Derivation of Equivalent

Derive $\min L \equiv \min Cost$

เรามี Lagrangian function $(L) = \sum_{j=0}^{p} w_j^2 + \frac{1}{\lambda}\big[\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 - c\big]$ และ $Cost = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \lambda \sum_{j=0}^{p} w_j^2$

เราต้องการแสดงว่าคำตอบที่ได้จากการ minimize L เป็นคำตอบเดียวกับที่ได้จากการ minimize $Cost$

พิจารณา $\frac{\partial L}{\partial w_d} = 0$ เมื่อ $d \in \{0, 1, ..., p\}$

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\begin{aligned}
\frac{\partial L}{\partial w_d} &= \frac{\partial\big(\sum_{j=0}^{p} w_j^2 + \frac{1}{\lambda}\big[\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 - c\big]\big)}{\partial w_d} \\
0 &= \frac{\partial \sum_{j=0}^{p} w_j^2}{\partial w_d} + \frac{1}{\lambda}\frac{\partial[\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 - c]}{\partial w_d} \\
0 &= 2w_d + \frac{1}{\lambda}\Big[\sum_{i=1}^{n}\frac{(y_i - \hat{y}_i)^2}{\partial w_d} - \frac{\partial c}{\partial w_d}\Big] \\
0 &= 2w_d + \frac{1}{\lambda}\Big[\sum_{i=1}^{n}\frac{(y_i - \hat{y}_i)^2}{\partial w_d} - 0\Big] \\
0 &= 2w_d + \frac{1}{\lambda}\Big[\sum_{i=1}^{n}\frac{(y_i - \hat{y}_i)^2}{\partial(y_i - \hat{y}_i)} \cdot \frac{(y_i - \hat{y}_i)}{\partial \hat{y}_i} \cdot \frac{\partial \hat{y}_i}{\partial w_d}\Big] \\
0 &= 2w_d + \frac{1}{\lambda}\Big[\sum_{i=1}^{n} 2(y_i - \hat{y}_i)(-1)\frac{\partial \hat{y}_i}{\partial w_d}\Big] \\
0 &= 2w_d + \frac{1}{\lambda}\Big[\sum_{i=1}^{n} -2(y_i - \hat{y}_i)\frac{\partial(w_0 + w_1 x_{1,i} + \cdots + w_d x_{d,i} + \cdots + w_p x_{p,i})}{\partial w_d}\Big] \\
0 &= 2w_d + \frac{1}{\lambda}\Big[\sum_{i=1}^{n} -2(y_i - \hat{y}_i)x_{i,d}\Big] \\
0 &= 2w_d - \frac{2}{\lambda}\sum_{i=1}^{n}(y_i - \hat{y}_i)x_{i,d} \\
0 &= w_d - \frac{1}{\lambda}\sum_{i=1}^{n}(y_i - \hat{y}_i)x_{i,d} \\
0 &= \lambda w_d - \sum_{i=1}^{n}(y_i - \hat{y}_i)x_{i,d} \\
0 &= -\sum_{i=1}^{n}(y_i - \hat{y}_i)x_{i,d} + \lambda w_d
\end{aligned}
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\begin{aligned}
&= -\sum_{i=1}^{n}(y_i - \hat{y}_i)x_{i,d} + \lambda w_d = 0 \\
&= -2\sum_{i=1}^{n}(y_i - \hat{y}_i)x_{i,d} + 2\lambda w_d = 0 \\
&= \sum_{i=1}^{n}(-2)(y_i - \hat{y}_i)x_{i,d} + 2\lambda w_d = 0
\end{aligned}
$$

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\begin{aligned}
&= \sum_{i=1}^{n} 2(y_i - \hat{y}_i)(-1)\frac{\partial(w_0 + w_1 x_{1,i} + \cdots + w_d x_{d,i} + \cdots + w_p x_{p,i})}{\partial w_d} + 2\lambda w_d = 0 \\
&= \sum_{i=1}^{n} 2(y_i - \hat{y}_i)(-1)\frac{\partial \hat{y}_i}{\partial w_d} + 2\lambda w_d = 0 \\
&= \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{\partial(y_i - \hat{y}_i)} \cdot \frac{(y_i - \hat{y}_i)}{\partial \hat{y}_i} \cdot \frac{\partial \hat{y}_i}{\partial w_d} + 2\lambda w_d = 0 \\
&= \sum_{i=1}^{n} \frac{\partial(y_i - \hat{y}_i)^2}{\partial w_d} + 2\lambda w_d = 0 \\
&= \sum_{i=1}^{n} \frac{\partial(y_i - \hat{y}_i)^2}{\partial w_d} + \lambda \frac{\partial \sum_{j=0}^{p} w_j^2}{\partial w_d} = 0 \\
&= \frac{\partial \sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\partial w_d} + \frac{\partial \lambda \sum_{j=0}^{p} w_j^2}{\partial w_d} = 0 \\
&= \frac{\partial}{\partial w_d}\left(\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \lambda \sum_{j=0}^{p} w_j^2\right) = 0 \\
&= \frac{\partial Cost}{\partial w_d} = 0
\end{aligned}
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