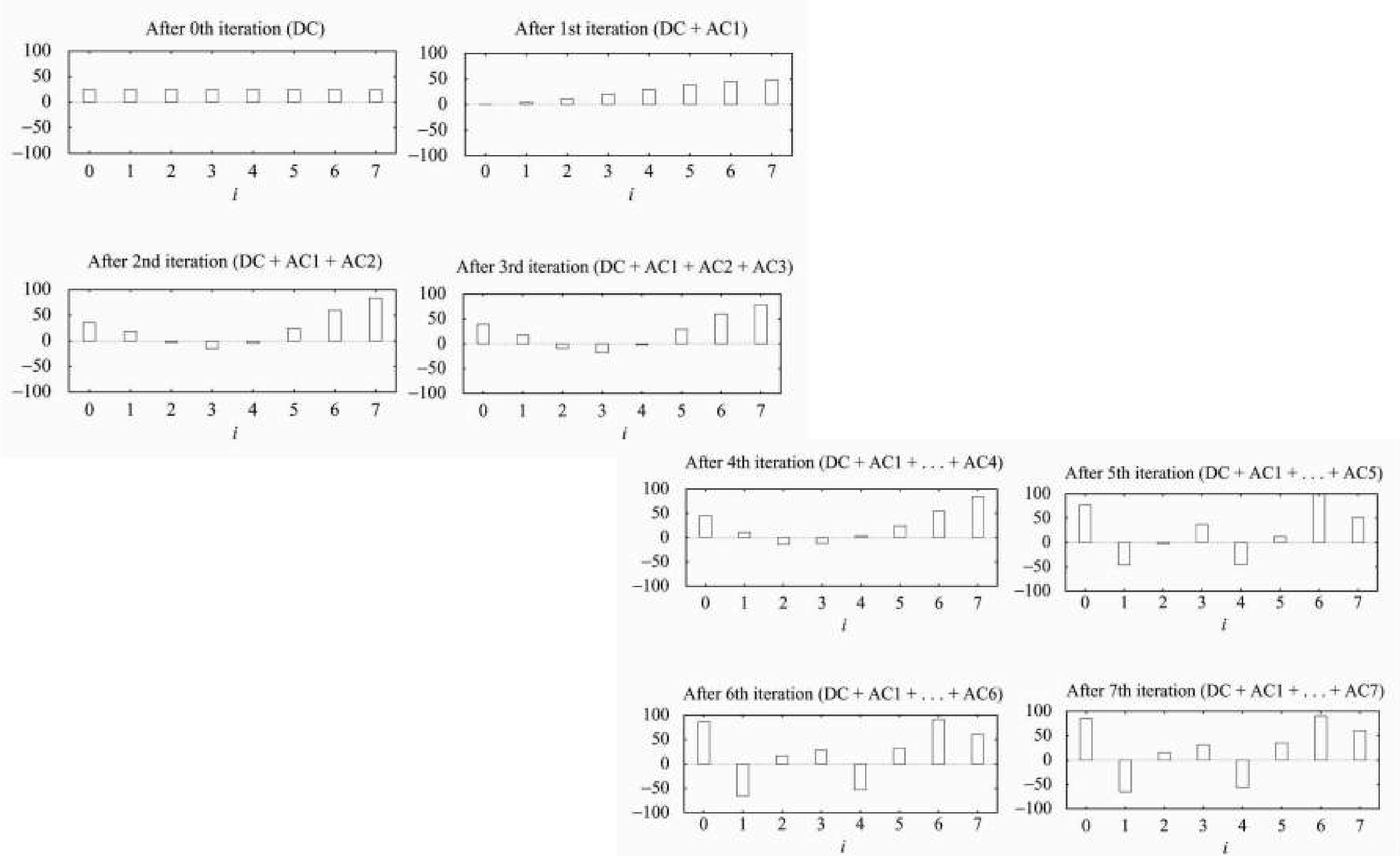
FIGURE 8.7: Examples of ID Discrete (c) (a)+(b); (d) an arbitrary signal

- Characteristics of the DCT can be summarized as follows:
  - The DCT produces the frequency spectrum F(u) corresponding to the s patial signal f(i).
  - The 0th DCT coefficient F(0) is the DC component of the signal f(i). F(0) equals the average magnitude of the signal.
  - The other seven DCT coefficients reflect the various changing (i.e., AC) components of the signal f(i) at different frequencies.
  - The DCT is a linear transform.

$$\mathcal{T}(\alpha p + \beta q) = \alpha \mathcal{T}(p) + \beta \mathcal{T}(q) \tag{8.21}$$

where **α and β** are constants, and p and q are any functions, variables or constant s.

- One-dimensional Inverse DCT



- Cosine basis functions
  - For a better decomposition, the basis functions should be orthogonal, so as to have the least redundancy amongst them.
  - Functions $\mathbf{B_p(i)}$ and $\mathbf{B_q(i)}$ are **orthogonal** if

$$\sum_i [B_p(i) \cdot B_q(i)] = 0 \qquad if \quad p \neq q \qquad (8.22)$$

  - Functions $\mathbf{B_p(i)}$ and $\mathbf{B_q(i)}$ are **orthonormal** if they are orthogonal and

$$\sum_i [B_p(i) \cdot B_q(i)] = 1 \qquad if \quad p = q \qquad (8.23)$$

  - For examples

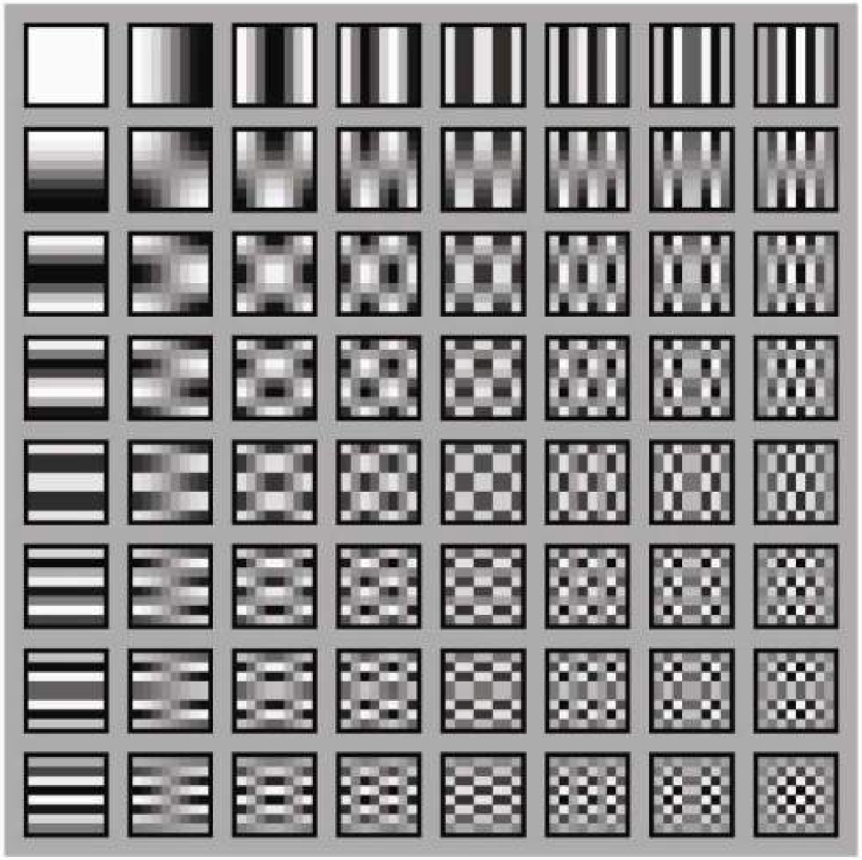
$$\sum_{i=0}^{7} \left[\cos\frac{(2i+1)\cdot p\pi}{16} \cdot \cos\frac{(2i+1)\cdot q\pi}{16}\right] = 0 \qquad if \quad p \neq q$$

$$\sum_{i=0}^{7} \left[\frac{C(p)}{2}\cos\frac{(2i+1)\cdot p\pi}{16} \cdot \frac{C(q)}{2}\cos\frac{(2i+1)\cdot q\pi}{16}\right] = 1 \qquad if \quad p = q$$

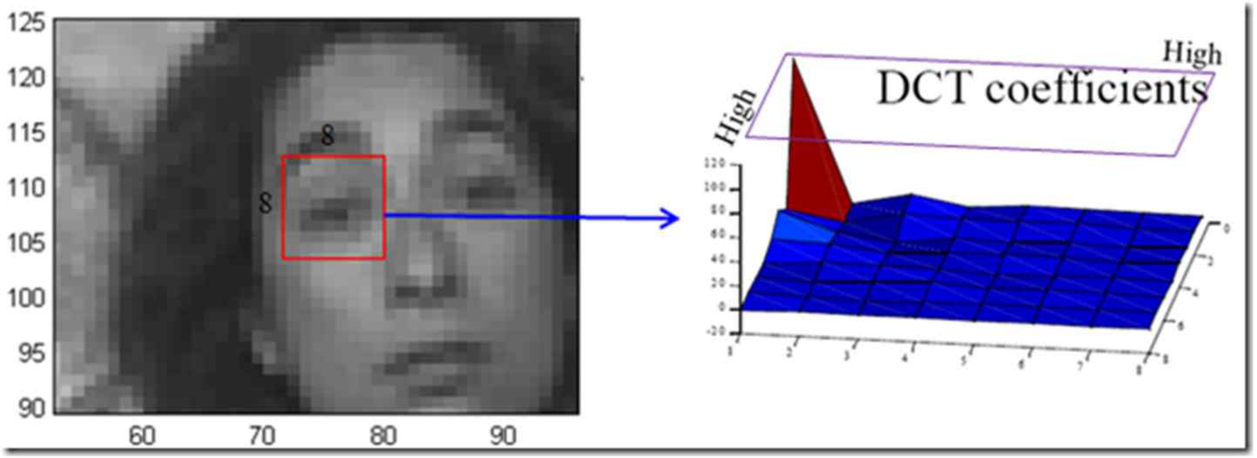
- 2D DCT
  - 8x8 images
  - White: positive numbers
  - Black: negative numbers

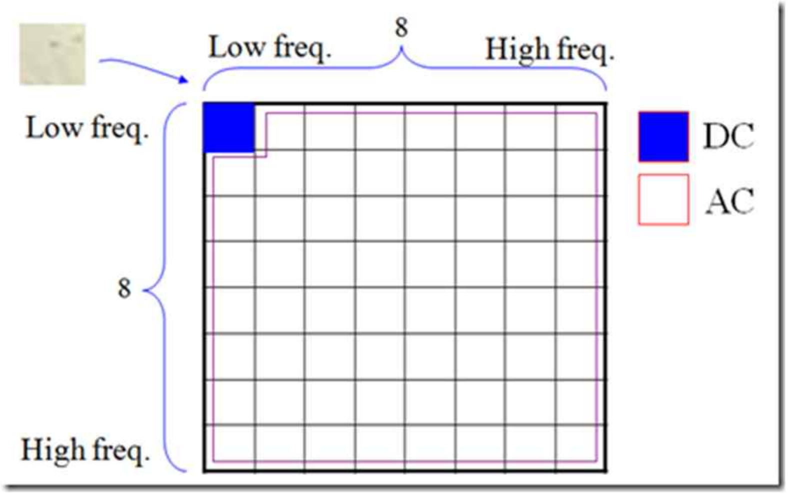
$$G(i,v) = \frac{1}{2}C(v)\sum_{j=0}^{7}\cos\frac{(2j+1)v\pi}{16}f(i,j) \qquad (8.24)$$

$$F(u,v) = \frac{1}{2}C(u)\sum_{i=0}^{7}\cos\frac{(2i+1)u\pi}{16}G(i,v) \qquad (8.25)$$



- Discrete Cosine Transform
  - 이산 코사인 변환
  - Spatial domain의 값을 frequency domain 값으로 변환





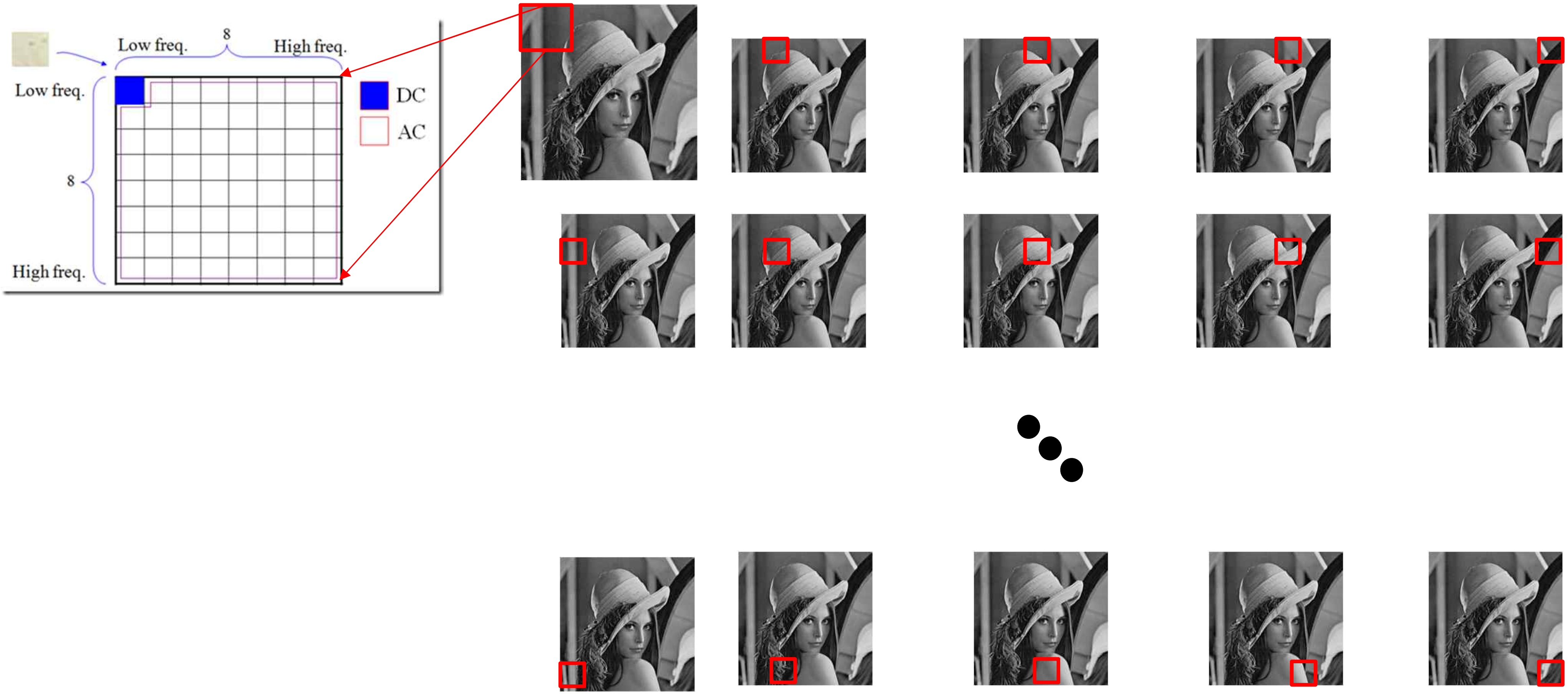
- **수식**

$$C(\xi) = \begin{cases} \frac{\sqrt{2}}{2} & if \quad \xi = 0, \\ 1 & otherwise. \end{cases}$$

$$F(u,v) = \frac{2\,C(u)\,C(v)}{\sqrt{MN}} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \cos\frac{(2i+1)\cdot u\pi}{2M} \cdot \cos\frac{(2j+1)\cdot v\pi}{2N} \cdot f(i,j)$$

▪ DCT 및 IDCT 구현 예제

- C++ 사용
- 8*8 Macroblock 단위에서 DCT 수행 후 IDCT 수행



```
void DCT_1D(double* data, double* dct)
{
int N = 8;
int mcrNb = DATALENGTH / N;

double sum = 0;

int u, n;

for (int mcr = 0; mcr < mcrNb; mcr++)
 {
   for (int k = 0; k < N; k++) {
    u = mcr * N + k;
    sum = 0;
       for (int i = 0; i < N; i++) {
       n = mcr * N + i;
       double theta = (double)(2. * i + 1)*k*PI / (2. * N);
       sum += (double)cos(theta)*data[n];
       }
        double ck = (k) ? 0.5: sqrt((double)1.0 / (double)N); // C(u) ->> C(k)
        dct[k] = ck * sum;
   }

 }
}
```

- 1D DCT

$$F(u) = \frac{C(u)}{2}\sum_{i=0}^{7}\cos\frac{(2i+1)u\pi}{16}f(i) \qquad (8.19)$$

- 1D IDCT

$$\tilde{f}(i) = \sum_{u=0}^{7}\frac{C(u)}{2}\cos\frac{(2i+1)u\pi}{16}F(u) \qquad (8.20)$$

$$C(\xi) = \begin{cases} \frac{\sqrt{2}}{2} & if \quad \xi = 0, \\ 1 & otherwise. \end{cases}$$

IDCT

```
void IDCT_1D(double* dct, double* result_data)
{
int N = 8;
int mcrNb = DATALENGTH / N;

double sum = 0;

int u, n;

for (int mcr = 0; mcr < mcrNb; mcr++)
 {
   for (int i = 0; i < N; i++) {
   n = mcr * N + i;
   sum = 0;
      for (int k = 0; k < N; k++) {
      u = mcr * N + k;
      double theta = (double)(2. * i + 1)*k*PI / (2. * N);
      double ck = (k) ? 0.5: sqrt((double)1.0 / (double)N); // C(u) ->> C(k)
      sum += ck * cos(theta)*dct[u];
      }
   result_data[i] = sum;
   }
 }
}
```

- 1D DCT

$$F(u) = \frac{C(u)}{2} \sum_{i=0}^{7} \cos \frac{(2i+1)u\pi}{16} f(i) \qquad (8.19)$$

- 1D IDCT

$$\tilde{f}(i) = \sum_{u=0}^{7} \frac{C(u)}{2} \cos \frac{(2i+1)u\pi}{16} F(u) \qquad (8.20)$$

$$C(\xi) = \begin{cases} \frac{\sqrt{2}}{2} & if \quad \xi = 0, \\ 1 & otherwise. \end{cases}$$

			0	0	0	0	0
			0	0	0	0	0
			0	0	0	0	0
			0	0	0	0	0
			0	0	0	0	0

[DCT 8x8]

각각의 DCT 결과에서
"주황색 영역"의 값만 **보존**, **흰색 영역**의 값은 **0**

- 2D DCT

$$F(u,v) = \frac{C(u)\,C(v)}{4}\sum_{i=0}^{7}\sum_{j=0}^{7}\cos\frac{(2i+1)u\pi}{16}\cos\frac{(2j+1)v\pi}{16}f(i,j) \quad (8.17)$$

- 2D IDCT

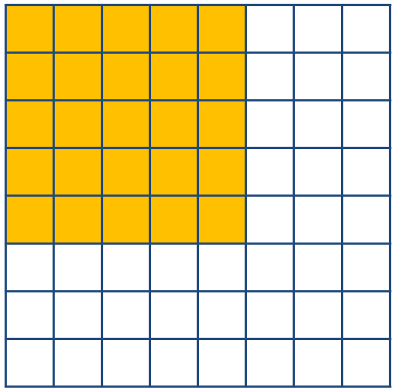
$$\tilde{f}(i,j) = \sum_{u=0}^{7}\sum_{v=0}^{7}\frac{C(u)\,C(v)}{4}\cos\frac{(2i+1)u\pi}{16}\cos\frac{(2j+1)v\pi}{16}F(u,v) \quad (8.18)$$

IDCT한 후 새로운 이미지를 생성

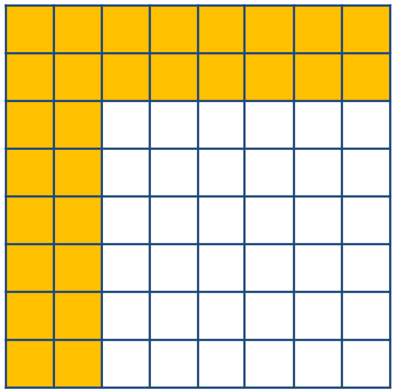
원본 이미지

IDCT 이미지

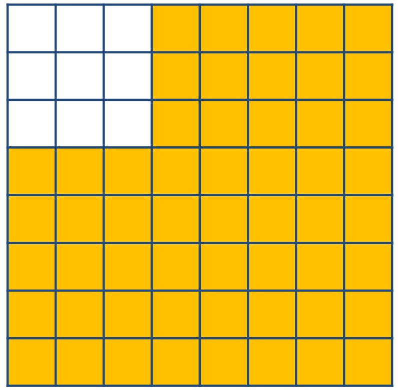
Assignment



[Case 1]



[Case 2]



[Case 3]

[DCT를 이용한 영상 Filtering]

각각의 DCT 결과에서 주황색 영역의 값만 보존, 나머지 값은 0 -> IDCT한 후 새로운 이미지를 생성

제출 파일
- .cpp파일, 보고서 파일
- Case 1 결과 image
- Case 2 결과 image
- Case 3 결과 image

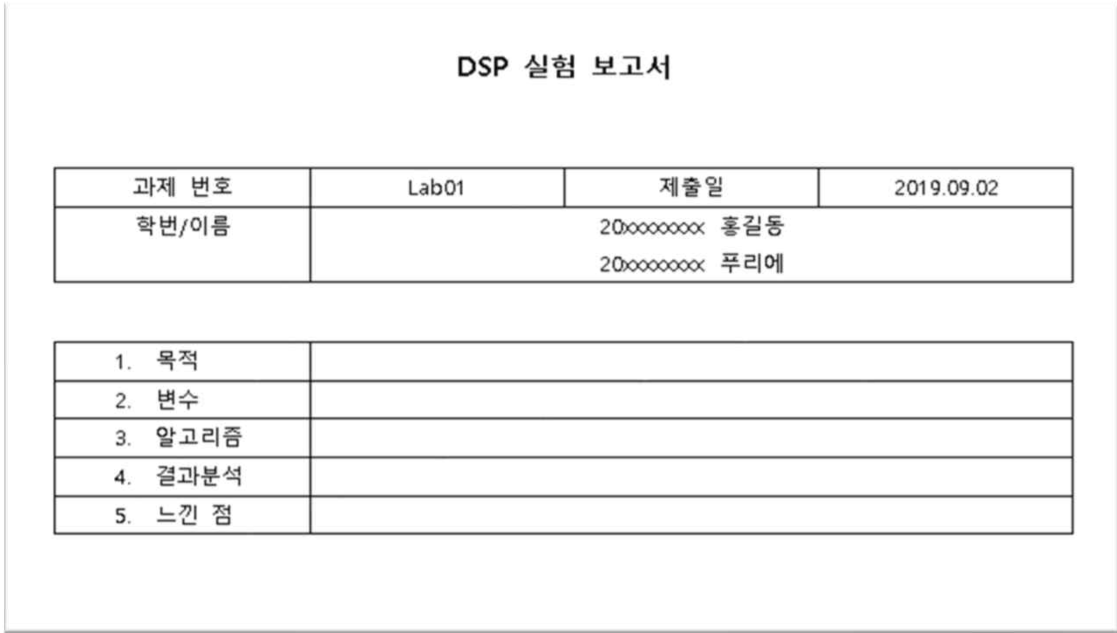
Assignment Rule

"KLAS에 제출할 때 다음 사항을 꼭 지켜주세요"

1. 파일명 : "Lab00_요일_대표자이름.zip"

Ex) Lab01_목_홍길동.zip **(압축 툴은 자유롭게 사용)**

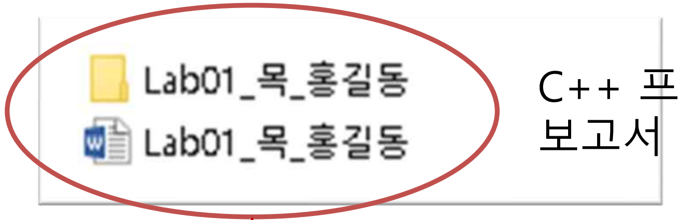
2. 제출 파일 **(보고서와 프로그램을 압축해서 제출)**
   - 보고서 파일 (hwp, word): 이름, 학번, 목적, 변수, 알고리즘(순서), 결과 분석, 느낀 점
   - 프로그램



DSP 실험 보고서

과제 번호	Lab01	제출일	2019.09.02
학번/이름	20xxxxxxxx 홍길동 20xxxxxxxx 푸리에		

1. 목적	
2. 변수	
3. 알고리즘	
4. 결과분석	
5. 느낀 점	



C++ 프로그램 파일
보고서

압축하여 제출