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Patterns in high-frequency FX data: discovery of 12 empirical scaling laws

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We have discovered 12 independent new empirical scaling laws in foreign exchange data series that hold for close to three orders of magnitude and across 13 currency exchange rates. Our statistical analysis crucially depends on an event-based approach that measures the relationship between different types of events. The scaling laws give an accurate estimation of the length of the price-curve coastline, which turns out to be surprisingly long. The new laws substantially extend the catalogue of stylized facts and sharply constrain the space of possible theoretical explanations of the market mechanisms.

Keywords: Power laws; Foreign exchange markets; Empirical time series analysis; Financial time series

1. Introduction

The global financial system has recently been rocked by losses that could total four trillion USD (International Monetary Fund 2009). The crisis is seriously undermining the functioning of the financial system, the backbone of the global economy. This suggests an acute deficiency in our understanding of how markets work. Are there 'laws of nature' to be discovered in financial systems, giving us new insights? We approach this question by identifying key empirical patterns, namely scaling-law relations. We believe that these universal laws have the potential to significantly enhance our understanding of the markets.

Scaling laws establish invariance of scale and play an important role in describing complex systems (e.g., West *et al.* 1997, Barabási and Albert 1999 and Newman 2005). In finance, there is one scaling law that has been widely reported (Müller *et al.* 1990, Mantegna and Stanley 1995, Galluccio *et al.* 1997, Guillaume *et al.* 1997, Ballocchi *et al.* 1999, Dacorogna *et al.* 2001, Corsi *et al.* 2001, Di Matteo *et al.* 2005): the size of the average absolute price change (return) is scale-invariant to the time interval of its occurrence. This scaling law has been applied to risk management and volatility modeling (Ghashghaie *et al.* 1996, Sornette 2000, Gabaix *et al.* 2003, Di Matteo 2007) even though there has been no consensus amongst researchers for why the scaling law exists (e.g., Bouchaud 2001, Barndorff-Nielsen and Prause 2001, Doyne Farmer and Lillo 2004, Lux 2006 and Joulin *et al.* 2008).

In the challenge of identifying new scaling laws, we analyse the price data of the foreign exchange (FX) market, a complex network consisting of interacting agents: corporations, institutional and retail traders, and brokers trading through market makers, who themselves form an intricate web of interdependence. With a daily turnover of more than 3 trillion USD (Bank for International Settlements 2007) and with price changes nearly every second, the FX market offers a unique opportunity to analyze the functioning of a highly liquid, over-the-counter market that is not constrained by specific exchange-based rules. In this study we consider five years of tick-by-tick data for 13 exchange rates through November 2007 (see section 3.1 for a description of the data set).

It is a common occurrence for an exchange rate to move by 10 to 20% within a year. However, since the seminal work of Mandelbrot (1963) we know about the fractal nature of price curves. The coastline, roughly being the sum of all price moves of a given threshold, at fine levels of resolution may be far longer than one might intuitively think. But how many times longer? The scaling

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laws described in this paper provide a surprisingly accurate estimate and highlight the importance of not only considering tail events (Sornette 2002), but set these in perspective with the remarkably long coastline of price changes preceding them. It should be noted that our study is not related to the analysis of lead–lag relationships.

The remainder of the paper is organized as follows. Our main results are presented in section 2. We start by enumerating the empirical scaling laws, then cross-check our results by establishing quantitative relations amongst them and discuss the coastline. In section 3 the methods and the data are described and we conclude with some final remarks in section 4. Finally, appendix A contains tables with all the estimated scaling-law parameters.

2. The laws and beyond

2.1. The new scaling laws

Interest in scaling relations in FX data was sparked in 1990 by a seminal paper relating the mean absolute change of the logarithmic mid-prices, sampled at time intervals Δt over a sample of size $n\Delta t$, to the size of the time interval (Müller *et al.* 1990)

$$\langle|\Delta\chi|\rangle_p = \left(\frac{\Delta t}{C_\chi(p)}\right)^{E_\chi(p)}, \tag{0a}$$

where $\Delta\chi_i = \chi_i - \chi_{i-1}$, $\chi_i = \chi(t_i) = (\ln \mathrm{bid}_i + \ln \mathrm{ask}_i)/2$ is the logarithmic mid-price of a currency pair at time t_i, and $E_\chi(p)$ and $C_\chi(p)$ are the scaling-law parameters. The averaging operator is $\langle x\rangle_p = (1/n\sum_{j=1}^n x_j^p)^{1/p}$, usually with $p \in \{1, 2\}$, and p is omitted if equal to one. Note that for law (0a) the data is sampled at fixed time intervals $t_i = i\Delta t$. This requires a time interpolation scheme (described in section 3.1) which we will also employ when necessary in the following. Throughout the paper we consider a simpler definition of the price given by $x_i = (\mathrm{bid}_i + \mathrm{ask}_i)/2$ where price moves are defined as $\Delta x_i = (x_i - x_{i-1})/x_{i-1}$. Although the definition of x_i loses the mathematical feature of χ_i of behaving anti-symmetrically under price inversions (e.g. $\chi_i^{\mathrm{EUR-USD}} = -\chi_i^{\mathrm{USD-EUR}}$) it is more natural as, practically, percentages are more intuitive to manipulate than differences between logarithmic values. However, considering either χ_i or x_i leads to very similar results even for large spread values. Note that the logarithmic mid-price is equivalent to the logarithm of the geometric mean of the bid and ask prices $\chi_i = \ln\sqrt{\mathrm{bid}_i\ \mathrm{ask}_i}$, whereas x_i is the arithmetic mean.

Later, in 1997, a second scaling law was reported by Guillaume *et al.* (1997), relating the number $\mathsf{N}(\Delta\chi_{\mathrm{dc}})$ of so-called directional changes to the directional-change sizes $\Delta\chi_{\mathrm{dc}}$

$$\mathsf{N}(\Delta\chi_{\mathrm{dc}}) = \left(\frac{\Delta\chi_{\mathrm{dc}}}{C_{\mathsf{N},\mathrm{dc}}}\right)^{E_{\mathsf{N},\mathrm{dc}}}. \tag{0b}$$

In financial markets, the flow of time is discontinuous: over weekends trading comes to a standstill or, inversely, at news announcements there are spurts of market activity. In law (0a), the confinement of analysing returns as observed in physical time is overly restrictive. Law (0b) is a first attempt at establishing a new paradigm by looking beyond such constraints within financial data, constituting an event-driven approach, where patterns emerge for successions of events at different magnitudes. This alternative approach defines an activity-based time-scale called intrinsic time.

Extending this event-driven paradigm further enables us to observe new, stable patterns of scaling and reduces the level of complexity of real-world time series. In detail, the fixed event thresholds of different sizes define focal points, blurring out irrelevant details of the price evolution. Figure 1 depicts how the price curve is dissected into so-called directional-change and overshoot sections. The dissection algorithm measures occurrences of a price change Δx_{dc} from the last high or low (i.e. extrema), if it is in an up or down mode, respectively. At each occurrence of a directional change, the overshoot associated with the previous directional change is determined as the difference between the price level at which the last directional change occurred and the extrema, i.e. the high when in up mode or low when in down mode. The high and low price levels are then reset to the current price and the mode alternates. In section 3.2 the pseudocode for the directional-change count is provided in algorithm **2**.

Here we confirm laws (0a) and (0b) considering x_i (see figures 3(a)–(c)), and report on 12 new independent scaling laws holding across 13 exchange rates and for close to three orders of magnitude. Appendix A provides tables of the estimated parameter values for all the laws and for the 13 exchange rates as well as for a Gaussian random walk (GRW) model, described in section 3.1. In addition, every table lists the average parameter values over all 13 currency pairs and their sample standard deviations. Table 1 shows the estimated scaling-law parameters for EUR–USD. We start the enumeration of the laws by a generalization of equation (0b) that relates the average number of ticks observed during a price move of Δx to the size of this threshold

$$\langle\mathsf{N}(\Delta x_{\mathrm{tck}})\rangle = \left(\frac{\Delta x}{C_{\mathsf{N},\mathrm{tck}}}\right)^{E_{\mathsf{N},\mathrm{tck}}}, \tag{1}$$

where a tick is defined as a price move larger than (in absolute value) $\Delta x_{\mathrm{tck}} = 0.02\%$. The definition of a tick can, however, be altered without destroying the scaling-law relation. In essence, this law counts the average number of ticks observed during every price move Δx. Law (1) is plotted in figure 2. The second law counts the average yearly number $\mathsf{N}(\Delta x)$ of price moves of size Δx

$$\mathsf{N}(\Delta x) = \left(\frac{\Delta x}{C_{\mathsf{N},x}}\right)^{E_{\mathsf{N},x}}. \tag{2}$$

The computation of this law is provided in algorithm **1**. We annualize the number of observations of laws (0b) and (2) by dividing them by 5, the number of years in our data sample. Law (2) and all the following scaling laws are given in figure 3. The next scaling law relates the average maximal price range $\Delta x_{\max}$, defined as the difference

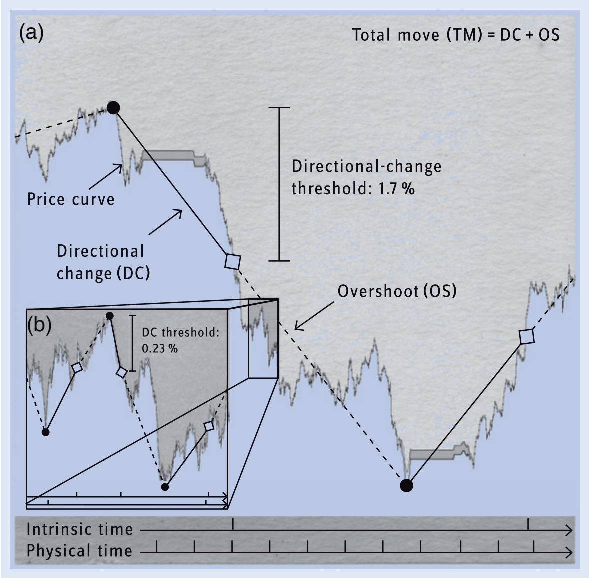

Figure 1. Projection of (a) a two-week and (b) a zoomed-in 36 hour price sample onto a reduced set of so-called directional-change events defined by a threshold: (a) $\Delta x_{dc} = 1.7\%$, (b) $\Delta x_{dc} = 0.23\%$. These directional-change events (diamonds) act as natural dissection points, decomposing a total-price move between two extremal price levels (bullets) into so-called directional-change (solid lines) and overshoot (dashed lines) sections. The directional-change computation is detailed in algorithm **2** of section 3.2. Note the increase of the spread size during the two weekends with no price activity. Time scales depict physical time ticking evenly across different price-curve activity regimes, whereas *intrinsic time* triggers only at directional-change events, independent of the notion of physical time.

between the high and low price levels, during a time interval Δt, to the size of that time interval

$$\langle \Delta x_{\max} \rangle_p = \left(\frac{\Delta t}{C_{\max}(p)} \right)^{E_{\max}(p)}, \tag{3}$$

where $\Delta x_{\max} = \max\{x(\tau);\ \ \tau \in [t - \Delta t; t]\} - \min\{x(\tau); \tau \in [t - \Delta t; t]\}$ and (3) holds for $p = 1, 2$.

We have also discovered laws relating the time during which events happen to the magnitude of these events. Law (4) relates the average time interval $\langle \Delta t_x \rangle$ for a price change of size Δx to occur to the size of the threshold

$$\langle \Delta t_x \rangle = \left(\frac{\Delta x}{C_{t,x}} \right)^{E_{t,x}}, \tag{4}$$

and, similarly, considering directional changes of threshold Δx_{dc}

$$\langle \Delta t_{dc} \rangle = \left(\frac{\Delta x_{dc}}{C_{t,dc}} \right)^{E_{t,dc}}. \tag{5}$$

Thus laws (4) and (5) relate the average numbers of seconds that elapse between consecutive price moves or directional changes, respectively.

Next we unveil a set of scaling laws emerging from the identification of directional-change events (see figure 1 and algorithm **2**) that make up the so-called total-move (TM) segments, which themselves decompose into directional-change (DC) and overshoot (OS) parts. The total price move, waiting time, and number of ticks can then be written as

$$\langle |\Delta x^{tm}| \rangle = \langle |\Delta x^{dc}| \rangle + \langle |\Delta x^{os}| \rangle, \tag{6}$$

$$\langle \Delta t^{tm} \rangle = \langle \Delta t^{dc} \rangle + \langle \Delta t^{os} \rangle, \tag{7}$$

$$\langle \mathsf{N}(\Delta x^{tm}_{tck}) \rangle = \langle \mathsf{N}(\Delta x^{dc}_{tck}) \rangle + \langle \mathsf{N}(\Delta x^{os}_{tck}) \rangle. \tag{8}$$

This decomposition leads to nine additional scaling laws, where the average values are functions of the directional-change thresholds Δx_{dc}

$$\langle |\Delta x^{*}| \rangle = \left(\frac{\Delta x_{dc}}{C_{x,*}} \right)^{E_{x,*}}, \tag{9}$$

$$\langle \Delta t^{*} \rangle = \left(\frac{\Delta x_{dc}}{C_{t,*}} \right)^{E_{t,*}}, \tag{10}$$

$$\langle \mathsf{N}(\Delta x^{*}_{tck}) \rangle = \left(\frac{\Delta x_{dc}}{C_{\mathsf{N},*}} \right)^{E_{\mathsf{N},*}}, \tag{11}$$

where $*$ stands for {tm, dc, os}. Note that $\langle |\Delta x^{dc}| \rangle = \Delta x_{dc}$ holds by construction. The actual deviation to $E_{x,dc} = 1$ and $C_{x,dc} = 1$, as seen in table A11, is given by the increasing noise for small thresholds, as the impact of the effect of a tick exceeding the exact threshold systematically overestimates $\langle |\Delta x^{dc}| \rangle$ (see figure 3(m)). The average parameter values (across the 13 currency pairs given in appendix A) of law (9) display a peculiar feature: on average, a directional change Δx_{dc} is followed by an overshoot of the same magnitude $\langle |\Delta x^{os}| \rangle \approx \Delta x_{dc}$ ($E^{av}_{x,os} \approx 1.04$ and $C^{av}_{x,os} \approx 1.06$), making the total move double the size of the directional-change threshold $\langle |\Delta x^{tm}| \rangle \approx 2\Delta x_{dc}$ ($E^{av}_{x,to} \approx 0.99$ and $C^{av}_{x,to} \approx 0.51$). This result is also found by computing the probable path of the price within a binomial tree as $0.5\Delta x + 0.5^2\, 2\Delta x + 0.5^3\, 3\Delta x + \cdots = \Delta x \sum_i^n i \xrightarrow{n \to \infty} 0.5^i 2\Delta x$. A similar feature holds for the waiting times and number of ticks: $\langle |\Delta t^{os}| \rangle \approx 2 \langle |\Delta t^{dc}| \rangle$ and $\langle \mathsf{N}(\Delta x^{os}_{tck}) \rangle \approx 2 \langle \mathsf{N}(\Delta x^{dc}_{tck}) \rangle$. So although in terms of size the overshoot price move is approximately as big as the direction-change threshold, it contains roughly twice as many ticks and takes twice as long to unfold.

Considering cumulative price moves instead of the averages in laws (9) leads to another triplet of laws

$$\Delta x^{*}_{cum} = \sum_{i=1}^{n} |\Delta x^{*}_i| = \left(\frac{\Delta x_{dc}}{C_{cum,*}} \right)^{E_{cum,*}}. \tag{12}$$

This concludes the presentation of 17 new scaling laws: we count equation (3) twice for $p = 1, 2$, and omit the trivial scaling law $\langle |\Delta x^{dc}| \rangle \propto \Delta x_{dc}$. In the next section, we will address the question of how many of these laws are independent and hence can be understood as primary laws. Our results show that most of the currency pairs exhibit similar average behavior. This does not, however, appear to be true in most of the laws for EUR–CHF, as seen in figure 3.

Table 1. Estimated scaling law parameter values considering EUR–USD.

Name	Equation	Table	E	C
Tick count	(1)	A2	1.93	$2.1 \cdot 10^{-2}$
Price move count	(2)	A3	−1.93	$9.5 \cdot 10^{0}$
Maximum price move	(3) ($p=1$)	A6	0.52	$1.9 \cdot 10^{5}$
Maximum price move	(3) ($p=2$)	A7	0.49	$1.3 \cdot 10^{5}$
Time of price move	(4)	A8	1.93	$1.2 \cdot 10^{-3}$
Time of directional change	(5)	A9	1.88	$1.1 \cdot 10^{-3}$
Total price move	(9)	A10	0.98	$4.9 \cdot 10^{-1}$
Overshoot move	(9)	A12	1.0	$9.9 \cdot 10^{-1}$
Time of total move	(10)	A13	1.89	$1.1 \cdot 10^{-3}$
Time of directional change	(10)	A14	1.85	$1.6 \cdot 10^{-3}$
Time of overshoot	(10)	A15	1.91	$1.4 \cdot 10^{-3}$
Total-move tick count	(11)	A16	1.89	$1.9 \cdot 10^{-2}$
Directional-change tick count	(11)	A17	2.02	$4.2 \cdot 10^{-2}$
Overshoot tick count	(11)	A18	1.87	$2.3 \cdot 10^{-2}$
Cumulative total move	(12)	A19	−0.94	$2.0 \cdot 10^{2}$
Cumulative total move with costs	(12)	A20	−0.98	$1.5 \cdot 10^{2}$
Cumulative directional change	(12)	A21	−0.95	$8.8 \cdot 10^{1}$
Cumulative overshoot	(12)	A22	−0.92	$1.1 \cdot 10^{2}$

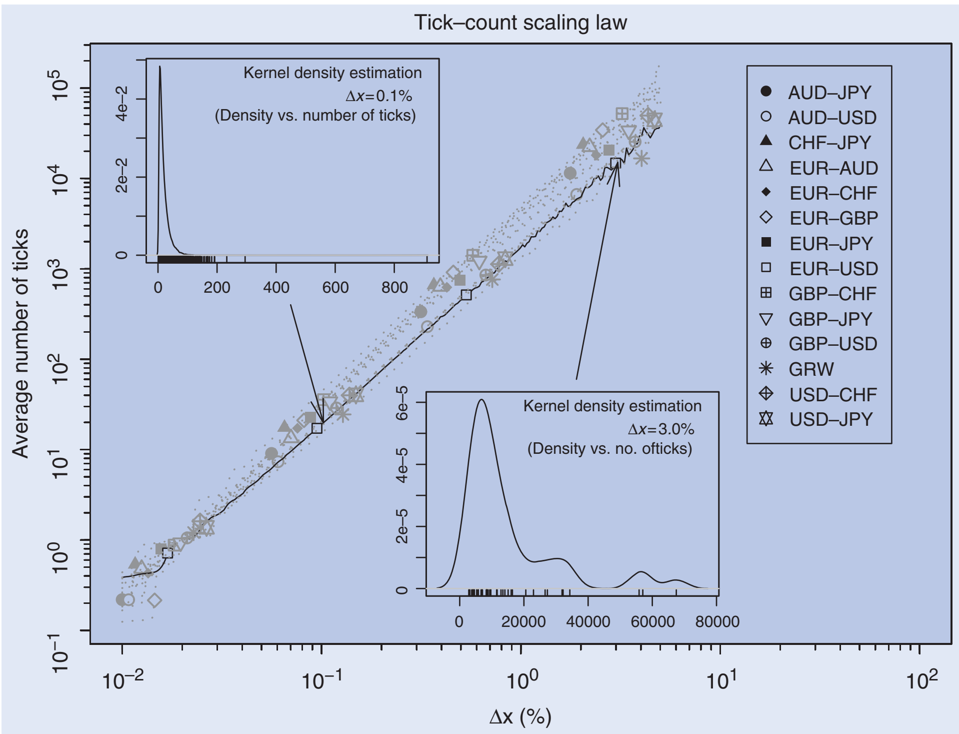

Figure 2. Scaling law (1) plotted where the x axis shows the price move thresholds of the observations and the y axis the average tick numbers. A tick is defined as a price move of 0.02%. The solid line shows the raw data for EUR–USD. For the remaining 12 currency pairs and the Gaussian random walk benchmark model the raw data is displayed with dots. Insets show the distribution of the EUR–USD observations (drawn above their x axis) for selected threshold values of 0.1% and 3.0%. See appendix A for the values of the estimated scaling-law parameters.

It is well known that the statistical properties of a GRW are different from those observed in empirical data (Mandelbrot and Hudson 2004). However, it is striking to observe how close this simple model can be to the average properties of the market data. Notable differences are seen in law (3) (see figures 3(e) and (f)), which reveal an unintuitive result: the bell-curve distribution of price moves leads to an average maximal price move that is roughly eight times larger than observed for the empirical data. It should also be noted that any realization of a GRW has an additional degree of freedom next to the specification of the price-move distribution $\Delta x_i \sim \mathcal{N}(\mu, \sigma^2)$, namely the distribution of the time intervals Δt. We employ unitary intervals Δt of one second (see section 3.1). It could therefore be argued that the additional deviations between the empirical data and

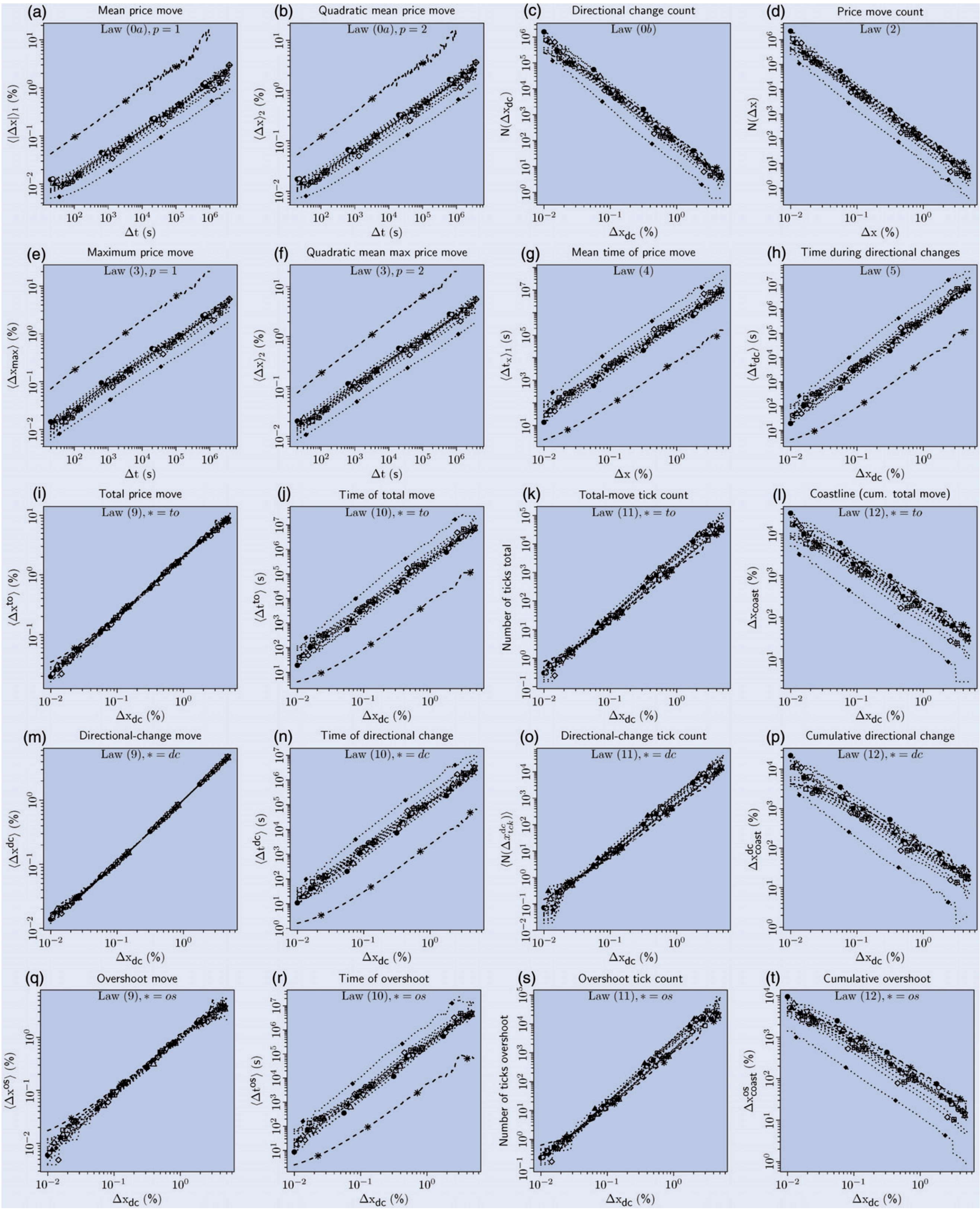

Figure 3. Plots of all scaling laws described in the text. Symbols are as in figure 2. The raw data is plotted for the 13 currency pairs with dots and for the Gaussian random walk model with dashes. See appendix A for the values of the estimated scaling-law parameters.

the GRW (seen in figures 3(a), (b), (g), (h), (j), (n), and (r)) are an artifact of this simplistic choice, as they are related to laws sensitive to the time-spacing of ticks.

It is often claimed that scaling laws with exponent 1/2 are expected for a GRW (Guillaume *et al.* 1997) and hence observing this in empirical data would comply with the efficient market hypothesis (Fama 1970). It should be noted, however, that this is only the case for law (0a) with $p = 2$, where the exponent of 1/2 can be analytically derived for a GRW. The value we actually measure is indeed 0.500 (see table A5). Yet for this law the real market data show an exponent averaging around 0.457. Furthermore, it is incorrect to assume an exponent of 1/2 (or 2) for the other laws. Indeed, our realizations of a GRW show exponents for all the laws but (0a) for $p=2$ to actually differ from 1/2 (and 2).

2.2. *Exploring the space of scaling laws*

The scaling laws do not represent isolated patterns but are in fact related to each other. We show that not only consistency requirements link the various laws, but that they can be combined to yield new scaling-law relations.

To cross-check the laws we first compare laws (2) and (4), and use the fact that the average price-move time equals the sample length divided by the number of observations

$$\langle \Delta t_x \rangle = Y/\mathsf{N}(\Delta x), \tag{13}$$

where Y is the number of seconds in a year. This implies that

$$E_{t,x} \leftrightarrow -E_{\mathsf{N},x}, \quad C_{t,x} \leftrightarrow Y^{1/E_{\mathsf{N},x}} C_{\mathsf{N},x}. \tag{14}$$

The estimated EUR–USD parameters from table 1 allow us to verify equation (14) as $E_{t,x}=1.93=-E_{\mathsf{N},x}$ and $C_{t,x}=1.23\cdot 10^{-3}=Y^{1/E_{\mathsf{N},x}}\ C_{\mathsf{N},x}$, where $Y=31{,}553{,}280$ seconds. Similarly, equivalent relations hold between laws (0b) and (5): $E_{t,\mathrm{dc}}=1.88\approx 1.91=-E_{\mathsf{N},\mathrm{dc}}$ and $C_{t,\mathrm{dc}}=1.05\cdot 10^{-3}\approx 1.11\cdot 10^{-3}=Y^{1/E_{\mathsf{N},\mathrm{dc}}}\ C_{\mathsf{N},\mathrm{dc}}$.

Furthermore, laws (0a) for $p=1$ and (4) are inverse relations of each other, implying

$$E_{t,x} \leftrightarrow -E_x^{-1}(1), \quad C_{t,x} \leftrightarrow C_x(1)^{-E_x(1)}. \tag{15}$$

We indeed have $E_{t,x}=1.93\approx 2.01=1/E_x(1)$ and $C_{t,x}=1.23\cdot 10^{-3}\approx 1.28\cdot 10^{-3}=C_x(1)^{-E_x(1)}$. There is a similar relationship to equation (13) that must hold for the cumulative and dissected average moves

$$\langle |\Delta x^*| \rangle = \Delta x^*_{\mathrm{cum}}/\mathsf{N}(\Delta x_{\mathrm{dc}}), \tag{16}$$

where the asterisk indicates $\{\mathrm{tm}, \mathrm{dc}, \mathrm{os}\}$. We find for the total move, considering EUR–USD, with a threshold of $\Delta x_{\mathrm{dc}}=0.1\%$, $\langle |\Delta x^{\mathrm{tm}}| \rangle = 0.2132 \approx 0.2129 = 1244.2/5843.4 = \Delta x^{\mathrm{tm}}_{\mathrm{cum}}/\mathsf{N}(\Delta x_{\mathrm{dc}})$. Similarly, considering the directional-change and the overshoot part results in errors smaller than 0.5%. In addition, $\Delta x^{\mathrm{tm}}_{\mathrm{cum}} = \Delta x^{\mathrm{dc}}_{\mathrm{cum}} + \Delta x^{\mathrm{os}}_{\mathrm{cum}}$ agrees to within an error of 0.4%. The discrepancies seen in the consistency checks are in line with the fitting errors (see appendix A) and give us confidence to proceed further in exploring the space of scaling laws.

The scaling laws can be assembled to produce additional laws. As an example, laws (0a) and (1) can be used to relate the average number of ticks to a time interval Δt

$$\langle \mathsf{N}(\Delta x_{\mathrm{tck}}) \rangle = \left(\frac{\Delta t}{C_x(1) C_{\mathsf{N},\mathrm{tck}}^{1/E_x(1)}} \right)^{E_x(1) E_{\mathsf{N},\mathrm{tck}}} = \left(\frac{\Delta t}{C_{t,\mathrm{tck}}} \right)^{E_{t,\mathrm{tck}}}, \tag{17}$$

where the empirical values lead to $E_{t,\mathrm{tck}}=0.96$ and $C_{t,\mathrm{tck}}=279$. This means that a tick is expected every 279 seconds. This expectation is compared with law (4), which indicates a move of 0.02% (i.e. a tick) every 258 seconds.

In summary, we have uncovered 18 novel empirical scaling-law relations. They provide a general self-consistent rendering of stochastic time series, and specifically give a glimpse of the bare bones of the market structure. There are 12 main independent scaling laws: (1), (2), (3) ($p=1$, 2), (9) (tm, os), (10) (tm, dc, os), and (11) (tm, dc, os). From these, plus laws (0a) and (0b), six additional laws can be derived: (4), (5), (12) (tm, dc, os), and the relation given in equation (17). Note that our classification is somewhat arbitrary, as, for example, law (9) (tm) could be derived from law (12) (tm).

2.3. *The coastline*

We now have the necessary tools in hand to come back to the measurement of the length of the coastline. The total-move scaling law (12) allows us to estimate its size as a function of the resolution defined by the directional-change threshold. Considering thresholds of 0.01%, 0.1%, 1% and 5%, one finds the average lengths of the annualized coastline to be 22,509%, 2046%, 186% and 34.8%, respectively. So by decreasing the threshold of the resolution 500-fold, the length of the coastline decreases by a factor of 650. Similarly, looking at the GRW we find 14,361%, 1946%, 264% and 65.2%, respectively. The 500-fold decrease in resolution entails a coastline decrease by a factor of only 220, highlighting the fact that GRW has fewer small moves and more middle-sized moves than the empirical price curves. Not surprisingly, taking transaction costs into account breaks the scaling law for small thresholds. However, it is still possible to evaluate the length of the coastline by employing the scaling relation for the interval [0.1%, 5%] and measuring it for 0.05%. Thus, for the thresholds 0.05%, 0.1%, 1% and 5% the new average coastline lengths are now 1604%, 1463%, 161% and 34.5%. For the 0.05% threshold (which occurs on average every 15 minutes), we measure an average daily move of 6.4%. The range of these average daily coastline lengths is from 1.8% for EUR–CHF to 9.1% for AUD–JPY.

3. Methods and data

3.1. *The data set*

We use a tick-by-tick database composed of 13 currency pairs spanning five years, from December 1, 2002 to December 1, 2007. The following currency pairs are considered with the total number of ticks given in parentheses: AUD–JPY (15,286,858), AUD–USD (7,037,203), CHF–JPY (17,081,987), GBP–CHF (27,141,146), GBP–JPY (26,423,199), GBP–USD (13,918,523), EUR–AUD (19,111,129), EUR–GBP (13,847,688), EUR–CHF (9,912,921), EUR–JPY (22,594,396), EUR–USD (13,093,081), USD–CHF (13,812,055) and USD–JPY (13,507,173). The difference in the number of ticks is due to the varying liquidity and the fact that some exchange rates are synthetically generated from two data streams, i.e. market makers quote prices based on two feeds creating cross-rates. As an example, GBP–JPY is derived from GBP–USD and USD–JPY. This, however, does not mean that the GBP–JPY data is artificial as it is also real published market data. The approximately 26.5 million ticks in the GBP–JPY cross-rate are derived from the ticks in GBP–USD (13.9 million) and USD–JPY (13.5 million).

The data are filtered as reoccurring ticks showing the same price as the last registered tick is omitted. As the timing of price quotes does not usually coincide with the fixed sampling times implied by the interval Δt, for all scaling laws proportional to a power of Δt we use an interpolation scheme that considers the last quoted price. To facilitate the reproduction of our results, we provide the USD–JPY and EUR–USD data sets that were used in our analysis at www.olsen.ch/more/datasets/.

In addition to the empirical data, a simple Gaussian random walk (GRW) model consisting of one million ticks is considered as a benchmark

$$\Delta x_i = x(t_i + \Delta t) - x(t_i) \sim \mathcal{N}(0, \sigma^2), \tag{18}$$

where $x_0 = 1.336723$, $\Delta t = 1$ second, and $\sigma = 1/6769.6$. This setting is arbitrarily chosen so as to mimic realistic FX behavior.

Algorithm 1: priceMoveCount(x)
Require: initialize variables ($n^{\uparrow} = n^{\downarrow} = 0$, $x^{\text{ext}} = x_0$, $\Delta x \geq 0$ fixed)

```
if (x − x^ext)/x^ext ≥ Δx then
  n↑ ← n↑ + 1
  x^ext ← x
else if (x − x^ext)/x^ext ≤ −Δx then
  n↓ ← n↓ + 1
  x^ext ← x
end if
```

Algorithm 2: ! directionalChangeCount(x)
Require: initialize variables ($n^{\uparrow} = n^{\downarrow} = 0$, $x^{\text{ext}} = x_0$, *mode* is up, $\Delta x_{\text{dc}} \geq 0$ fixed)

```
if mode is down then
  if x < x^ext then
    x^ext ← x
  else if (x − x^ext)/x^ext ≥ Δx then
    n↑ ← n↑ + 1
    x^ext ← x
    mode ← up
  end if
else if mode is up then
  if x > x^ext then
    x^ext ← x
  else if (x − x^ext)/x^ext ≤ −Δx then
    n↓ ← n↓ + 1
    x^ext ← x
    mode ← down
  end if
end if
```

3.2. Pseudocode

Algorithm **1** computes the number of price moves $n = n^{\uparrow} + n^{\downarrow}$ for a fixed percentage threshold Δx. Applying this computation to an array of thresholds yields law (2). Similarly, algorithm **2** counts the number of directional changes given a threshold Δx_{dc}. Performing this calculation for multiple thresholds recovers law (0b). Excluding law (3), the computation of all the other new scaling laws featured in this study relies on the detection of price moves and directional changes.

3.3. Data fitting

It is worth noting that we do not attempt to fit power-law distributions to empirical data (Clauset *et al.* 2007). We actually make no claim on how the data is distributed for each predefined point of observation. As seen in the insets of figure 2 it is unclear to what family of distributions they belong. Rather, we detect scaling-law relations for the average and cumulative values of various quantities uncovered in the empirical data.

We select 250 measurement points for laws proportional to price thresholds, and the range is from 0.01% to 5.05% in logarithmic steps: in log-space the difference of the threshold values is always 0.025. For laws depending on time intervals, there are 245 observations, and the range is from 20 to 3,975,783 seconds (which is 46 days, 23 minutes and 3 seconds). The logarithmic steps are always 0.05.

We assume a linear relationship between the response variable Y and the random variables X, or $Y = A + BX$, where A and B are the unknown parameters to be estimated. The actual fitting is done using the R programming language's† linear model function. In addition to the linear model, a quadratic model is tested as an alternative hypothesis, i.e. $Y = A + BX + CX^2$, to detect systematic curvature in the fitted data.

The tables provided in appendix A give the estimated scaling-law parameters for all 13 currency pairs and a GRW model, plus their errors. In addition, we report the adjusted R^2 values of the fits. For some of the plots a slight curvature can be discerned (e.g. for EUR–CHF in figures 3(a) and (b)). As scaling-law relations are identified from their linear dependence, we systematically test if a quadratic model yields a better fit. In the last column we check for any curvature by comparing the quadratic model's R^2 value to the previously reported linear value, i.e. $R^2_{\text{quad}} - R^2_{\text{lin}}$. The quadratic model mostly yields a marginally improved fit only for the GRW data. Finally, the last row shows the average parameter values for the currency pairs and in parentheses the sample standard deviations.

†www.r-project.org

From the linear model, it is straightforward to retrieve a scaling law relation:

$$y = \left(\frac{x}{C}\right)^{E}, \tag{19}$$

where $y = \exp Y$, $x = \exp X$, $E = B$, and $C = \exp(-A/B)$. To see how the error of C propagates, we assume $A, B \sim \mathcal{N}(\mu_{A,B}, \sigma^2_{A,B})$ and use the approximation

$$\mathrm{Var}[C] \approx \left(\frac{\partial f}{\partial A}\bigg|_{\mu_A,\mu_B} \sigma_A\right)^2 + \left(\frac{\partial f}{\partial B}\bigg|_{\mu_A,\mu_B} \sigma_B\right)^2, \tag{20}$$

where $f(A, B) = \exp(-A/B)$.

4. Conclusions

We have enlarged the catalogue of FX stylized facts by observing 12 independent new scaling laws, in addition to six secondary ones, holding for close to three orders of magnitude and across 13 currency pairs. Our analysis relies heavily on understanding the empirical time series as an event-based process, instead of focusing on their stochastic nature. The relationships amongst the scaling laws can be derived and their combinations yield new laws. Considering a 0.05% threshold, and taking costs into account, the coastline measures on average 6.4% *per day*. This is astonishingly long, and, to our knowledge, has not been reported in the literature. In contrast, on average across all currency pairs there is a mean maximal move of 0.60% to be observed within 24 hours (law (3), $E^{\mathrm{av}}_{\max}(1)$, $C^{\mathrm{av}}_{\max}(1)$), and on average it takes 220 days for a move of 6.4% to be measured (law (4)). This indicates the importance of considering not only the tail events associated with a crisis, but also accounting for the numerous smaller events that precede the event.

In finance, where frames of reference and fixed points are hard to come by and often illusory, the new scaling laws provide a reliable framework. We believe they can enhance our study of the dynamic behavior of markets and improve the quality of the inferences and predictions we make about the behavior of prices. The new laws represent the foundation of a completely new generation of tools for studying volatility, measuring risk, and creating better forecasting and trading models.

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Appendix A: Tables

Table A1. Directional change count, law (0b).

Currency	$E_{N,dc}$	$\Delta E_{N,dc}$	$C_{N,dc}$	$\Delta C_{N,dc}$	Adj. R^2	$R^2_{quad} - R^2_{lin}$
AUD–JPY	−2.046	±4.6e−03	1.116e+01	±8.3e−02	0.99877	2.775e−04
AUD–USD	−1.949	±4.4e−03	1.136e+01	±8.7e−02	0.99873	4.807e−04
CHF–JPY	−2.067	±4.7e−03	8.699e+00	±6.3e−02	0.99871	1.068e−04
EUR–AUD	−2.133	±5.8e−03	8.233e+00	±7.0e−02	0.99818	6.540e−05
EUR–CHF	−2.158	±5.4e−03	3.218e+00	±2.1e−02	0.99844	2.642e−04
EUR–GBP	−2.178	±7.6e−03	5.430e+00	±5.5e−02	0.99696	1.004e−03
EUR–JPY	−2.002	±2.9e−03	9.083e+00	±4.2e−02	0.99949	1.090e−04
EUR–USD	−1.908	±5.0e−03	9.422e+00	±8.1e−02	0.99827	1.031e−03
GBP–CHF	−2.131	±3.1e−03	6.406e+00	±2.7e−02	0.99949	4.152e−05
GBP–JPY	−2.017	±3.4e−03	9.440e+00	±5.1e−02	0.99931	−2.733e−06
GBP–USD	−1.904	±3.2e−03	8.947e+00	±4.8e−02	0.99931	2.562e−04
GRW	−1.797	±9.3e−03	1.528e+01	±2.8e−01	0.99337	5.137e−03
USD–CHF	−1.908	±3.3e−03	1.070e+01	±6.1e−02	0.99928	3.056e−04
USD–JPY	−1.928	±4.6e−03	9.841e+00	±7.7e−02	0.99857	7.382e−04
Currency average	−2.03	(1.0e−01)	8.61e+00	(2.3e+00)		

Table A2. Tick count, law (1), $\Delta x_{tck} = 0.02\%$.

Currency	$E_{N,tck}$	$\Delta E_{N,tck}$	$C_{N,tck}$	$\Delta C_{N,tck}$	Adj. R^2	$R^2_{quad} - R^2_{lin}$
AUD–JPY	2.051	±4.0e−03	1.879e−02	±1.7e−04	0.99906	4.957e−05
AUD–USD	1.970	±3.8e−03	2.146e−02	±1.9e−04	0.99906	−3.722e−06
CHF–JPY	2.085	±5.4e−03	1.663e−02	±2.0e−04	0.99833	1.392e−04
EUR–AUD	2.134	±4.2e−03	2.028e−02	±1.8e−04	0.99904	1.879e−04
EUR–CHF	2.120	±5.2e−03	2.053e−02	±2.3e−04	0.99848	7.283e−04
EUR–GBP	2.185	±9.4e−03	2.150e−02	±4.2e−04	0.99541	1.699e−03
EUR–JPY	1.997	±3.0e−03	1.864e−02	±1.3e−04	0.99946	1.960e−04
EUR–USD	1.928	±3.2e−03	2.099e−02	±1.6e−04	0.99933	1.827e−04
GBP–CHF	2.122	±3.0e−03	1.931e−02	±1.3e−04	0.99950	1.265e−04
GBP–JPY	2.027	±2.7e−03	1.920e−02	±1.2e−04	0.99955	−1.793e−06
GBP–USD	1.932	±2.2e−03	2.035e−02	±1.1e−04	0.99967	6.484e−05
GRW	1.864	±6.0e−03	2.112e−02	±3.1e−04	0.99740	1.838e−03
USD–CHF	1.945	±3.2e−03	2.027e−02	±1.5e−04	0.99932	2.199e−04
USD–JPY	1.975	±3.9e−03	2.206e−02	±2.0e−04	0.99901	4.497e−04
Currency average	2.04	(8.6e−02)	2.0e−02	(1.5e−03)		

Table A3. Price move count, law (2).

Currency	$E_{N,x}$	$\Delta E_{N,x}$	$C_{N,x}$	$\Delta C_{N,x}$	Adj. R^2	$R^2_{quad} - R^2_{lin}$
AUD–JPY	−2.051	±4.0e−03	1.111e+01	±7.2e−02	0.99907	4.533e−05
AUD–USD	−1.973	±3.8e−03	1.125e+01	±7.3e−02	0.99907	−2.901e−06
CHF–JPY	−2.092	±5.4e−03	8.095e+00	±6.5e−02	0.99837	8.115e−05
EUR–AUD	−2.135	±4.1e−03	7.912e+00	±4.8e−02	0.99907	1.727e−04
EUR–CHF	−2.138	±4.8e−03	3.171e+00	±1.9e−02	0.99875	3.725e−04
EUR–GBP	−2.187	±9.3e−03	5.141e+00	±6.3e−02	0.99547	1.638e−03
EUR–JPY	−2.001	±2.8e−03	9.137e+00	±4.1e−02	0.99951	1.469e−04
EUR–USD	−1.930	±3.2e−03	9.469e+00	±5.1e−02	0.99932	2.033e−04
GBP–CHF	−2.124	±3.0e−03	6.210e+00	±2.6e−02	0.99951	1.162e−04
GBP–JPY	−2.029	±2.8e−03	9.142e+00	±4.1e−02	0.99953	−3.668e−07
GBP–USD	−1.936	±2.3e−03	8.758e+00	±3.3e−02	0.99965	9.540e−05
GRW	−1.866	±6.1e−03	1.419e+01	±1.6e−01	0.99737	1.907e−03
USD–CHF	−1.946	±3.2e−03	1.022e+01	±5.6e−02	0.99931	2.329e−04
USD–JPY	−1.978	±4.0e−03	9.048e+00	±5.9e−02	0.99897	4.988e−04
Currency average	−2.04	(8.8e−02)	8.36e+00	(2.3e+00)		

Table A4. Mean price move during Δt, law (0a), $p = 1$.

Currency	$E_x(1)$	$\Delta E_x(1)$	$C_x(1)$	$\Delta C_x(1)$	Adj. R^2_{lin}	$R^2_{quad} - R^2_{lin}$
AUD–JPY	0.462	±1.3e−03	4.783e+05	±2.2e+04	0.99809	1.261e−03
AUD–USD	0.473	±1.8e−03	4.779e+05	±3.0e+04	0.99646	2.108e−03
CHF–JPY	0.461	±1.1e−03	7.962e+05	±3.1e+04	0.99872	6.185e−05
EUR–AUD	0.451	±1.3e−03	8.584e+05	±4.1e+04	0.99802	1.233e−03
EUR–CHF	0.450	±2.0e−03	6.362e+06	±5.1e+05	0.99538	3.115e−03
EUR–GBP	0.456	±1.8e−03	1.825e+06	±1.2e+05	0.99630	2.198e−03
EUR–JPY	0.483	±7.6e−04	6.899e+05	±1.8e+04	0.99940	1.606e−04
EUR–USD	0.497	±1.1e−03	6.632e+05	±2.5e+04	0.99875	7.131e−04
GBP–CHF	0.462	±8.8e−04	1.317e+06	±4.3e+04	0.99913	3.825e−04
GBP–JPY	0.476	±7.7e−04	6.637e+05	±1.8e+04	0.99936	2.521e−04
GBP–USD	0.496	±9.9e−04	7.517e+05	±2.5e+04	0.99903	5.506e−04
GRW	0.510	±3.1e−03	1.070e+04	±8.4e+02	0.99200	8.416e−04
USD–CHF	0.493	±9.6e−04	5.516e+05	±1.8e+04	0.99907	2.802e−04
USD–JPY	0.482	±9.3e−04	6.949e+05	±2.2e+04	0.99909	3.420e−04
Currency average	0.47	(1.7e−02)	1.24e+06	(1.6e+06)		

Table A5. Quadratic mean price move (historical volatility) during Δt, law (0a), $p = 2$.

Currency	$E_x(2)$	$\Delta E_x(2)$	$C_x(2)$	$\Delta C_x(2)$	Adj. R^2_{lin}	$R^2_{quad} - R^2_{lin}$
AUD–JPY	0.454	±9.5e−04	2.274e+05	±7.5e+03	0.99892	3.195e−04
AUD–USD	0.458	±1.2e−03	2.616e+05	±1.1e+04	0.99822	8.464e−04
CHF–JPY	0.449	±8.6e−04	4.225e+05	±1.3e+04	0.99910	−2.042e−06
EUR–AUD	0.441	±1.1e−03	4.467e+05	±1.8e+04	0.99852	8.995e−04
EUR–CHF	0.430	±1.4e−03	3.888e+06	±2.3e+05	0.99727	1.685e−03
EUR–GBP	0.440	±1.5e−03	1.018e+06	±5.7e+04	0.99730	1.473e−03
EUR–JPY	0.467	±6.3e−04	3.852e+05	±8.4e+03	0.99955	−1.412e−06
EUR–USD	0.473	±7.6e−04	3.843e+05	±1.0e+04	0.99936	6.888e−05
GBP–CHF	0.450	±6.3e−04	7.212e+05	±1.7e+04	0.99951	7.869e−05
GBP–JPY	0.463	±6.2e−04	3.588e+05	±7.8e+03	0.99956	5.572e−06
GBP–USD	0.475	±6.8e−04	4.399e+05	±1.0e+04	0.99950	9.130e−05
GRW	0.500	±2.3e−03	7.133e+03	±4.1e+02	0.99548	1.762e−05
USD–CHF	0.472	±7.6e−04	3.131e+05	±8.1e+03	0.99937	−1.410e−06
USD–JPY	0.463	±6.5e−04	3.912e+05	±8.9e+03	0.99952	3.957e−05
Currency average	0.46	(1.4e−02)	7.12e+05	(9.8e+05)		

Table A6. Maximal price move during Δt, law (3), $p = 1$.

Currency	$E_{max}(1)$	$\Delta E_{max}(1)$	$C_{max}(1)$	$\Delta C_{max}(1)$	Adj. R^2	$R^2_{quad} - R^2_{lin}$
AUD–JPY	0.479	±1.1e−03	9.743e+04	±3.4e+03	0.99867	5.190e−04
AUD–USD	0.511	±1.2e−03	1.106e+05	±3.9e+03	0.99867	8.429e−04
CHF–JPY	0.478	±7.1e−04	1.515e+05	±3.4e+03	0.99947	3.451e−04
EUR–AUD	0.464	±1.1e−03	1.523e+05	±5.4e+03	0.99872	3.016e−04
EUR–CHF	0.466	±4.2e−04	1.084e+06	±1.7e+04	0.99980	7.757e−06
EUR–GBP	0.467	±4.6e−04	3.410e+05	±5.4e+03	0.99977	4.654e−06
EUR–JPY	0.495	±7.8e−04	1.572e+05	±3.8e+03	0.99939	2.617e−04
EUR–USD	0.521	±1.1e−03	1.676e+05	±5.7e+03	0.99884	8.795e−04
GBP–CHF	0.469	±7.2e−04	2.528e+05	±6.1e+03	0.99942	1.830e−04
GBP–JPY	0.487	±9.5e−04	1.426e+05	±4.3e+03	0.99908	3.303e−04
GBP–USD	0.522	±1.2e−03	1.873e+05	±6.5e+03	0.99880	8.679e−04
GRW	0.513	±1.0e−03	2.996e+03	±7.2e+01	0.99914	6.057e−05
USD–CHF	0.516	±1.1e−03	1.355e+05	±4.4e+03	0.99893	8.976e−04
USD–JPY	0.508	±1.2e−03	1.579e+05	±5.7e+03	0.99865	1.020e−03
Currency average	0.49	(2.2e−02)	2.41e+05	(2.6e+05)		

Table A7. Quadratic maximal price move during Δt, law (3), $p = 2$.

Currency	$E_{\max}(2)$	$\Delta E_{\max}(2)$	$C_{\max}(2)$	$\Delta C_{\max}(2)$	Adj. R^2	$R^2_{\text{quad}} - R^2_{\text{lin}}$
AUD–JPY	0.470	±7.5e−04	6.850e+04	±1.6e+03	0.99939	2.267e−04
AUD–USD	0.486	±7.5e−04	8.513e+04	±2.0e+03	0.99942	2.839e−04
CHF–JPY	0.466	±6.0e−04	1.198e+05	±2.4e+03	0.99959	2.618e−04
EUR–AUD	0.450	±6.7e−04	1.191e+05	±2.7e+03	0.99947	2.459e−05
EUR–CHF	0.446	±3.8e−04	9.439e+05	±1.4e+04	0.99982	2.594e−06
EUR–GBP	0.450	±4.0e−04	2.806e+05	±4.0e+03	0.99980	3.721e−06
EUR–JPY	0.481	±6.4e−04	1.229e+05	±2.5e+03	0.99957	2.387e−04
EUR–USD	0.494	±1.1e−03	1.337e+05	±4.4e+03	0.99887	8.416e−04
GBP–CHF	0.456	±5.6e−04	2.050e+05	±3.9e+03	0.99963	1.333e−04
GBP–JPY	0.475	±7.4e−04	1.106e+05	±2.6e+03	0.99940	2.845e−04
GBP–USD	0.495	±9.7e−04	1.529e+05	±4.6e+03	0.99907	6.398e−04
GRW	0.510	±1.0e−03	2.739e+03	±6.5e+01	0.99914	1.415e−04
USD–CHF	0.493	±1.1e−03	1.080e+05	±3.8e+03	0.99871	1.042e−03
USD–JPY	0.485	±1.0e−03	1.259e+05	±4.1e+03	0.99888	8.555e−04
Currency average	0.4728	(1.8e−0)	1.98e+05	(2.3e+05)		

Table A8. Time of price move, law (4).

Currency	$E_{t,x}$	$\Delta E_{t,x}$	$C_{t,x}$	$\Delta C_{t,x}$	Adj. R^2	$R^2_{\text{quad}} - R^2_{\text{lin}}$
AUD–JPY	2.051	±4.0e−03	2.477e−03	±3.1e−05	0.99906	4.705e−05
AUD–USD	1.972	±3.8e−03	1.774e−03	±2.3e−05	0.99907	−3.584e−06
CHF–JPY	2.088	±5.4e−03	2.109e−03	±3.6e−05	0.99835	1.145e−04
EUR–AUD	2.134	±4.2e−03	2.451e−03	±3.1e−05	0.99905	1.803e−04
EUR–CHF	2.125	±5.1e−03	9.598e−04	±1.7e−05	0.99858	6.277e−04
EUR–GBP	2.184	±9.4e−03	1.905e−03	±5.5e−05	0.99538	1.725e−03
EUR–JPY	1.999	±2.9e−03	1.627e−03	±1.6e−05	0.99947	1.794e−04
EUR–USD	1.928	±3.2e−03	1.227e−03	±1.4e−05	0.99933	1.853e−04
GBP–CHF	2.123	±3.0e−03	1.825e−03	±1.7e−05	0.99951	1.237e−04
GBP–JPY	2.028	±2.7e−03	1.841e−03	±1.7e−05	0.99954	−1.336e−06
GBP–USD	1.932	±2.2e−03	1.162e−03	±9.5e−06	0.99967	6.721e−05
GRW	1.864	±6.0e−03	8.640e−03	±1.5e−04	0.99740	1.843e−03
USD–CHF	1.945	±3.2e−03	1.428e−03	±1.6e−05	0.99932	2.213e−04
USD–JPY	1.977	±4.0e−03	1.461e−03	±2.0e−05	0.99898	4.814e−04
Currency average	2.04	(8.6e−02)	1.71e−03	(4.7e−04)		

Table A9. Time between directional changes, law (5).

Currency	$E_{t,\text{dc}}$	$\Delta E_{t,\text{dc}}$	$C_{t,\text{dc}}$	$\Delta C_{t,\text{dc}}$	Adj. R^2	$R^2_{\text{quad}} - R^2_{\text{lin}}$
AUD–JPY	2.046	±4.5e−03	2.439e−03	±3.5e−05	0.99877	2.749e−04
AUD–USD	1.948	±4.4e−03	1.608e−03	±2.5e−05	0.99873	4.641e−04
CHF–JPY	2.063	±4.8e−03	2.053e−03	±3.2e−05	0.99863	1.438e−04
EUR–AUD	2.132	±5.8e−03	2.531e−03	±4.4e−05	0.99817	6.042e−05
EUR–CHF	2.111	±6.8e−03	9.819e−04	±2.3e−05	0.99741	1.586e−03
EUR–GBP	2.162	±8.3e−03	1.907e−03	±4.9e−05	0.99635	1.536e−03
EUR–JPY	2.001	±2.9e−03	1.629e−03	±1.6e−05	0.99947	1.180e−04
EUR–USD	1.884	±3.9e−03	1.050e−03	±1.6e−05	0.99893	3.450e−04
GBP–CHF	2.127	±3.2e−03	1.926e−03	±1.9e−05	0.99944	7.375e−05
GBP–JPY	2.016	±3.4e−03	1.804e−03	±2.0e−05	0.99930	−2.835e−06
GBP–USD	1.899	±3.2e−03	1.020e−03	±1.3e−05	0.99928	1.643e−04
GRW	1.790	±9.1e−03	6.953e−03	±1.9e−04	0.99361	4.807e−03
USD–CHF	1.904	±3.1e−03	1.247e−03	±1.5e−05	0.99932	2.326e−04
USD–JPY	1.927	±4.6e−03	1.266e−03	±2.1e−05	0.99857	7.180e−04
Currency average	2.02	(9.8e−02)	1.65e−03	(5.2e−04)		

Table A10. Total price move, law (9), $* = \mathrm{tm}$.

Currency	$E_{x,\mathrm{tm}}$	$\Delta E_{x,\mathrm{tm}}$	$C_{x,\mathrm{tm}}$	$\Delta C_{x,\mathrm{tm}}$	Adj. R^2	$R^2_{\mathrm{quad}} - R^2_{\mathrm{lin}}$
AUD–JPY	1.001	±2.3e−03	4.998e−01	±2.8e−03	0.99872	4.740e−04
AUD–USD	0.990	±2.0e−03	4.876e−01	±2.4e−03	0.99899	2.066e−04
CHF–JPY	0.995	±2.3e−03	5.211e−01	±2.9e−03	0.99868	2.848e−05
EUR–AUD	0.997	±2.4e−03	5.232e−01	±3.1e−03	0.99851	3.814e−04
EUR–CHF	1.006	±2.3e−03	5.299e−01	±3.0e−03	0.99865	2.258e−04
EUR–GBP	1.004	±3.0e−03	5.388e−01	±3.9e−03	0.99782	2.652e−04
EUR–JPY	1.001	±1.3e−03	5.001e−01	±1.6e−03	0.99956	6.695e−06
EUR–USD	0.976	±1.7e−03	4.871e−01	±2.1e−03	0.99921	2.324e−04
GBP–CHF	0.993	±1.6e−03	5.358e−01	±2.2e−03	0.99932	4.097e−05
GBP–JPY	0.996	±1.5e−03	5.039e−01	±1.9e−03	0.99940	2.969e−05
GBP–USD	0.981	±1.2e−03	4.885e−01	±1.5e−03	0.99963	3.725e−05
GRW	0.943	±3.4e−03	4.708e−01	±4.2e−03	0.99670	2.060e−03
USD–CHF	0.973	±1.4e−03	4.984e−01	±1.7e−03	0.99950	−1.570e−06
USD–JPY	0.969	±1.8e−03	5.028e−01	±2.3e−03	0.99913	1.081e−04
Currency average	0.99	(1.2e−02)	5.10e−01	(1.9e−02)		

Table A11. Directional-change move, law (9), $* = \mathrm{dc}$.

Currency	$E_{x,\mathrm{dc}}$	$\Delta E_{x,\mathrm{dc}}$	$C_{x,\mathrm{dc}}$	$\Delta C_{x,\mathrm{dc}}$	Adj. R^2	$R^2_{\mathrm{quad}} - R^2_{\mathrm{lin}}$
AUD–JPY	0.941	±2.5e−03	9.847e−01	±6.1e−03	0.99828	1.033e−03
AUD–USD	0.937	±2.4e−03	9.814e−01	±5.9e−03	0.99838	9.673e−04
CHF–JPY	0.949	±2.6e−03	9.898e−01	±6.3e−03	0.99816	8.473e−04
EUR–AUD	0.946	±2.0e−03	9.843e−01	±5.0e−03	0.99884	8.851e−04
EUR–CHF	0.964	±1.5e−03	9.907e−01	±3.7e−03	0.99937	3.488e−04
EUR–GBP	0.948	±2.6e−03	9.827e−01	±6.2e−03	0.99818	5.941e−04
EUR–JPY	0.961	±1.7e−03	9.921e−01	±4.0e−03	0.99926	4.974e−04
EUR–USD	0.959	±1.9e−03	9.918e−01	±4.6e−03	0.99902	6.268e−04
GBP–CHF	0.961	±1.6e−03	9.918e−01	±3.8e−03	0.99934	4.941e−04
GBP–JPY	0.959	±1.7e−03	9.926e−01	±4.1e−03	0.99922	5.991e−04
GBP–USD	0.961	±1.5e−03	9.918e−01	±3.6e−03	0.99940	4.778e−04
GRW	0.937	±2.6e−03	9.943e−01	±6.5e−03	0.99805	1.656e−03
USD–CHF	0.958	±1.9e−03	9.913e−01	±4.6e−03	0.99901	6.267e−04
USD–JPY	0.954	±2.3e−03	9.934e−01	±5.6e−03	0.99858	8.996e−04
Currency average	0.95	(8.6e−03)	9.89e−01	(4.2e−03)		

Table A12. Overshoot move, law (9), $* = \mathrm{os}$.

Currency	$E_{x,\mathrm{os}}$	$\Delta E_{x,\mathrm{os}}$	$C_{x,\mathrm{os}}$	$\Delta C_{x,\mathrm{os}}$	Adj. R^2	$R^2_{\mathrm{quad}} - R^2_{\mathrm{lin}}$
AUD–JPY	1.081	±2.5e−03	1.012e+00	±5.5e−03	0.99865	−4.901e−06
AUD–USD	1.060	±2.8e−03	9.788e−01	±6.1e−03	0.99823	1.273e−04
CHF–JPY	1.064	±4.9e−03	1.098e+00	±1.2e−02	0.99478	3.123e−03
EUR–AUD	1.066	±3.8e−03	1.121e+00	±9.3e−03	0.99692	−9.706e−06
EUR–CHF	1.071	±6.0e−03	1.118e+00	±1.5e−02	0.99228	3.963e−03
EUR–GBP	1.103	±7.4e−03	1.164e+00	±1.8e−02	0.98905	7.059e−03
EUR–JPY	1.052	±2.9e−03	1.004e+00	±6.4e−03	0.99815	1.110e−03
EUR–USD	0.996	±2.2e−03	9.879e−01	±5.1e−03	0.99880	1.930e−05
GBP–CHF	1.041	±4.0e−03	1.168e+00	±1.1e−02	0.99629	2.002e−03
GBP–JPY	1.042	±2.7e−03	1.027e+00	±6.3e−03	0.99830	3.072e−04
GBP–USD	1.003	±2.0e−03	9.871e−01	±4.6e−03	0.99900	1.002e−04
GRW	0.945	±4.7e−03	9.812e−01	±1.1e−02	0.99390	2.167e−03
USD–CHF	0.990	±2.8e−03	1.045e+00	±7.0e−03	0.99798	9.718e−04
USD–JPY	0.987	±2.6e−03	1.067e+00	±6.6e−03	0.99825	1.625e−04
Currency average	1.04	(3.8e−02)	1.06e+00	(6.8e−02)		

Table A13. Time of total move, law (10), $*=\text{tm}$.

Currency	$E_{t,\text{tm}}$	$\Delta E_{t,\text{tm}}$	$C_{t,\text{tm}}$	$\Delta C_{t,\text{tm}}$	Adj. R^2	$R^2_{\text{quad}} - R^2_{\text{lin}}$
AUD–JPY	2.049	±4.6e−03	2.450e−03	±3.5e−05	0.99876	3.127e−04
AUD–USD	1.950	±4.5e−03	1.616e−03	±2.5e−05	0.99870	5.183e−04
CHF–JPY	2.066	±4.8e−03	2.063e−03	±3.1e−05	0.99868	1.143e−04
EUR–AUD	2.133	±5.8e−03	2.534e−03	±4.4e−05	0.99816	6.282e−05
EUR–CHF	2.103	±8.5e−03	9.655e−04	±2.9e−05	0.99594	2.089e−03
EUR–GBP	2.167	±8.1e−03	1.926e−03	±4.8e−05	0.99653	1.311e−03
EUR–JPY	2.004	±2.8e−03	1.638e−03	±1.6e−05	0.99952	8.980e−05
EUR–USD	1.886	±3.9e−03	1.053e−03	±1.6e−05	0.99892	3.705e−04
GBP–CHF	2.124	±3.3e−03	1.919e−03	±2.0e−05	0.99941	8.651e−05
GBP–JPY	2.018	±3.4e−03	1.809e−03	±2.0e−05	0.99931	−2.651e−06
GBP–USD	1.902	±3.2e−03	1.026e−03	±1.3e−05	0.99928	1.987e−04
GRW	1.791	±9.3e−03	6.967e−03	±2.0e−04	0.99338	4.870e−03
USD–CHF	1.898	±3.1e−03	1.229e−03	±1.4e−05	0.99934	1.430e−04
USD–JPY	1.925	±4.6e−03	1.260e−03	±2.1e−05	0.99857	6.307e−04
Currency average	2.02	(9.8e−02)	1.65e−03	(5.3e−04)		

Table A14. Time of directional change, law (10), $*=\text{dc}$.

Currency	$E_{t,\text{dc}}$	$\Delta E_{t,\text{dc}}$	$C_{t,\text{dc}}$	$\Delta C_{t,\text{dc}}$	Adj. R^2	$R^2_{\text{quad}} - R^2_{\text{lin}}$
AUD–JPY	1.996	±4.5e−03	3.654e−03	±5.0e−05	0.99875	2.448e−04
AUD–USD	1.871	±4.1e−03	2.280e−03	±3.2e−05	0.99882	4.129e−04
CHF–JPY	2.015	±5.5e−03	3.096e−03	±5.3e−05	0.99812	5.073e−04
EUR–AUD	2.102	±4.7e−03	3.789e−03	±5.1e−05	0.99875	1.251e−04
EUR–CHF	2.066	±7.1e−03	1.463e−03	±3.5e−05	0.99709	2.023e−03
EUR–GBP	2.135	±6.3e−03	2.838e−03	±5.3e−05	0.99786	4.578e−04
EUR–JPY	1.941	±4.1e−03	2.415e−03	±3.3e−05	0.99887	6.149e−04
EUR–USD	1.846	±3.4e−03	1.636e−03	±2.1e−05	0.99915	3.813e−05
GBP–CHF	2.099	±3.9e−03	2.874e−03	±3.4e−05	0.99914	1.035e−04
GBP–JPY	1.958	±3.1e−03	2.626e−03	±2.7e−05	0.99938	1.054e−04
GBP–USD	1.866	±2.4e−03	1.596e−03	±1.4e−05	0.99960	6.737e−05
GRW	1.774	±9.8e−03	1.235e−02	±3.4e−04	0.99240	6.439e−03
USD–CHF	1.888	±2.8e−03	2.016e−03	±2.0e−05	0.99945	5.768e−05
USD–JPY	1.914	±4.1e−03	2.057e−03	±2.9e−05	0.99885	5.935e−04
Currency average	1.98	(1.0e−01)	2.49e−03	(7.5e−04)		

Table A15. Time of overshoot, law (10), $*=\text{os}$.

Currency	$E_{t,\text{os}}$	$\Delta E_{t,\text{os}}$	$C_{t,\text{os}}$	$\Delta C_{t,\text{os}}$	Adj. R^2	$R^2_{\text{quad}} - R^2_{\text{lin}}$
AUD–JPY	2.079	±5.4e−03	3.241e−03	±5.2e−05	0.99831	2.681e−04
AUD–USD	1.997	±5.5e−03	2.273e−03	±4.1e−05	0.99810	3.617e−04
CHF–JPY	2.093	±5.8e−03	2.698e−03	±4.7e−05	0.99812	4.505e−05
EUR–AUD	2.150	±6.8e−03	3.274e−03	±6.4e−05	0.99750	2.799e−05
EUR–CHF	2.119	±1.0e−02	1.234e−03	±4.2e−05	0.99441	2.222e−03
EUR–GBP	2.190	±1.1e−02	2.519e−03	±7.8e−05	0.99420	2.342e−03
EUR–JPY	2.035	±3.1e−03	2.182e−03	±2.1e−05	0.99944	4.337e−06
EUR–USD	1.906	±5.2e−03	1.411e−03	±2.7e−05	0.99816	6.454e−04
GBP–CHF	2.136	±4.9e−03	2.469e−03	±3.7e−05	0.99869	1.094e−04
GBP–JPY	2.049	±4.4e−03	2.435e−03	±3.4e−05	0.99884	2.919e−05
GBP–USD	1.921	±4.4e−03	1.372e−03	±2.2e−05	0.99871	2.923e−04
GRW	1.796	±9.6e−03	8.900e−03	±2.5e−04	0.99295	3.823e−03
USD–CHF	1.903	±4.0e−03	1.597e−03	±2.3e−05	0.99891	2.088e−04
USD–JPY	1.930	±5.5e−03	1.624e−03	±3.2e−05	0.99796	6.314e−04
Currency average	2.04	(1.0e−01)	2.18e−03	(6.9e−04)		

Table A16. Total-move tick count, law (11), $* = \text{tm}$.

Currency	$E_{N,tm}$	$\Delta E_{N,tm}$	$C_{N,tm}$	$\Delta C_{N,tm}$	Adj. R^2	$R^2_{quad} - R^2_{lin}$
AUD–JPY	2.048	±4.6e−03	1.861e−02	±1.9e−04	0.99877	3.049e−04
AUD–USD	1.949	±4.4e−03	2.008e−02	±2.1e−04	0.99873	4.784e−04
CHF–JPY	2.063	±4.9e−03	1.661e−02	±1.8e−04	0.99862	1.473e−04
EUR–AUD	2.131	±5.8e−03	2.098e−02	±2.6e−04	0.99813	5.135e−05
EUR–CHF	2.121	±6.3e−03	2.166e−02	±2.9e−04	0.99778	1.058e−03
EUR–GBP	2.177	±7.7e−03	2.225e−02	±3.5e−04	0.99687	9.997e−04
EUR–JPY	2.003	±2.8e−03	1.864e−02	±1.2e−04	0.99950	1.014e−04
EUR–USD	1.893	±4.1e−03	1.931e−02	±1.9e−04	0.99884	5.552e−04
GBP–CHF	2.121	±3.4e−03	2.024e−02	±1.5e−04	0.99937	1.176e−04
GBP–JPY	2.016	±3.4e−03	1.909e−02	±1.5e−04	0.99927	−2.765e−06
GBP–USD	1.902	±3.2e−03	1.883e−02	±1.4e−04	0.99931	2.122e−04
GRW	1.792	±9.3e−03	1.767e−02	±4.3e−04	0.99338	4.899e−03
USD–CHF	1.898	±3.0e−03	1.866e−02	±1.4e−04	0.99937	1.604e−04
USD–JPY	1.922	±4.6e−03	2.047e−02	±2.2e−04	0.99858	5.640e−04
Currency average	2.02	(1.0e−02)	1.97e−02	(1.5e−03)		

Table A17. Directional-change tick count, law (11), $* = \text{dc}$.

Currency	$E_{N,dc}$	$\Delta E_{N,dc}$	$C_{N,dc}$	$\Delta C_{N,dc}$	Adj. R^2	$R^2_{quad} - R^2_{lin}$
AUD–JPY	2.002	±7.5e−03	2.974e−02	±4.7e−04	0.99647	7.871e−04
AUD–USD	1.904	±7.8e−03	3.375e−02	±5.7e−04	0.99583	2.323e−04
CHF–JPY	2.009	±8.1e−03	2.652e−02	±4.6e−04	0.99600	−4.305e−06
EUR–AUD	2.128	±4.6e−03	3.406e−02	±3.0e−04	0.99884	9.624e−06
EUR–CHF	2.198	±1.0e−02	3.896e−02	±7.4e−04	0.99442	2.203e−03
EUR–GBP	2.205	±1.5e−02	3.748e−02	±1.0e−03	0.98832	1.535e−03
EUR–JPY	2.040	±9.4e−03	3.465e−02	±6.6e−04	0.99471	1.755e−03
EUR–USD	2.017	±1.2e−02	4.187e−02	±1.0e−03	0.99088	2.739e−03
GBP–CHF	2.181	±6.4e−03	3.548e−02	±4.2e−04	0.99786	7.460e−04
GBP–JPY	2.048	±5.2e−03	3.436e−02	±3.6e−04	0.99839	5.759e−04
GBP–USD	1.985	±7.0e−03	3.782e−02	±5.3e−04	0.99694	9.365e−04
GRW	1.809	±6.6e−03	3.350e−02	±5.0e−04	0.99669	2.568e−03
USD–CHF	1.991	±9.4e−03	3.772e−02	±7.2e−04	0.99451	7.606e−04
USD–JPY	2.052	±9.9e−03	4.266e−02	±8.1e−04	0.99427	9.073e−04
Currency average	2.06	(9.2e−02)	3.58e−02	(4.5e−03)		

Table A18. Overshoot tick count, law (11), $* = \text{os}$.

Currency	$E_{N,os}$	$\Delta E_{N,os}$	$C_{N,os}$	$\Delta C_{N,os}$	Adj. R^2	$R^2_{quad} - R^2_{lin}$
AUD–JPY	2.081	±4.7e−03	2.395e−02	±2.4e−04	0.99873	5.969e−05
AUD–USD	1.978	±4.5e−03	2.573e−02	±2.5e−04	0.99873	4.303e−04
CHF–JPY	2.101	±6.0e−03	2.140e−02	±2.7e−04	0.99799	5.063e−04
EUR–AUD	2.137	±7.9e−03	2.602e−02	±4.2e−04	0.99658	3.846e−05
EUR–CHF	2.089	±7.9e−03	2.554e−02	±4.2e−04	0.99642	1.126e−03
EUR–GBP	2.182	±8.9e−03	2.743e−02	±4.8e−04	0.99587	1.462e−03
EUR–JPY	1.998	±4.0e−03	2.246e−02	±2.0e−04	0.99902	−8.981e−07
EUR–USD	1.868	±6.6e−03	2.277e−02	±3.6e−04	0.99687	1.731e−03
GBP–CHF	2.095	±6.0e−03	2.416e−02	±3.0e−04	0.99796	7.889e−05
GBP–JPY	2.006	±5.3e−03	2.306e−02	±2.7e−04	0.99828	4.535e−05
GBP–USD	1.878	±5.2e−03	2.239e−02	±2.8e−04	0.99806	7.896e−04
GRW	1.783	±1.1e−02	2.194e−02	±6.2e−04	0.99034	5.591e−03
USD–CHF	1.871	±4.5e−03	2.220e−02	±2.4e−04	0.99855	4.614e−04
USD–JPY	1.884	±6.8e−03	2.405e−02	±3.8e−04	0.99678	1.210e−03
Currency average	2.01	(1.1e−01)	2.39e−02	(1.8e−03)		

Table A19. Cumulative total move (coastline), law (12), $* = \mathrm{tm}$.

Currency	$E_{\mathrm{cum,tm}}$	$\Delta E_{\mathrm{cum,tm}}$	$C_{\mathrm{cum,tm}}$	$\Delta C_{\mathrm{cum,tm}}$	Adj. R^2	$R^2_{\mathrm{quad}} - R^2_{\mathrm{lin}}$
AUD–JPY	−1.048	±2.6e−03	2.139e+02	±3.1e+00	0.99850	1.949e−04
AUD–USD	−0.961	±2.7e−03	2.861e+02	±4.9e+00	0.99805	1.054e−03
CHF–JPY	−1.076	±2.5e−03	1.154e+02	±1.4e+00	0.99868	1.307e−04
EUR–AUD	−1.140	±3.7e−03	9.003e+01	±1.5e+00	0.99738	−7.231e−06
EUR–CHF	−1.189	±5.5e−03	1.312e+01	±2.1e−01	0.99461	1.357e−04
EUR–GBP	−1.184	±4.9e−03	3.711e+01	±6.6e−01	0.99579	1.347e−03
EUR–JPY	−1.005	±1.6e−03	1.601e+02	±1.4e+00	0.99938	2.140e−04
EUR–USD	−0.937	±3.8e−03	2.009e+02	±4.8e+00	0.99583	2.997e−03
GBP–CHF	−1.145	±2.1e−03	5.351e+01	±4.5e−01	0.99916	−3.260e−06
GBP–JPY	−1.024	±1.9e−03	1.605e+02	±1.7e+00	0.99910	4.353e−07
GBP–USD	−0.929	±2.4e−03	1.878e+02	±2.8e+00	0.99832	9.695e−04
GRW	−0.868	±6.8e−03	6.157e+02	±3.3e+01	0.98509	1.284e−02
USD–CHF	−0.939	±2.7e−03	2.515e+02	±4.3e+00	0.99796	1.580e−03
USD–JPY	−0.963	±3.3e−03	1.921e+02	±3.8e+00	0.99701	2.282e−03
Currency average	−1.04	(9.7e−02)	1.51e+02	(8.4e+01)		

Table A20. Cumulative cost-adjusted total move, law (12), $* = \mathrm{tm}$, fitted from 0.2%.

Currency	$E_{\mathrm{cum,tm}}$	$\Delta E_{\mathrm{cum,tm}}$	$C_{\mathrm{cum,tm}}$	$\Delta C_{\mathrm{cum,tm}}$	Adj. R^2	$R^2_{\mathrm{quad}} - R^2_{\mathrm{lin}}$
AUD–JPY	−0.941	±2.9e−03	3.378e+02	±6.2e+00	0.99874	3.358e−05
AUD–USD	−0.943	±2.0e−03	2.920e+02	±3.6e+00	0.99940	7.101e−05
CHF–JPY	−0.956	±3.4e−03	1.854e+02	±3.5e+00	0.99838	1.087e−03
EUR–AUD	−0.970	±4.9e−03	1.652e+02	±4.3e+00	0.99678	1.822e−03
EUR–CHF	−1.201	±2.0e−02	1.254e+01	±5.5e−01	0.96693	1.789e−02
EUR–GBP	−1.034	±7.9e−03	5.743e+01	±1.8e+00	0.99249	3.734e−03
EUR–JPY	−0.915	±2.0e−03	2.454e+02	±2.9e+00	0.99941	1.810e−04
EUR–USD	−0.980	±8.2e−03	1.524e+02	±6.5e+00	0.99102	5.138e−03
GBP–CHF	−1.059	±7.4e−03	6.783e+01	±2.1e+00	0.99366	3.061e−03
GBP–JPY	−0.912	±2.3e−03	2.666e+02	±3.8e+00	0.99919	9.254e−06
GBP–USD	−0.916	±4.2e−03	1.903e+02	±4.6e+00	0.99730	7.322e−04
GRW	−0.994	±9.2e−03	2.588e+02	±1.4e+01	0.98897	3.848e−03
USD–CHF	−0.949	±5.4e−03	2.214e+02	±6.9e+00	0.99582	3.302e−03
USD–JPY	−0.979	±4.4e−03	1.650e+02	±3.8e+00	0.99744	1.522e−03
Currency average	−0.98	(7.9e−02)	1.82e+02	(9.5e+01)		

For the GRW a constant spread of 0.02% was introduced.

Table A21. Cumulative directional change, law (12), $* = \mathrm{dc}$.

Currency	$E_{\mathrm{cum,dc}}$	$\Delta E_{\mathrm{cum,dc}}$	$C_{\mathrm{cum,dc}}$	$\Delta C_{\mathrm{cum,dc}}$	Adj. R^2	$R^2_{\mathrm{quad}} - R^2_{\mathrm{lin}}$
AUD–JPY	−1.108	±2.8e−03	8.654e+01	±1.1e+00	0.99843	2.727e−05
AUD–USD	−1.014	±2.3e−03	1.075e+02	±1.3e+00	0.99867	2.578e−04
CHF–JPY	−1.121	±4.0e−03	5.387e+01	±9.0e−01	0.99678	1.674e−03
EUR–AUD	−1.191	±4.8e−03	4.376e+01	±7.9e−01	0.99590	1.308e−05
EUR–CHF	−1.232	±6.2e−03	7.196e+00	±1.1e−01	0.99367	1.967e−04
EUR–GBP	−1.240	±7.0e−03	1.940e+01	±4.1e−01	0.99213	4.493e−03
EUR–JPY	−1.044	±3.0e−03	6.849e+01	±9.5e−01	0.99797	1.399e−03
EUR–USD	−0.954	±3.8e−03	8.805e+01	±1.8e+00	0.99597	1.953e−03
GBP–CHF	−1.178	±3.1e−03	2.851e+01	±3.0e−01	0.99831	5.737e−04
GBP–JPY	−1.061	±2.9e−03	7.116e+01	±9.5e−01	0.99813	3.460e−04
GBP–USD	−0.948	±2.2e−03	8.117e+01	±9.2e−01	0.99872	2.150e−04
GRW	−0.874	±7.6e−03	2.630e+02	±1.4e+01	0.98147	1.385e−02
USD–CHF	−0.954	±1.9e−03	1.143e+02	±1.2e+00	0.99901	1.745e−04
USD–JPY	−0.978	±2.7e−03	9.014e+01	±1.3e+00	0.99811	7.933e−04
Currency average	−1.08	(1.1e−01)	6.62e+01	(3.4e+01)		

Table A22. Cumulative overshoot, law (12), $* = \text{os}$.

Currency	$E_{\text{cum,os}}$	$\Delta E_{\text{cum,os}}$	$C_{\text{cum,os}}$	$\Delta C_{\text{cum,os}}$	Adj. R^2	$R^2_{\text{quad}} - R^2_{\text{lin}}$
AUD–JPY	−0.968	±3.5e−03	1.609e+02	±3.2e+00	0.99678	1.365e−03
AUD–USD	−0.892	±4.3e−03	2.054e+02	±5.7e+00	0.99436	4.187e−03
CHF–JPY	−1.006	±3.3e−03	7.612e+01	±1.2e+00	0.99731	1.683e−03
EUR–AUD	−1.071	±3.2e−03	5.888e+01	±8.3e−01	0.99775	3.374e−04
EUR–CHF	−1.124	±6.7e−03	7.751e+00	±1.4e−01	0.99117	3.547e−03
EUR–GBP	−1.084	±3.6e−03	2.503e+01	±3.3e−01	0.99729	9.155e−04
EUR–JPY	−0.954	±1.7e−03	1.010e+02	±9.3e−01	0.99921	3.324e−04
EUR–USD	−0.917	±4.2e−03	1.061e+02	±2.6e+00	0.99474	4.471e−03
GBP–CHF	−1.098	±3.4e−03	3.117e+01	±4.0e−01	0.99766	1.305e−03
GBP–JPY	−0.978	±2.4e−03	9.847e+01	±1.2e+00	0.99852	5.018e−04
GBP–USD	−0.906	±3.3e−03	9.983e+01	±1.9e+00	0.99667	2.486e−03
GRW	−0.866	±6.6e−03	2.803e+02	±1.3e+01	0.98589	1.256e−02
USD–CHF	−0.922	±4.5e−03	1.277e+02	±3.3e+00	0.99413	5.365e−03
USD–JPY	−0.945	±4.7e−03	9.785e+01	±2.5e+00	0.99380	5.314e−03
Currency average	−0.99	(8.0e−02)	9.20e+01	(5.5e+01)		