Stochastic Processes

CSE-5605

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Lecture 2

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Bernoulli Trial

- Experiment with only two outcomes
- Either success or failure
  - Flip a coin
  - Take a penalty shot on goal
  - Test a randomly selected circuit to see whether it is defective
  - Roll a die and determine whether it is a 6 or not
  - Determine whether there was flooding this year at Warsak

Source: *http://www.zweigmedia.com/RealWorld/Summary6.html*

The Binomial Probability Law

- Sequence of independent Bernoulli trials
  - k number of successes
  - n number of independent Bernoulli trials
- k successes in n trials
- Probabilities of k given by binomial probability law

$$p_n(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- Properties:
  - 2^n possible outcomes of experiments with n Bernoulli trials
  - Binomial probabilities sum to 1
- Graphical representation online tool

http://www.zweigmedia.com/RealWorld/stats/bernoulli.html

Example

- A communication system transmits binary information over a channel that introduces random bit errors with probability $\varepsilon = 0.1$. The transmitter transmits each information bit three times, and a decoder takes a majority vote of the received bits to decide on what the transmitted bit was. Find the probability that the receiver will make an incorrect decision.

Geometric Probability Law

- Independent Bernoulli trials till occurrence of first success
- Success probability p
- $m-1$ trials result in failure
- m^{th} trial results in success
- Probability of such event is

$$p(m) = P\left[A_1^c A_2^c \ldots A_{m-1}^c A_m\right] = (1-p)^{m-1} p$$

- If more than K trials required before a success
- Probability of performing at least K trials before a success

$$P[\{m > K\}] = q^K$$

Sequences of Dependent Experiments

- Sequence or "chain" of subexperiments
- Outcome of a given subexperiment determines
  - Which subexperiment is performed next

$$P[\{s_0\}\cap\{s_1\}\cap\{s_2\}] = P[\{s_2\}\,|\,\{s_0\}\cap\{s_1\}].P[\{s_0\}\cap\{s_1\}]$$

$$= P[\{s_2\}\,|\,\{s_0\}\cap\{s_1\}].P[\{s_1\}\,|\,\{s_0\}].P[\{s_0\}]$$

- If next subexperiment depends only on last outcome
  - The sequence is called "Markov chain"

$$P[\{s_0\}\cap\{s_1\}\cap\{s_2\}] = P[\{s_2\}\,|\,\{s_1\}].P[\{s_1\}\,|\,\{s_0\}].P[\{s_0\}]$$

Random Variable

- Random Variable or RV

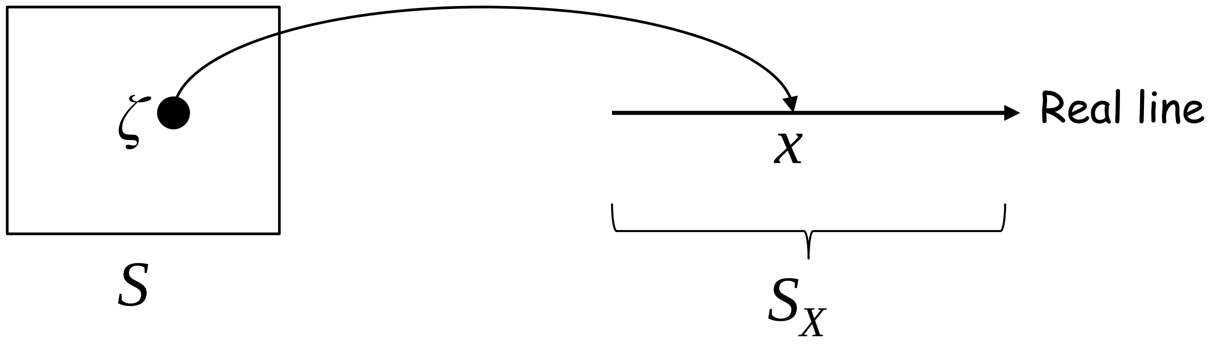
- "A function for assigning a number (numerical value) to each outcome of a random experiment"

- Outcome of random experiment not always a number
- Outcome has some measurement or numerical attribute
  - Interest in number related to outcome, called **value**

- Notations
  - Capital letters for RVs $(X, Y, \ldots)$
  - Small letters for values $(x, y, \ldots)$

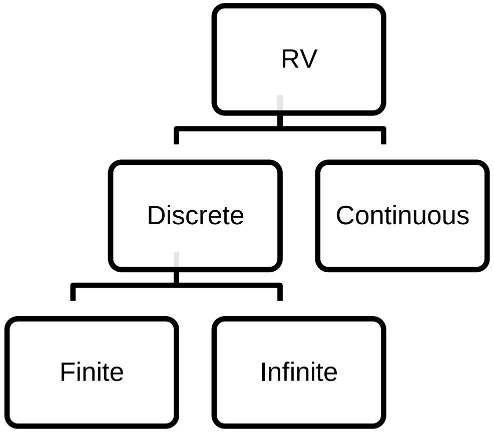
Random Variable (cont.)

- RV X assigns number $X(\zeta) = x$, to each outcome ζ in the sample space of a random experiment



Discrete and Finite RVs

- Discrete RV has only specific, isolated numerical values
  - Finite discrete RV has finite possible values
    - E.g. outcome of roll of a dice
  - Infinite discrete RV has unlimited number of values
    - E.g. number of stars in universe
- Continuous RV can have any values within a continuous range or an interval
  - E.g. temperature in lab 1, height of a person in cm



Source: *http://www.zweigmedia.com/ThirdEdSite/tutstats/frames8_1.html*

Probability Mass Function

- pmf of a discrete RV X is

$$p_X(x) = P[X = x] = P[\{\zeta : X(\zeta) = x\}]$$

- Properties

$$p_X(x) \geq 0$$

$$\sum_{x \in S_X} p_X(x) = 1$$

$$P[X \text{ in } B] = \sum_{x \in B} p_X(x) \text{ where } B \subset S_X$$