

Übungen zu Algorithmen und Programmentwicklung für die Biologische Chemie

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(based on slides by Sven Findeiß)

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Dynamic Programming

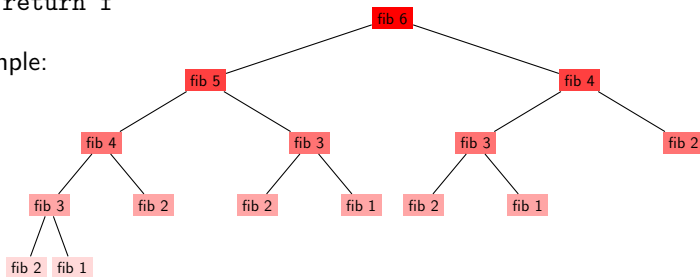
- Solving recursion equations (with overlapping subproblems) efficiently
- Optimization employing 'optimal substructure'

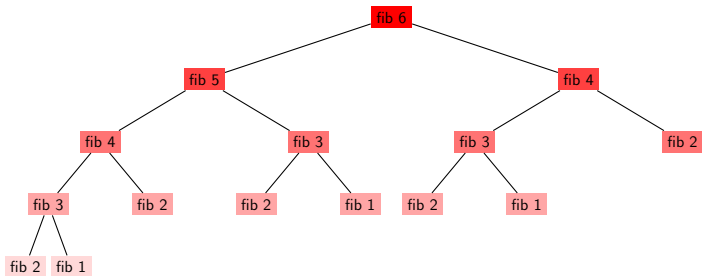
Example 1: Fibonacci Series

The Fibonacci Series of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, ... can be defined recursively:

```
def fib(n):  
    if n <= 2:  f = 1  
    else:       f = fib(n-1) + fib(n-2)  
    return f
```

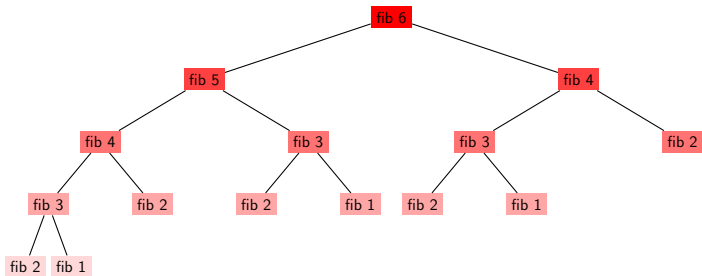
Example:





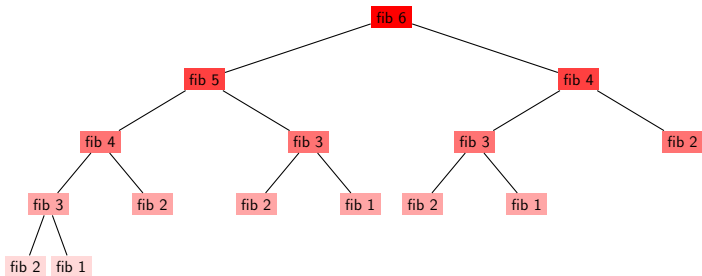
Building the solution space we realize:

- 1 The run-time of the recursion scales exponentially ($\sim 2^n$) with the input size.
- 2 Many (sub)solutions are calculated over and over again ($5 \times \text{fib } 2$, $3 \times \text{fib } 3$, $2 \times \text{fib } 4$).
- 3 Avoid redundancy: tabulate subsolutions!



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memo = {}  
def fib(n):  
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        return memo[n]  
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Stop and think:

- How much faster can one compute fib using DP?
- How much space do we need for DP-fib?

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Recursive (top-down) vs. iterative (bottom-up) Fibonacci

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DP = Order + Tabulation (Reuse)

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Optimization by DP: Levenshtein Distance

Definition: *Levenshtein distance of strings a and b* $:=$ minimal cost of transforming a into b by edit operations “replace”, “insert”, “delete” each of cost 1.

Example: $\text{AUTO} \Rightarrow \text{RAD}$, $\text{MOTORRAD} \Rightarrow \text{FAHRRAD}$

As edit sequence:

$\text{AUTO} \rightarrow \text{AUT} \rightarrow \text{AUD} \rightarrow \text{AD} \rightarrow \text{RAD}$ (4)

$\text{MOTORRAD} \rightarrow \text{MOTHRRAD} \rightarrow \text{MOAHRAD} \rightarrow \text{MFAHRAD} \rightarrow \text{FAHRRAD}$ (4)

or as alignment:

- AUTO	MOTORRAD
RA-D-	-FAHRRAD

NOTE: “Levenshtein Distance” has *optimal substructure*:

If ~~-~~AUTO is optimal, then it's subsolution ~~-~~AUT (of

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subproblem 'AUT' vs. 'RAD') must be optimal.

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Calculating the Levenshtein distance of a and b

- build $n \times m$ -matrix D
where: $D_{ij} :=$ distance of a_1, \dots, a_i and b_1, \dots, b_j
- calculate each D_{ij} from optimal partial solutions

Recursion:

$$D_{0,0} = 0; D_{0,j} = j; D_{i,0} = i$$
$$D_{i,j} = \min \begin{cases} D_{i-1,j-1} + \begin{cases} 1 & \text{if } a_i \neq b_j \\ 0 & \text{otherwise} \end{cases} \\ D_{i-1,j} + 1 \\ D_{i,j-1} + 1 \end{cases}$$

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i		-	R	A	D
0	-	0	1	2	3
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$D_{n,m}$ contains the distance of a and b .

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$D_{n,m}$ contains the distance of a and b .

Tracing back the optimal choices from $D_{n,m}$ to $D_{0,0}$ yields *some* optimum alignment of a and b .

Assignment A3: Optimization by DP

Go to:

<https://github.com/TBIAPBC/APBC2025/tree/master/A3>

Happy hacking!