JC3504 Robot Technology

Lecture 3: Mechanical Manipulator Introduction, Spatial Descriptions and Transformations

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Outline

Part 1: Mechanical Manipulator Introduction

- Robotic Arm Architecture
- Types of Robotic Arms
- Degrees of Freedom

Part 2: Spatial Descriptions and Transformations

- Position, Orientation and Frame
- Frame Transformations
- Transformation Operators
- Transform Equations

Mechanical Manipulator Introduction

Mechanical Manipulator

A mechanical manipulator, often simply referred to as a robotic arm, is a mechanically sophisticated device designed to perform a wide range of tasks with high precision and flexibility. It is essentially an artificial limb, usually programmable, with similar functionalities to a human arm. The manipulator can have multiple joints and links, which can be hydraulic, pneumatic, or electrically powered, allowing it to move in several directions and perform various operations.

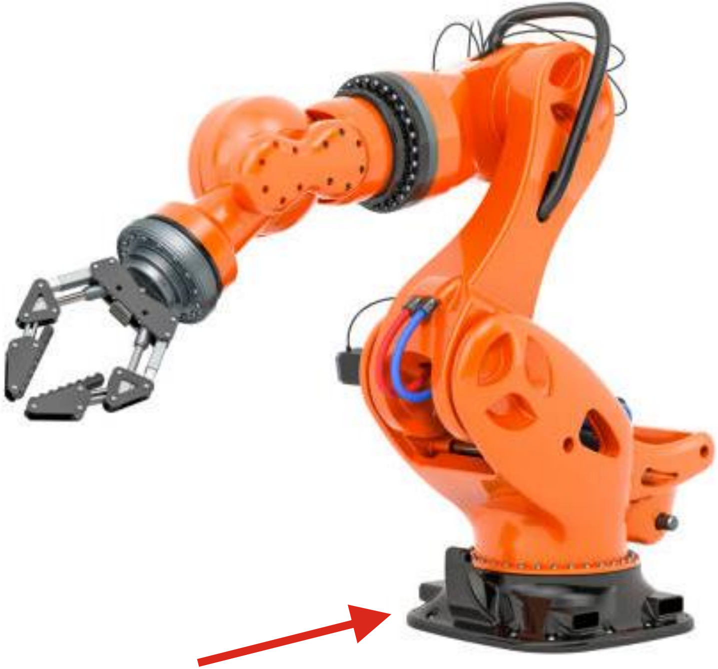
Robotic Arm Architecture

1. Base
2. Joints
3. Links
4. End Effector
5. Actuators
6. Sensors

Robotic Arm Architecture

1. Base

The base serves as the foundation of the robotic arm, anchoring it to a surface or platform. It provides stability and support for the entire structure.



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Robotic Arm Architecture

2. Joints

Joints are pivotal elements that allow for the flexibility and mobility of a robotic arm. They can be rotary or linear, enabling the arm to rotate, extend, retract, and move in various directions.

Robotic Arm Architecture

3. Links

Links are the rigid segments connected by joints. They form the arm's structure and determine its reach and range of motion.

Robotic Arm Architecture

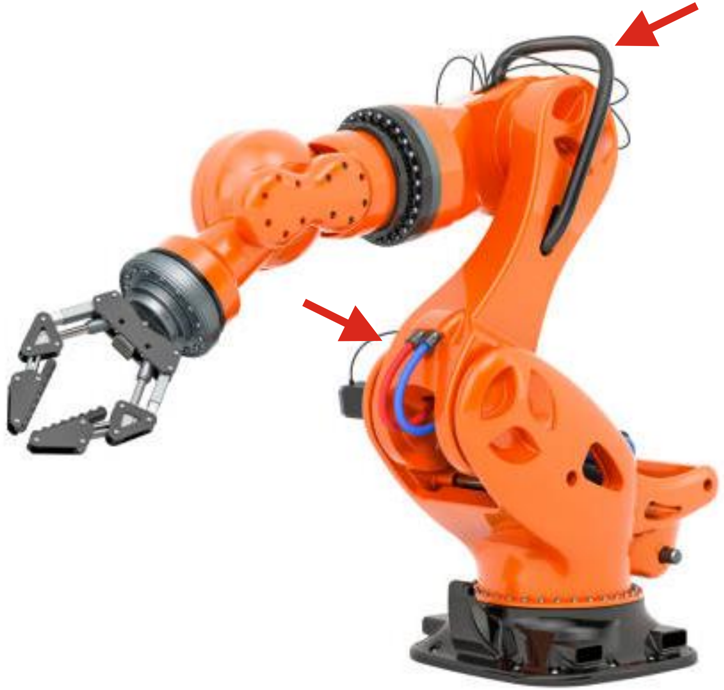
4. End Effector

The end effector is the component that interacts with the environment, such as a gripper, tool, or sensor. The design of the end effector is highly application-specific, enabling tasks like welding, picking, placing, or assembly.

Robotic Arm Architecture

5. Actuators

Actuators provide the force and movement necessary for the robotic arm's operation. They can be electric motors, pneumatic or hydraulic cylinders, depending on the required strength, precision, and speed.



Robotic Arm Architecture

6. Sensors

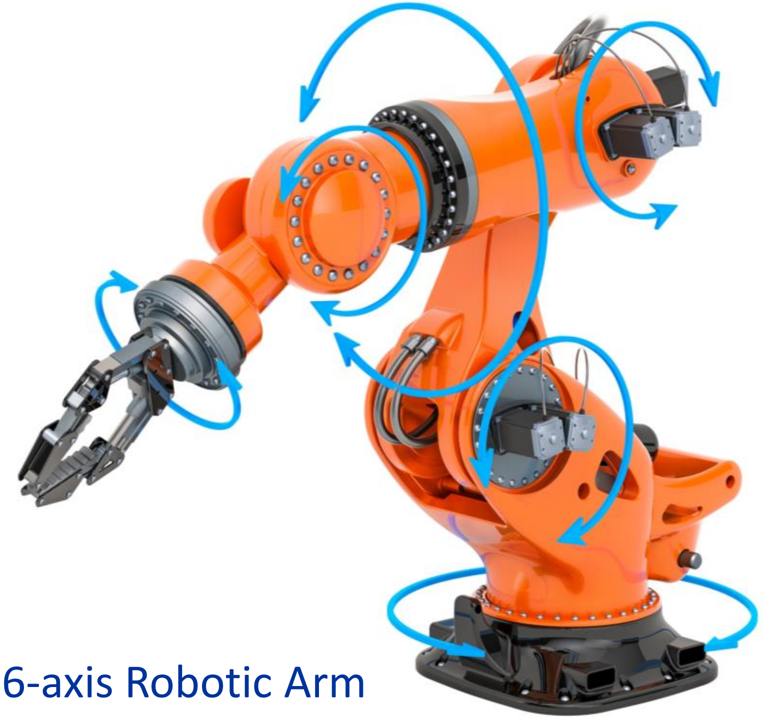
Sensors collect data about the environment and the arm's status. This includes position, velocity, force, and proximity sensors, which feed information back to the control system to facilitate accurate and responsive movements.

6-axis Force/Torque Sensor

Axis

An axis of a robotic arm refers to the degrees of freedom it has, which determine the directions in which it can move or rotate. Each axis represents a potential movement pivot point or joint on the robotic arm, allowing it to extend, bend, rotate, or twist.

The number of axes a robotic arm has directly affects its versatility and ability to manipulate objects or perform tasks in three-dimensional space.



6-axis Robotic Arm

Types of Robotic Arms

- Articulated Robotic Arms
- SCARA (Selective Compliance Assembly Robot Arm)
- Cartesian Robots
- Delta Robots
- ……

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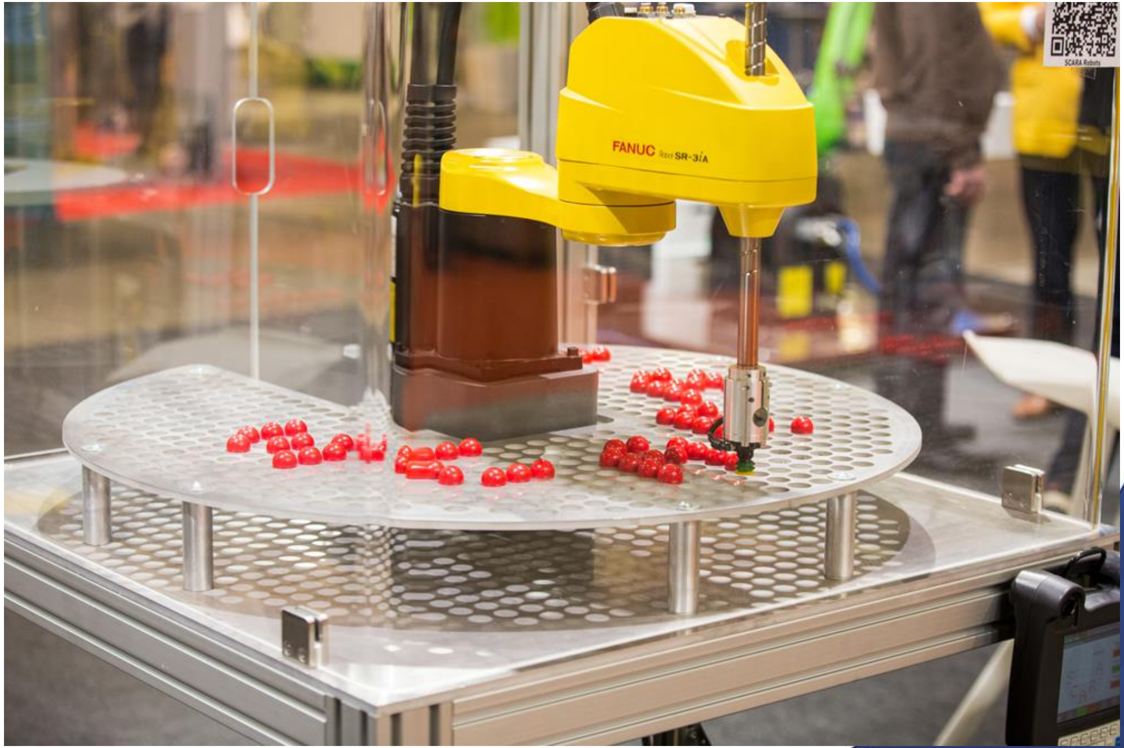

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Types of Robotic Arms

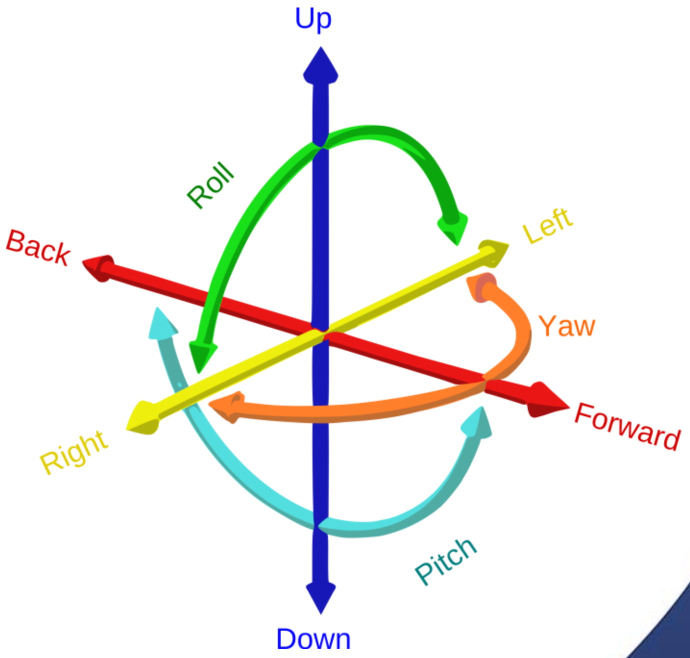
- Other types of robotic arms

Degrees of Freedom (DoF)

Degrees of Freedom (DoF)

The degrees of freedom (DOF) of a robot refer to the total number of independent movements or motions it can perform, dictating its ability to position itself and manipulate objects within its environment.

An object in the physical world can have up to **six** Cartesian degrees of freedom (6DoF), namely forward/backward, sideways, and up/down as well as rotations around those axes.



Wheel DoF

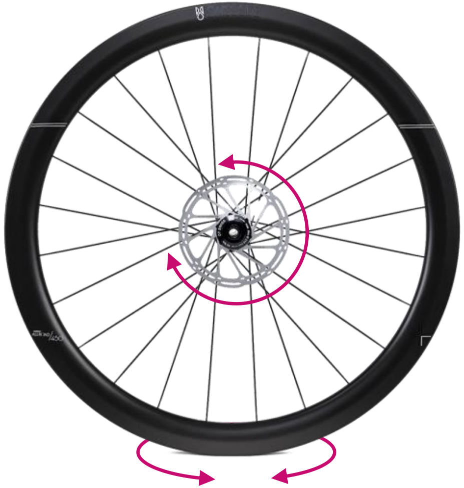
Only robots that use exclusively wheels with three degrees-of-freedom (3-DoF wheels) will be able to freely move on a plane. This is because the pose of a robot on a plane is fully given by three number: its position (two values) and its orientation (one value).

Robots that don’t have wheels with three degrees of freedom will have kinematic constraints that prevent them from reaching every possible point at every possible orientation.

2-DoF Wheels

Usually a wheel has two degrees of freedom, such as the front wheel of a bicycle.

- Rotation around the wheel axle
- Rotation around its contact point with the ground

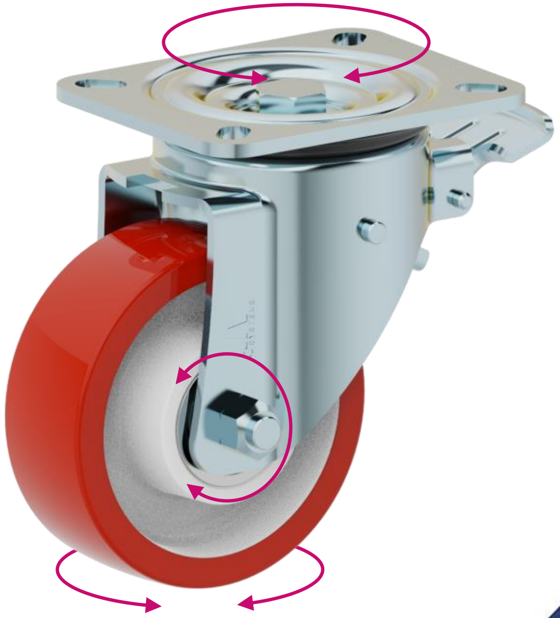


3-DoF Wheels

There are specifically designed 3-DoF wheels.

Caster wheel

- Rotation around the wheel axle
- Rotation around its contact point with the ground
- Rotation around the caster axis

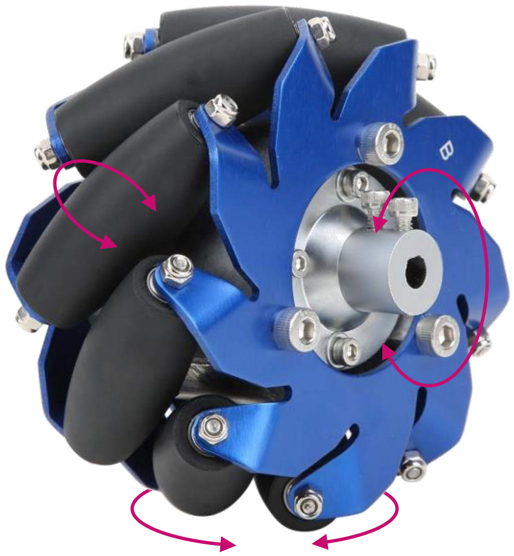


3-DoF Wheels

There are specifically designed 3-DoF wheels.

Swedish wheel

- Rotation around the wheel axle
- Rotation around its contact point with the ground
- Rotation around the caster axis

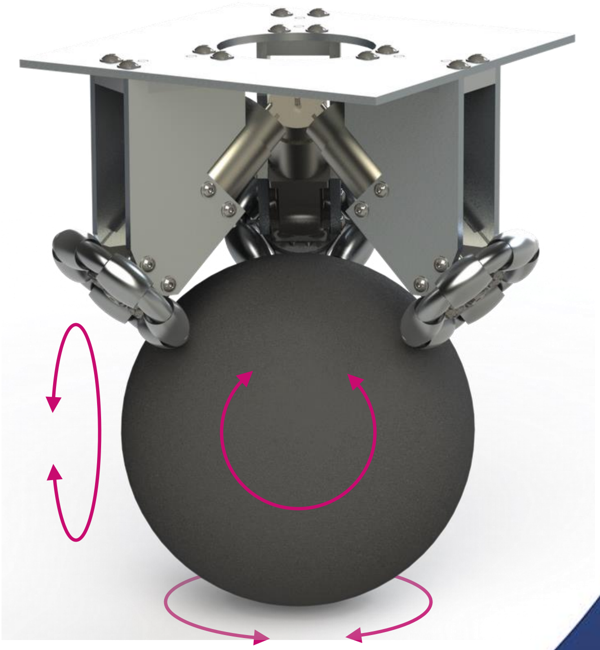


3-DoF Wheels

There are specifically designed 3-DoF wheels.

Spherical wheel

- Rotation in any direction
- Rotation around its contact point



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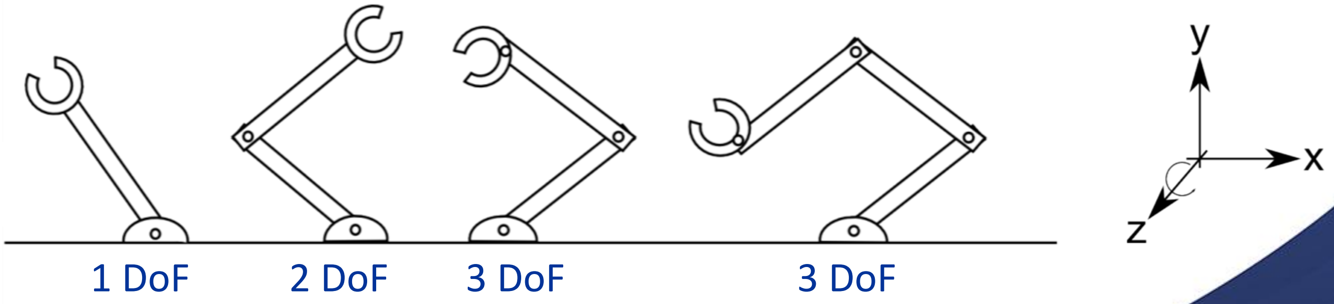
Manipulating Arms DoF

For manipulating arms, DoF usually refer to the positions and orientations, i.e., rotations around the primary axes, the end-effector can reach. As a rule of thumb, each joint usually adds a degree of freedom unless they are redundant, that is, moving in the same direction.

The figure shows a series of manipulators operating in a plane. By this, the degrees of freedom of the end-effector are limited to moving up and down, sideways, and rotating around its pivot point. As a plane only has those three degrees of freedom, adding additional joints cannot increase the degrees of freedom unless they allow the robot to also move in and out of the plane.



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Spatial Descriptions and Transformations

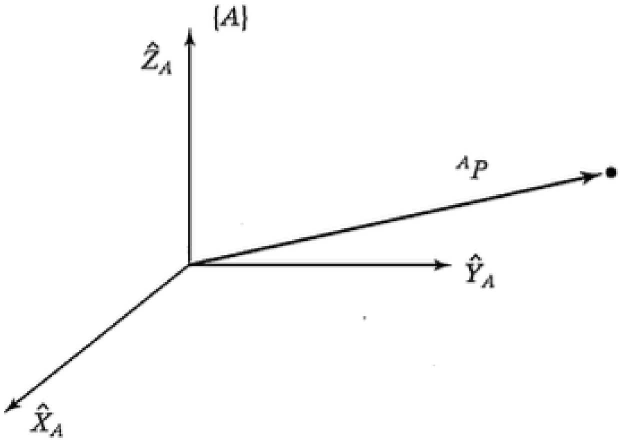
Description: Position, Orientation and Frame

A description is used to specify attributes of various objects with which a manipulation system deals. These objects are parts, tools, and the manipulator itself. In this section, we discuss the description of positions, of orientations, and of an entity that contains both of these descriptions: the frame (coordinate system).

Description of a Position

Once a coordinate system is established, we can locate any point in the universe with a 3×1 position vector.

In the slides, vectors are written with a leading superscript indicating the coordinate system to which they are referenced (unless it is clear from context).



$$^AP = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

Description of an Orientation

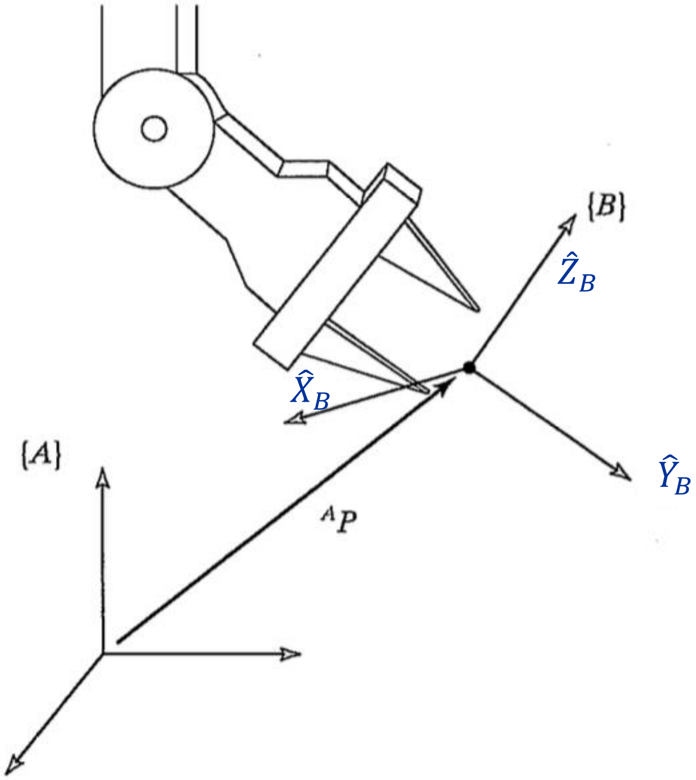
In order to describe the orientation of a body, we will attach a coordinate system {B} to the body and then give a description of {B} relative to the reference system {A}.

One way to describe {B} (i.e. the body attached coordinate system) is to a rotation matrix (${}^{A}_{B}R$), which consists of the unit vectors of its three principal axes in terms of the coordinate system {A}:

$$ {}^{A}_{B}R = [\hat{X}_B, \hat{Y}_B, \hat{Z}_B] $$

We use ${}^{A}_{B}R$ to denote the orientation of the body.

NB: the orientation is NOT affected ${}^{A}P$ (its' position).



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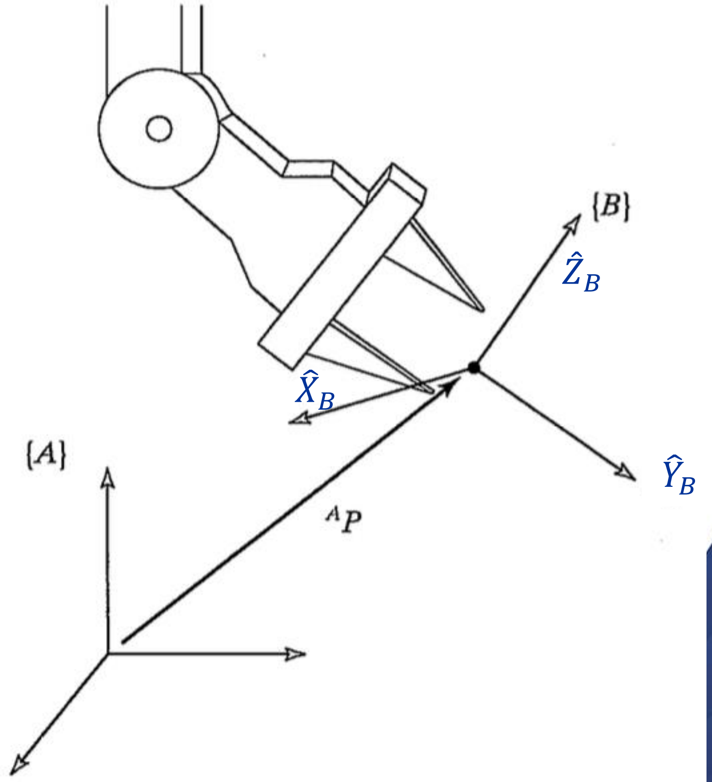
Properties of Rotation Matrix

- ${}^{A}_{B}R$, the rotation matrix which converts vectors of {B} to the vectors of {A}.
- ${}^{A}_{B}R$ is orthogonal, because according to the definition: (1) $\hat{X}_B, \hat{Y}_B, \hat{Z}_B$ are unit vectors and (2) they are perpendicular to each other.
- So, ${}^{A}_{B}R$ is invertible, and

$$ {}^{A}_{B}R^{-1} = {}^{A}_{B}R^{T} = {}^{B}_{A}R $$

$$ {}^{B}_{A}R\,{}^{A}_{B}R = {}^{A}_{B}R\,{}^{B}_{A}R = I $$

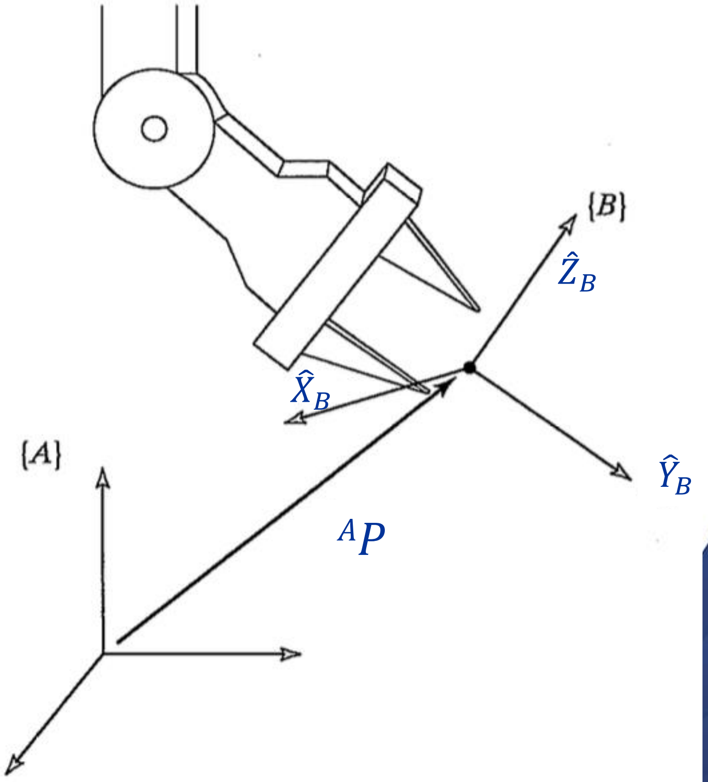


Description of a Frame

The information needed to completely specify the whereabouts of the manipulator hand is a position and an orientation. The point on the body whose position we describe could be chosen arbitrarily, however, for convenience, the point whose position we will describe is chosen as the origin of the body-attached frame.

The situation of a position and an orientation pair arises so often in robotics that we define an entity called a frame, which is a set of four vectors giving position and orientation information.

$$\{B\} = \{{}^{A}_{B}R, {}^{A}P\}$$

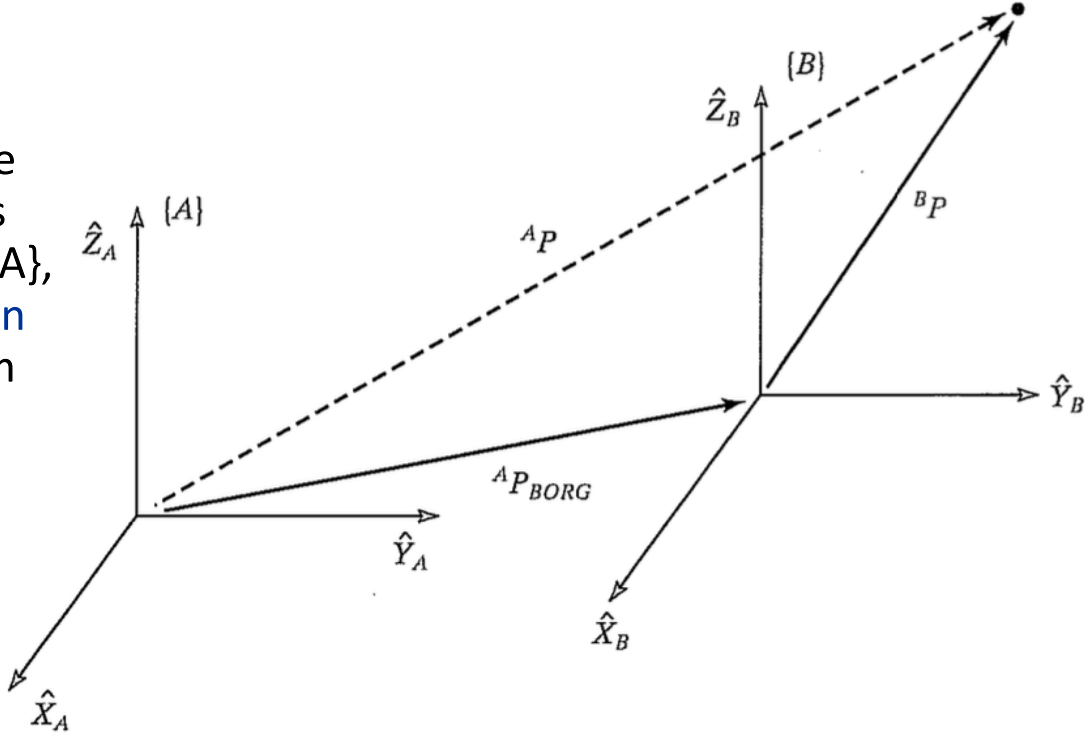


Changing Descriptions form Frame to Frame

Mappings involving translated frames

We have a position defined by the vector ${}^{B}P$. We wish to express this point in space in terms of frame {A}, when {A} has the same orientation as {B}. In this case, {B} differs from {A} only by a translation, which is given by ${}^{A}P_{BORG}$:

$${}^{A}P = {}^{B}P + {}^{A}P_{BORG}$$



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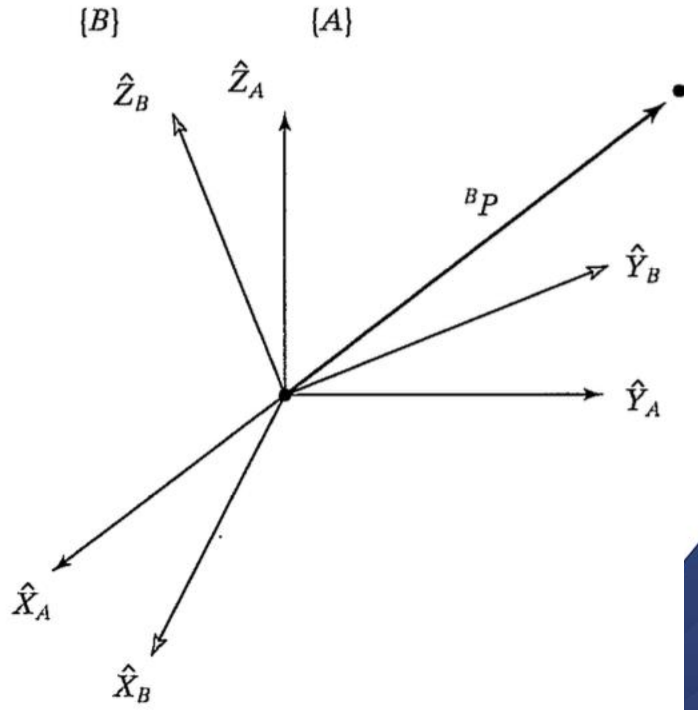
Changing Descriptions form Frame to Frame

Mappings involving rotated frames

We have a position defined by the vector ${}^{B}P$. We wish to express this point in space in terms of frame {A}, where the origins of the two frames are coincident, and the orientation {B} is given by the rotation matrix ${}^{A}_{B}R$:

$${}^{A}P = {}^{A}_{B}R\,{}^{B}P$$



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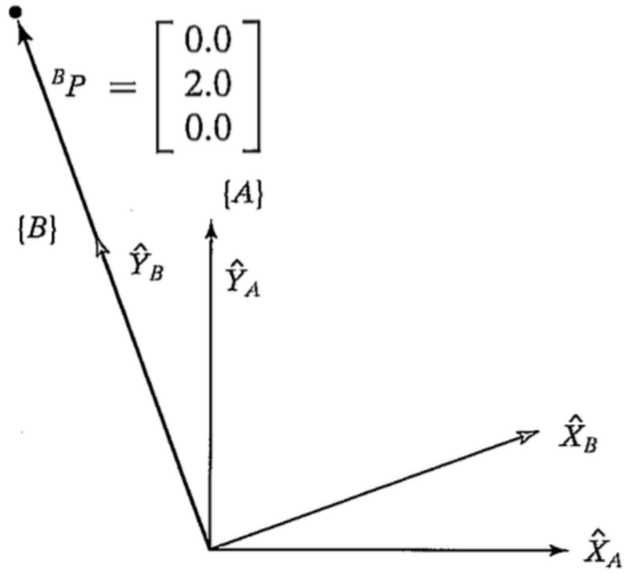
Example

A frame {B} is rotated relative to frame {A} about $\hat{Z}$ by 30 degrees. Here, $\hat{Z}$ is pointing out of the page.

- Writing the unit vectors of {B} in terms of {A} and the rotation matrix ${}^{A}_{B}R$.

$$
{}^{A}_{B}R = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}
$$

$$
{}^{A}P = {}^{A}_{B}R\,{}^{B}P = \begin{bmatrix} -1.000 \\ 1.732 \\ 0.000 \end{bmatrix}
$$



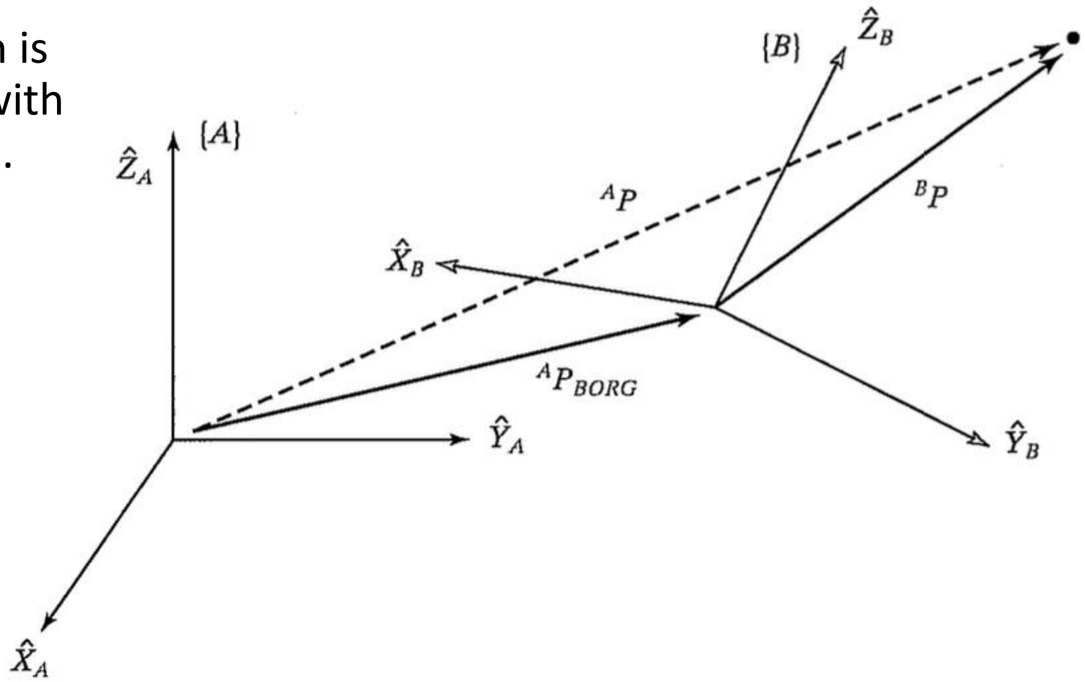
Changing Descriptions form Frame to Frame

Mappings involving general frames

The vector that locates {B}'s origin is called ${}^{A}P_{BORG}$. Also, {B} is rotated with respect to {A}, as described by ${}^{A}_{B}R$.

Given ${}^{B}P$, we wish to compute ${}^{A}P$:

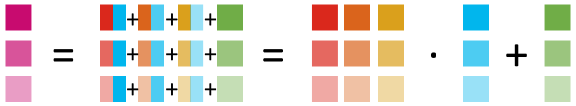
$$ {}^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG} $$

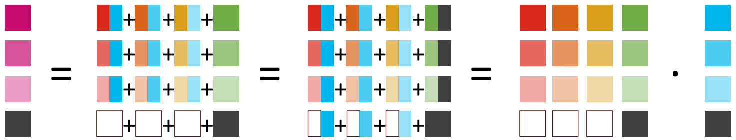


Changing Descriptions form Frame to Frame

We would like to think of a mapping from one frame to another as one operator in matrix form by using homogeneous transform.

$$ {}^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG} $$





$$ \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} $$

$$ {}^{A}P = {}^{A}_{B}T\,{}^{B}P $$

Example

Given ${}^{B}P = [3.0 \quad 7.0 \quad 0.0]^T$ and $\{B\} = \{{}^{A}_{B}R, {}^{A}P_{BORG}\}$ where

$${}^{A}_{B}R = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix} \text{ and } {}^{A}P_{BORG} = [10.0 \quad 5.0 \quad 0.0]$$

find ${}^{A}_{B}T$ and ${}^{A}P$.

$${}^{A}_{B}T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 10.0 \\ 0.500 & 0.866 & 0.000 & 5.0 \\ 0.000 & 0.000 & 1.000 & 0.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}P = {}^{A}_{B}T\,{}^{B}P = \begin{bmatrix} 9.098 \\ 12.562 \\ 0.000 \end{bmatrix}$$

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Operators

The same mathematical forms used to map points between frames can also be interpreted as operators (i.e. ${}^{X}_{Y}T$) that translate points, rotate vectors, or do both.

- Translational operators
  A translation moves a point in space a finite distance along a given vector direction (no rotation).
- Rotational operators
  Only do rotation with no translation
- Transformation operators
  Do both translation and rotation in one operator

Translational Operators

Recall that the position translation: ${}^{A}P = {}^{B}P + {}^{A}Q$, where ${}^{A}Q = {}^{A}P_{BORG}$

If we use a variable q to represent the extend of ${}^{A}Q$, we get:

$$ {}^{A}P = {}^{B}P + {}^{A}Q(q), \text{ where } {}^{A}Q(q) = {}^{A}Q \cdot q $$

then, ${}^{A}Q$ or ${}^{A}Q(\cdot)$ is the translational operator.

We can also use homogeneous transform to represent ${}^{A}Q(\cdot)$ by $D_Q(\cdot)$

$$ D_Q(q) = \begin{bmatrix} 1 & 0 & 0 & {}^{A}Q_x \\ 0 & 1 & 0 & {}^{A}Q_y \\ 0 & 0 & 1 & {}^{A}Q_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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Rotational Operators

A rotational operator $R_v(\theta)$ can be defined by a given vector $v = [v_x \quad v_y \quad v_z]^T$ with a parameter θ.

In this notation, $R_v(\theta)$ performs a rotation about the axis direction v by θ degrees. This operator can be written accord to *Rodrigues' rotation formula*:

$$R_v(\theta) = I + (\sin\theta)K + (1 - \cos\theta)K^2$$

where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $K = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$

The homogeneous transform is:

$$\begin{bmatrix} R_v(\theta) & 0 \\ 0 & 1 \end{bmatrix}$$

NB: when a rotation matrix is shown as an operator, no sub- or superscripts appear, because it is not viewed as relating two frames.

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Rotational Operators about $\hat{X}$

Specifically, when $v = \hat{X} \equiv [1 \quad 0 \quad 0]^T$,

$$\Rightarrow K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow R_X(\theta) = I + (\sin\theta)K + (1-\cos\theta)K^2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sin\theta \\ 0 & \sin\theta & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \cos\theta - 1 & 0 \\ 0 & 0 & \cos\theta - 1 \end{bmatrix}$$

$$\Rightarrow R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotational Operators about $\hat{Y}$

When $v = \hat{Y} \equiv \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$,

$$\Rightarrow K = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow R_Y(\theta) = I + (\sin\theta)K + (1 - \cos\theta)K^2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \sin\theta \\ 0 & 0 & 0 \\ -\sin\theta & 0 & 0 \end{bmatrix} + \begin{bmatrix} \cos\theta - 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \cos\theta - 1 \end{bmatrix}$$

$$\Rightarrow R_Y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Rotational Operators about $\hat{Z}$

When $v = \hat{Z} \equiv [0 \quad 0 \quad 1]^T$,

$$\Rightarrow K = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow R_Z(\theta) = I + (\sin\theta)K + (1 - \cos\theta)K^2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\sin\theta & 0 \\ \sin\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \cos\theta - 1 & 0 & 0 \\ 0 & \cos\theta - 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow R_Z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformation Operators

As with vectors and rotation matrices, a frame has another interpretation as a transformation operator. In this interpretation, only one coordinate system is involved, and so the symbol T is used without sub- or superscripts. The operator T rotates and translates a vector ${}^{A}P$, to compute a new vector, i.e.:

$$ {}^{A}P_2 = T({}^{A}P_1) = TAP_1 $$

Here, the mathematics described T and ${}^{Y}_{X}T$ is the same, only our interpretation is different. This fact also allows us to see how to obtain homogeneous transforms that are to be used as operators:

The transformation that rotates by R and translates by Q is the same as the transform that describes a frame rotated by R and translated by Q relative to the reference frame.

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Compound Transformations

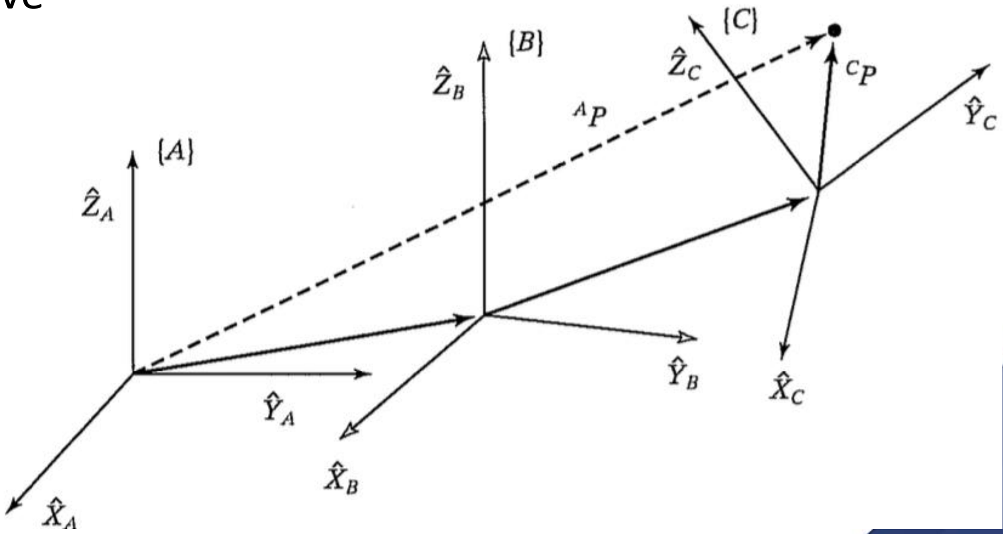
In the figure, we have ${}^{C}P$ and wish to find ${}^{A}P$, given frame {C} is known relative to frame {B}, and frame {B} is known relative to frame {A}.

$${}^{B}P = {}^{B}_{C}T\,{}^{C}P$$

$${}^{A}P = {}^{A}_{B}T\,{}^{B}P = {}^{A}_{B}T\,{}^{B}_{C}T\,{}^{C}P$$

from which we could define:

$${}^{A}_{C}T = {}^{A}_{B}T\,{}^{B}_{C}T$$



Inverting a Transform

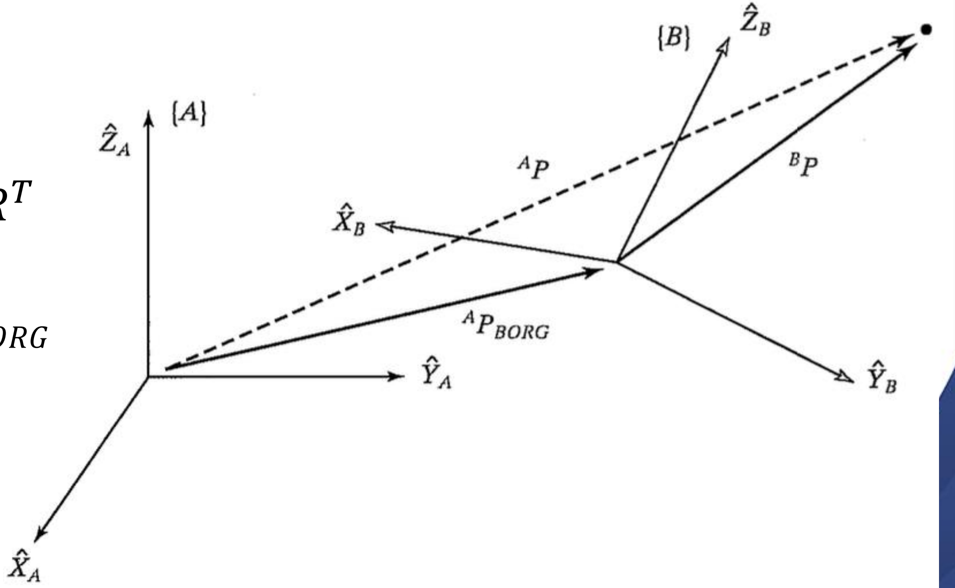
Suppose we have ${}^{A}P$ and wish to find ${}^{B}P$, given ${}^{A}_{B}T$

$$ {}^{B}P = {}^{B}_{A}T\,{}^{A}P $$

Recall that ${}^{B}_{A}T = \begin{bmatrix} {}^{B}_{A}R & {}^{B}P_{BORG} \\ 0 & 1 \end{bmatrix}$, ${}^{B}_{A}R = {}^{A}_{B}R^{T}$

$$ {}^{B}P_{BORG} = -{}^{B}_{A}R\,{}^{A}P_{BORG} = -{}^{A}_{B}R^{T}\,{}^{A}P_{BORG} $$

So, ${}^{B}_{A}T = \begin{bmatrix} {}^{B}_{A}R^{T} & -{}^{A}_{B}R^{T}\,{}^{A}P_{BORG} \\ 0 & 1 \end{bmatrix} = {}^{B}_{A}T^{-1}$



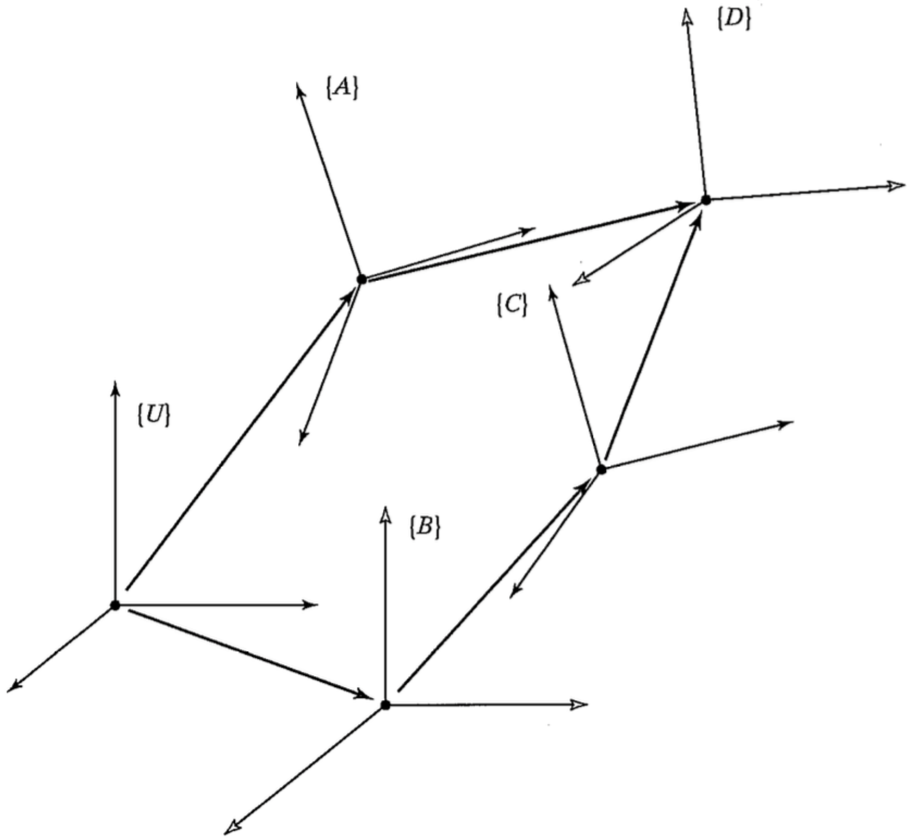
Transform Equations

A frame {D} can be expressed as products of transformations in two different ways.

1. ${}^{U}_{D}T = {}^{U}_{A}T\,{}^{A}_{D}T$, 2. ${}^{U}_{D}T = {}^{U}_{B}T\,{}^{B}_{C}T\,{}^{C}_{D}T$

This derives that: ${}^{U}_{A}T\,{}^{A}_{D}T = {}^{U}_{B}T\,{}^{B}_{C}T\,{}^{C}_{D}T$

Also, ${}^{B}_{C}T = {}^{U}_{B}T^{-1}\,{}^{U}_{A}T\,{}^{A}_{D}T\,{}^{C}_{D}T^{-1}$



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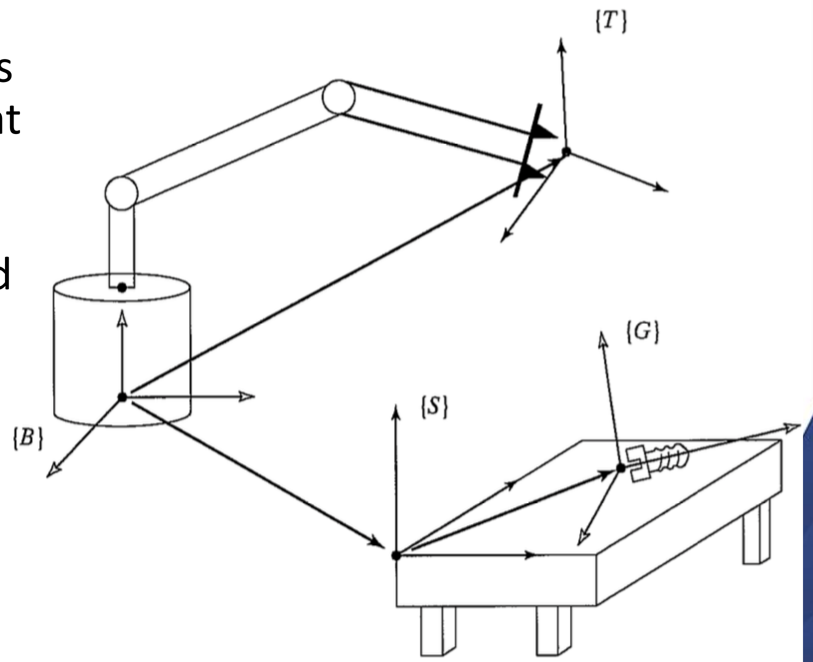
Example

Assume that we know the transform ${}^{B}_{T}T$, which describes the frame at the manipulator's fingertips {T} relative to the base of the manipulator, {B}, that we know where the tabletop is located in space relative to the manipulator's base (because we have a description of the frame {S} that is attached to the table as shown, ${}^{B}_{S}T$, and that we know the location of the frame attached to the bolt lying on the table relative to the table frame i.e. ${}^{S}_{G}T$. Calculate the position and orientation of the bolt relative to the manipulator’s hand, ${}^{T}_{G}T$.



$$ {}^{T}_{G}T = {}^{B}_{T}T^{-1}\,{}^{B}_{S}T\,{}^{S}_{G}T $$

Conclusion

Reviewed the fundamentals of Mechanical Manipulators, including the architecture and various types of robotic arms, and the concept of degrees of freedom is introduced.

Explored Spatial Descriptions and Transformations, covering essential concepts such as position, orientation, frame transformations, operators, and equations, crucial for understanding how robots interact with their environment.

Highlighted the importance of these concepts in designing and controlling robotic systems, enabling precise movement and manipulation within three-dimensional space.