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Experiment8

Aim:ToLearnCalculuswithSageMath

7.1ThesyntaxforEvaluationofIntegrationisProblem:

Evaluate $\int \frac{\cos(x)}{\sqrt{\sin(x)+1}}dx$

In [1]:

```
f(x) = cos(x)/sqrt(1+sin(x))
```

In [2]:

```
show(integrate(f(x),x))
```

$$2\sqrt{\sin(x)+1}$$

Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{\sqrt{\sin(x)+1}}dx$

In [3]:

```
show(integrate(f(x),x,0, pi/2))
```

$$2\sqrt{2}-2$$

Evaluate $\int_0^{\infty} e^{-x^2}dx$

In [4]:

```
integral(e^(-x^2),x,0, infinity)
```

Out[4]:

1/2*sqrt(pi)

Exercise7.1

Evaluatefollowingintegrals

$$(1).\int \frac{-4}{\sqrt{1-x^2}}dx$$

In [5]:

```
f(x)=-4/sqrt(1-x^2)
show(f(x))
```

$$-\frac{4}{\sqrt{-x^2+1}}$$

In [6]:

```
show(integrate(f(x),x,0, pi))
```

$$-4\arcsin(\pi)$$

$$(2)\ \int \sin^5(x)\cos^2(x)dx$$

In [7]:

```
f(x)=(sin(x)^5)*(cos(x)^5)
show(f(x))
```

$$\cos(x)^5\sin(x)^5$$

In [8]:

```
show(integrate(f(x),x,0, pi))
```

$$0$$

$$(3).\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{1+\sin(x)-\cos(x)}$$

In [10]:

```
f(x)=1/(1+sin(x)-cos(x))
```

In [11]:

```
show(f(x))
```

$$-\frac{1}{\cos(x)-\sin(x)-1}$$

In [12]:

```
show(integrate(f(x),x,pi/3, pi/2))
```

$$\frac{1}{2}\log(3)-\log(2)+\log\left(\frac{1}{3}\sqrt{3}+1\right)$$

7.2.AverageValueofafunction

Averagevalueoffunctionf(x)oninterval[a,b]isgivenbyAvg= $\frac{1}{b-a}\int_a^b f(x)dx$

Problem:Acartravelswithvelocityy=4t+10betweent=0andt=5.Findtheaveragevelocity.

In [9]:

```
var('t')
a,b = 0,  5
v(t) = 4*t+10
avg_vel = 1/(b-a)*integral(v(t),t,a,b)
print(avg_vel)
```

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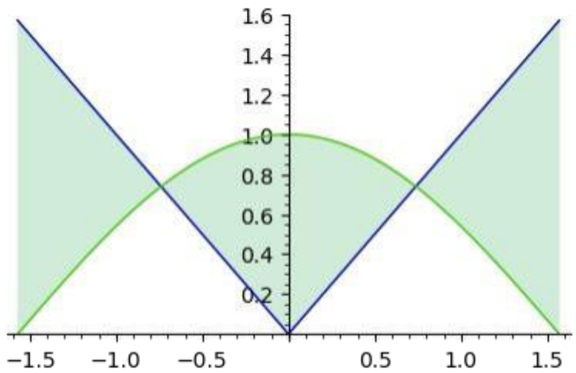
7.3ApplicationsofIntegrals

7.3.1Areas

Problem:Findtheareaenclosed between |x| andcos(x)haa

In [30]:

```
## Area between two curves
f (x ) = abs  (x)
g(x) =  cos(x)
a, b = -pi/2,pi/2
h = plot ((f(x), g(x)),x, a,b, figsize =4, fill ={0:g,1:f})
show(h)
```



In [31]:

```
c1 = find_root(f(x) -g(x),-1,0)
c2 = find_root(f(x)-g(x),0,1)
c1,c2
```

Out[31]:

(-0.7390851332151559, 0.7390851332151559)

In [32]:

```
I1 = integral(f(x)-g(x) ,x ,a,c1)
I2 = integral(g(x)-f(x) ,x ,c1,c2)
I3 = integral (f(x)-g(x) ,x ,c2 , b)
(I1+I2+I3).n()
```

Out[32]:

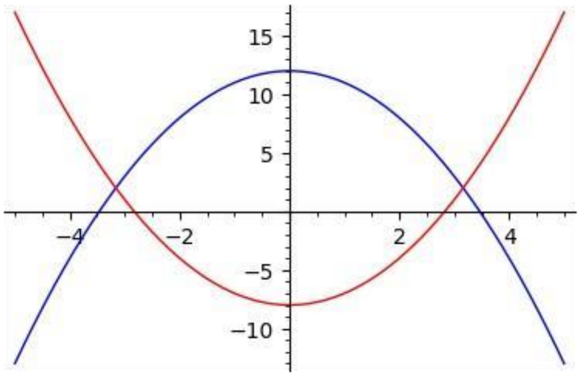
2.06935554872585

Problem:Findtheareaenclosedbetweentwocurvesy=12−x ^{2} andy=x ^{2} −8.

In [33]:

```
f(x)=12-x^2
g(x) = x^2-8
plot(f(x),-5,5)+plot(g(x),-5,5,color='red',figsize=4)
```

Out[33]:



In [34]:

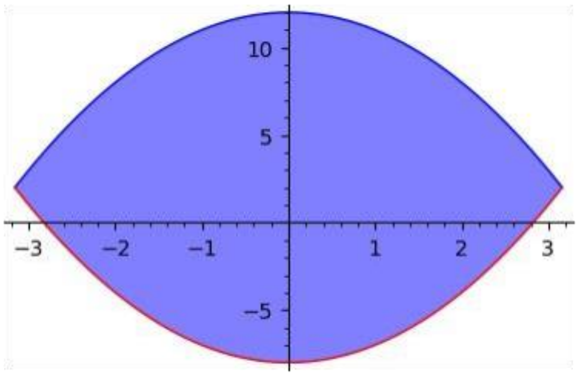
```
S = solve(f(x)==g(x),x,solution_dict=True)
a,b = S[0][x],S[1][x]
a,b
```

Out[34]:

(-sqrt(10), sqrt(10))

In [35]:

```
p1 = plot(f(x),(x,a,b),fill=g(x),fillcolor='blue')
p2 = plot(g(x),(x,a,b),color='red')
show(p1+p2,figsize=4)
```



In [36]:

```
A = integral((f(x)-g(x)),x,a,b)
show(A.n())
```

84.3274042711568

7.3.2.ArcLength

Ifthecurveisgivenbyy=f(x),a≤x≤b.Thenarclengthofcurveisgivenby

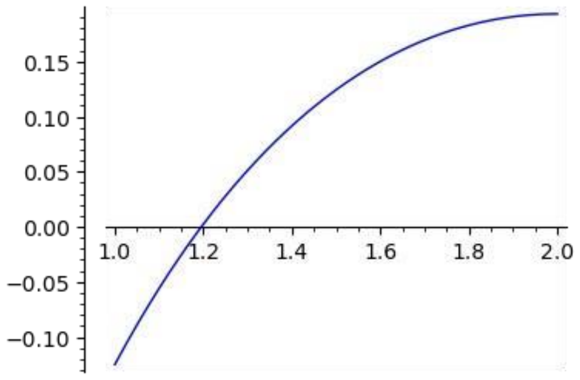
$$L = \int_a^b \sqrt{1+ \left(\frac{dy}{dx}\right)^2} dx$$

Problem:Findarclengthofthecurvey=log(x)− $\frac{x^2}{8}$, 1≤x≤2.

In [37]:

```
f(x)=ln(x)-x^2/8
plot(f(x),(x,1,2),figsize=4)
```

Out[37]:



In [38]:

```
integral(sqrt(1+derivative(f,x)^2),x,1,2)
```

Out[38]:

log(2) + 3/8

7.3.3. AreasofSurfacesofSurfaceofRevolution

·Thesurfaceareaofthesurfaceofrevolutionobtatinedbyrotatingthecurvey=f(x),a≤x≤b,aboutthex-axis isgivenby

$$S=\int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx.$$

·Thesurfaceareaofthesurfaceofrevolutionobtatinedbyrotatingthecurvex=f(y),c≤y≤b,aboutthey-axisisgivenby

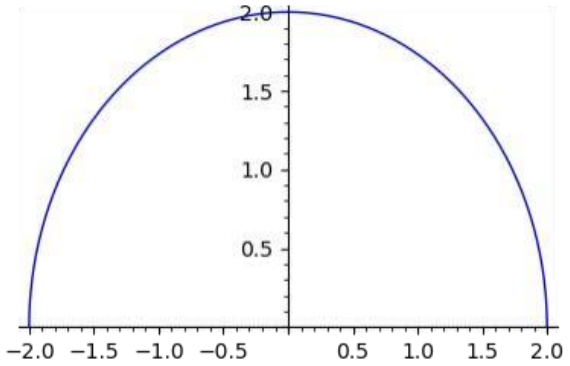
$$S=\int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}dy.$$

Problem:FindthesurfaceareaoftheSphere $x^2+y^2+z^2=r^2$

In [39]:

```
var('x,y,r')
r=2
f(x) = sqrt(r^2-x^2)
plot(f(x),-r,r,figsize=4,aspect_ratio='automatic')
```

Out[39]:



In [40]:

```
revolution_plot3d(f, (x,-r,r),show_curve=True,opacity=0.3,parallel_axis='x',aspect_ratio='automatic')
```

Out[40]:

In [41]:

```
## Surface area of a sphere of radius r
var('r')
f(x)=sqrt(r^2-x^2)
integral(2*pi*f(x)*sqrt(1+derivative(f,x)^2),x,-r,r)
```

Out[41]:

4*pi*r^2

Problem:Findthesurfaceareaofthesurfaceofrevolutionobtainedberotationf(x)=x+cos(x)betweenx=0 andx=πaboutthex-axis

In [42]:

```
f(x)=x+cos(x)
a,b = 0, pi
surf = revolution_plot3d(f, (x,a,b),show_curve=True,opacity=0.6,parallel_axis='x')
show(surf)
```

In [43]:

```
A = integral(2*pi*f(x)*sqrt(1+f.diff()(x)^2),(x,a,b))
A.n()
```

Out[43]:

34.1375871765462

7.3.4. Volumesofsolidofrevolution

· Thevolumeofthesolidofrevolutionaboutthex-axisofthesolidobtainedbyrevolvingaregionbetween $y=f(x), a\le x\le b$,aboutthex-axisisgiven

by

$$V=\int_a^b \pi f(x)^2 dx$$

· Thevolumeofthesolidofrevolutionaboutthex-axisofthesolidobtainedbyrevolvingaregionbetween$x=f(y), c\le x\le d$,aboutthex-axisisgivenby

$$V=\int_c^d \pi f(y)^2 dy$$

· Iftheregionboundedbytwocurves$y=f(x), y=g(x)$andthelines$x=a, x=b$with$g(x)\le f(x)$,thevolumeofthesolidofrevolutnobtainedbyrevolvinga regionaboutthex-axisisgivenby

$$V=\int_a^b \pi[f(x)^2-g(x)^2]dx$$

In [44]:

```
## Volume of sphere of radius r
var('x,r')
f(x) = sqrt(r^2-x^2)
V = integral(pi*f(x)^2,x,-r,r)
V
```

Out[44]:

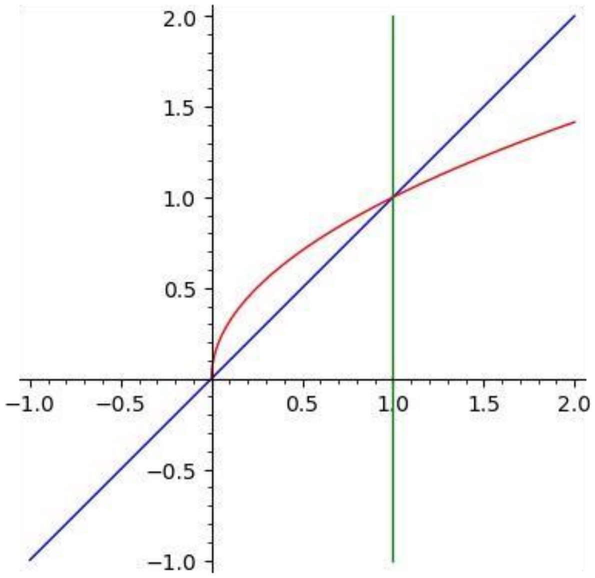
4/3*pi*r^3

Problem:Findthevolumeofthesolidobtainedbyrotationtheregionboundedby$y+x, y=\sqrt{x}$abouttheline $x=1$.

In [45]:

```
f(x)=x
g(x)=sqrt(x)
plot(f(x),(x,-1,2))+plot(g(x),0,2,color='red')+parametric_plot((1,y),(y,-1,2),color='green')
```

Out[45]:



In [46]:

```
S1 = revolution_plot3d(f, (x,0,1),show_curve=True,opacity=0.5,parallel_axis='z',color='green',axis=(1,0))
S2 = revolution_plot3d(g, (x,0,1),show_curve=True,opacity=0.3,parallel_axis='z',color='blue',axis=(1,0))
show(S1+S2,figsize=3)
```

ⓘ

In [47]:

```
A(y)=(1-y^2)^2-(1-y)^2
V = integral(pi*A(y),y,0,1)
V
```

Out[47]:

1/5*pi

Exercise5.6

1.Findtheareabetweenf(x) $= \sin(x) - xe^{-x^2}$ andg(x) $= \cos(x) - xe^{-x^2}$ andx = 1andx = 3.5.

In [49]:

```
f(x)=sin(x)-x*e^(-x^2)
show(f(x))
g(x)=cos(x)-x*e^(-x^2)
show(g(x))
```

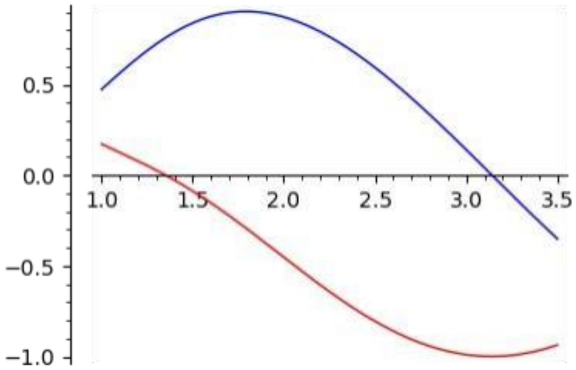
$$-xe^{\left(-x^2\right)} + \sin(x)$$

$$-xe^{\left(-x^2\right)} + \cos(x)$$

In [50]:

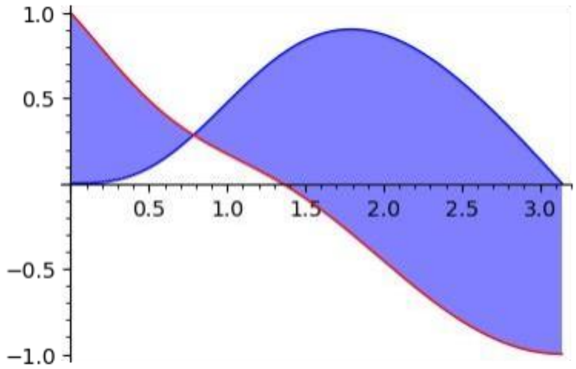
```
plot(f(x),x,1,3.5)+plot(g(x),x,1,3.5,color='red',figsize=4)
```

Out[50]:



In [51]:

```
p1 = plot(f(x),(x,a,b),fill=g(x),fillcolor='blue')
p2 = plot(g(x),(x,a,b),color='red')
show(p1+p2,figsize=4)
```



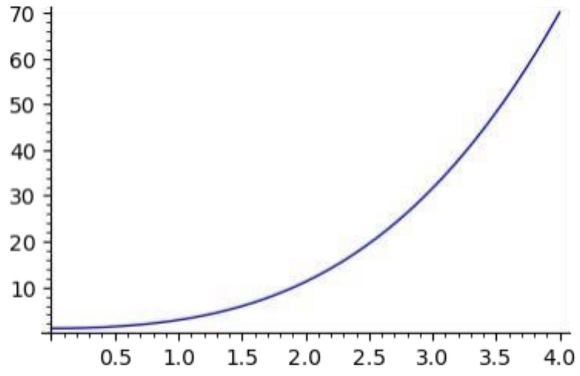
2. Graphthecurvey $= (1+x^2)^{\frac{3}{2}}$ for0 $\leq x \leq 4$andhencefinditsarclength.

In [56]:

```
f(x)=(1+x^2)^(3/2)

plot(f(x),(x,0,4),figsize=4)
```

Out[56]:



In [57]:

```
integral(sqrt(1+derivative(f,x)^2),x,0,4)
```

Out[57]:

integrate(sqrt(9*(x^2 + 1)*x^2 + 1), x, 0, 4)

In [ ]:

3.Findthevolumeofsolidrevolutionofthecurvey=cos(x),0≤x≤πrevolveabouttheaxisy=1.

In [58]:

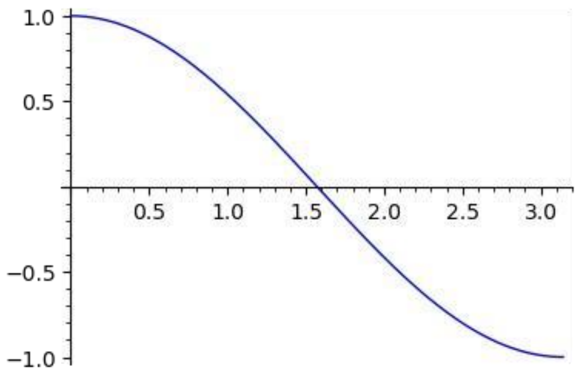
```
f(x)=cos(x)
show(f(x))
```

$$\cos(x)$$

In [59]:

```
plot(f(x),(x,0,pi),figsize=4)
```

Out[59]:



In [61]:

```
V = integral(pi*f(x)^2,x,0,pi)
show(V)
```

$$\frac{1}{2}\pi^2$$

In [ ]: