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Group choices

Every team (group) needs a way of making decisions that apply to the whole team. Sometimes the way to do this is to defer to an 'expert' opinion about which is the best of the options available or to follow the decision made by institutional authority. However, there are always decisions that are not, and cannot, be made outside the team. Hence, a properly structured team will have an **agreed way to make collective decisions**. Taking a vote is widely accepted and a well defined set of rules (system) for voting is needed and should be established before any decision needs to be made. (Such a set of rules is often termed a *constitution*.) The problem is how to identify the 'best' voting system for the team, as there is no single 'best' system for voting. As will be discussed, this depends on the sort of decisions to be made.

Unfortunately most of the texts which discuss voting do so in the context of elections. Groups rarely have elections (except for a Chair) compared to other decision-making needs. Also, a group is almost always concerned with identifying a single winning option, rather than multiple winners (as in some elections). In a few cases, most noticeably in selecting someone for a position, the group needs to express a collective ranking for all options being considered.

possible counting systems As a way of introducing the problems associated with voting, consider a simple case. Suppose a team of 15 needs to decide between 4 options w, x, y, z and members of the team have individual preferences expressed as follows:

7 people with $w > z > y > x$
4 people with $x > y > z > w$
3 people with $y > x > z > w$
1 person with $z > x > y > w$

The simplest system is **simple majority** (aka **first-past-the-post**). Option w with 7 first preferences outscores all of the others and so is given the highest collective ranking, i.e. is the team's choice.
However, you can see that 8 of 15 (more than half) ranked w last and that all other options would have beaten w in a two-way vote. In order to get an *absolute majority* when more than 2 options exist, a form of **preferential voting** is often used. In this case, z, with the lowest first preference score is eliminated, and its vote transferred to x, its next highest preference; likewise for y; x becomes the group's choice, winning 8 to 7 over w.
However, x would lose in a simple 2-way contest with either y or z. For this reason, other counting systems might also be used. The Marquis de **Condorcet** (1743-94), who was one

of the earliest to use formal mathematical methods to consider voting systems, specified that the system consider all the individual 2-way contests implicit in the preferences expressed and then count the number of such contests won by each option, as in a round-robin sporting competition. In the above case, $x > w, y > w, z > w, y > x, z > x, z > y$ meaning that z, which wins 3 contests, is the overall collective choice. The **Kemeny-Young** method, which was first examined in 1959, is a more elaborate method that takes account of the margins with which options would win the respective 2-way votes, and makes use of the increase available in computing power.
The **Borda count** might also be used. (This system is named for Jean-Charles de Borda (1733-99), despite being used for votes by ancient Rome's senate.) In a count with n options, a kth preference gets $(n-k)$ points and totalling the points gives the group's collective ranking. In our example, the scores are $w : 7 \times 3 = 21$, $x : \times 3 + 4 \times 2 = 20$, $y : 3 \times 3 + 4 \times 2 + 8 = 25$, and $z : 1 \times 3 + 7 \times 2 + 7 = 24$ so that the overall collective preference is y, followed by z.

Four different counting methods have given 4 different results. The latter two are more concerned with identifying a broadly supported option, closer to a consensus option.

Of course, a voting pattern such as that above would leave a team badly divided, whatever counting system is used.

approval voting An alternative to expressing preferences is to indicate whether an option is **acceptable or not**. Each voter indicates all options which are acceptable so that it becomes possible to identify the options that are at least acceptable to everyone. For example, in the votes shown above, those who listed w as last may have known something the others didn't and to them w was unacceptable. If w were eliminated, z would win on all ways of counting.
Approval votes are also a way of culling a list before final voting takes place.

"assessment" voting One can consider approval as assessing the options with a binary marking scale: 1 is 'in' or 0 is 'out.' A more complex system of voting (e.g. as used in marking your thesis!) involves each voter marking each option on a bigger scale, typically out of 10. The total marks are tallied, and the aggregate score determines the collective choice. Normalising an individual's scores often presents a practical difficulty. Nevertheless, this procedure is often used when a committee (team) judges a competition.

deadlock A voting system may not be able to decide the collective choice in all situations. This is the (Codorcet) *paradox of voting*:
Even if every person individually ranks all options, the collective ranking may remain indeterminate.
The simplest case showing this is two voters with reversed preferences, but it happens whenever a group expresses *cyclic/intransitive preferences*. In the case of the 15 voters

above, consider the situation with

5 people with $x > y > z > w$
5 people with $y > z > x > w$
5 people with $z > x > y > w$

No set of rules can distinguish between x, y, z. Such cyclic preferencing is a feature of the psychology of human choices.
Nevertheless, the team would need to make a decision and so this is why a voting system sometimes includes a *casting vote*, i.e. a designated person gets a special vote to break a deadlock. The person may be external to the group or someone with special status within it, e.g. the Chair.

vetos There is a role for **vetos**, which is when the vote of one person (or possibly subgroup) that, by itself, has the power to prevent an option from being chosen, irrespective of the preferences of all other voters. Often, in the engineering context, a veto may arise by default, when the rest of a team defers to the opinion of someone seen to have greater expertise (or maybe institutional authority).
A veto may be appropriate for someone, but only in certain circumstances: when an option specially affects that person or is related to his/her special expertise.
If everyone has a veto, then the situation is back to *consensus*.

some general theory In the latter half of the 20th century, *game theorists* used mathematical tools to explore the properties of systems for groups to make collective choices, i.e. to identify an agreed preference. Consider a group of people $\{A_1, \ldots A_n\}$ needing to choose its preferred outcome from a set of options $\{p_1, \ldots p_k\}$. Person α has a personal ranking $\ldots > p_\alpha > p_\beta \ldots$.

The first formal analysis was Condorcet's insisting that a system must satisfy what is now called the *Condorcet criterion*:
If an option beats every other one in pairwise comparison, i.e. $p_\alpha > p_\beta$ for all voters, then the collective ranking has $p_\alpha > p_\beta$.

Kenneth Arrow (1921-) proved that there is **no voting system** that satisfies all the desired criteria (his *impossibility theorem*), unless only 1 person votes (a **dictatorship** or only 2 options can ever be considered. The criteria are now codified as *Arrow's axioms*:

(i) The system must accept all possible preference arrangements. [*univeresal scope*]

(ii) If everyone individually puts $p_\alpha > p_\beta$, then the collective ranking is $p_\alpha > p_\beta$. [*unanimity*]

(iii) The collective ranking of options p_j, p_k depends only on the ndividual relative rankings of p_j, p_k.

(iv) For each pair of options p_j, p_k, a collective ranking exists, although it may be indifference meaning that they are ranked the same. [*completeness*]

(v) If the collective rankings are $p_j \geq p_k$ and $p_k \geq p_\ell$, then the collective choice is $p_j \geq p_\ell$. (*transitivity*)

Of course, remember that any voting system for a team also needs to be *quick and simple to use* and should be transparent enough for participating people to accept!

None of the above analysis examines how different voting systems can or cannot be manipulated. Generally, in the context of successful teamwork, the scope and desire to do this should be absent. The inappropriate use of decision-making systems is a matter of ethics. Borda himself observed[1] that voting is "designed only for honest people."

Finally, it of interest that we could use a voting system in our *engineering designs*. Remember that design usually wants conflicting criteria optimised and some trade-off is needed. If you consider each 'voter' to be a different criterion and its corresponding 'preferences' as the order of the design options, when assessed against this criterion, then a suitable voting system can be used to choose the overall best design.

refer: *Wikipedia* provides easy to read discussion of several *voting systems*, with extensive worked examples, but concentrating on implications for elections not teams (committees).
For those with an interest, more detailed/rigorous explanations exist, e.g. Blair & Pollak (1983), "Rational collective choice", *Sci Am.* Aug, p76.
The next level of treatment is for specialists (game theorists, psychologists, political scientists, etc).

[1] "Mon scrutin n'est fait que pour d'honnêtes gens." quoted in J. Mascart, D. Lieppe, É. Taillemite (2000), *La vie et les travaux du chevalier Jean-Charles de Borda (1733-1799)*, 2nd edn, p130.