SHRI RAMDEOBABA COLLEGE OF ENGINNERING AND MANAGEMENT,NAGPUR

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BATCH:C3

ROLL NO:65

EXPERIMENT NO:4

AIM:BASIC OF LINEAR ALGEBRA

MATRIX ALGEBRA

In [2]:
```
#  DEFINE A MATRIX
A=matrix(3,3,[1,-1,3,2,1,4,0,3,8])
show('A=',A)
```

A= $\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 3 & 8 \end{pmatrix}$

In [3]:
```
#  DEFINE A MATRIX
B=matrix([[1,2,3],[3,4,5],[3,5,8]])
show('B=',B)
```

B= $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 3 & 5 & 8 \end{pmatrix}$

In [4]:
```
B=matrix(3,4,[1,-1,3,2,1,4,0,3,8,7,5,4])
show('B=',B)
C=matrix(3,4,[1,-1,3,2,1,4,0,3,8,7,5,4])
show('C=',C)
show('B+C=',B+C)
```

B= $\begin{pmatrix} 1 & -1 & 3 & 2 \\ 1 & 4 & 0 & 3 \\ 8 & 7 & 5 & 4 \end{pmatrix}$

C= $\begin{pmatrix} 1 & -1 & 3 & 2 \\ 1 & 4 & 0 & 3 \\ 8 & 7 & 5 & 4 \end{pmatrix}$

B+C= $\begin{pmatrix} 2 & -2 & 6 & 4 \\ 2 & 8 & 0 & 6 \\ 16 & 14 & 10 & 8 \end{pmatrix}$

In [5]:
```
B=matrix(3,4,[1,-1,3,2,1,4,0,3,8,7,5,4])
show('B=',B)
C=matrix(3,4,[6,-3,1,7,6,4,0,9,0,1,9,2])
show('C=',C)
show('B-C=',B-C)
```

B= $\begin{pmatrix} 1 & -1 & 3 & 2 \\ 1 & 4 & 0 & 3 \\ 8 & 7 & 5 & 4 \end{pmatrix}$

C= $\begin{pmatrix} 6 & -3 & 1 & 7 \\ 6 & 4 & 0 & 9 \\ 0 & 1 & 9 & 2 \end{pmatrix}$

B-C= $\begin{pmatrix} -5 & 2 & 2 & -5 \\ -5 & 0 & 0 & -6 \\ 8 & 6 & -4 & 2 \end{pmatrix}$

In [6]:
```
B=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('B=',B)
C=matrix(3,3,[6,5,4,7,3,2,9,1,0])
show('C=',C)
show('B*C=',B*C)
```

B= $\begin{pmatrix} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{pmatrix}$

C= $\begin{pmatrix} 6 & 5 & 4 \\ 7 & 3 & 2 \\ 9 & 1 & 0 \end{pmatrix}$

B*C= $\begin{pmatrix} 65 & 27 & 18 \\ 128 & 58 & 40 \\ 85 & 38 & 26 \end{pmatrix}$

In [7]:
```
B=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('B=',B)
show('Determinant of B=',det(B))
```

B= $\begin{pmatrix} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{pmatrix}$

Determinant of B=12

In [8]:
```
A=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('A=',A)
show('transpose=',A.transpose())
```

A= $\begin{pmatrix} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{pmatrix}$

transpose= $\begin{pmatrix} 2 & 6 & 3 \\ 5 & 8 & 7 \\ 2 & 4 & 2 \end{pmatrix}$

In [9]:
```
#inverse:
A=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('A=',A)
show('inverse=',A.inverse())
```

$$\mathtt{A=}\begin{pmatrix} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{pmatrix}$$

$$\mathtt{inverse=}\begin{pmatrix} -1 & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{1}{6} & \frac{1}{3} \\ \frac{3}{2} & \frac{1}{12} & -\frac{7}{6} \end{pmatrix}$$

In [10]:
```
#adjugate:
A=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('A=',A)
show('adjugate=',A.adjugate())
```

$$\mathtt{A=}\begin{pmatrix} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{pmatrix}$$

$$\mathtt{adjugate=}\begin{pmatrix} -12 & 4 & 4 \\ 0 & -2 & 4 \\ 18 & 1 & -14 \end{pmatrix}$$

In [11]:
```
#trace:
A=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('A=',A)
show('trace=',A.trace())
```

$$\mathtt{A=}\begin{pmatrix} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{pmatrix}$$

$\mathtt{trace=}12$

In [12]:
```
#rank:
A=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('A=',A)
show('rank=',A.rank())
```

$$\mathtt{A=}\begin{pmatrix} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{pmatrix}$$

$\mathtt{rank=}3$

In [13]:
```
#minors of order 1
A=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('A=',A)
show('minors of order 1=',A.minors(1))
```

A= $\left(\begin{array}{rrr} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{array}\right)$

minors of order 1= $[2, 5, 2, 6, 8, 4, 3, 7, 2]$

In [14]:
```
#minors of order 2
A=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('A=',A)
show('minors of order 2=',A.minors(2))
```

A= $\left(\begin{array}{rrr} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{array}\right)$

minors of order 2= $[-14, -4, 4, -1, -2, -4, 18, 0, -12]$

In [15]:
```
#minors of order 3
A=matrix(3,3,[2,5,2,6,8,4,3,7,2])
show('A=',A)
show('minors of order 3=',A.minors(3))
```

A= $\left(\begin{array}{rrr} 2 & 5 & 2 \\ 6 & 8 & 4 \\ 3 & 7 & 2 \end{array}\right)$

minors of order 3= $[12]$

In [16]:
```
#real no. only entry:
A=matrix(RR,3,3,[2.87,5.67,2.456,pi,sqrt(4),4,3,7,2])
show('A=',A)
```

A= $\left(\begin{array}{rrr} 2.87000000000000 & 5.67000000000000 & 2.45600000000000 \\ 3.14159265358979 & 2.00000000000000 & 4.00000000000000 \\ 3.00000000000000 & 7.00000000000000 & 2.00000000000000 \end{array}\right)$

In [17]:
```
A=matrix(ZZ,3,3,[2,5,2,6,8,4,3,7,2])
```

In [18]:
```
A=matrix(QQ,3,3,[2,5,2,6,8,4,3,7,2])
```

In [26]:
```
A=matrix(CC,3,3,[2,5,2,6,8,4,3,7,2])
```

In [19]:
```
A=matrix(CDF,3,3,[2,5,2,6,8,4,3,7,2])
```

In [20]:
```
#showing matrix with variables:
var('x,y,z')
V=matrix(3,3,[1,z,-y,-z,1,x,y,-z,1])
show('V=',V)
```

V= $\left(\begin{array}{rrr} 1 & z & -y \\ -z & 1 & x \\ y & -z & 1 \end{array}\right)$

In [21]:
```
#determinant of above matrix:
var('x,y,z')
V=matrix(3,3,[1,z,-y,-z,1,x,y,-z,1])
show('V=',det(V))
```

V= $(xz + y)y - (yz - z)z + xz + 1$

In [22]:
```
#transpose of above matrix:
var('x,y,z')
V=matrix(3,3,[1,z,-y,-z,1,x,y,-z,1])
show('V=',V.transpose())
```

V= $\left(\begin{array}{rrr} 1 & -z & y \\ z & 1 & -z \\ -y & x & 1 \end{array}\right)$

In [23]:
```
#differentiating w.r.t x:
var('x,y,z')
V=matrix(3,3,[1,z,-y,-z,1,x,y,-z,1])
diff(V,x)
```

Out[23]:
```
[0 0 0]
[0 0 1]
[0 0 0]
```

In [24]:
```
#identity matrix
I=identity_matrix(4)
show(I)
```

$\left(\begin{array}{rrrr} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$

In [25]:
```
#zero matrix
R=zero_matrix(3,5)
show(R)
```

$\left(\begin{array}{rrrrr} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

In [26]:
```
#ones matrix
S=ones_matrix(4,5)
show(S)
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In [27]:
```
#random values
T=random_matrix(ZZ,2,3)
show(T)
```

$$\begin{pmatrix} -11 & -1 & 1 \\ 37 & 3 & 2 \end{pmatrix}$$

In [28]:
```
#random values between x and y:
T=random_matrix(ZZ,2,3,x=5,y=10)
show(T)
```

$$\begin{pmatrix} 9 & 7 & 5 \\ 5 & 7 & 9 \end{pmatrix}$$

In [29]:
```
#random values between x and y:
T=random_matrix(ZZ,2,3,x=-5,y=10)
show(T)
```

$$\begin{pmatrix} -1 & 9 & -4 \\ 3 & 3 & -1 \end{pmatrix}$$

In [30]:
```
#ones matrix
X=ones_matrix(5,6)
show(X)
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In [31]:
```
# ENTRY OF A NO. IN MATRIX:
X[3,1]=1000 #index begin with zero
show(X)
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1000 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In [32]:
```
X[2,4]=500
show(X)
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 500 & 1 \\ 1 & 1000 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In [33]:
```
X[:,3]=-500
show(X)
```

$$\begin{pmatrix} 1 & 1 & 1 & -500 & 1 & 1 \\ 1 & 1 & 1 & -500 & 1 & 1 \\ 1 & 1 & 1 & -500 & 500 & 1 \\ 1 & 1000 & 1 & -500 & 1 & 1 \\ 1 & 1 & 1 & -500 & 1 & 1 \end{pmatrix}$$

In [34]:
```
X[2,:]=20
show(X)
```

$$\begin{pmatrix} 1 & 1 & 1 & -500 & 1 & 1 \\ 1 & 1 & 1 & -500 & 1 & 1 \\ 20 & 20 & 20 & 20 & 20 & 20 \\ 1 & 1000 & 1 & -500 & 1 & 1 \\ 1 & 1 & 1 & -500 & 1 & 1 \end{pmatrix}$$

In [35]:
```
X[3:5,4:6]=-1
show(X)
```

$$\begin{pmatrix} 1 & 1 & 1 & -500 & 1 & 1 \\ 1 & 1 & 1 & -500 & 1 & 1 \\ 20 & 20 & 20 & 20 & 20 & 20 \\ 1 & 1000 & 1 & -500 & -1 & -1 \\ 1 & 1 & 1 & -500 & -1 & -1 \end{pmatrix}$$

solution of system of equation:

In [36]:
```
#unique solution
var('x,y,z')
solve([5*x+3*y-2*z==4,x-y+5*z==3,2*x+y-3*z==-1],x,y,z)
```

Out[36]: `[[x == 0, y == 2, z == 1]]`

In [37]:
```
A=matrix(3,3,[5,3,-2,1,-1,5,2,1,-3])
show('A=',A)
B=matrix(3,1,[4,3,-1])
show('B=',B)
```

$$A=\begin{pmatrix} 5 & 3 & -2 \\ 1 & -1 & 5 \\ 2 & 1 & -3 \end{pmatrix}$$

$$B=\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$$

In [38]:
```
C=A.augment(B)
show('C=',C)
```

$$C=\begin{pmatrix} 5 & 3 & -2 & 4 \\ 1 & -1 & 5 & 3 \\ 2 & 1 & -3 & -1 \end{pmatrix}$$

In [39]:
```
show('Rank of A=',A.rank())
```

Rank of A=3

In [40]:
```
show('Rank of C=',C.rank())
```

Rank of C=3

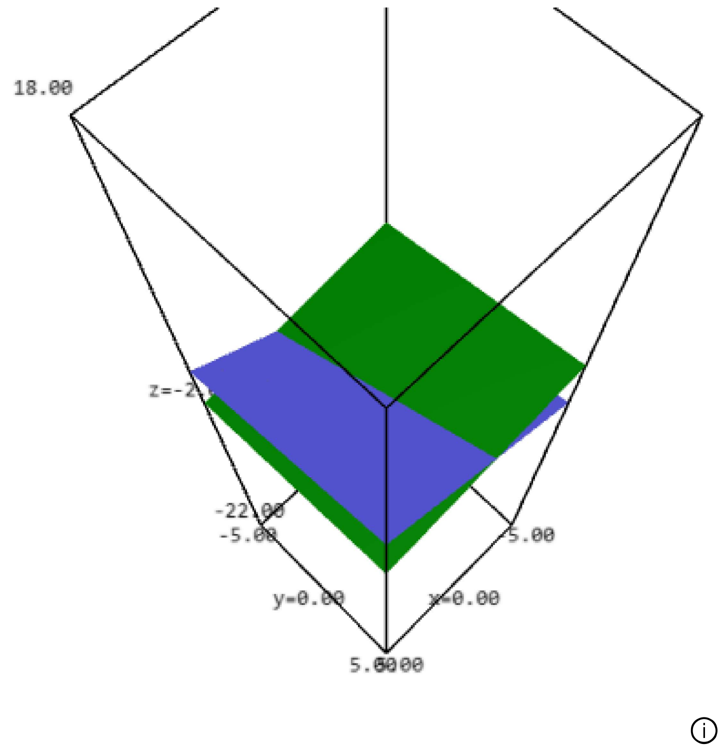
In [41]:
```
#r1 can be any real number because it has infinitely many solution
var('x,y,z')
solve([5*x+3*y-2*z==4,x-y+5*z==3,2*x-2*y+10*z==6],x,y,z)
```

Out[41]:
```
[[x == -13/8*r1 + 13/8, y == 27/8*r1 - 11/8, z == r1]]
```

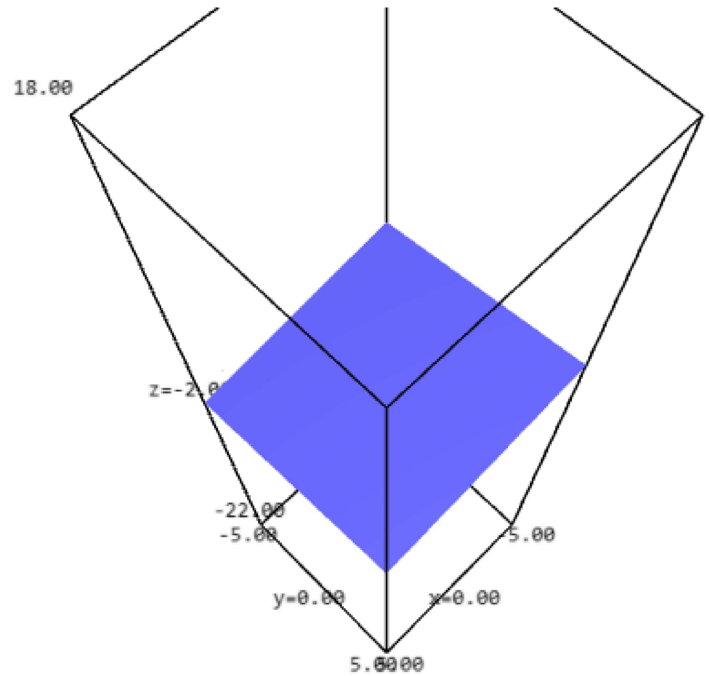
In [43]:
```
#no solution of the equation:
var('x,y,z')
solve([5*x+3*y-2*z==4,x-y+5*z==3,2*x-2*y+10*z==-1],x,y,z)
```

Out[43]:
```
[]
```

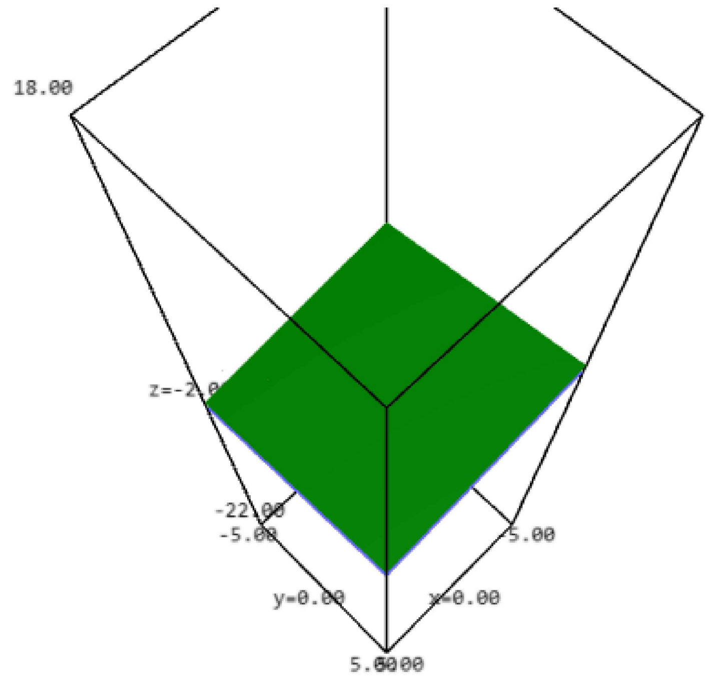
In [45]:
```
var('x,y,z')
#([5*x+3*y-2*z==4,x-y+5*z==3,2*x+y-3*z==-1],x,y,z)
# unique solution
z1=(-4+5*x+3*y)/2
z2=(3-x+y)/5
z3=(1+2*x+y)/3
p1=plot3d(z1,(x,-5,5),(y,-5,5),color='pink')
p2=plot3d(z2,(x,-5,5),(y,-5,5),color='green')
p3=plot3d(z3,(x,-5,5),(y,-5,5))
show(p1+p2+p3)
```



In [1]:
```
var('x,y,z')
#([5*x+3*y-2*z==4,x-y+5*z==3,2*x-2*y+10*z==6],x,y,z)
#infinitely many solution
z1=(4-5*x-3*y)/(-2)
z2=(3-x+y)/5
z3=(6-2*x+2*y)/10
p1=plot3d(z1,(x,-5,5),(y,-5,5),color='pink')
p2=plot3d(z2,(x,-5,5),(y,-5,5),color='green')
p3=plot3d(z3,(x,-5,5),(y,-5,5))
show(p1+p2+p3)
```



In [44]:
```
#no solution of the equation:
var('x,y,z')
#([5*x+3*y-2*z==4,x-y+5*z==3,2*x-2*y+10*z==-1],x,y,z)
z1=(4-5*x-3*y)/(-2)
z2=(3-x+y)/5
z3=(-1-2*x+2*y)/10
p1=plot3d(z1,(x,-5,5),(y,-5,5),color='pink')
p2=plot3d(z2,(x,-5,5),(y,-5,5),color='green')
p3=plot3d(z3,(x,-5,5),(y,-5,5))
show(p1+p2+p3)
```



EXERCISE QUESTIONS:

EX. 2.1

In [47]:
```
A=matrix(3,3,[2,-2,2,1,1,1,1,3,-1])
show('A=',A)
```

A= $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$

In [49]:
```
# (i)
p(x)=A.characteristic_polynomial(x)
show('p(x)=',p(x))
```

a(x)= $x^3 - 2\,x^2 - 4\,x + 8$

In [51]:
```
#(ii)
p(x).roots()
```
Out[51]: `[(-2, 1), (2, 2)]`

In [52]:
```
A.eigenvalues()
```
Out[52]: `[-2, 2, 2]`

In [53]:
```
# (iii)
h=A.eigenvectors_right()
show(h)
```
$$\left[\left(-2, \left[\left(1, \frac{1}{4}, -\frac{7}{4}\right)\right], 1\right), \left(2, \left[(0, 1, 1)\right], 2\right)\right]$$

In [54]:
```
#(iv)
A.trace()
```
Out[54]: `2`

In [55]:
```
sum(A.eigenvalues())
```
Out[55]: `2`

In [57]:
```
#(v)
A.det()
```
Out[57]: `-8`

In [ ]:
```
product(A.eigenvalues())
```

EX. 2.2

In [63]:
```
var('t')
A=matrix(2,2,[cos(t),sin(t),-sin(t),cos(t)])
show('A=',A)
p(x)=A.characteristic_polynomial(x)
show('p(x)=',p(x))
```
$$\text{A=}\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$$

$$\text{p(x)=}x^2 - 2\,x\cos(t) + \cos(t)^2 + \sin(t)^2$$

In [78]:
```
coeff=p.coefficients()
show(coeff)
```
$$\left[\left[x \mapsto x^2 - 2\,x\cos(t) + \cos(t)^2 + \sin(t)^2, x \mapsto 0\right]\right]$$

In [ ]:

Therefore, matrix A satisfies its char. eqn.

What we learnt: From this experiment ,we learnt the principles, concepts, working and application of Linear Algebra

In [ ]:

In [ ]: