UNIVERSITY OF ABERDEEN 1495

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ABERDEEN 2040

Association Rule Learning (1)

Data Mining & Visualisation

Lecture 19

2025

Road Map

- Basic concepts
- Apriori algorithm
- Different data formats for mining
- Mining with multiple minimum supports
- Mining class association rules
- Summary

Association Rule Mining

- Proposed by Agrawal et al in 1993.
- It is an important data mining model studied extensively by the database and data mining community.
- Assume all data are categorical.
- No good algorithm for numeric data.
- Initially used for Market Basket Analysis to find how items purchased by customers are related.

Bread $\rightarrow$ Milk [sup = 5%, conf = 100%]

The model: data

- $I = \{i_1, i_2, \ldots, i_m\}$: a set of *items*.
- Transaction *t* :
  - *t* a set of items, and $t \subseteq I$.
- Transaction Database *T*: a set of transactions $T = \{t_1, t_2, \ldots, t_n\}$.

Transaction data: supermarket data

- Market basket transactions:

  t1: {bread, cheese, milk}

  t2: {apple, eggs, salt, yogurt}

  … …

  tn: {biscuit, eggs, milk}

- Concepts:
  - An *item*: an item/article in a basket
  - *I*: the set of all items sold in the store
  - A *transaction*: items purchased in a basket; it may have TID (transaction ID)
  - A *transactional dataset*: A set of transactions

Transaction data: a set of documents

- **A text document data set. Each document is treated as a “bag” of keywords**

doc1: Student, Teach, School

doc2: Student, School

doc3: Teach, School, City, Game

doc4: Baseball, Basketball

doc5: Basketball, Player, Spectator

doc6: Baseball, Coach, Game, Team

doc7: Basketball, Team, City, Game

The model: rules

- A transaction *t* contains *X*, a set of items (itemset) in *I*, if $X \subseteq t$.
- An association rule is an implication of the form:

  $$X \to Y, \text{ where } X, Y \subset I, \textit{ and } X \cap Y = \emptyset$$

- An itemset is a set of items.
  - E.g., X = {milk, bread, cereal} is an itemset.
- A *k*-itemset is an itemset with *k* items.
  - E.g., {milk, bread, cereal} is a 3-itemset

Rule strength measures

- Support: The rule holds with support *sup* in *T* (the transaction data set) if sup% of transactions contain $X \cap Y$.
  - $sup = \Pr(X \cap Y)$.
- Confidence: The rule holds in *T* with confidence *conf* if *conf*% of tranactions that contain *X* also contain *Y*.
  - $conf = \Pr(Y \mid X)$
- An association rule is a pattern that states when *X* occurs, *Y* occurs with certain probability.

Support and Confidence

Support count: The support count of an itemset *X*, denoted by *X.count*, in a data set *T* is the number of transactions in *T* that contain *X*. Assume *T* has *n* transactions.

Then,

$$support = \frac{(X \cap Y).count}{n}$$

$$confidence = \frac{(X \cap Y).count}{X.count}$$

Goal and key features

Goal: Find all rules that satisfy the user-specified *minimum support* (minsup) and *minimum confidence* (minconf).

Key Features

- Completeness: find all rules.
- No target item(s) on the right-hand-side
- Mining with data on hard disk (not in memory)

An example

t1:	Beef, Chicken, Milk
t2:	Beef, Cheese
t3:	Cheese, Boots
t4:	Beef, Chicken, Cheese
t5:	Beef, Chicken, Clothes, Cheese, Milk
t6:	Chicken, Clothes, Milk
t7:	Chicken, Milk, Clothes

Transaction data

Assume:

minsup = 30%

minconf = 80%

An example frequent *itemset*:

{Chicken, Clothes, Milk} [sup = 3/7]

Association rules from the itemset:

Clothes $\rightarrow$ Milk, Chicken [sup = 3/7, conf = 3/3]

… …

Clothes, Chicken $\rightarrow$ Milk, [sup = 3/7, conf = 3/3]

Transaction data representation

- A simplistic view of shopping baskets,
- Some important information not considered. E.g,
    - the quantity of each item purchased and
    - the price paid.

Many mining algorithms

- There are a large number of them!!
- They use different strategies and data structures.
- Their resulting sets of rules are all the same.
  - Given a transaction data set *T*, and a minimum support and a minimum confident, the set of association rules existing in *T* is uniquely determined.
- Any algorithm should find the same set of rules although their computational efficiencies and memory requirements may be different.
- We study only one: the Apriori Algorithm

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The Apriori algorithm

Probably the best known algorithm

Two steps:

- Find all itemsets that have minimum support (*frequent itemsets*, also called large itemsets).
- Use frequent itemsets to generate rules.

E.g., a frequent itemset

{Chicken, Clothes, Milk} [sup = 3/7]

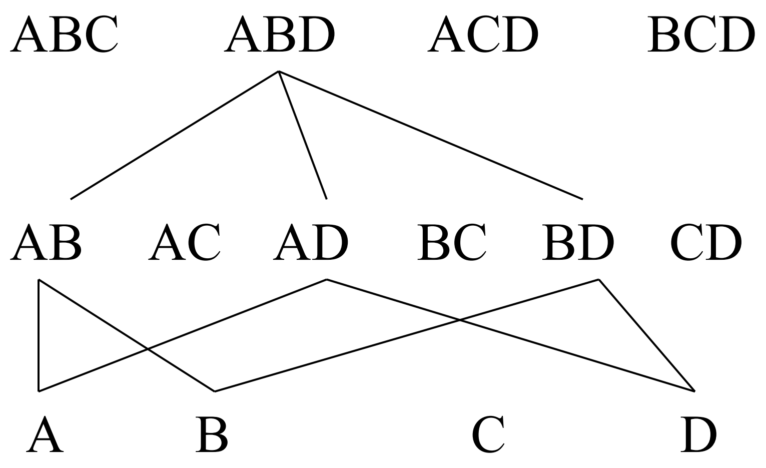
and one rule from the frequent itemset

Clothes $\rightarrow$ Milk, Chick [sup = 3/7, conf = 3/3]

Step 1: Mining all frequent itemsets

A frequent *itemset* is an itemset whose support is ≥ minsup.

Key idea: The apriori property (downward closure property): any subsets of a frequent itemset are also frequent itemsets



The Algorithm

Iterative algo. (also called level-wise search): Find all 1-item frequent itemsets; then all 2-item frequent itemsets, and so on.

- In each iteration *k*, only consider itemsets that contain some *k*-1 frequent itemset.

Find frequent itemsets of size 1: F_1

From $k = 2$

- C_k = candidates of size k: those itemsets of size k that could be frequent, given F_{k-1}
- F_k = those itemsets that are actually frequent, $F_k \subseteq C_k$ (need to scan the database once).

Example – Finding frequent itemsets

Dataset T
minsup=0.5

TID	Items
T100	1, 3, 4
T200	2, 3, 5
T300	1, 2, 3, 5
T400	2, 5

itemset:count

1. scan T ➔ C_1: {1}:2, {2}:3, {3}:3, {4}:1, {5}:3

➔ F_1: {1}:2, {2}:3, {3}:3, {5}:3

➔ C_2: {1,2}, {1,3}, {1,5}, {2,3}, {2,5}, {3,5}

2. scan T ➔ C_2: {1,2}:1, {1,3}:2, {1,5}:1, {2,3}:2, {2,5}:3, {3,5}:2

➔ F_2: **{1,3}**:2, **{2,3}**:2, **{2,5}:**3, **{3,5}**:2

➔ C_3: {2, 3,5}

3. scan T ➔ C_3: **{2, 3, 5}**:2 ➔ F_3: **{2, 3, 5}**

Details: ordering of items

- The items in I are sorted in lexicographic order (which is a total order).
- The order is used throughout the algorithm in each itemset.
- $\{w[1], w[2], \ldots, w[k]\}$ represents a k-itemset w consisting of items $w[1], w[2], \ldots, w[k]$, where $w[1] < w[2] < \ldots < w[k]$ according to the total order.

Details: the algorithm

Algorithm Apriori(T)

$C_1 \leftarrow$ init-pass(T);

$F_1 \leftarrow \{f \mid f \in C_1, f.\text{count}/n \geq minsup\}$; // n: no. of transactions in T

for (k = 2; $F_{k-1} \neq \emptyset$; k++) **do**

$C_k \leftarrow$ candidate-gen(F_{k-1});

for each transaction $t \in T$ **do**

for each candidate $c \in C_k$ **do**

if c is contained in t **then**

$c.count$++;

end

end

$F_k \leftarrow \{c \in C_k \mid c.count/n \geq minsup\}$

end

return $F \leftarrow \bigcup_k F_k$;

Apriori candidate generation

The candidate-gen function takes $F_{k\text{-}1}$ and returns a superset (called the candidates) of the set of all frequent k-itemsets. It has two steps

- *join* step: Generate all possible candidate itemsets C_k of length k
- *prune* step: Remove those candidates in C_k that cannot be frequent.

Candidate generation function

```
Function candidate-gen(F_{k-1})
  C_k ← ∅;
  forall f_1, f_2 ∈ F_{k-1}
        with f_1 = {i_1, … , i_{k-2}, i_{k-1}}
        and f_2 = {i_1, … , i_{k-2}, i'_{k-1}}
        and i_{k-1} < i'_{k-1} do
    c ← {i_1, …, i_{k-1}, i'_{k-1}};          // self-join f_1 and f_2
    C_k ← C_k ∪ {c};
    for each (k-1)-subset s of c do
        if (s ∉ F_{k-1}) then
            delete c from C_k;                 // prune
    end
  end
  return C_k;
```

How Pruning Works

1. **Candidate Generation:**
    1. Initially, all individual items (1-itemsets) are evaluated against the minimum support threshold.
    2. Frequent items that pass this threshold are combined to form 2-itemsets, which are then tested against the minimum support.
2. **Elimination Process:**
    1. Any itemsets (1-itemsets, 2-itemsets, etc.) that do not meet the minimum support threshold are "pruned" or discarded from further consideration.
    2. For instance, if a 1-itemset is infrequent, all 2-itemsets containing that 1-itemset can be safely excluded.
3. **Further Iterations:**
    1. The surviving frequent 2-itemsets are combined to form 3-itemsets, which are also tested against the minimum support.
    2. This process continues iteratively for larger itemsets until no more frequent itemsets can be formed.

Benefit of Pruning

•**Reduced Computation:** Eliminates unnecessary checks for itemsets that cannot meet the minimum support, saving computational resources.
•**Efficient Searches:** Focuses only on candidate itemsets that have the potential to be frequent.

Example

- $F_3 = \{\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\},$
  $\{1, 3, 5\}, \{2, 3, 4\}\}$

- After join
  - $C_4 = \{\{1, 2, 3, 4\}, \{1, 3, 4, 5\}\}$
- After pruning:
  - $C_4 = \{\{1, 2, 3, 4\}\}$
    because {1, 4, 5} is not in F_3 ({1, 3, 4, 5} is removed)

Step 2: Generating rules from frequent itemsets

- Frequent itemsets $\neq$ association rules
- One more step is needed to generate association rules
- For each frequent itemset *X*,

  For each proper nonempty subset *A* of *X*,
  - Let *B* = X - *A*
  - A $\rightarrow$ B is an association rule if
    - Confidence(A $\rightarrow$ B) $\geq$ minconf,

      support(A $\rightarrow$ B) = support(A$\cup$B) = support(X)

      confidence(A $\rightarrow$ B) = support(A $\cup$ B) / support(A)

Generating rules: an example

Suppose {2,3,4} is frequent, with sup=50%

- Proper nonempty subsets: {2,3}, {2,4}, {3,4}, {2}, {3}, {4}, with sup=50%, 50%, 75%, 75%, 75%, 75% respectively
- These generate these association rules:
  - 2,3 $\rightarrow$ 4, confidence=100%
  - 2,4 $\rightarrow$ 3, confidence=100%
  - 3,4 $\rightarrow$ 2, confidence=67%
  - 2 $\rightarrow$ 3,4, confidence=67%
  - 3 $\rightarrow$ 2,4, confidence=67%
  - 4 $\rightarrow$ 2,3, confidence=67%
  - All rules have support = 50%

Generating rules: summary

To recap, in order to obtain A $\rightarrow$ B, we need to have support(A $\cup$ B) and support(A)

All the required information for confidence computation has already been recorded in itemset generation. No need to see the data *T* any more.

This step is not as time-consuming as frequent itemsets generation.

On Apriori Algorithm

Seems to be very expensive

Level-wise search

K = the size of the largest itemset

It makes at most K passes over data

In practice, K is bounded (10).

The algorithm is very fast. Under some conditions, all rules can be found in linear time.

Scale up to large data sets

More on association rule mining

Clearly the space of all association rules is exponential, $O(2^m)$, where m is the number of items in *I*.

The mining exploits sparseness of data, and high minimum support and high minimum confidence values.

Still, it always produces a huge number of rules, thousands, tens of thousands, millions, ...