Notes on Bernoulli Numbers for Evan

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1 Note on subscripts

(This section was originally titled 'Quick Note on Subscripts,' but that name has become increasingly inaccurate, so I've changed it. You can see the progression in the git commits.)

Bernoulli numbers have been subscripted in three different ways:

1. Where $B_1 = -\frac{1}{2}$. This is now standard (?), and is what I use in my dissertation. You probably learned it this way. If not, let me know. That's what I'll use unless I hear otherwise from you.

2. Where $B_1 = \frac{1}{2}$. This was common in algebraic number theory pre-1980, e.g. Iwasawa or Washington. B_n for $n \neq 1$ agrees entirely with 1., above, so it only differs in this one location.

3. Where $B_1 = \frac{1}{6}$. In this case, B_n of this kind = B_{2n} of either 1. or 2., above. This was common in Algebraic Topology, e.g. Milnor (one of the true giants of 20^{th}-century mathematics, still alive and well in NJ as far as I know) and Lance (my advisor, who you see mentioned several times in my dissertation).

In a line of the Preface that made me laugh out loud when I read it (though I doubt it will for you), Introduction to Cyclotomic Fields, by Larry Washington of University of Maryland said:"At Serge Lang's urging I have let the first Bernoulli number be $B_1 = -\frac{1}{2}$ rather than $+\frac{1}{2}$. This disagrees with Iwasawa [Washington's advisor at Princeton] and several of my papers, but conforms to what is becoming standard usage." Serge Lang was well-known for churning out huge textbooks in almost any field of graduate-level mathematics, whether he was an expert in that field or not. (This is likely connected to Lang's membership in the Bourbaki, which, if you don't know the story of Nicholas Bourbaki, you should look it up or ask me). So of course Lang would have done this. Lang also famously traveled with a delegation to the Republic of South Africa where many thousands of people were dying in an AIDS epidemic; they successfully convinced the government there that the HIV virus did not cause AIDS, and that preventing transmission of HIV would not slow the epidemic. This was disastrous, and the policies of the RSA government following this resulted in much loss of life there.

Just banging around doing a little research, it seems that this is not so fully resolved/ standard as I had thought, and is still in debate: http://luschny.de/math/zeta/The-Bernoulli-Manifesto.html: link to Conversation between Peter Luschny and Donald Knuth on this topic. Note that Donald Knuth is the same guy who, back in the 1970's, got so fed up with trying to type a mathematical paper for submission to a journal that he decided to create the first version of TEX so that he could properly typeset it.

I think it is good for you as a young mathematician, to look at a debate on the definition of something (in this case, the Bernoulli numbers), and see what goes into it. When you study mathematics, typically, you just have a definition in a book, and it's sitting there, complete and perfect, and you memorize and then you learn what it really means, and then you learn to use it (which are all different things). But, there was a long road to get to the point where it was decided what should be in that definition and what should not be in that definition. And there are other formulations that are logically equivalent, and therefore, are in some sense, no different, but that lead mathematicians in other directions, or are more complex to understand, or are harder to learn to use, and so have been rejected. There is psychology in the mathematics in this case – we choose the version that helps people learn the mathematics, or the one that is most elegant, or the one that makes the most connections. Often, this is not a controversial process, and mathematicians settle pretty quickly on something, but sometimes this conversation serves a guide to the entire branch of mathematics, its development and its history. The terms "regular prime/irregular prime" have this to some extent, and I talk really briefly about this in my dissertation also.