Notes on Bernoulli Numbers for Evan

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1 Quick Note on subscripts

Bernoulli numbers have been subscripted in three different ways:

1. Where $B_1 = -\frac{1}{2}$. This is now standard, and is what I use in my dissertation. You probably learned it this way.

2. Where $B_1 = \frac{1}{2}$. This was common in algebraic number theory pre-1980, e.g. Iwasawa or Washington. B_n for $n \neq 1$ agrees entirely with 1., above, so it only differs in this one location.

3. Where $B_1 = \frac{1}{6}$. In this case, B_n of this kind $= B_{2n}$ of either 1. or 2., above. This was common in Algebraic Topology, e.g. Milnor (one of the true giants of 20^{th}-century mathematics, still alive and well in NJ as far as I know) and Lance (my advisor, who you see mentioned several times in my dissertation).

In a line of the Preface that made me laugh out loud when I read it (though I doubt it will for you), Introduction to Cyclotomic Fields, by Larry Washington of University of Maryland said: "At Serge Lang's urging I have let the first Bernoulli number be $B_1 = -\frac{1}{2}$ rather than $+\frac{1}{2}$. This disagrees with Iwasawa [Washington's advisor at Princeton] and several of my papers, but conforms to what is becoming standard usage." Serge Lang was well-known for churning out huge textbooks in almost any field of graduate-level mathematics, whether he was an expert in that field or not. So of course Lang would have done this. Lang also famously traveled with a delegation to the Republic of South Africa where many thousands of people were dying in an AIDS epidemic; they successfully convinced the government there that the HIV virus did not cause AIDS, and that preventing transmission of HIV would not slow the epidemic. This was disastrous, and the policies of the RSA government following this resulted in much loss of life there.