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Rumor propagation model with consideration of scientific knowledge level and social reinforcement in heterogeneous network

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ABSTRACT

In the Internet age, rumors bring great panic, research on the mechanism of propagation will help mitigate the bad influence of rumors. In this paper, we propose a modified rumor propagation model with consideration of scientific knowledge level and social reinforcement, and derive the mean-field equations that describe the dynamics of rumor propagation process. We obtain the rumor propagation threshold, which is closely related to scientific knowledge level and social reinforcement. The threshold of rumor propagation is increased by the scientific knowledge level, and the critical threshold of scientific knowledge level is also affected by the rumor propagation probability The results show that rumor propagates more quickly and more widely in people without scientific knowledge, while rumor propagates more slowly and the final size of the rumor is smaller in people with scientific knowledge. Positive social reinforcement will reduce the propagation threshold of rumor propagation, increase the propagation rate, and finally increase the rumor scale. Negative social reinforcement has the opposite effect on rumor. These results are verified by numerical simulations on scale-free networks. Research results also provide a good reference for future studies on how to control rumor propagation.

1. Introduction

Rumor refers to the information that appears and circulates in the society and has not been publicly confirmed by the authoritative. The propagation of rumor has brought great trouble to daily life of people. After the rise of the Internet, rumor propagation is more convenient and has greater influence by means of the Internet, which has aroused the general attention of the society. It is necessary to study the dynamic mechanism of rumor propagation from different aspects to reduce the harm of rumor.

Due to the similarity between rumor propagation and disease, epidemic model is used as the basis for the study of rumor propagation. In the 1960s, Daley and Kendall proposed the mathematical model (DK model) of rumor propagation by referring to the infectious disease model, which was widely used in the quantitative research of rumor propagation [1], and many variants such as the Maki–Thompson model (MK model) were generated [2]. Through the communication with spreader and others, rumors are propagated among people in accordance with the rules of mass action. The complex topological structure of large social networks plays an important role in the dynamic propagation process. Zanette simulated the deterministic MK model on the small-world networks [3]. Moreno et al. studied the random version of

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MK model on scale-free networks through simulation and numerical solutions of a group of mean field equations [4]. These studies revealed the complex interaction between network topology and rumor model rules, highlighting the huge influence of network heterogeneity on rumor propagation dynamics. After the pioneering work of Pastor-Satorraset al. [5], propagation dynamics had received more and more attention. In the present study, many modified models have been proposed to describe different propagation mechanisms based on the traditional DK model, for example, the SIS model [6], the SIR model [7], the SIR–UA model [8], the SEIR model [9], the ILSR model [10], and so on.

In fact, rumor propagation is closely related to the corresponding propagation behavior in the real world. Li et al. built an I2S2R model in complex networks to analyze the impact of model parameters on rumor propagation [11]. Liu et al. newly developed a dynamic system to model rumor propagation dynamics by the compartment method [12]. In addition, scientists have begun to seriously consider the role of human behavior in rumor propagation. Wang et al. [13] proposed a new ISR model by introducing the trust mechanism between ignorant nodes and spreader nodes. They concluded that the introduction of the trust mechanism not only significantly reduced the maximum impact of rumors, but also delayed the end of rumors. Deng et al. took forgetting and remembering rates into account when studying rumor propagation model [14]. Xu et al. proposed a new SHPRS rumor propagation model based on scale-free network, taking into account people's hesitation, forgetting and other psychological factors as well as the heterogeneity of networks [15]. Jin et al. used the forgetting factor α and the recall factor β to characterize the oblivion–recall mechanism [16]. Zhang et al. found that the couple reinforcements and the attractiveness of rumor and behavior are crucial factors affecting the interactive propagation processes [17]. In the real world, rumor propagation can be affected by different factors, and it is very important to study the effects of these factors on rumor. The factors affecting rumor propagation mainly include human behavior and social influences from the outside world, which are called internal and external factors affecting rumor propagation. In the past studies, the research of factors affecting rumor propagation mainly unilateral analysis of internal or external causes, without combining the two, but rumor propagation is actually affected by both internal and external factors.

From previous studies, it can be seen that there are many kinds of human behaviors that affect rumor propagation. In this paper, we consider the influence of scientific knowledge level on rumor propagation. This is because scientific knowledge is personal accomplishment. Individuals may learn different amounts of knowledge due to their different knowledge levels and living environments [18], which leads to their different cognitive attitudes towards science, thus exerting a certain impact on the rumor propagation process. In the process of rumor propagation, people with high level of scientific knowledge seldom take the initiative to propagate rumors. On the contrary, people with low level of scientific knowledge may actively accept and spread rumors due to lack of relevant knowledge [19]. In fact, after the rumor comes into being, the main reason why some people propagate rumor is that they lack scientific knowledge [20]. For example, after the nuclear power plant explosion in Japan in 2011 [21], rumors began to propagate that iodized salt could resist radiation, which caused people to buy and store a lot of salt. Later, the authority organized the propaganda of the scientific knowledge about nuclear radiation [22]. After the public understood the relevant knowledge, the buying trend of iodized salt subsided. Therefore, according to these studies, this paper adds the factor of scientific knowledge level into the rumor propagation model to study its influence on the rumor propagation process.

In addition, outside information will also have an impact on the rumor propagation, the existing results show that external public opinion plays an important role in people behavior [23]. This external public opinion is called social reinforcement, and the so-called social reinforcement mechanism refers to the superposition of information from neighbors and even social groups before individuals take actions [24]. Most of the existing studies only consider the influence of single social reinforcement on rumor propagation, for example, the positive reinforcement [25], the negative reinforcement, and the coupled reinforcements [17]. However, due to the strong ambiguity of rumors and the life experience, education and experience of different people, it is easy to produce two opposing views after rumor propagation, and thus form positive and negative social reinforcement. So we studied the effects of these two kinds of social reinforcement on rumor propagation.

To sum up, this article considers the influence of internal and external factors, on the basis of traditional ISR model [26], a new rumor propagation model is proposed. In the process of analyzing this rumor propagation model, we discuss the influences of scientific knowledge level and social reinforcement on the process of rumor propagation, including their influences on the threshold of rumor propagation. The structure of this paper is as follows. In the second part, the scientific knowledge level and social reinforcement are added into the model to conduct the further study on the rumor propagation dynamics. In the third part, we use the mean field method to derive the corresponding equations of the model. The steady-state analysis of the model in heterogeneous networks is in the fourth section, and the influences of internal and external factors on rumors are further analyzed. And in the fifth section, we carry out numerical research to verify the theoretical analysis. Finally, the thesis is summarized.

2. Rumor propagation model

In the traditional ISR model of rumor propagation [26], nodes in social networks are divided into three categories: ignorant(I), spreader(S) and stifler(R). In the social networks, rumors are propagating among people in accordance with the rules of mass action. The ignorance node has not contacted the rumor and will not propagate it, but it may become the spreader by contacting the spreader nodes. The spreader node has accepted the rumor and propagates it. The stifler node finally decides to stop propagating rumors.

Considering the influence of scientific knowledge level on rumor propagation, we divided people into two groups according to their scientific knowledge level in our model. Before the rumor appeared, some people had studied the scientific knowledge related to the rumor, which is called people with scientific knowledge, denoted by$K$; Some people had never learned the scientific knowledge of the rumor, which is called people without scientific knowledge, denoted by U. In this way, individuals in the group can be divided into: ignorant with scientific knowledge (I_K), ignorant without scientific knowledge (I_U), spreader with scientific knowledge (S_K), spreader without scientific knowledge (S_U), stifler with scientific knowledge (R_K), stifler without scientific knowledge (R_U).

According to the characteristics of knowledge transmission, knowledge transmission requires a certain amount of contact time or 'finite time commitment' [27]. In a group, if there are few or even zero individuals with knowledge around people, such knowledge will be gradually forgotten over time. Studies have shown that the knowledge-believed node may forget the information and return to the unknown state after a considerable period of time [28]. Therefore, the groups with scientific knowledge and the groups without scientific knowledge can be transformed into each other. The people with scientific knowledge may become the people without scientific knowledge due to forgetting, while the people without scientific knowledge can become the people with scientific knowledge through learning and training [29]. People with scientific knowledge would think about rumors and decide whether to propagate them. People without scientific knowledge are easily influenced and blindly choose to propagate rumors. So rumors are less likely to spread in people with scientific knowledge. The rumor propagation process can be shown in Fig. 1.

The rumor propagation process can be described as follows:

(1) As a result of forgetting or the influence of others, people with scientific knowledge may forget or doubt their own scientific knowledge about the rumor, then turn to the people without scientific knowledge with probability α. On the contrary, by accepting the propaganda of scientific knowledge, people without scientific knowledge will be transformed into people with scientific knowledge with the probability $1-\alpha$.
(2) An ignorant with scientific knowledge only interacts with the spreader without scientific knowledge and then it will become a spreader with scientific knowledge with probability p_m. A spreader with scientific knowledge only contact with other spreader or stifler neighbors it links to directly, who have no scientific knowledge, then it will become a stifler with scientific knowledge with probability β.
(3) An ignorant without scientific knowledge will contact with all spreader neighbors and it will become a spreader without scientific knowledge with probability p_m. A spreader without scientific knowledge will contact with all spreader or stifler neighbors it links to directly, then it will become a stifler without scientific knowledge with probability β.
(4) In addition, considering the forgetting mechanism, the spreader with scientific knowledge and the spreader without scientific knowledge may change their status into stifler at the rate δ, which is the rate to stop spreading rumor spontaneously.

The reaction process can be schematically represented by the following:

(i) Behavior state changes

$$\begin{aligned}
&I_K \xrightarrow{\alpha} I_U\\
&S_K \xrightarrow{\alpha} S_U\\
&R_K \xrightarrow{\alpha} R_U\\
&I_U \xrightarrow{1-\alpha} I_K\\
&S_U \xrightarrow{1-\alpha} S_K\\
&R_U \xrightarrow{1-\alpha} R_K
\end{aligned} \tag{1}$$

(ii) State changes with scientific knowledge

$$\begin{aligned}
&I_K + S_U \xrightarrow{p_m} S_K + S_U\\
&S_K + S_U \xrightarrow{\beta} R_K + S_U\\
&S_K + R_U \xrightarrow{\beta} R_K + R_U
\end{aligned} \tag{2}$$

(iii) State changes without scientific knowledge

$$\begin{aligned}
&I_U + S_K \xrightarrow{p_m} S_U + S_K\\
&I_U + S_U \xrightarrow{p_m} S_U + S_U\\
&S_U + S_K \xrightarrow{\beta} R_U + S_K\\
&S_U + S_U \xrightarrow{\beta} R_U + S_U\\
&S_U + R_K \xrightarrow{\beta} R_U + R_K\\
&S_U + R_U \xrightarrow{\beta} R_U + R_U
\end{aligned} \tag{3}$$

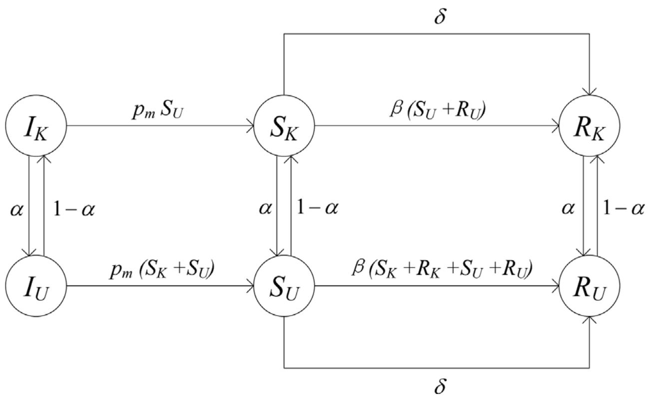

Fig. 1. The dynamic of rumor propagation.

(iv) Stop propagation a rumor spontaneously

$$\begin{aligned} &S_K \xrightarrow{\delta} R_K \\ &S_U \xrightarrow{\delta} R_U \end{aligned} \tag{4}$$

In the experiment of Centola, it is found that social reinforcement plays a crucial role in the propagation of network behavior [30]. The social reinforcement is defined as the amount of information a person needs from neighbors before accepting an opinion or behavior [31]. The experiment shows that when a person receives multiple signals [32], he is more likely to accept information and take actions [33]. Under the complexity of human activities and interactions, individuals' behaviors and choices are influenced by their surroundings all the time [34]. If an individual accepts a rumor more than once, he is likely to believe it, which can be called positive social reinforcement. For reason such as educational or rationality analysis or information redundancy, if an individual receives the same information again, he may doubt the rumor. Therefore, social reinforcement will be significantly weakened. We can call it negative social reinforcement. Positive social reinforcement promotes rumor propagation, while negative social reinforcement inhibits rumor propagation. Under the influence of positive and negative social reinforcement, the ignorant becomes the spreader with a certain probability. When the ignorant node first hears the rumor, it will spread the rumor with probability $p_1 = \lambda(1+b) - h\lambda(1+b)$. Where $b \in (0, 1)$ is positive social reinforcement, $h \in (0, 1)$ is negative social reinforcement, λ is the rumor propagation probability, which represents the probability of an ignorant node propagating rumors when it hears the rumor for the first time without social reinforcement. When a node hears the rumor twice or three times, it will propagate the rumor with the probability p_2 or p_3, respectively. By analogy, if the ignorant node hears a rumor m times, the probability that it will propagate the rumor is p_m, as follows:

$$\begin{aligned} &p_1 = \lambda(1+b) - h\lambda(1+b) \\ &p_2 = p_1(1+b) - hp_1(1+b) \\ &\qquad\qquad \vdots \\ &p_m = p_{m-1}(1+b) - hp_{m-1}(1+b) \end{aligned} \tag{5}$$

The above formula can be simplified to obtain the transition probability p_m, that is, when the ignorant node hears the rumor $m$times, the probability that it becomes the spreader under the influence of social reinforcement as follows:

$$p_m = \lambda[(1+b)(1-h)]^m, 0 \le b, h \le 1, m \ge 1 \tag{6}$$

This model introduces the scientific knowledge level, emphasizes rumor sensitivity, positive social reinforcement and negative social reinforcement effect, and makes the rumor propagation process closer to the real network.

3. Mean-field equations analysis

In this section, we consider the mean-field equations of above propagation dynamic model on networks. In this model, each node represents an individual whose state is in six types, such as ignorant with scientific knowledge, ignorant without scientific knowledge, spreader with scientific knowledge, spreader without scientific knowledge, stifler with scientific knowledge and stifler without scientific knowledge, and each link represents contact between individuals. To give a mean-field analysis, we divide all nodes into different classes according to their degree following the conventional approach of propagation dynamic studies [35]. Number of nodes in each class is denoted by $N(k, t)$. Then, let us represent $I(k, t)$, $S(k, t)$, $R(k, t)$ as expected number of nodes with degree k at time t in ignorant, spreader or stifler state, and $\rho^I(k, t)$, $\rho^S(k, t)$, $\rho^R(k, t)$ as their density respectively. Obviously, $\rho^I(k, t) = I(k, t)/N$, $\rho^S(k, t) = S(k, t)/N$, $\rho^R(k, t) =$

$R(k, t)/N$. As an ignorant without scientific knowledge, node i can interact with any spreader neighbors with transition probability p_m. Given that node i has $g(g \leq k)$ spreader neighbors at time t, the probability of i to stay in ignorant state in the time interval$[t, t + \Delta t]$ is

$$p_{II}^i = \prod_{m=1}^{g}(1 - p_m \Delta t) = \prod_{m=1}^{g}\{1 - \lambda[(1 + b)(1 - h)]^m \Delta t\} \tag{7}$$

The probability that node i has g spreader neighbors is

$$N_i^S(g, t) = C_k^g \theta(k, t)^g [1 - \theta(k, t)]^{k-g} \tag{8}$$

Where $\theta(k, t)$ is the probability that an edge emanating from an ignorant node with k links point to a spreader node at time t.

$$\theta(k, t) = \sum_{k'} P(k'/k)\rho^S(k', t) \tag{9}$$

In this equation $P(k'/k)$ is the probability of a node with degree k linking to a node with degree k'. And $\rho^S(k', t)$ is the density of spreader nodes which belong to connectivity class k at time t. By removing all higher-order terms, then the probability that ignorant without scientific knowledge keeps its state unchanged for arbitrary g at time t is

$$\begin{aligned} t_{II}^U(k, t) &= \sum_{g=0}^{k} C_k^g \prod_{i}^{g} 1 - \lambda[(1 + b)(1 - h)]^i \Delta t \theta(k, t)^g [1 - \theta(k, t)]^{k-g} \\ &\approx \sum_{g=0}^{k} C_k^g 1 - \lambda(1 + b)(1 - h)\Delta t \frac{1 - [(1 + b)(1 - h)]^g}{1 - (1 + b)(1 - h)} \theta(k, t)^g [1 - \theta(k, t)]^{k-g} \\ &= 1 - \frac{\lambda(1 + b)(1 - h)\Delta t}{1 - (1 + b)(1 - h)} + \frac{\lambda(1 + b)(1 - h)\Delta t}{1 - (1 + b)(1 - h)} 1 + [(1 + b)(1 - h) - 1]\theta(k, t)^k \\ &= 1 - (1 + b)(1 - h)\lambda k \theta(k, t)\Delta t \end{aligned} \tag{10}$$

As an ignorant with scientific knowledge, node i can only interacts with its spreader neighbors without scientific knowledge. We denote $g(g \leq k)$ as the number of spreader neighbors without scientific knowledge of node i at time t. Then the probability that node i has g spreader neighbors without scientific knowledge at time t will be

$$N_i^S(g, t) = C_k^g [\alpha\theta(k, t)]^g [1 - \alpha\theta(k, t)]^{k-g} \tag{11}$$

Similarly as above circumstance, the probability that ignorant with scientific knowledge with degree k keeps its state unchanged for arbitrary g at time t is

$$\begin{aligned} t_{II}^K(k, t) &= \sum_{g=0}^{k} C_k^g \prod_{i}^{g} 1 - \lambda[(1 + b)(1 - h)]^i \Delta t [\alpha\theta(k, t)]^g [1 - \alpha\theta(k, t)]^{k-g} \\ &= 1 - (1 + b)(1 - h)\lambda k \alpha\theta(k, t)\Delta t \end{aligned} \tag{12}$$

Based on above analysis, the probability of an ignorant node $i$with degree k to stay in ignorant state at time t is

$$\begin{aligned} &t_{II} = \alpha t_{II}^U + (1 - \alpha)t_{II}^K \\ &= \alpha[1 - (1 + b)(1 - h)\lambda k \theta(k, t)\Delta t] + (1 - \alpha)[1 - (1 + b)(1 - h)\lambda k \alpha\theta(k, t)\Delta t] \\ &= 1 - (2\alpha - \alpha^2)(1 + b)(1 - h)\lambda k \theta(k, t)\Delta t \\ &= 1 - (2\alpha - \alpha^2)(1 + b)(1 - h)\lambda k \Delta t \sum_{k'} P(k'/k)\rho^S(k', t) \end{aligned} \tag{13}$$

Similarly, we can deduce the expression for the probability t_{SS} that spreader keeps its state unchanged during $[t, t+\Delta t]$. Following the steps above, we can get

$$\begin{aligned} &t_{SS} = \alpha t_{SS}^U + (1 - \alpha)t_{SS}^K \\ &= [1 - (2\alpha - \alpha^2)k\beta\Delta t \sum_{k'} P(k'/k)(\rho^S(k', t) + \rho^R(k', t))](1 - \delta\Delta t) \end{aligned} \tag{14}$$

By using the probability of Eq. (13), the changing rate of ignorant nodes of k degree class during $[t, t + \Delta t]$ can be expressed as

$$\begin{aligned} I(k, t + \Delta t) &= I(k, t) - I(k, t)(1 - t_{II}) \\ &= I(k, t) - I(k, t)(2\alpha - \alpha^2)(1 + b)(1 - h)\lambda k \Delta t \sum_{k'} P(k'/k)\rho^S(k', t) \end{aligned} \tag{15}$$

Similarly, we can get the changing rate of spreader nodes and stifler nodes during $[t, t + \Delta t]$ as well:

$$\begin{aligned} S(k, t+\Delta t) &= S(k,t) + I(k,t)(1-t_{II}) - S(k,t)(1-t_{SS}) \\ &= S(k,t) + I(k,t)(2\alpha-\alpha^2)(1+b)(1-h)\lambda k \Delta t \sum_{k'} P(k'/k)\rho^S(k',t) \\ &- S(k,t)(2\alpha-\alpha^2)\beta k \Delta t \sum_{k'} P(k'/k)(\rho^S(k',t)+\rho^R(k',t)) - \delta \Delta t S(k,t) \end{aligned} \tag{16}$$

$$\begin{aligned} R(k, t+\Delta t) &= R(k,t) + S(k,t)(1-t_{SS}) \\ &= R(k,t) + S(k,t)(2\alpha-\alpha^2)\beta k \Delta t \sum_{k'} P(k'/k)(\rho^S(k',t)+\rho^R(k',t)) \\ &+ \delta \Delta t S(k,t) \end{aligned} \tag{17}$$

Then, in the limit$\Delta t \to 0$, we can obtain

$$\frac{\partial \rho^I(k,t)}{\partial t} = -(2\alpha-\alpha^2)(1+b)(1-h)\lambda k \rho^I(k,t) \sum_{k'} P(k'/k)\rho^S(k',t) \tag{18}$$

$$\begin{aligned} \frac{\partial \rho^S(k,t)}{\partial t} &= (2\alpha-\alpha^2)(1+b)(1-h)\lambda k \rho^I(k,t) \sum_{k'} P(k'/k)\rho^S(k',t) \\ &- (2\alpha-\alpha^2)\beta k \rho^S(k,t) \sum_{k'} P(k'/k)(\rho^S(k',t)+\rho^R(k',t)) - \delta \rho^S(k,t) \end{aligned} \tag{19}$$

$$\frac{\partial \rho^R(k,t)}{\partial t} = (2\alpha-\alpha^2)\beta k \rho^S(k,t) \sum_{k'} P(k'/k)(\rho^S(k',t)+\rho^R(k',t)) + \delta \rho^S(k,t) \tag{20}$$

4. Steady-state analysis in heterogeneous networks

In uncorrelated heterogeneous networks, the degree correlation can be written as [36]:

$$P(k'/k) = q(k') = \frac{k'P(k')}{\langle k \rangle} \tag{21}$$

Where $P(k)$ is degree distribution function of the network.$\langle k \rangle$ is the average degree. With this representation and the initial condition $\rho^S(k,t) = 0$, Eq. (18) can be solved exactly as

$$\rho^I(k,t) = e^{-(2\alpha-\alpha^2)(1+b)(1-h)\lambda k \phi(t)} \tag{22}$$

Where $\phi(t)$ is an auxiliary function.

$$\phi(t) = \int_0^t \sum_k q(k)\rho^S(k,t')dt' = \int_0^t \langle\langle \rho^S(k,t') \rangle\rangle dt' \tag{23}$$

In the above equation and hereafter we use the shorthand notation

$$\langle\langle O(k) \rangle\rangle = \sum_k q(k) O(k) \tag{24}$$

It is necessary to figure out$\phi(t)$ to work out the expression of final size of the rumor, R. We can obtain a differential equation for this quantity by multiplying Eq. (19) with $q(k)$ and summing over$k$ and then integrate the equation. This yields after some elementary manipulations:

$$\begin{aligned} \frac{d\phi}{dt} &= 1 - \langle\langle e^{-(2\alpha-\alpha^2)(1+b)(1-h)\lambda k \phi(t)} \rangle\rangle - \delta \phi(t) \\ &-(2\alpha-\alpha^2)\beta \int_0^t \langle\langle k\rho^S(k,t') \rangle\rangle (1 - \langle\langle e^{-(2\alpha-\alpha^2)(1+b)(1-h)\lambda k \phi(t')} \rangle\rangle) dt' \end{aligned} \tag{25}$$

Without loss of generality, we assume an initial distribution of ignorant $\rho^I(k,t) \approx 0$. In the limit $t \to \infty$, $d\phi/dt = 0$, $\phi_\infty = \lim_{t\to\infty} \phi(t)$, then Eq. (25) becomes

$$\begin{aligned} 0 &= 1 - \langle\langle e^{-(2\alpha-\alpha^2)(1+b)(1-h)\lambda k \phi_\infty} \rangle\rangle - \delta \phi_\infty \\ &-(2\alpha-\alpha^2)\beta \int_0^\infty \langle\langle k\rho^S(k,t') \rangle\rangle (1 - \langle\langle e^{-(2\alpha-\alpha^2)(1+b)(1-h)\lambda k \phi(t')} \rangle\rangle) dt' \end{aligned} \tag{26}$$

For $\beta \neq 0$ we can solve Eq. (26) to leading order in β. And we can integrating Eq. (19) to zero order inβ as following:

$$\rho^S(k,t) = 1 - e^{-(2\alpha-\alpha^2)(1+b)(1-h)\lambda k\phi(t)} - \delta\int_0^t [1 - e^{-(2\alpha-\alpha^2)(1+b)(1-h)\lambda k\phi(t')}]e^{\delta(t'-t)}dt' + o(\beta) \tag{27}$$

Because the critical threshold both $\phi(t)$ and ϕ_∞ are small, we write $\phi(t) = \phi_\infty f(t)$, where $f(t)$ is a finite function. Then working to leading order in ϕ_∞, we obtain

$$\rho^S(k,t) \approx -\delta(2\alpha-\alpha^2)(1+b)(1-h)\lambda k\phi_\infty\int_0^t f(t')e^{\delta(t'-t)}dt' + o(\phi_\infty^2) + o(\beta) \tag{28}$$

Inserting Eq. (28) in Eq. (26) and expanding ϕ_∞ to the relevant order we find

$$\begin{aligned}0 &= \phi_\infty\{(2\alpha-\alpha^2)(1+b)(1-h)\lambda\langle\langle k\rangle\rangle - \delta \\ &- [(2\alpha-\alpha^2)(1+b)(1-h)\lambda]^2\phi_\infty\langle\langle k^2\rangle\rangle[\frac{1}{2} + (2\alpha-\alpha^2)\beta\langle\langle k\rangle\rangle I]\} + o(\beta^2) + o(\phi_\infty^3)\end{aligned} \tag{29}$$

Where $I = -\int_0^\infty [\int_0^t f(t')e^{\delta(t'-t)}d\delta(t'-t)]f(t)dt$, which is a finite and positive-defined integral. In above equation, one solution is$\phi_\infty = 0$ and the non-trivial solution is given by

$$\phi_\infty = \frac{(2\alpha-\alpha^2)(1+b)(1-h)\lambda\langle\langle k\rangle\rangle - \delta}{[(2\alpha-\alpha^2)(1+b)(1-h)\lambda]^2\langle\langle k^2\rangle\rangle[\frac{1}{2} + (2\alpha-\alpha^2)\langle\langle k\rangle\rangle\beta I]} \tag{30}$$

Noting that$\langle\langle k\rangle\rangle = \langle k^2\rangle/\langle k\rangle$ and $\langle\langle k^2\rangle\rangle = \langle k^3\rangle/\langle k\rangle$we obtain

$$\phi_\infty = \frac{2[(2\alpha-\alpha^2)(1+b)(1-h)\lambda\langle k^2\rangle\langle k\rangle - \delta\langle k\rangle^2]}{[(2\alpha-\alpha^2)(1+b)(1-h)\lambda]^2\langle k^3\rangle[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I]} \tag{31}$$

This yields a positive value for ϕ_∞provided that

$$\frac{\lambda}{\delta} \geq \frac{\langle k\rangle}{\langle k^2\rangle(2\alpha-\alpha^2)(1+b)(1-h)} \tag{32}$$

Thus, to leading order in α, the critical rumor propagation threshold is independent of the stifling mechanism and is the same as the ISR model [37]. The critical rumor propagation threshold is given by

$$\lambda_c = \frac{\delta\langle k\rangle}{\langle k^2\rangle(2\alpha-\alpha^2)(1+b)(1-h)} \tag{33}$$

When$\alpha < 1 - \sqrt{1 - \delta\langle k\rangle/\langle k^2\rangle(1+b)(1-h)}$, λ_c will be greater than 1. Note that$\lambda_c$does not always make sense. That is to say, when the scientific knowledge level is under a certain threshold, the rumor propagation scale will be relatively small whatever the rumor propagation probability is. And from this, the threshold of scientific knowledge level can be obtained

$$\alpha_c = 1 - \sqrt{1 - \frac{\delta\langle k\rangle}{\lambda\langle k^2\rangle(1+b)(1-h)}} \tag{34}$$

Similarly, the threshold of positive social reinforcement and negative social reinforcement is

$$b_c = \frac{\delta\langle k\rangle}{\lambda\langle k^2\rangle(2\alpha-\alpha^2)(1-h)} - 1 \tag{35}$$

$$h_c = 1 - \frac{\delta\langle k\rangle}{\lambda\langle k^2\rangle(2\alpha-\alpha^2)(1+b)} \tag{36}$$

To further understand the impact of critical thresholds, we try to approximate the final size of the rumor with critical rumor propagation threshold, the threshold of scientific knowledge level, the threshold of positive social reinforcement and negative social reinforcement.

$$\begin{aligned}\phi_\infty &= \frac{2\langle k^2\rangle^2\lambda_c}{\lambda\langle k^3\rangle\delta[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I]}[1 - \frac{\lambda_c}{\lambda}] \\ &= \frac{2\langle k^2\rangle^2[1-(1-\alpha_c)^2]}{\langle k^3\rangle(2\alpha-\alpha^2)\delta[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I]}[1 - \frac{1-(1-\alpha_c)^2}{(2\alpha-\alpha^2)}] \\ &= \frac{2\langle k^2\rangle^2(1+b_c)}{\langle k^3\rangle\delta[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I](1+b)}[1 - \frac{(1+b_c)}{(1+b)}] \\ &= \frac{2\langle k^2\rangle^2(1-h_c)}{\langle k^3\rangle\delta[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I](1-h)}[1 - \frac{(1-h_c)}{(1-h)}]\end{aligned} \tag{37}$$

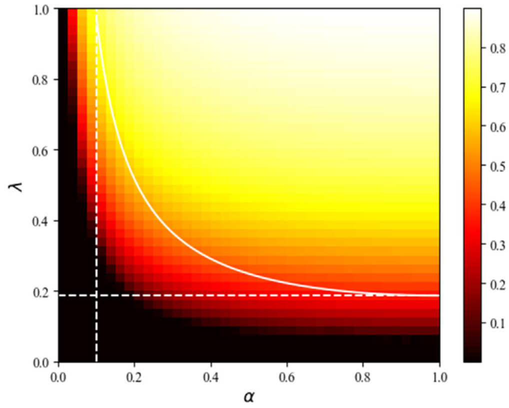

Fig. 2. Critical phenomenon of final size of the rumor R with different λ and α (colors represent final size of the rumor), while $\beta = 0.1$, $b = 0.1$, $h = 0.8$, $\delta = 0.2$.

Note that these two forms of ϕ_∞ are valid only when$\lambda \geq \lambda_c$ and $\alpha \geq \alpha_c$ respectively. Consider that $\rho^S(k,\infty) \to 0$, the final size of the rumor $R = \rho^R(\infty)$ can be represented as

$$R = 1 - \rho^I(\infty) = \sum_k P(k)(1 - e^{-(2\alpha-\alpha^2)(1+b)(1-h)\lambda k\phi_\infty}) \tag{38}$$

By inserting Eq. (37) and expanding the exponential part, we can get

$$\begin{aligned} R &= \frac{2\langle k^2\rangle\langle k\rangle^2}{\langle k^3\rangle[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I]}[1 - \frac{\lambda_c}{\lambda}] \\ &= \frac{2\langle k^2\rangle\langle k\rangle^2}{\langle k^3\rangle[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I]}[1 - \frac{1-(1-\alpha_c)^2}{(2\alpha-\alpha^2)}] \\ &= \frac{2\langle k^2\rangle\langle k\rangle^2}{\langle k^3\rangle[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I]}[1 - \frac{(1+b_c)}{(1+b)}] \\ &= \frac{2\langle k^2\rangle\langle k\rangle^2}{\langle k^3\rangle[\langle k\rangle + 2(2\alpha-\alpha^2)\langle k^2\rangle\beta I]}[1 - \frac{(1-h_c)}{(1-h)}] \end{aligned} \tag{39}$$

This formula respectively represents the relationship between the final size of the rumor and rumor propagation probability, scientific knowledge level, positive social reinforcement and negative social reinforcement.

5. Numerical simulation

In this section, we will conduct numerical simulation on heterogeneous networks to verify the theoretical prediction in the previous section. In following part of this section, the size of the network is considered as 10^6, the average degree is fixed at $\langle k\rangle = 1.62$, and secondary moment is $\langle k^2\rangle = 7.93$. As far as heterogeneous network is concerned, the degree correlation is given by Eq. (21), $P(k'/k) = k'P(k)/\langle k\rangle$, and the degree distribution is $P(k) = (1+\gamma)m^{1+\gamma}k^{-2-\gamma}$ where we set$\gamma = 1$ and $m = 1$. In addition, the following simulations based on Eqs.(15)–(17), the initial spreader is randomly selected to start the rumor in each simulation, and all results are average of 40 runs with different initial spreaders. This part is divided into three sections to study the influence of α, b and $h$on the rumor propagation probability and the final size of the rumor, respectively.

5.1. Influence of scientific knowledge level on the final threshold

Fig. 2 is a heat map of the final size of the rumor under different rumor propagation probability λ and scientific knowledge level α, which shows obvious critical phenomenon. There is a clearly critical curve composed by critical rumor propagation probability and critical scientific knowledge level. The solid white line in Fig. 2 accords with $\lambda = 0.205 * 0.2/(2\alpha-\alpha^2)(1+0.1)(1-0.8)$ where $\langle k\rangle/\langle k^2\rangle = 0.205$, $b = 0.1$, $h = 0.8$ and $\delta = 0.2$, which matches the critical curve of the heat map well. The horizontal dashed line corresponds $\lambda = 0.186$, below which the rumor cannot propagating in large scale of the network. The vertical white dotted line corresponds $\alpha = 1 - \sqrt{1 - \delta\langle k\rangle/\langle k^2\rangle(1+b)(1-h)} = 0.1$, which implies that the rumor propagation dynamic will limited in a small area even the rumor propagation probability $\lambda = 1$.

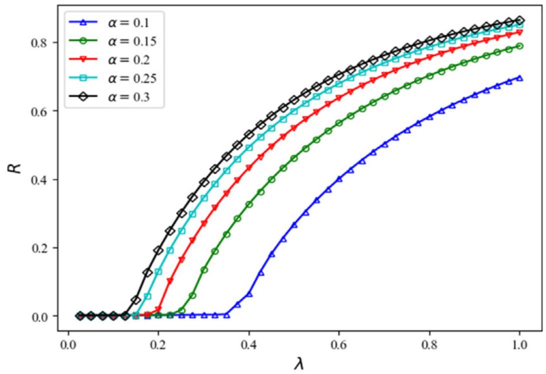

Fig. 3. Relationship with the final size of propagation dynamics R and rumor propagation probability λ for several values of scientific knowledge level α.

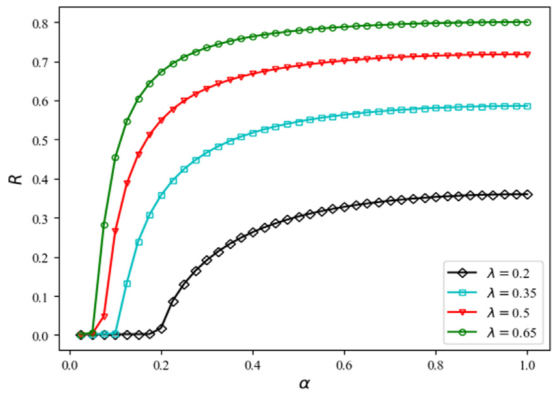

Fig. 4. Relationship with the final size of the rumor R and scientific knowledge level α for several values of rumor propagation probability λ.

In Fig. 3, R is plotted as a function of rumor propagation probability λ and for several different values of scientific knowledge level α. It is found that the critical rumor propagation threshold is highly relevant to α, and lower scientific knowledge level leads to higher critical rumor propagation threshold. This is because lower scientific knowledge level indicates a lower probability that people with scientific knowledge will become people without scientific knowledge. In Fig. 4, R is plotted as a function of scientific knowledge level α and for several different values of rumor propagation probability λ. It shows that the critical scientific knowledge level is highly relevant to λ, and lower rumor propagation probability leads to higher critical scientific knowledge level. Figs. 3 and 4 show that the final size of the rumor R is related to rumor propagation probability λ and scientific knowledge level α, which validate our prediction of Eq. (39).

In addition to studying the critical phenomenon, it is also meaningful to understand how the time-dependent behavior of the final size of the rumor is affected by scientific knowledge level. In Figs. 5 and 6, we display the time evolution of spreader nodes and stifler nodes at different scientific knowledge level. Fig. 5 shows that low scientific knowledge level results in not only slow increment of spreader node but also small fraction of maximum$S(t)$. It can be observed that the rumor propagation lasts longer with slower velocity at a lower scientific knowledge level α. In Fig. 6, both increment speed and the final size of the rumor in steady state are dependent on scientific knowledge level. Lower scientific knowledge level results in smaller final size of the rumor with longer time and slower velocity. These results well explain the influence of scientific knowledge level on rumor propagation.

5.2. Influence of positive social reinforcement on the final threshold

Fig. 7 shows the heat map of the final size of the rumor under different rumor propagation probability λ and positive social reinforcement b, which exists obvious critical phenomenon, that is, a clearly critical curve composed by critical rumor propagation probability and critical positive social reinforcement. The solid white line in Fig. 7 accords with $\lambda = 0.205 * 0.2/0.51(1+b)(1-0.8)$ where $\langle k\rangle/\langle k^2\rangle = 0.205$, $\alpha = 0.3$, $h = 0.8$ and $\delta = 0.2$, which fits the critical curve of the heat map well. The horizontal dashed line corresponds $\lambda = 0.2$, below which the scale of rumor propagation will be small.

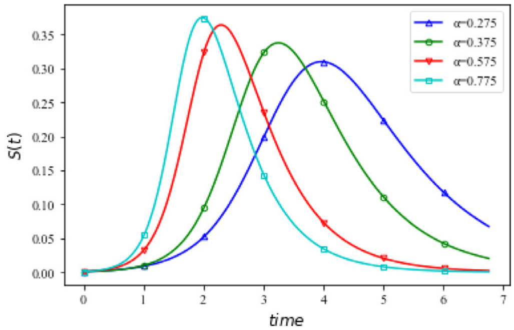

Fig. 5. Time evolution of the fraction of spreader nodes when the dynamics starts with a random selected node for several values of scientific knowledge level α.

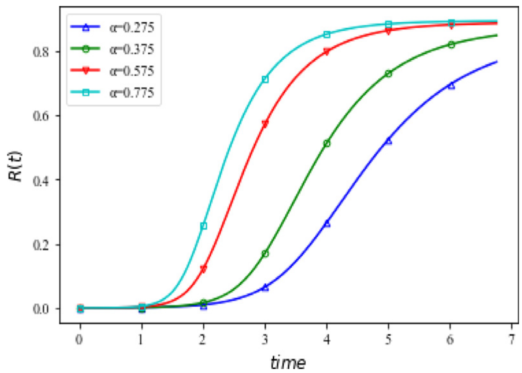

Fig. 6. Time evolution of the fraction of stifler nodes when the dynamics starts with a random selected node for several values of scientific knowledge level α.

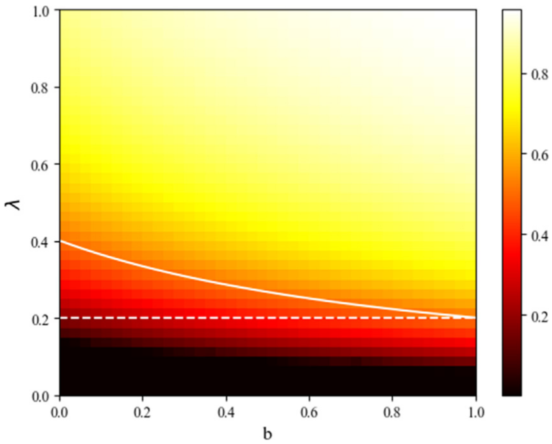

Fig. 7. Critical phenomenon of final size of the rumor R with different λ and b (colors represent final size of the rumor), while $\beta = 0.1, \alpha = 0.3, h = 0.8, \delta = 0.2$.

In Fig. 8, we plot the final size of the rumor R as a function of rumor propagation probability λ and for several different values of positive social reinforcement b. It is found that the critical rumor propagation threshold is highly relevant to b, and lower positive social reinforcement leads to higher critical rumor propagation threshold. In Fig. 9, R is plotted as a function of positive social reinforcement and for several different values of rumor propagation probability. It shows that the critical positive social reinforcement is highly relevant to λ, and higher rumor propagation probability leads to lower critical positive social reinforcement. Figs. 8 and 9 show that the final size of the rumor R is related to rumor propagation probability λ and positive social reinforcement b. This result provides further support for our prediction of previous researches.

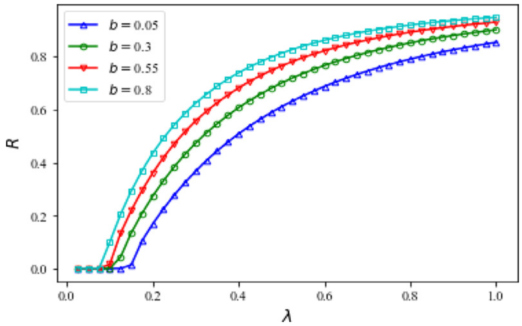

Fig. 8. Relationship with the final size of the rumor R and rumor propagation probability λ for several values of positive social reinforcement b.

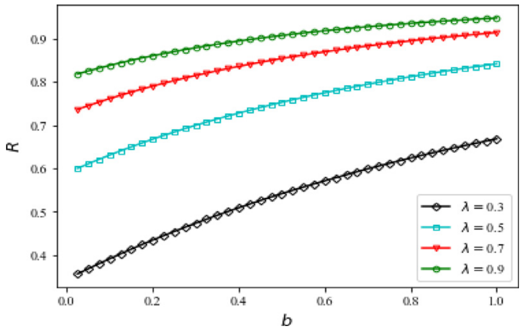

Fig. 9. Relationship with the final size of the rumor R and positive social reinforcement b for several values of rumor propagation probability λ.

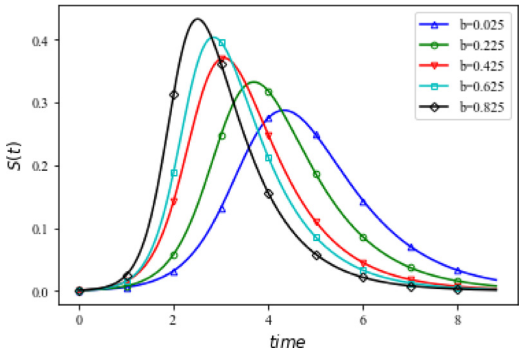

Fig. 10. Time evolution of the fraction of spreader nodes when the dynamics starts with a random selected node for several values of positive social reinforcement b.

Fig. 10 illustrates the density of spreader versus time varying different positive social reinforcement b. It shows that lower positive social reinforcement results in not only slow increment of spreader node but also small fraction of maximum$S(t)$. It can be observed that the rumor propagation lasts longer with slower velocity at a lower positive social reinforcement. Fig. 11 illustrates the density of stifler versus time varying different positive social reinforcement. It can be observed that positive social reinforcement determines the increment speed as well as final size of the rumor. Higher positive social reinforcement will result in larger final size of the rumor with shorter time and faster velocity. These results explain the influence of positive social reinforcement on rumor propagation.

5.3. Influence of negative reinforcement on the final threshold

Fig. 12 shows the heat map of the final size of the rumor under different rumor propagation probability λ and negative social reinforcement h, which exhibits obvious critical phenomenon. There is a clearly critical curve composed by critical rumor propagation probability and critical negative social reinforcement. The solid white line in Fig. 12 accords with $\lambda = 0.205 * 0.3/0.51(1+0.1)(1-h)$ where $\langle k\rangle/\langle k^2\rangle = 0.205$, $\alpha = 0.3$, $b = 0.1$ and $\delta = 0.2$, which matches the critical

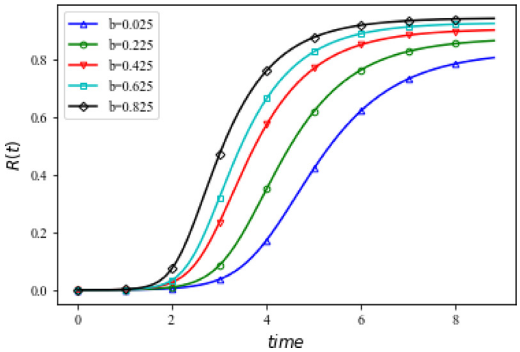

Fig. 11. Time evolution of the fraction of stifler nodes when the dynamics starts with a random selected node for several values of positive social reinforcement b.

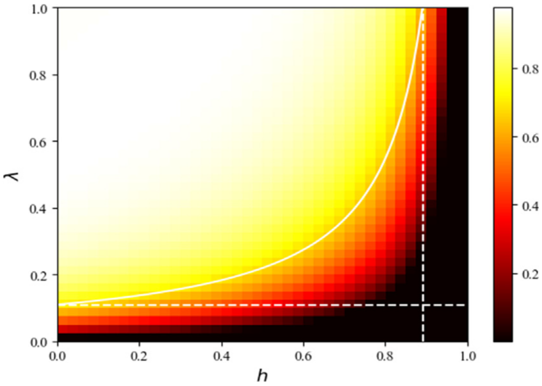

Fig. 12. Critical phenomenon of final size of the rumor R with different λ and h (colors represent final size of the rumor), while $\beta = 0.1, \alpha = 0.3, b = 0.1, \delta = 0.2$.

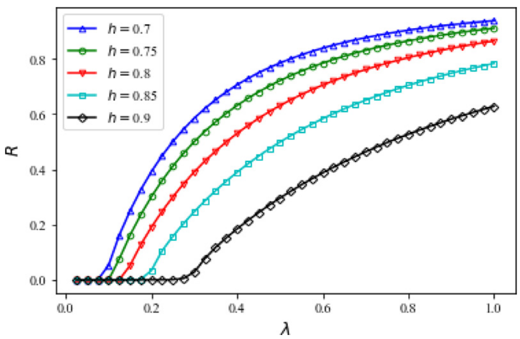

Fig. 13. Relationship with the final size of the rumor R and rumor propagation probability λ for several values of negative social reinforcement h.

curve of the heat map well. The horizontal dashed line corresponds $\lambda = 0.109$, below which the rumor cannot propagating in large scale of the network. The vertical white dotted line corresponds $h_c = 1 - \delta\langle k\rangle/\langle k^2\rangle(2\alpha - \alpha^2)(1+b) = 0.891$, and which implies that the rumor propagation dynamic will limited in a small area even with maximum rumor propagation probability $\lambda = 1$.

To show critical phenomenon of λ and h, we plot the final size of the rumor R as a function of rumor propagation probability λ and for several different values of negative social reinforcement$h$ in Fig. 13. It shows that the critical rumor propagation threshold is highly relevant to h, and higher negative social reinforcement leads to higher critical rumor propagation threshold. Fig. 14 shows the dependence of negative social reinforcement on rumor propagation probability. It shows that the critical negative social reinforcement is highly relevant to λ, and higher rumor propagation probability leads to higher critical negative social reinforcement. Figs. 13 and 14 show that the final size of the rumor R is related to rumor propagation probability λ and negative social reinforcement h. The result is consistent with the previous analysis.

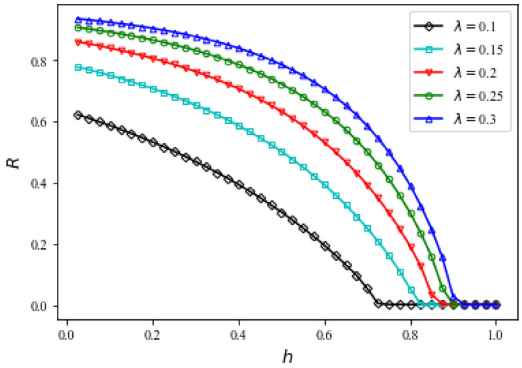

Fig. 14. Relationship with the final size of the rumor R and negative social reinforcement h for several values of rumor propagation probability λ.

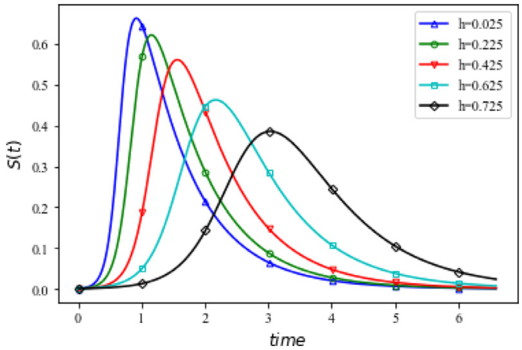

Fig. 15. Time evolution of the fraction of spreader nodes when the dynamics starts with a random selected node for several values of negative social reinforcement h.

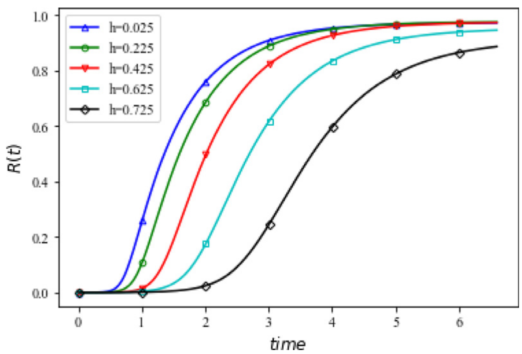

Fig. 16. Time evolution of the fraction of stifler nodes when the dynamics starts with a random selected node for several values of negative social reinforcement h.

In Figs. 15 and 16, the number of spreader nodes and stifler nodes over time are expressed under different negative social reinforcement respectively. Fig. 15 shows that large negative social reinforcement results in not only slow increment of spreader node but also small fraction of maximum$S(t)$. It can be observed that the rumor propagation lasts longer with slower velocity at high negative social reinforcement h. In Fig. 6, both increment speed and the final size of the rumor in steady state are dependent on negative social reinforcement. Lower negative social reinforcement will results in larger final size of the rumor with shorter time and faster velocity. These results further explain the influence of negative social reinforcement on rumor propagation.

6. Conclusion

In this paper, we introduce a model with scientific knowledge level and social reinforcement based on the traditional ISR model. In this model, we have divided the population into two groups, that is, people with scientific knowledge and people without scientific knowledge. Forgetfulness is an inherent characteristic of knowledge. In a group, if there are fewer individuals have knowledge, knowledge will gradually be forgotten. Therefore, people with scientific knowledge may forget the knowledge and transform into people without scientific knowledge, while people without scientific knowledge can be transformed into people with scientific knowledge through learning. For people with scientific knowledge, they can only be affected by those who do not have scientific knowledge. For people without scientific knowledge, they may be affected by the neighbors of all status, then changing states. Secondly, human behavior is complex, and rumor propagation is easily affected by external factors. In this paper, positive and negative social reinforcement are added into the model to comprehensively consider the impact of these two kinds of reinforcement on rumor. We used the mean field method to analyze the behavior of the model over heterogeneous networks. Through analysis, it found that the threshold of rumor propagation is increased by the scientific knowledge level, and the critical threshold of scientific knowledge level is also affected by the rumor propagation probability. In addition, positive social reinforcement and negative social reinforcement have a significant impact on the process of rumor propagation. Positive social reinforcement makes rumor propagation faster, wider in scope and smaller in critical threshold. Negative social reinforcement has the opposite effect. At the same time, the final size of the rumor is also affected by the scientific knowledge level, positive social reinforcement and negative social reinforcement to different degrees. These results are verified by numerical simulations on scale-free networks.

CRediT authorship contribution statement

Liang'an Huo: Conceptualization, Methodology, Supervision, Writing - review & editing, Formal analysis, Funding acquisition, Investigation, Project administration. **Sijing Chen:** Data curation, Writing - original draft, Visualization, Writing - review & editing, Investigation, Software, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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